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The relation between mathematics research activity and the design of resources for teaching at the university

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We study the relation between research and teaching practices of teachers-researchers at university. We examine this issue from the documentational approach point of view that focuses on the interactions between resources and mathematicians by considering their research activities and teaching practices. We suggest indeed theoretical and methodological developments to take into account, from the documentational approach to didactics, the interactions with resources during the research activities of the mathematicians. The data collection consists in audio-recorded interviews. We identify three forms of use of research resources in teaching practices.

Keywords: resources in university mathematics education, teachers' practices at university level, relation between research activity and teaching practices.

INTRODUCTION AND CONTEXT OF THE STUDY

The professional activity of a university teacher usually involves teaching activity and research activity. In France, among the university teachers there are teachers-researchers: they do research and have to teach at different levels (Tertiary level and Master's degree). At the early post-bachelor years, some universities offer to teach the basics of classical mathematics. Some of teachers-researchers consider those kinds of courses as important and necessary to students but unfortunately too basic regarding their field of research. The present paper constitutes a part of our research interest that concerns the understanding of the relation between teaching and research activities, and this by highlighting the disciplinary specificities. Indeed, we aim to highlight aspects that might be considered to characterize the factors underpinning it.

We are particularly interested in the study of the relation between research activity and teaching practices through the lens of interaction with resources. As Adler (2000), we give to the "resource" here, a meaning related to the verb "re-source", to source again or differently. We are conducting exploratory studies related to this issue considering different aspects:

- Considering two disciplines mathematics and physics, work that allows us to characterize factors determining the relation between research activity and teaching practices, either related to the epistemology of the discipline or not (Sabra & El Hage, 2018).
- Setting a contemporary field in mathematics (graph theory) and varying the institutions of teaching (Tabchi's PhD work, in progress) (Tabchi, 2018).
- Setting an institutional context – Engineering Education – and considering teachers of mathematics, coming from different research disciplines (physicist,

mathematician, and engineer), study that allows us to characterize factors that enhance the design and use of resources in terms of the personal relationship to mathematics and his/her (researcher) domain of research (Sabra, 2019).

The present paper constitutes a contribution to this research work. We provide a case study of the research activity of three mathematicians through the lens of the interactions with resources. We particularly dwell upon the place of research resources and their impacts on the designing and the use of the resources in and for teaching. Indeed, our general question is: how do the resources coming from research activity are related to the teacher's capacity to re-design them for his/her teaching work?

We present some theoretical and methodological development based on the *Documentational Approach to Didactics* (DAD) (Gueudet, Pepin, & Trouche, 2012).

RELATION BETWEEN RESEARCH ACTIVITY AND TEACHING IN THE RESEARCH LITERATURE

Some researches in science education attempted to find evidence of “positive” or “negative” correlations between research and teaching without taking into account a specific discipline (Elton, 1986; Neumann, 1992). They tried to characterize the relation that may occur between teaching activity and research activity (symbiosis, conflict, tension, etc.). Neumann (1992) presents three aspects of what he calls “nexus” that can exist between teaching and research: 1) the tangible aspects, generally linked to an articulation between content transfer of knowledge from research in teaching; 2) intangible aspects, which relate to the actions of the researcher in the teaching activity and vice versa); 3) the global aspect, which relates to nexus between teaching institution and research institution. In a more recent study, Elton (2001) examined the reasons behind the presence or absence of the relation between teaching and research in the practice of university teachers. In a perspective of transformation of practice, he suggests ways that could reinforce “positive” articulations between the two kinds of activities.

The question of the correlations between the two activities of a university teacher has been studied recently depending on the discipline involved. As an example, Madsen and Winsløw (2009) emphasize that the relation between research and teaching in the case of mathematics significantly differs from the physical geography discipline. In their comparative study between teachers in geography and mathematics, they emphasized the fact that the forms of relation between teaching and research strongly depend on the disciplinary specificities (institutional and epistemological characteristics of the discipline). They also stressed that the relation that can take place between both teaching and research activities depend on the perception of university teachers on the specificities of their disciplines.

Other comparative study based on the interviews with teachers-researchers in physics and mathematics, emphasizes the place of what they called *professional identity* of university professors (Lebrun et al., 2018). They highlight that the professional identity

of the teachers-researchers in both disciplines seems to be in tension due to the epistemology of the discipline; interviewed professors from both disciplines highlight the importance to teach following methods derived from research activities (group work, problem solving, modelling, etc.). However, they raise organisational constraints that prohibit applying them, particularly the assessment practices and the limited time.

Therefore, we claim to understand the relation between teaching and research within the mathematics discipline through the lens of interaction with resources. This interaction can take place at different moments of teaching practices in: the design of the classroom sessions, the choice of the contents, the implementation of resources in the classroom, and in the evaluation of learning. In addition, university teachers could use the same resources in their teaching practices and their research activities (Broley, 2016).

DOCUMENTATIONAL WORK IN RESEARCH AND TEACHING INSTITUTIONS

The DAD considers the activity of the teacher as a continuous process. In the DAD, there is a distinction between resources and documents. We define here resources as all the things that could re-source a university teacher activity (research and teaching). The interaction with the resources generates a document, which is the association of resources and a scheme of use of these resources. We can assume that in the case of university teachers the research resources re-source particularly the research activity. However, this dimension is not investigated here. We are interested in how research resources influence the design of resources for teaching.

A scheme is used here as defined by Vergnaud (1998) as the invariant organization of conduct for a set of situations having the same aim. According to Vergnaud (1998), a scheme is a dynamic structure that has four interacting components: aim, rules of actions, operational invariants, and possibilities of inferences. A class of situations includes all the situations having the same aim.

A university teacher develops a professional experience by interacting with the teaching institution and the research institution simultaneously (Madsen & Winsløw, 2009). The interaction with resources in each of the institutions are related on the one hand to the specific classes of situations (research classes of situations, teaching classes of situations) and on the other hand to the specificities of the discipline. The relation between research and teaching could take place as a migration and adaptation of the resources between institutions, or also like a dissemination by a university teacher of the professional knowledge and mode of teaching (the “operational invariants” component of scheme of use resources, Gueudet & Trouche, 2009).

We distinguish between: 1) the *teaching document* (aims related to the teaching class of situation, resources for teaching, rules of action and operational invariants) in the meaning of (Gueudet, 2017); 2) the *research document* (aims related to research classes of situation, resources for research, rules of action and operational invariants). Each

kind of document is considered in its institution with corresponding conditions and constraints. Gueudet (2017) notices that university teachers develop a resources system for research in the research institution and a resources system for teaching in the teaching institution. The study of both resources systems and their interaction requires new theoretical and methodological developments. Given the background, we have explored the process of interaction between both systems from the point of view of “pivotal” resources in research activities of the university teacher.

The concept of “pivotal resources” is characterized in the previous studies using DAD since resources that intervene in several classes of situations (Gueudet, 2017). In this paper, the “pivotal resources” are considered in the teaching documentation work. In our contribution, we define a “pivotal resource” as a resource that contributes for a given teacher to the construction of many research documents in an institution. We consider that a pivotal resource is used in several class of research situations. Using frequently a pivotal resource could influence a part of the research activity. For us, if there are relations between research and teaching activities, it will take place in terms of the classes of situations where pivotal resources are mobilized. We hypothesized that there is at least one pivotal resource in the research work of a given mathematician. It could be a software of numerical computation, a founding book in his/her field of research, or others. Consequently, our general question turns out to be as follow: How do the pivotal resources coming from research institution enrich the teacher’s capacity to re-design and use them for his teaching work?

CONTEXT AND METHODOLOGY OF THE STUDY

As an exploration of different facets of the issue of relation between research activity and teaching practices, we present here a study based on three interviews with French teachers-researchers (see Table 1. for the profiles). To keep the anonymity we will call them M1, M2 and M3. A university teacher in France must teach at different levels, a variety of subjects and topics ranging from the basic level in a discipline to very specialized courses in her fields of research.

| | Research experience | Research domain | Teaching experience | Teaching level |
|----|---------------------|--|---------------------|--|
| M1 | 16 years | Mathematical modelling of physical phenomena | 16 years | Undergraduate degree (Mathematics and computer sciences) and Master degree (applied Mathematics) |
| M2 | 6 years | Mathematical modelling of scientific phenomena | 6 years | Undergraduate degree (Mathematics) |

| | | | | |
|----|----------|--|----------|--|
| M3 | 17 years | Theorist mathematician (number theory) | 17 years | Undergraduate (Mathematics) and Master degree (pure Mathematics) |
|----|----------|--|----------|--|

Table 1: The profiles of the three university teachers.

We elaborated the interviews guidelines from two distinct parts: research activity part and teaching activity part. We did not ask direct questions about resources so that the interviewed could express themselves freely about their research and teaching activities. This choice allowed us to identify the resources quoted in their answers that we considered as a pivotal resource. The interviews lasted between an hour and an hour and a half; they were semi-structured; each interview took place in the office of the university teachers. All the interviews were recorded and conducted in French.

The transcripts of the interviews were coded according to the theoretical framework and our development/adaptation in order to build for each interview two tables: the teaching documents table corresponding to the teaching work, and the research documents table corresponding to the research activity (see Table 2). The tables allowed us to consider the list of documents in each of both institutions: research institution and teaching institution.

| | | | |
|---------------------------|-----------|-----------------|------------------------|
| Research documents tables | | | |
| Research aims | Resources | Rules of action | Operational Invariants |
| | | | |
| Teaching documents tables | | | |
| Teaching aims | Resources | Rules of action | Operational Invariants |
| | | | |

Table 2: Presentation of the research documents table and the teaching documents table.

To build the teaching documents tables, we proceeded in the same way as (Gueudet, 2017). Actually, we tracked in the transcript of the teaching part of each interview the given answer of the aim of the teaching activity mentioned by the university teacher. For each aim, we added the resources explicitly mentioned in the transcribed declaration. Then, we identified stable elements in the way these resources were used (rules of actions). Concerning stability, we relied on the teacher's declarations (e.g., "for ..., we always start by..."). Finally, we noted the operational invariants (this corresponds to statements in the interview such as: "I do this way Because I think that ...").

We proceeded in the same way for the research part of the interview, which concerns research in order to build the research documents tables. First, we defined a research aim. Then we added resources, we identified rules of actions in the declaration. Finally, we noted the operational invariants.

Once both tables were built, we first identified the pivotal resources in the research documents table (see table 3).

| Research documents table | | | |
|--------------------------|------------------------|-----------------------|-----------------------------|
| Aims (A_i) | Resources | Rules of actions (RA) | Operational invariants (OI) |
| A_1 | Resource 1 | RA_1 | OI_1 |
| A_2 | Resource 2 | RA_2 | OI_2 |
| A_3 | Resource 3, Resource 1 | RA_3 | OI_3 |
| ... | ... | ... | ... |
| A_n | Resource 4, Resource 1 | RA_n | OI_n |
| Teaching documents table | | | |
| Aims (A_i) | Resources | Rules of actions (RA) | Operational invariants (OI) |
| A_1 | Resource 1 | RA_1 | OI_1 |
| A_2 | Resource 5 | RA_2 | OI_2 |
| A_3 | Resource 6, Resource 1 | RA_3 | OI_3 |
| ... | ... | ... | ... |
| A_n | Resource 7, Resource 8 | RA_n | OI_n |

Table 3: Identifying pivotal resource, which is Resource 1 in this research documents table. Resource 1 appears also in the teaching documents table.

Then we checked whether the pivotal resource in the research documents table (Resource 1 in table 3) was mentioned or not in the teaching documents table. When it was the case, we took into account the teaching document where this resource appears (the table line corresponding to the document). If not, we tried to understand the reason behind the lack of this resource regarding the operational invariant in research institution and/or the consideration of constraints in the teaching institution.

This methodology enables to question the resource mobilization process from research institutions to teaching institutions, by considering a horizontal analysis of each document in each institution.

FORMS OF RELATION BETWEEN RESEARCH AND TEACHING IN TERMS OF RESOURCES

By our analysis, we identified three forms of relation between research and teaching in terms of resources.

First form: research resource in instantiation processes

In the case of M1, we identified seven aims in the research institution, in which the software (Matlab, Maple, etc.) is fundamental in numerical modelling research (6 aims over 7). M1 uses the software to conjecture, validate (a conjecture or a modelling method). The place of the software occupies the main line of his research approach. In the teaching institution, we identified two teaching documents where the software is used. He uses the software with the Master's degree students in order to sensitize students to the characteristics of the software in the activity of mathematical modelling (see table 4 for an example of those documents).

| | |
|--|---|
| | M1- teaching document |
| Aims | Sensitizing students to the characteristics of software in the activity of mathematical modelling. |
| Resources | Software of numerical computing. Resources from previous teaching years that contains problem to solve. |
| Rules of action (way to use the resources) | Choosing software used in the research. Choosing and adapting a problem solving that permit a manipulation, an observation and the interface of software and experiment with it. |
| Operational Invariants (reasons for using them this way) | The modelling activity in mathematics is exploratory and experimental. |

Table 4: Presentation of a teaching document where the pivotal resource is used.

In this case (table 4), we qualify the use of pivotal research resource in teaching institution as an action of instantiation of it. The instantiation of this resource consists in the mobilization of the research resource from research institution in the teaching institution in, as far as possible, the similar situations and in the similar role in both institutions, but in a more restricted domain of validity.

Second form: research resource to scaffold the learning of a given content

In the case of M2, we identified six aims related to his research activities, in which the software (Matlab, Maple, Scilab, etc.) is fundamental in numerical computation and graphical simulations (3 aims over 6). His research activities using a software particularly consists in analyzing, modelling biological phenomena, validating the experimental results, and communicating results to the biologists he works with. In the teaching institution, the software of numerical simulations appears in two teaching documents. We develop, in the table 5, one of them which corresponds to the aim "designing session to experiment and discover mathematical properties with software".

| | |
|---|---|
| | M2 – teaching document |
| Aims | Designing session to experiment and discover mathematical properties with software. |
| Resources | Software of numerical computation. Resources corresponding to the course in question. |
| Rules of action (way to use the resources) | Select a phenomenon of stability of differential equation. Show the stability on a graphical representation. Offer the possibility to vary values and parameters in order to lead a discussion about hidden properties. |
| Operational Invariants (reasons for using them this way) | A software is a tool that gives the results in a visual way and hides the properties. We have to stimulate the spirit of imagination to make links between representations and mathematical properties underpinning. |

Table 5: Presentation a teaching document related to the aim “designing session to experiment and discover mathematical properties with software”.

M2 assigns the same role to the software in the construction knowledge in both institutions (research and teaching), while the operational invariants show that M2 uses software in teaching institution to scaffold contents, in the design of the resource as well as in the implementation.

Third case: the no relation form in terms of resources

In the case of M3, there is a pivotal resource in the research documents table; however, it is not mentioned in the teaching documents table. This result is strengthened by the words of M3 during the interview acknowledging that there is a gap between mathematics research activity and mathematics teaching activity. From his point of view, if there is a link it will be in the way of teaching (Operational Invariant). He teaches the proof following the same process lived in his research: he makes hypotheses then he determines the properties to be mobilized. There are no resources in common between teaching institution and research institution. He has a perception of “divorce” between the two institutions. He does not place his students in research situations. According to him, to be able to do this, the whole community of the class does not have to know how to solve tasks. The relations that can exist are not tangible (Neumann, 1992). They correspond to for instance, the relations between the way of teaching “to follow the same approach of research” in the treatment of a proof.

We can deduce that there is a relation between teaching and research which could be seen through the process of using the resource in the classroom and not only as a

process of migration of resources from research institution to teaching institution. This result meets the ones identified by Tabchi (2018) in the case of teachers-researchers in graph theory. We qualify the interactions between research and teaching institutions as an action of spreading scientific attitude (research process) in teaching practices.

FINDINGS, DISCUSSION AND PERSPECTIVES

It appears mainly that the relation maintained between research and teaching depends closely on the university teachers' perceptions of his/her research resources. We remind that our methodological choice requires to identify the pivotal research resources of university teachers and then study the relation between research and teaching. The analysis results support our hypothesis that the pivotal resources influence an important part of the research activity and thus if there are relations between research and teaching activity, it might take place in terms of the classes of situations where these resources are mobilized.

The documentational approach offers a possibility to characterize *tangible nexus* (Neumann, 1992) between research and teaching (via the kind of interaction with resources), but also *intangible nexus* (Neumann, 1992) related to the interaction links to the specific professional knowledge of the university teachers; the operational invariants resulting from the research activity partly determine teaching practices. Therefore, schemes (Vergnaud, 1998) in the interactions with resources are challenging to infer. One source of complexity of the scheme concept is the component 'operational invariant', which is invisible and not always conscious to the teacher. From a methodological point of view, it is a matter of inferring schemes by cross-referencing data from different tools and sources: interviews, observation of teachers, and so forth.

The study of the relation between the research resources system and the teaching resources system deserves further study or even a long-term study that contains observations. In addition, A teacher may have two different forms of relation between teaching and research depending on the teaching aims (indeed, the associate classes of situation). This is a field to explore in order to understand the interactions between the teaching resources system and the research resources system.

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