

JJCR'2013

Mobile Robot Localization: A Set-Membership Approach

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Introduction

- Robot localization is an important issue of mobile robotics
- The robotics challenge called CAROTTE ¹
- The Simultaneous Localization And Mapping (SLAM) and the global localization problems
- In this presentation a set membership approach will be considered to deal with the global localization problem

1. CARtographie par ROBoT d'un TERRitoire (Robot Land Mapping) organized by the french ANR (National Research Agency) and the DGA (french army)

Summary

- 1 Interval Analysis
- 2 The Global Localization Problem
- 3 The Proposed Method
- 4 Experimental Results

Summary

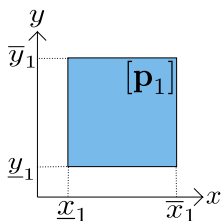
- 1 Interval Analysis
 - Interval Analysis
 - Constraint Satisfaction Problem
 - Q-Relaxed Intersection
- 2 The Global Localization Problem
- 3 The Proposed Method
- 4 Experimental Results

Definitions

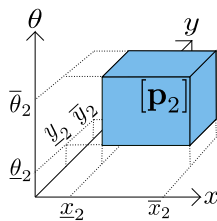
An Interval Vector

An *interval vector*, or a *box* $[\mathbf{p}]$ is defined as a closed subset of \mathbb{R}^n

$$[\mathbf{p}] = ([x], [y], \dots) = ([\underline{x}, \bar{x}], [\underline{y}, \bar{y}], \dots) \subset \mathbb{R}^n$$



$$[\mathbf{p}_1] = ([x_1], [y_1])$$



$$[\mathbf{p}_2] = ([x_2], [y_2], [\theta_2])$$

Interval Arithmetic

Definition

Any real number elementary operators such as $+$, $-$, \times , \div and functions such as exp , sin , sqr , $sqrt$, can be easily extended to intervals

Example

Be $[x]$ and $[y]$ two intervals, we define

$$\rightarrow [x] + [y] = [\underline{x} + \underline{y}, \bar{x} + \bar{y}]$$

$$\rightarrow [x] \times [y] = [\min(\underline{x}\underline{y}, \bar{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\bar{y}), \max(\underline{x}\underline{y}, \bar{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\bar{y})]$$

Be $f \in \{cos, sin, exp, tan, log, sqrt, sqr\}$, we define

$$\rightarrow f([x]) = \{f(x) \text{ with } x \in [x]\}$$

Definitions

Constraint Satisfaction Problem (CSP)

A CSP is defined by three sets. A set of **variables** \mathcal{V} , a set of **domains** \mathcal{D} for those variables and a set of **constraints** \mathcal{C} connecting the variables together

$$\text{CSP} : \left\{ \begin{array}{l} \mathcal{V} = \{x_1, x_2, \dots, x_n\} \\ \mathcal{D} = \{[x_1], [x_2], \dots, [x_n]\} \\ \mathcal{C} = \{c_1, c_2, \dots, c_m\} \end{array} \right\}$$

Constraint propagation

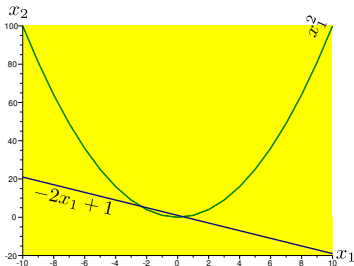
This kind of problem can be solved using **constraint propagation**. Constraint propagation consist in reducing the variable domains by using **contractors** C_{c_i} associated to each constraints c_i

Example

Example of CSP

Let x_1 and x_2 be two variables with $[x_1] = [-10, 10]$ and $[x_2] = [-20, 100]$ their domains. We consider the constraints

$$c_1 : x_2 = x_1^2 \text{ and } c_2 : x_2 = -2x_1 + 1$$

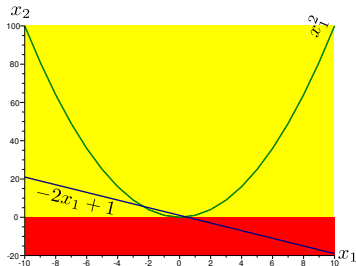


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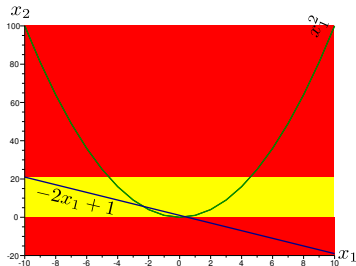
$$x_2 = x_1^2$$

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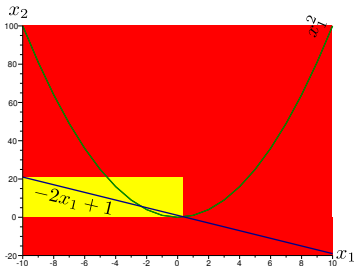
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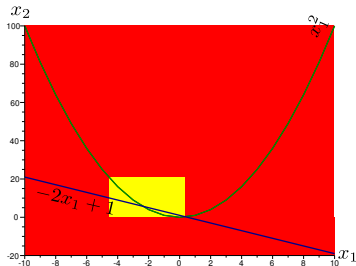
$$x_1 = (1 - x_2)/2$$

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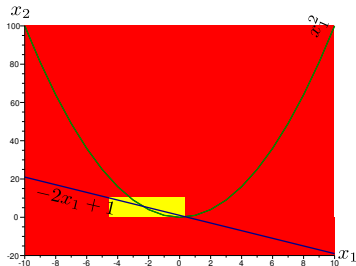
$$|x_1| = \sqrt{x_2}$$

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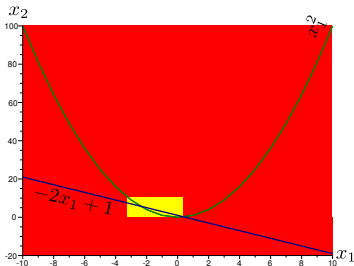
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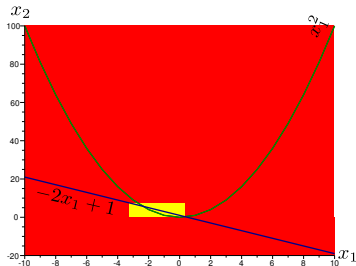
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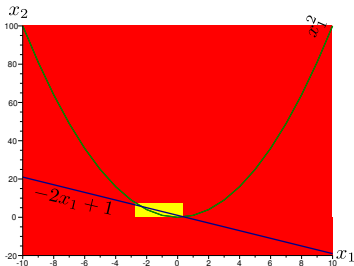
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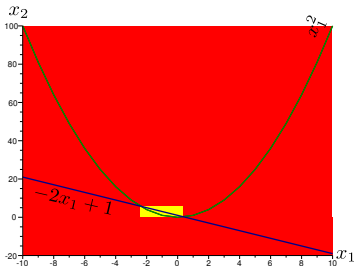
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The result of the
constraint propagation

Q-Relaxed Intersection

Definition

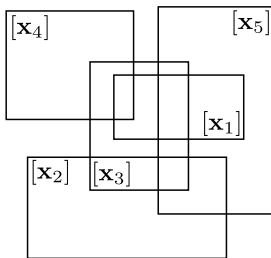
Let be m interval vectors $[\mathbf{x}_1], \dots, [\mathbf{x}_m]$ of \mathbb{R}^n . The q -relaxed intersection $\bigcap^{\{q\}}([\mathbf{x}_i])$ is defined as all the $\mathbf{x} \in \mathbb{R}^n$ that are in all of the $[\mathbf{x}_i]$ excepted q at most

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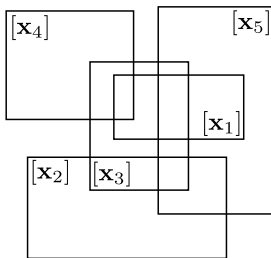


We consider five interval vectors : $[\mathbf{x}_1], \dots, [\mathbf{x}_5]$

Q-Relaxed Intersection

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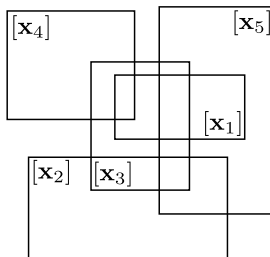


$$\bigcap_{i=1, \dots, 5}^{\{0\}} [\mathbf{x}_i]$$

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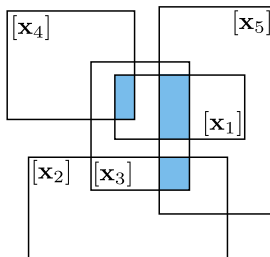


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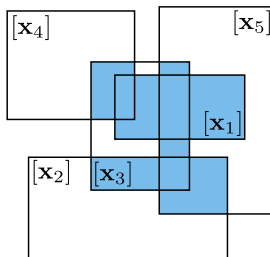


$$\bigcap_{i=1, \dots, 5}^{\{2\}} [\mathbf{x}_i]$$

Q-Relaxed Intersection

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$$\bigcap_{i=1, \dots, 5}^{\{3\}} [\mathbf{x}_i]$$

Summary

- 1 Interval Analysis
- 2 The Global Localization Problem
 - The Robot
 - The Environment
 - The Objective
- 3 The Proposed Method
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The Robot

The considered robot

We consider a mobile wheeled robot with a LIDAR^a sensor. Its pose is defined by $\mathbf{p} = (x, y, \theta)$, with (x, y) its localization and θ its orientation

a. Light Detection And Ranging

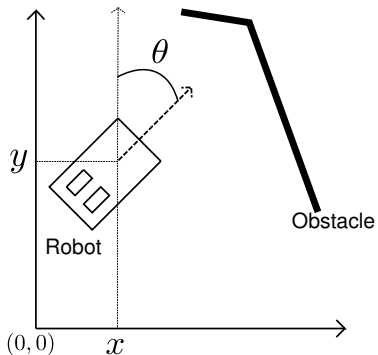
The measurements

The sensor provides a set of measurements :

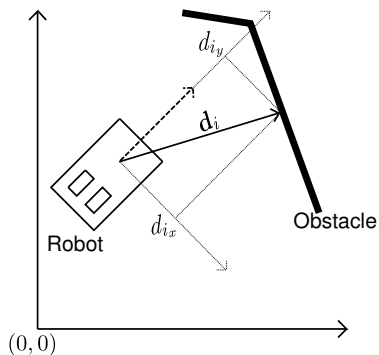
$$\mathbb{D} = \{\mathbf{d}_i = (d_{i_x}, d_{i_y})\}, i = 1, \dots, n$$



The Robot



The robot pose



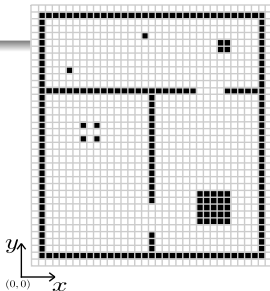
A measurement \mathbf{d}_i

The Environment

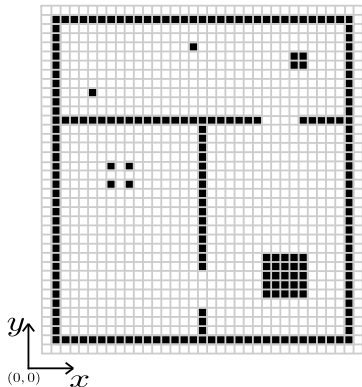
The map

The known environment $\mathbb{E} \in \mathbb{R}^2$ is discretized with a resolution δ_x, δ_y and this lead to a grid \mathbb{G} composed of $n \times m$ cells (i,j) . At each cell (i,j) is associated $g_{i,j} \in \{0,1\}$

$$g_{i,j} = \begin{cases} 1 & \text{if there is an obstacle in the cell } (i,j) \\ 0 & \text{else} \end{cases}$$



The Objective

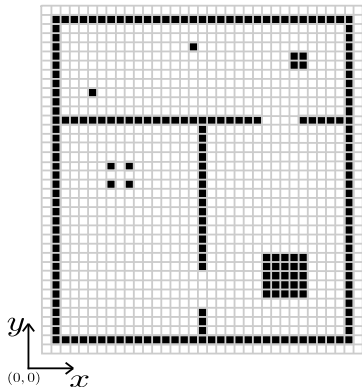


Hypotheses

- Bounded error context
- Outliers can be considered

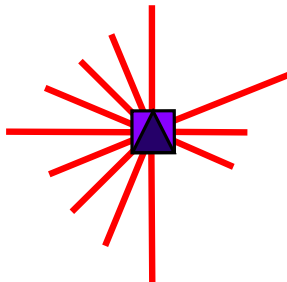


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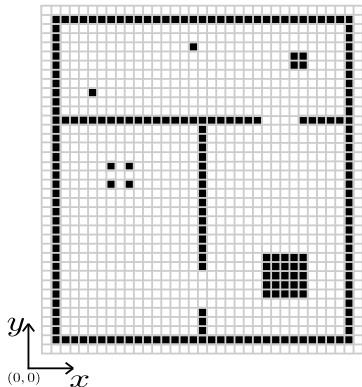


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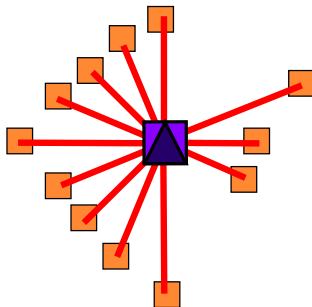


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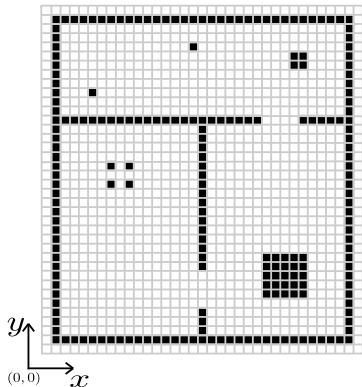


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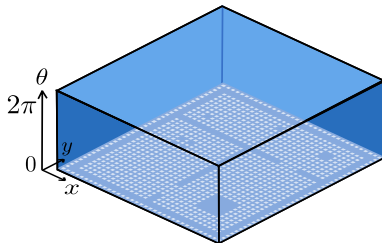


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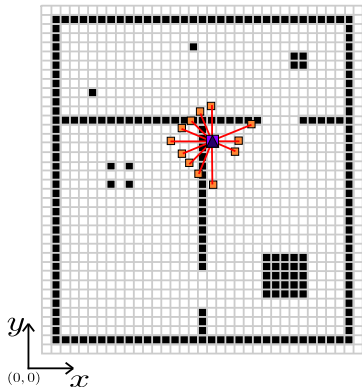
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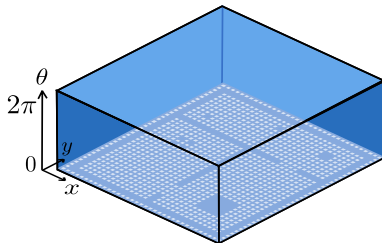
The Objective

(JJCR'2013)

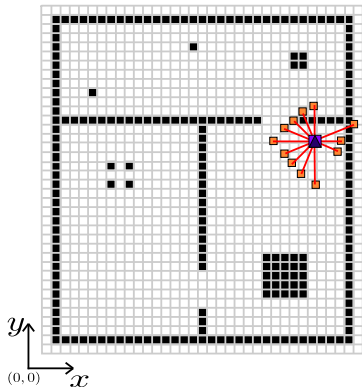


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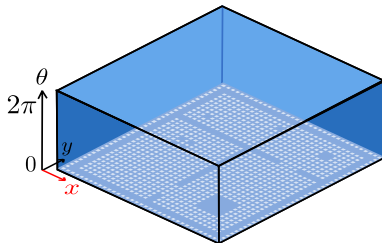


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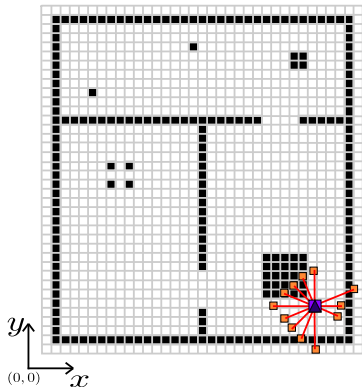


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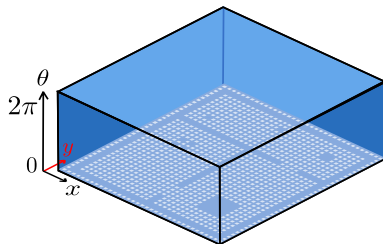


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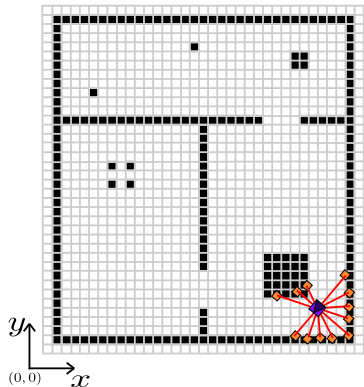


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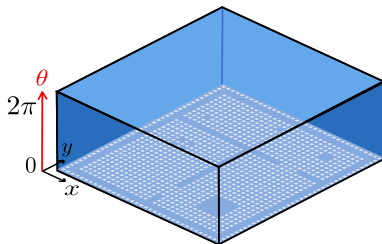


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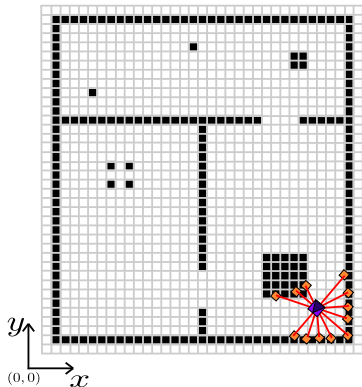


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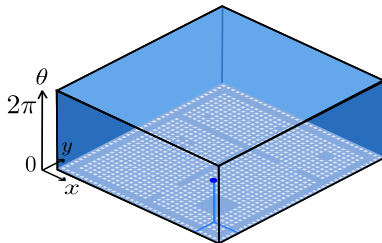


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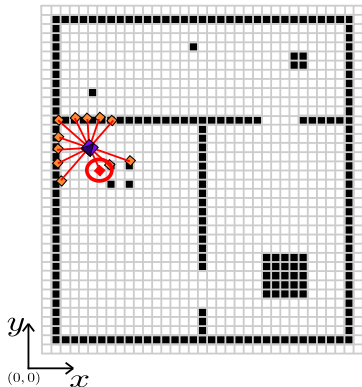


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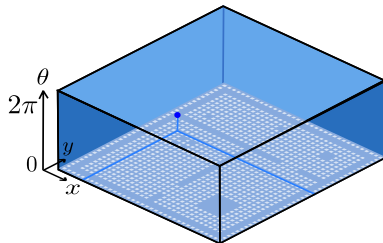


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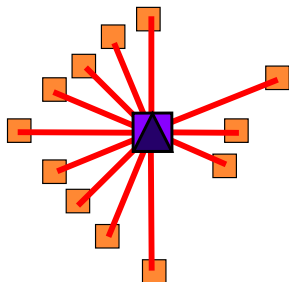
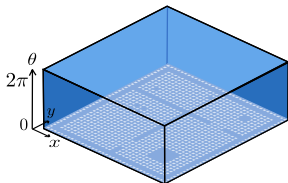
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 - The Measurement CSP
 - The Localization Algorithm
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The Measurement CSP

The Context

Let $\mathbf{p} = ([x], [y], [\theta])$ be an initial domain that encloses the robot pose (x, y, θ) and $\mathbb{D} = \{\mathbf{d}_i\}, i = 1, \dots, n$ a set of n telemeter measurements

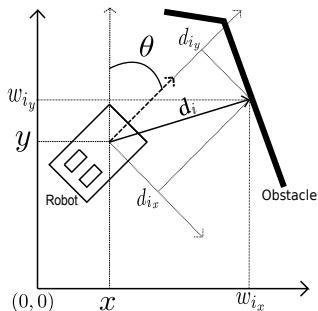


The Measurement CSP

The Measurements Coordinates

The coordinates (w_{i_x}, w_{i_y}) of an obstacle in the map are defined by

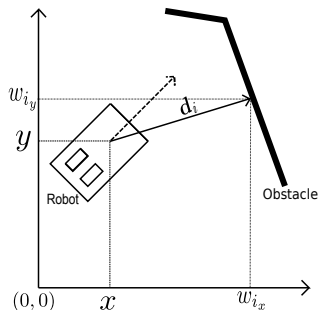
$$\begin{pmatrix} w_{i_x} \\ w_{i_y} \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} d_{i_x} \\ d_{i_y} \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$$



The Measurement CSP

The Robot-Measurement Distance

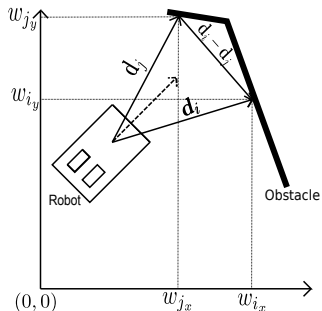
$$\|\mathbf{d}_i\|^2 = (x - w_{i_x})^2 + (y - w_{i_y})^2$$



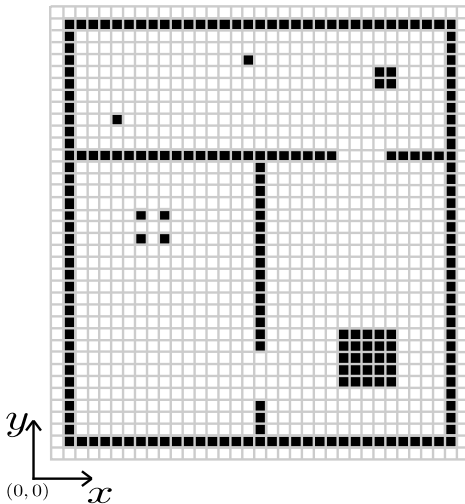
The Measurement CSP

The Measurement-Measurement Distance

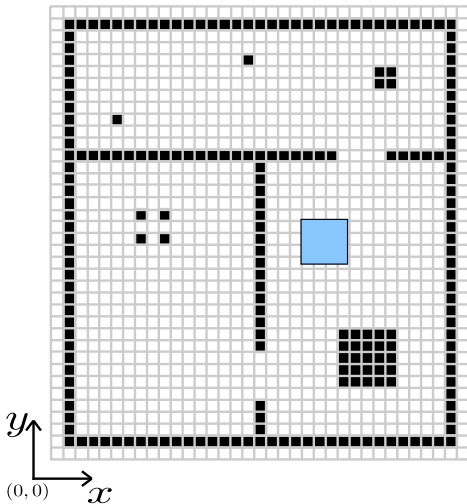
$$\|\mathbf{d}_i - \mathbf{d}_j\|^2 = (w_{i_x} - w_{j_x})^2 + (w_{i_y} - w_{j_y})^2$$



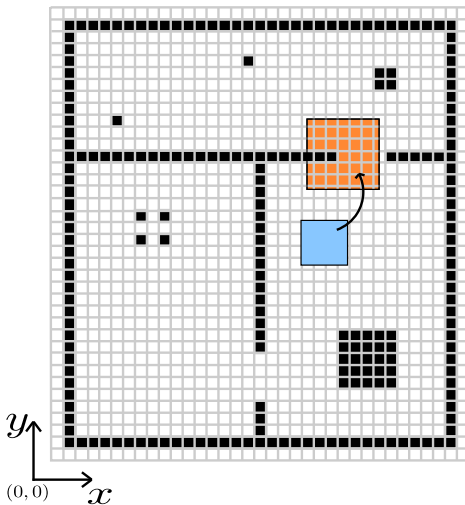
The Measurement CSP



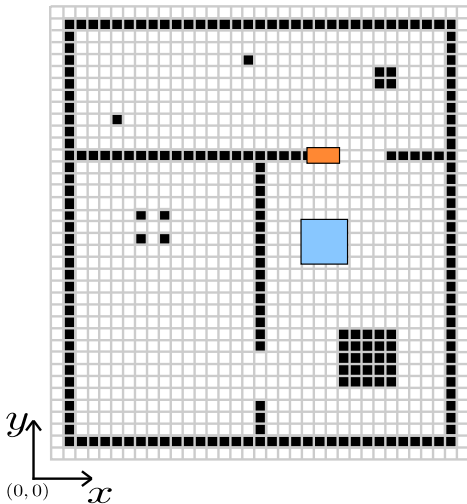
The Measurement CSP



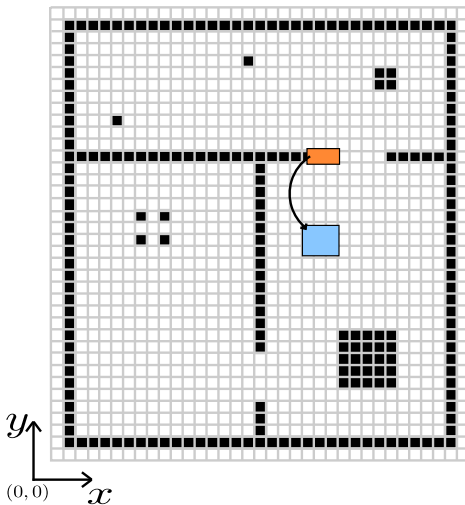
The Measurement CSP



The Measurement CSP



The Measurement CSP



The Measurement CSP

The map constraint

We define c_G the constraint which says that the measurement has to be consistent with the map.

The map contractor

We define C_G the contractor of the constraint c_G

The Measurement CSP

Considered Constraint Satisfaction Problem

$$\mathcal{V} = \{x, y, \theta, \mathbf{d}_i = (d_{i_x}, d_{i_y}), w_{i_x}, w_{i_y}\}$$

$$\mathcal{D} = \{$$

$$[x] = [-\infty, +\infty], [y] = [-\infty, +\infty], [\theta] = [0, 2\pi]$$

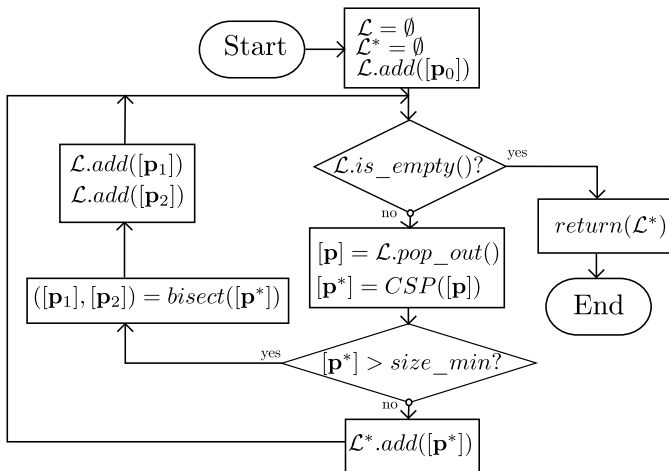
$$[w_{i_x}] = [-\infty, +\infty], [w_{i_y}] = [-\infty, +\infty]$$

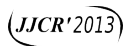
$$[d_{i_x}] = \text{obtained from the sensor}$$

$$[d_{i_y}] = \text{obtained from the sensor}\}$$

$$C = \begin{cases} c_{w_{i_x}} : d_{i_x} \cos(\theta) + d_{i_y} \sin(\theta) + x \\ c_{w_{i_y}} : -d_{i_x} \sin(\theta) + d_{i_y} \cos(\theta) + y \\ c_{d_i} : \|\mathbf{d}_i\|^2 = (x - w_{i_x})^2 + (y - w_{i_y})^2 \\ c_{d_{i,j}} : \|\mathbf{d}_i - \mathbf{d}_j\|^2 = (w_{i_x} - w_{j_x})^2 + (w_{i_y} - w_{j_y})^2 \\ c_{\mathbb{G}} : \text{to be consistent with the map (using } C_{\mathbb{G}}) \end{cases}$$

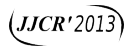
The Localization Algorithm





Summary

- 1 Interval Analysis
- 2 The Global Localization Problem
- 3 The Proposed Method
- 4 Experimental Results**



Experimental Results



Conclusion

- During this presentation a global localization method has been presented
- This method is
 - guaranteed (interval analysis)
 - robust (consideration of outliers)
 - efficient (contractors)
- Future work : implementation of the algorithm in a MiniRex robot

Thank you for your attention

