

# Stochastic population dynamics applied to ovarian follicles development

Romain Yvinec

### ▶ To cite this version:

Romain Yvinec. Stochastic population dynamics applied to ovarian follicles development. BioHasard 2019, Aug 2019, Rennes, France. hal-03115063

## HAL Id: hal-03115063 https://hal.science/hal-03115063

Submitted on 19 Jan 2021

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## Stochastic population dynamics applied to ovarian follicles development

#### Romain Yvinec

#### Physiologie de la Reproduction et des Comportements INRA Tours



#### Acknowledgements

- \* INRIA Saclay : Frédérique Clément, Frédérique Robin
- \* INRA PRC : Team BIOS, BINGO (D. Monniaux, V. Cadoret, R. Dalbies-Tran)
- \* INRA LPGP (J. Bobe, V. Thermes)
- \* INERIS (R. Beaudouin)
- Céline Bonnet (CMAP, X),
   Kerloum Chahour (U. Côte d'Azur)
- \* INRA phase (Crédit incitatif)







maîtriser le risque pour un développement durable

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#### Population dynamics and ovarian follicles development





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 Morphogenesis and maturation of ovarian follicles somatic and germ (egg) cells
 ⇒ Somatic cell division and germ cell growth up to ovulation



Fig. 1. Illustrations of follicle types: (a) Type B,  $\times$  570; (b) Type B/C,  $\times$  570; (c) Type C,  $\times$  570; (d) Type D,  $\times$  410.

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Gougeon & Chainy, J. Reprod. Fert. 1987

- Morphogenesis and maturation of ovarian follicles somatic and germ (egg) cells
- Pool of Quiescent follicles static reserve (perinatal in most mammals) Slow activation



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Monniaux, Theriogenology 2016

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- Morphogenesis and maturation of ovarian follicles somatic and germ (egg) cells
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- Basal growth
   Dynamic reserve (starting at birth)
   Spanning over several ovarian cycles



Monniaux, Theriogenology 2016

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- Pool of Quiescent follicles static reserve (perinatal in most mammals) Slow activation
- Basal growth Dynamic reserve (starting at birth) Spanning over several ovarian cycles
- Terminal growth After puberty : ovulation within an ovarian cycle
- Interactions between all follicles via complex (neuro-) hormonal signals



Ovarian reserves of follicles and their regulations



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Monniaux, Theriogenology 2016

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#### Follicle population in women



Scaramuzzi et al., Reprod.Fert. Dev. 2011

Image: A mathematical states and a mathem

#### Follicle population in women

- Quiescent follicles
  - $\begin{array}{ll} \mbox{peri-natal} & \approx 5 \cdot 10^6 \\ \mbox{At birth} & \approx 1 \cdot 10^6 \\ \mbox{At puberty} & 10^4 10^6 \\ \mbox{At menopause} & < 10^3 \\ \mbox{Activation rate} & "A few per days" \end{array}$
- Growing follicles Maturation time 120 - 180jBasal follicles  $10^3 - 10^4$ Terminal follicles  $10^2$ Pre-Ovulatory follicles a few Atresia Most of them !



Scaramuzzi et al., Reprod.Fert. Dev. 2011

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#### Follicle population in women

- Quiescent follicles  $pprox 5 \cdot 10^6$ peri-natal  $pprox 1 \cdot 10^6$ At birth  $10^4 - 10^6$ At puberty At menopause  $< 10^3$ Activation rate "A few per days"
- Growing follicles Maturation time 120 - 180i $10^3 - 10^4$ Basal follicles  $10^{2}$ Terminal follicles Pre-Ovulatory follicles a few Atresia Most of them ! -> Only 400 follicles will ever reach the pre-ovulatory stage



Scaramuzzi et al., Reprod.Fert, Dev. 2011

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Courtesy of Danielle Monniaux.

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#### Scientific and societal issues

## Understanding of a complex process of developmental biology, occuring during the whole lifespan

Numerous cell types involved, and various interactions Many different spatial and temporal scales Hormonal feedback (endocrine, paracrine, autocrine) Steric and biophysical constraint

#### Preserve the reproductive ability

latrogenic or physiological alterations Sensibility to environmental conditions Biodiversity preservation

#### Control of the reproduction function (in humans and animals)

Biotechnology of reproduction (*in vivo*, *ex vivo*, *in vitro*) Clinical, economical and environmental issues

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## Modeling ovarian folliculogenesis

#### Growth of a single follicle



Monniaux et al., M/S. 1999

• Thesis of F. Robin, (cosupervised. F. Clément)





Scaramuzzi et al., Reprod. Fert. Dev. 2011

 CEMRACS 2018 (summer school), C. Bonnet, K. Chahour

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#### Follicle initiation model



### Key features of follicle initiation

- Leave the quiescent phase (static reserve)
- A single layer of somatic cells
- Two types of cells : Flattened and Cuboid
- Irreversible transition from Flattened to Cuboid cells
- The follicle is "activated" when all cells have transitioned



Fig. 1. Illustrations of follicle types: (a) Type B,  $\times$  570; (b) Type B/C,  $\times$  570; (c) Type C,  $\times$  570; (d) Type D,  $\times$  410.

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Gougeon & Chainy, J. Reprod. Fert. 1987

## Stochastic model (CTMC)

- $\hookrightarrow$  Two cell populations : F (flattened) and C (cuboid)
- $\hookrightarrow$  Small number of cells : stochastic model with ponctual event, with density dependent rates.
- $\hookrightarrow$  Initial Condition :  $F = F_0$  (parameter), C = 0; Condition finale : F = 0,  $C(F = 0) \ge F_0$  (output)

Events	Reaction	Intensity function
differentiation	F  ightarrow C	$\alpha F + \beta \frac{FC}{F+C}$
proliferation	$C \rightarrow C + C$	$\gamma C$

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- Ex vivo data in sheep fetus ( Courtesy of K. McNatty) : WT (++) vs Mutant (BB)
- $\Rightarrow \text{ Proportion of} \\ \text{ cuboid cells} \\ p_C = C/(F+C) \text{ vs} \\ \text{ number of cuboid} \\ \text{ cells } C$



- Ex vivo data in sheep fetus WT vs BB
- Once activated, follicles have "fast" cell proliferation
- ⇒ Are both differentiation et de proliferation process concomitant or successive ?



- Ex vivo data in sheep fetus WT vs BB
- Proportion of cuboid cells seems higher in mutant than WT, for a given number of cuboid cells.
- ⇒ Is it coming from a kinetic difference?



- Ex vivo data in sheep fetus WT vs BB
- Regulatory mechanism for this process are barely known.
- ⇒ Is the transition of cell differentiation abrupt or more progressive ?

Events	Reaction	Intensity function
differentiation	F  ightarrow C	$\alpha F + \beta \frac{FC}{F+C}$
prolifération	$C \rightarrow C + C$	γC

- Theoretical study
  - $\Rightarrow$  Statistics of the "transition" time  $\tau$  to reach F = 0.
  - $\Rightarrow$  Variability of final cuboid cells ( $\mathbb{E}[C_{\tau}] < \infty$  if  $\gamma < \alpha + \beta$ )
  - ⇒ Impact of parameters e.g. on qualitative dynamics (progressive vs abrupt)



Robin et al. Stochastic nonlinear model for somatic cell population dynamics during ovarian follicle activation, (submitted) arXiv :1903.01316

Bio Initiation	Basal Antrum Ovog	genesis
Events	Reaction	Intensity function
differentiation	F  ightarrow C	$\alpha F + \beta \frac{FC}{F+C}$
prolifération	$C \rightarrow C + C$	γC

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• Theoretical study : stochastic bounds on  $\tau$  and finite state projection algorithm are obtained thanks to coupling arguments with linear processes



Events	Reaction	Intensity function
differentiation	F  ightarrow C	$\alpha F + \beta \frac{FC}{F+C}$
prolifération	$C \rightarrow C + C$	$\gamma C$

- Theoretical study
- Parameter calibration :
  - $\Rightarrow$  Adimensionalize parameter ( $\alpha = 1$ ,  $\beta \leftarrow \beta/\alpha$ ,  $\gamma \leftarrow \gamma/\alpha$ )
  - ⇒ Reformulation : cuboid cell number increase by 1 at each event -> it may be used as a "counter" instead of physical time :  $\mathbb{P}\{\exists t, (F(t), C(t)) = (f, c)\} = \mathbb{P}\{F(c) = f\}$ ⇒ Likelihood :  $\prod_{i=1}^{n} \mathbb{P}\{F(c_i) = f_i\}$



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Events	Reaction	Intensity function
differentiation	F  ightarrow C	$\alpha F + \beta \frac{FC}{F+C}$
prolifération	$C \rightarrow C + C$	γC

- Theoretical study
- Parameter calibration : lack of identifiability. Either  $\gamma<<1$  and  $\beta$  unconstrained, or  $\gamma>1$  and  $\beta/\gamma>>1$



Robin et al. Stochastic nonlinear model for somatic cell population dynamics during ovarian follicle activation, (submitted) arXiv :1903.01316

Image: A mathematical states and a mathem

#### Agreement to data



- $\Rightarrow$  The model can capture both data sets
- $\Rightarrow$  Lack of identifiability (non-conclusive on retro-action)
- $\Rightarrow$  First differentiation, then proliferation (sligtly more concomitant in mutant case)

#### Basal growth model



#### Key features of follicle basal growth

- Growth of a small follicle after initiation
- Spherical Symmetry
- Spatial structure of somatic cells in concentric layers
- Joint dynamic ovocyte growth Somatic cells Proliferation



Courtesy of Danielle Monniaux.

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#### Geometric model : spatial compartment in successive layers

- Spherical somatic cells (*d<sub>G</sub>*)
- Spherical ovocyte (*d*<sub>O</sub>)
- Finite number of Layers (*J*)
- Spherical Follicle  $(d_f)$  $\hookrightarrow d_f = d_0 + 2Jd_G$

Somatic cells are supposed incompressible and migrate to successive layers.



# Dynamical model (Multi-type Bellman-Harris Branching process)

- Age and position dependent division rate (cell cycle regulated by the ovocyte)
- At division, unidirectional motion centrifugal
- Cells are independant between each other (Unlimited layer capacity)



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- The geometrical model allows a simple spatial description
- The model is linear and decomposable : exponential growth (under appropriate assumption), with a stable asymptotic spatial profile (with analytical first two moments) : there exists a unique λ > 0 such that the process Z<sub>t</sub> verifies





$$\lim_{t\to\infty} Z_t e^{-\lambda t} = \hat{Z} \quad (\text{in law})$$

Clément et al. Analysis and Calibration of a Linear Model for Structured Cell Populations with Unidirectional Motion : Application to the Morphogenesis of Ovarian Follicles, SIAM App. math, 2019

#### Data

• We have counting data of somatic cells in snapshot data, morphological data (diameter) and *order of magnitude of transit times between follicle "type"* 

	t = 0	t = 20	t = 35
‡Data points	34	10	18
Total cell number	$113.89 \pm  57.76$	$885.75 \pm \scriptscriptstyle 380.89$	$2241.75 \pm {\scriptstyle 786.26}$
Oocyte diameter ( $\mu m$ )	$49.31 \pm 8.15$	$75.94 \pm \scriptscriptstyle 10.89$	$88.08 \pm \textbf{7.43}$
Follicle diameter ( $\mu m$ )	$71.68 \pm \texttt{13.36}$	$141.59\ \pm \ {}^{17.11}$	$195.36 \pm {\scriptstyle 23.95}$

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⇒ Can we explain proliferation in concentric layers by a simple model of "division-migration"? Or do physical constraint play important role?

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- $\Rightarrow$  Can we characterize the growth rate of a follicle and spatial repartition of somatic cells?
- $\Rightarrow$  What is the impact of spatial position of a somatic cell on its division rate?

#### Fitting results

#### $\Rightarrow$ **Exponential** growth dominated by the first cell layer





Clément et al. Analysis and Calibration of a Linear Model for Structured Cell Populations with Unidirectional Motion : Application to the Morphogenesis of Ovarian Follicles, SIAM App. math. 2019

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#### Fitting results

⇒ Parameter identifiability and doubling time quantification  $(\approx 16 \text{ days})$  : Cell-cycle time  $\nearrow$  with ovocyte distance



Clément et al. Analysis and Calibration of a Linear Model for Structured Cell Populations with Unidirectional Motion : Application to the Morphogenesis of Ovarian Follicles, SIAM App. math, 2019

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#### Fitting results

 $\Rightarrow$  Prediction of a stable spatial distribution (not observed in data)



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Clément et al. Analysis and Calibration of a Linear Model for Structured Cell Populations with Unidirectional Motion : Application to the Morphogenesis of Ovarian Follicles, SIAM App. math, 2019

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#### More realistic model?



 $\frac{\partial u}{\partial t} + div (\overrightarrow{v} u) = b(x)u(t,x)$ for  $x \in \Omega(t)$  and with  $\overrightarrow{v}$  linked to the negative gradient of the pressure, and the pressure related to the density... Under locally constant density and spherical geometry, one have :

$$\frac{d}{dt}\left(r_{F}(t)\operatorname{vol}(\partial\Omega(t)\setminus\partial\Omega_{O}(t))\right) = \gamma \int_{\Omega(t)} b(x)dx + \frac{d}{dt}\left(r_{O}(t)\operatorname{vol}(\partial\Omega_{O}(t))\right)$$

#### Terminal growth model (work in progress)



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#### Terminal growth model (work in progress)

- Lost of spherical symmetry
- Joint Dynamic Liquid-filled cavity formation and growth Proliferation and differential of somatic cells Morphogen gradient
- Morphodynamic of the Liquid-filled cavity formation?
- Differentiation vs proliferation : which regulation?
- Role of the Liquid-filled cavity?



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Tertiary ( antral) follicle

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### Antrum growth model

- Lost of spherical symmetry
- Joint Dynamic Liquid-filled cavity formation and growth Proliferation and differential of somatic cells Morphogen gradient

## $\Rightarrow$ "Advection-Diffusion-Reaction" PDE.

$$\begin{array}{rcl} \frac{\partial \phi_A}{\partial t} + D\Delta \phi_A &=& 0\,, x \in \Omega_A(t)\,,\\ \frac{\partial u_M}{\partial t} + div\left(\overrightarrow{v_M}u_M\right) &=& R_M(u_M)\,, x \in \Omega_M(t)\,,\\ \frac{\partial u_C}{\partial t} + div\left(\overrightarrow{v_C}u_C\right) &=& R_C(u_C)\,, x \in \Omega_C(t)\,, \end{array}$$

+ Boundary conditions and constitutive laws





work in progress...

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### Modeling ovarian folliculogenesis

#### Growth of a single follicle



Monniaux et al., M/S. 1999

• Thesis of F. Robin, (cosupervised. F. Clément)

#### **Populations of follicules**



Scaramuzzi et al., Reprod. Fert. Dev. 2011

 CEMRACS 2018 (summer school), C. Bonnet, K. Chahour

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#### Modeling ovarian folliculogenesis on a lifespan timescale

- Compartment based model (CTMC)
- Non-linear interaction between follicles populations (endocrine and paracrine) via λ's and μ's.
- Several time scales (slow initiation, fast growth)



Scaramuzzi et al., Reprod. Fert. Dev. 2011



#### Modeling ovarian folliculogenesis on a lifespan timescale

- Compartment based model (CTMC)
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- Several time scales (slow initiation, fast growth)



Scaramuzzi et al., Reprod. Fert. Dev. 2011



#### Modeling ovarian folliculogenesis on a lifespan timescale

⇒ When  $\varepsilon \rightarrow 0$ : (quasi-) stable distribution into maturity stages, driven by a slow deterministic decay of the static reserve (quiescent follicles)





Scaramuzzi et al., Reprod. Fert. Dev. 2011

Bonnet, et al. Multiscale population dynamics in reproductive biology : singular perturbation reduction in deterministic and stochastic models preprint version of a proceeding of CEMRACS 2018 : arXiv :1903.08555



#### Qualitative agreement with data







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#### Summary

## Population dynamics in ovarian folliculogenesis

- Somatic cells differentiation and proliferation during follicle initiation
- Somatic cells proliferation and migration during basal follicle growth
- Multiscale nonlinear dynamics shape the follicle population distribution into different maturity stages.



### Summary

## Population dynamics in ovarian folliculogenesis

- Somatic cells differentiation and proliferation during follicle initiation
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- Multiscale nonlinear dynamics shape the follicle population distribution into different maturity stages.

#### Thank you for your attention !



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#### Role of the Liquid-filled cavity?

Species	Oocyte radius (µm)	Follicle radius when antrum first appears (µm)	Reference	Estimated voidage
Rat	32	100	Parkes (1931)	0.09
Mouse	35	100	Parkes (1931)	0.09
Horse	36	110	Driancourt (1979)	0.14
Pig	38	200	Parkes (1931)	0.46
Human	40	200	McNatty (1978) and Peters & McNatty (1980)	0.46
Baboon	42	155	Parkes (1931)	0.33
Rabbit	42	125	Parkes (1931)	0.20
Sheep	47	150	Turnbull et al. (1977)	0.30
Buffalo	53	300	Kumar et al. (1997)	0.65
Ferret	54	115	Parkes (1931)	0.13
Range	32-54	100-300		0.09-0.65

Redding et al, *Mathematical modelling of oxygen transport-limited follicle growth*, Reproduction, 2007

- Antrum formation is driven by oxygen (and nutriment) accessibility of the oocyte
- Antrum size is driven by the need to increase the number of somatic cells (without increasing the thickness of the somatic layers) to supply estrogen production in agreement with body mass

#### Table I The diameter of the mature follicle and characteristic weight in different mammalian species.

Follicle (mm)	Weight (kg)	Species	Ref.
0.42	0.03	Mouse	Griffin et al. (2006)
0.55	0.2-0.25	Albino rat	Sangha and Guraya (1989)
0.64	0.2	Hamster	Griffin et al. (2006)
2.8	2.7	Rabbit	Osteen and Mills (1980)
4	3	Cat	Izumi et al. (2012)
6	9-10	Beagle (dog)	Reynaud et al. (2009)
6	23	Sheep	Aurich(2011)
7.5	35	Serrana goat	Simões et al. (2006)
8	150	Gilt	Chiou et al. (2004)
7-12	48-84	Alpaca	Bravo et al. (1991)
7-12	130-200	Llama	Bravo et al. (1991)
16	250-800	Water buffalo	Taneja et al. (1996)
20	700	Cow	Evans (2003)
20	2700	Elephant	Lueders et al. (2011)
23	60	Human	Evans (2003)
20-25	800	Summatran rhino ceros	Hermes et al. (2007)
27-38	300-550	Camel dromedarius	Manjunatha et al. (2012)
30-34	1800	White rhinoceros	Hermes et al. (2007)
50	1000	Black rhinoceros	Hermes et al. (2007)
55	450	Horse	Aurich (2011)
120	1900	Indian rhinoceros	Hermes et al. (2007)

Bächler et al, Species-specific differences in follicular antral sizes result from diffusion-based limitations on the thickness of the grahulosa cell Jayer, MHR, 2014