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Exploring the affordances of Numbas for mathematical learning: A case study

Said Hadjerrouit

University of Agder, Kristiansand, Norway, Said.Hadjerrouit@uia.no

The computer-based assessment system Numbas offers new learning possibilities in mathematics education by means of formative feedback. The paper proposes a theoretical framework that captures the affordances emerging from students' interactions with Numbas at the technological, mathematical, and assessment level. The aim of the paper is to explore the affordances that arise from these levels using the theoretical framework as a lens. Based on the results, preliminary conclusions and recommendations for future work are proposed.

Keywords: Affordance, feedback, interaction, mathematics learning, Numbas, teacher education.

INTRODUCTION

Although several research studies provide compelling examples of computer-based assessment systems in mathematics education, there has yet to be systematic investigations into how affordances of the systems might support mathematical learning. Numbas is a computer-based assessment system that provides formative feedback to students on whether their answer to a task is correct and reveals solutions to the students' submitted mathematical formulas (Perfect, 2015). Formative feedback allows students to see their own progress and change their mathematical thinking. Students' interactions with Numbas by means of formative feedback create affordances for learning at the technological, mathematical, and assessment level. This article uses Gibson's affordance theory to analyze the affordances that arise at these levels in a course on digital tools in mathematics teaching.

THE COMPUTER-BASED ASSESSMENT SYSTEM NUMBAS

Numbas is a computer-based assessment system with an emphasis on formative feedback. It is developed at the university of Newcastle (UK), and it is used at over 30 institutions around the world (Perfect, 2015). Formative feedback in mathematics education is normally given by a teacher, but it could be viewed as being the result of the student's interaction with any digital or non-digital learning milieu. If the student's action changes the milieu that provides feedback, this very change may cause the student to reconsider her action (Brousseau, 1997). Shute (2008) identified two main functions of formative feedback. Verification of whether an answer is correct, and elaboration to provide relevant cues to guide the learner towards the correct answer. Grounded on the idea of formative feedback, the primary design goal of Numbas is to enable a student to enter a mathematical answer in the form of an algebraic expression and submit the answer. Numbas provides feedback to the student on whether the

submitted answer is correct or incorrect and generates information according to the students' submitted expressions. Numbas user interface offers several options: *Enter an expression* in the input field, *“Submit all part”* of the answer or *“Submit part”* of the answer, *“Show steps”* of the solution, *“Try another question like this one”*, and *“Reveal answers”*, which gives the solution to the question. Numbas has also a marking scheme for the answer, e.g., *“1 mark”* for submitting part of the answer, and *“2 marks”* all parts (Figure 1 and 2).

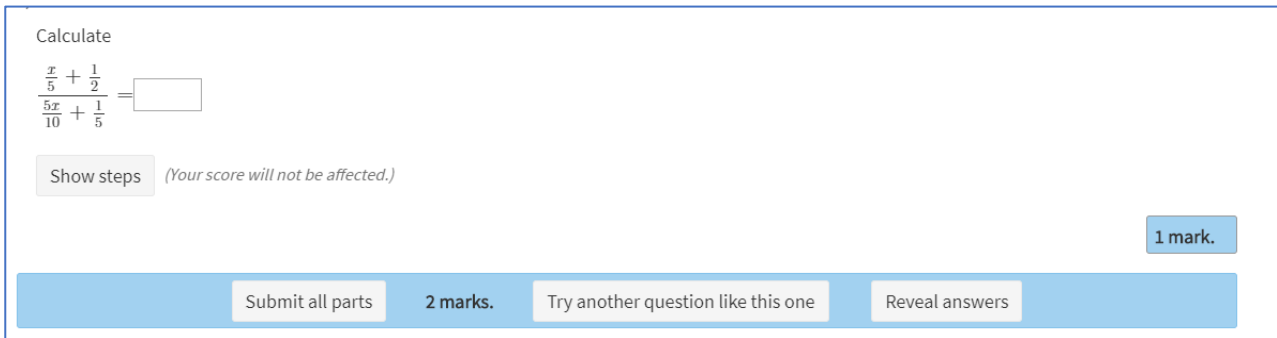


Figure 1: Numbas user interface showing a task at grade 8-13

Like many other computer-based assessment systems with automated feedback, Numbas involves a set of tasks or questions similar to paper-based tests (Van der Kleij et al., 2015). Numbas supports several question types, e.g., number entry, multiple choice/multiple response, and text entry. Questions are randomized according to a set of variables, defined by mathematical expressions (Perfect, 2015). A question consists of a statement, one or more parts, and an advice section, which contains a fully worked solution to the question and is revealed once the student has finished the question.

While many other mathematics assessment systems have a dependency on server-side computer algebra systems, Numbas runs entirely on the client-side, which means that the feedback is either immediate and very fast or with a small delay (Perfect, 2015). Numbas contains a computer algebra system written in JavaScript so that expressions can be interpreted and evaluated entirely on the client-side. Numbas uses a simple syntax similar to the one used on a calculator. Moreover, the student's expression is rendered in a mathematical notation beside the input field and updated immediately as the student types to make sure that Numbas has interpreted her expression as she intended (Figure 2).

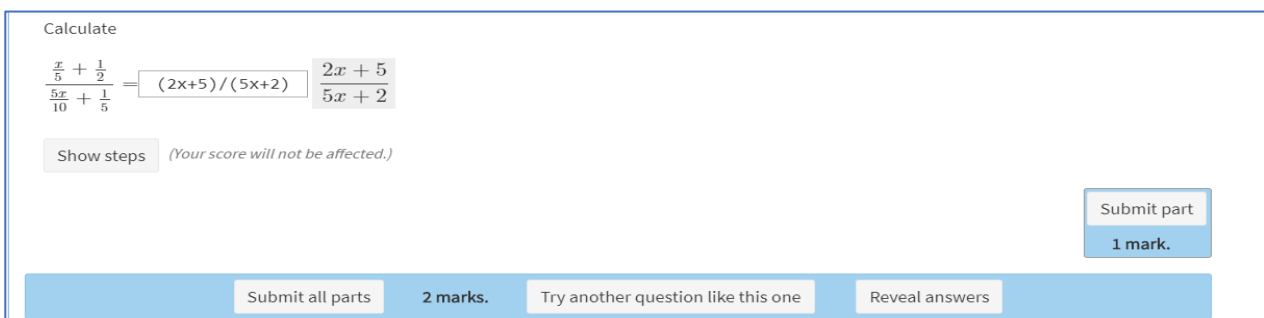


Figure 2: The student's answer is rendered immediately next to the input field

AFFORDANCE THEORY: A FRAMEWORK FOR ANALYSING STUDENTS' INTERACTIONS WITH NUMBAS

Affordance theory, originally proposed by Gibson (1977), refers to the relationship between an object's physical properties and the characteristics of a user that enables particular interactions between user and object. Affordance theory states that the world is perceived not only in terms of object properties but also in terms of object *action possibilities* offered to an animal by the environment with reference to the animal's action capabilities. A typical example is a tall tree that has the affordance of food for a giraffe because it has a long neck, or a chair that affords a human a possibility of sitting. Hence, affordances are not a set of characteristics that are inherent to the object and independent of the user. Rather, an affordance is ontologically neither an objective nor a subjective property, or in other words, affordances are *relational emergent properties of the user-object interaction*. Likewise, De Landa (2002) emphasizes that affordances are not intrinsic properties of the object or subject. Rather affordances emerge from the interaction between the subject and object and become actualized in a specific context.

Norman (1988) adapted the concept of affordance in the computer world. Accordingly, an affordance is the design aspect of an object which suggests how the object should be used and a visual clue to its function and use. Examples of affordances are user interface elements that directly suggest suitable actions such as clickable geometrical figures, draggable sliders, pressable buttons, selectable menus for figures, etc. Several research studies used the concept of affordance in various educational settings. For example, Turner and Turner (2002) specified a three-layer articulation of affordances: Perceived affordances, ergonomic affordances, and cultural affordances. Likewise, Kirchner et al. (2014) described a three-layer definition of affordances: Technological affordances that cover usability issues, educational affordances to facilitate teaching and learning, and, social affordances to foster social interactions. In mathematics education, Chiappini (2013) applied the notion of perceived, ergonomic, and cultural affordances to Alnuset, a digital tool for high school algebra. Finally, Hadjerrouit (2017) presented a model of affordances in teacher education.

Based on the literature on affordances and the features of Numbas described in the previous section, this work uses Gibson's affordance theory and proposes a set of affordances that may emerge from students' interactions with Numbas at the technological, mathematical, and assessment level. Considering affordances as emergent properties means that Numbas does not have any affordances, except in interaction with students.

Given this background, several technological affordances could emerge as students interact with Numbas. These are: a) Numbas is user-friendly and accessible at any time and place. b) Numbas facilitates accurate and quick completion of mathematical activities. c) Numbas helps to draw graphs and functions, solve equations, construct diagrams, figures and shapes.

Likewise, several affordances could potentially emerge from the mathematical level.

a) Numbas presents the mathematical content in several ways using text, graphs, symbols, interactive diagrams, and dynamic visualizations. b) Numbas facilitates the transformation of mathematical expressions that support conceptual understanding. c) Numbas supports various mathematical activities such as problem solving, multiple choice, and quizzes. d) Numbas mathematics is congruent with textbook mathematics. e) Numbas provides high quality of mathematical content and useful questions that foster higher-level mathematical thinking. f) Numbas displays formulas, functions, graphs, numbers, algebraic expressions, and geometrical figures correctly. g) Numbas simplifies mathematical expressions so they look as they are on paper.

Finally, several affordances could potentially emerge from the assessment level. These are: a) Numbas provides several assessment tests. b) The order and wording of the assessment questions are appropriate. c) Numbas questions are useful and relevant to test mathematical knowledge. d) Numbas gives immediate feedback. e) Numbas provides several types of feedback, e.g. expected answers and advices to the solution. f) Numbas feedback contains useful information to understand the tasks and answer the questions. g) Numbas gives hints in form to problem solving. h) Numbas is flexible to handle a wide range of assessment questions, answer to a question, and whether it is correct or not. i) Numbas provides a summary of the test, statistics on students' answers, what they have done wrong or right, their performances and grading.

THE STUDY

Context, participants and research questions

This study was conducted in a course on digital tools in mathematics teaching at the University of Agder in Kristiansand, Norway. The participants ($N=15$) were three categories of teacher students following teacher education programmes for grade 1-7, 5-10, and 8-13. None of the students had any prior experience with Numbas. The study addresses two main research questions:

- Which affordances emerge from students' interactions with Numbas at the technological, mathematical, and assessment level?
- How do students value the formative feedback provided by Numbas?

Methods

Both quantitative and qualitative methods were used to analyze the affordances that emerge from the students' interactions with Numbas:

- A survey questionnaire with a five-point Likert scale from 1 to 5, where 1 was coded as the highest and 5 as the lowest (1 = "Strongly Agree"; 2 = "Agree"; 3 = "Neither Agree nor Disagree"; 4 = "Disagree"; 5 = "Strongly Disagree"). The average score (MEAN) and the standard deviation (Std. Dev) were calculated
- Students' comments in their own words on each of the survey items, and qualitative analysis of the comments

The survey questionnaire was guided by the theoretical framework and the set of potential affordances described in the previous section. The qualitative analysis was

based on indicators of affordances from students' comments on the survey items in interaction with the affordances of the theoretical framework and search for meaningful interpretation of the results in the context of the course, the participants and the teacher.

Numbas mathematical tasks

Mathematics tasks at grade 1-7, 5-10, and 8-13 for a period of two weeks were given to the students using Numbas. These are some examples of tasks at grade 1-7:

Answer these mathematical expressions:

a) $3 - 6$	b) $3 - 9$	c) $3 + 6 - 9$
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These are some examples of tasks at grade 5-10:

You may write your expression in the text box. Remember to write for example a^3 to get a^3 and $2a-3b$ to get $2a - 3b$. Next to the text box, you will get an image of how Numbas reads your input. Make sure this is what you intended to answer!

a) Simplify $-2a - 3b + 4a + 7b$	b) Simplify $2a^2 + 6a + 2 + 7a^2 - 5a - 3$ c) Dissolve the parenthesis and simplify $(6a + 4b) - (5a - 5b)$
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These are some examples of tasks at grade 8-13 (See also figure 1 and 2):

Calculate and give your answer as a fraction or an algebra expression. Use / as the fraction line, $\frac{x-2}{3}$ is written as $(x-2)/3$ and $\frac{x+1}{x-4}$ as $(x+1)/(x-4)$

a) Calculate $\frac{\frac{x}{2} + \frac{1}{4}}{\frac{3x}{8} + \frac{1}{2}}$	b) Calculate $\frac{x+3}{5x} - \frac{2x-4}{4x}$
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RESULTS

Technological affordances

The results (Table 1) show that most students indicated that Numbas has a user-friendly interface and that it is easy to use (Item 1, 2). It has also a ready-made mathematical content, and it is accessible anytime and anyplace (Item 3, 4). This is reflected in many students' comments such as: "*easy and fine design*"; "*easy to find and navigate through the information*"; "*very positive that we get immediate feedback*".

	Mean	Std. Dev
1.Numbas is user friendly	1.80	0.561
2.Numbas is easy to use	1.73	0.704
3.Numbas is accessible anytime and anyplace	1.67	0.724
4.Numbas has a readily available content	1.80	0.775

Table 1: Technological affordances

Mathematical affordances

In terms of mathematical affordances (Table 2), most students agreed that Numbas provides a high quality of mathematical content, and that the questions are well-

designed and formulated (Item 1, 2). Likewise, more than the majority of the students found that Numbas displays mathematical notations correctly, and that it simplifies mathematical expressions (Item 7, 8). Many students also think that Numbas provides opportunities to foster mathematical thinking through various entry points, such as “Submit answer”, “Try another question like this one” or “Reveal answers” and choosing different questions (Item 6). Furthermore, less than the majority of the students answered positively item 3, 4, and 5, but there is a relatively big variation in their responses (Std. Dev=1.014 and 1.033).

A qualitative analysis of the students’ comments reveals two main themes, which are important to deepen some students’ responses to the survey (See Table 2, item 1, 3, 5, 7, 8). These are: congruence of Numbas with paper-pencil techniques and textbooks mathematics, and Numbas mathematical tasks and questions. Some relevant comments on congruence are:

“It was very good that one could enter the formulas in the fields and calculate the answer here”

“Very good that Numbas writes my answer as you see it on paper even though I write it differently”

“I use paper to figure out the answer in different steps and enter, not just the answer”

In terms of the questions and tasks provided by Numbas, students think that this issue is to a great degree dependent on the teacher and her knowledge and the way she designed the tasks and questions. Errors in Numbas may also foster reflections. Some selected students’ comments are:

“Depends on how the questions are asked”

“Again, it really depends on whether the teacher has designed and programmed the questions correctly. On the other hand, errors in the program can also help to stimulate reflection if they try to understand what has gone wrong”

“The degree to which Numbas responds to the questions above, is entirely dependent on the teacher who creates the questions, (...) the program is very flexible in terms of how to create these, as it has advanced features, (...)”

“(...) I think what is very positive is the given response to answers, the possibility of hints and of showing the solution. This information can help students to reinforce their understanding. The teacher has a lot of power to control how Numbas will affect the student.”

It appears from the students’ responses that mathematical affordances depend mostly on how the teacher has formulated the tasks, even though an intuitive user interface and the way feedback is designed are pre-requisites for mathematical affordances. The congruence of Numbas with paper-pencil techniques may also facilitate mathematical understanding as mentioned above. Moreover, errors in Numbas mathematics do not automatically hinder student learning if they are able to exploit the limitations of the system to provoke their mathematical thinking.

Mathematical affordances	Mean	Std. Dev
1.Numbas questions are well-designed and formulated	1.87	0.516
2.Numbas has a high quality of mathematical content	2.20	0.775
3.Numbas exercises and questions are useful to foster reflections, metacognition, and higher-level mathematical thinking	2.80	1.014
4.Numbas provides opportunities to exploit the constraints and limitations of the tool to provoke students' mathematical thinking	2.60	0.737
5.Numbas provides opportunities to focus on conceptual understanding of mathematics rather than procedural skills	2.93	1.033
6.Numbas provides opportunities to foster mathematical thinking through various entry points, different questions and exercises	2.27	0.799
7.Numbas is mathematically correct. It displays formulas, graphs, functions, numbers, expressions, and geometric figures correctly	2.07	0.884
8.Numbas simplifies mathematical expressions so they look as if we wrote them on paper	2.29	1.069

Table 2: Mathematical affordances

Affordances emerging from the assessment level

In terms of affordances emerging from the assessment level (Table 3), most students think that the order of the questions is appropriate when trying another one (Item 1). Likewise, the language and wording of the questions are understandable (Item 2), indicating overall satisfaction with the teacher who creates the questions. Most students also think that Numbas gives immediate feedback to the students, and it provides several types of feedback such as expected answers and advices to the solution (Item 4, 5). Likewise, for most students, Numbas provides a summary of the test, students' answers to questions, and what they have done wrong or right (Item 8), and in a lesser degree whether the answer is correct (Item 9), but there is an important variation in their answers (Std. Dev=1.033). Likewise, some students did not find that Numbas feedback contain useful information that helps them understand the tasks and answer the questions (Item 6). Hints in form of videos to problem solving were not always useful (Item 7), but there is a huge variation in their answers (Std. Dev=1.351). Moreover, most students think that Numbas provides a summary of the test (Item 10), and statistics on students' answers to questions and their performances and grading (Item 11). Finally, it should be noted that Numbas does not take the profile and knowledge level of the student into account or serve up appropriate questions (Item 8).

The most important themes that emerged from the qualitative analysis are the quality of the feedback of Numbas and the role of the teacher in formulating the questions and designing the feedback. The following comments show that some students were not satisfied with the quality of the feedback and the way it is implemented to support mathematical understanding.

“That said, the forms of feedback both to the students and to the teachers, as far as I can see, are not very good, and therefore should not be based on such tests alone”

“The fact that the students receive feedback right away is positive, which means that they can make self-assessments, but if the test is difficult, the result will not come from Numbas”

“To some extent, it might have been better that Numbas gives more concrete feedback if I had made an obvious mistake as for example, a wrong sign”

“Numbas measures right / wrong, and has little focus on process of conceptual understanding, even though one can object that if a student gave the correct answer, he might have understood the mathematical concept”

Beyond the feedback, some students emphasized the role of the teacher in providing meaningful mathematical tasks and questions, in line with students’ comments on mathematical affordances.

“In Numbas, I feel that the students get a little more control over their own test result as they can choose how much help and support they want themselves. Again, I think this depends much on the design of individual tests that determine the degree to which feedback satisfies the needs of individual students”

“Item 1 and 2 depend on the teacher who creates the questions, but it can be added that Numbas offers the possibility of letting the order of questions be random”

Affordances at the assessment level	Mean	Std. Dev
1.The order of the questions is appropriate	1.80	0.775
2.The language and wording of the questions are understandable	1.60	0.632
3.The questions are useful and relevant to test my own knowledge	2.20	1.014
4.Numbas gives immediate feedback to students’ answers	1.53	0.915
5.Numbas provides several types of feedback such as expected answers and advices to the solution	1.93	0.884
6.Numbas feedback contains useful information that helps me understand the exercises and answer the questions	2.47	0.915
7.If get stuck I can ask Numbas to give a hint in form of videos to problem solving step by step	2.14	1.351
8.Numbas takes the profile and knowledge level of the student into account and serves up appropriate questions	3.13	0.915
9.Numbas shows the answer to a question, and whether it is correct	1.93	1.033
10.Numbas provides a summary of the test, students’ answers to questions, and what they have done wrong or right	1.47	0.743
11.Numbas provides statistics on students’ answers to questions and their performances and grading	1.77	0.832

Table 3: Affordances at the assessment level

DISCUSSION AND PRELIMINARY CONCLUSIONS

It is acknowledged that this study cannot be generalized due to the limited number of participating students. However, some answers to the research questions and

preliminary conclusions can be drawn. First, affordance theory revealed to be a useful framework to analyse affordances that arise from students' interactions with Numbas.

Second, several affordances have emerged from the technological, mathematical, and assessment level, but not all potential affordances were actualized. Another educational context might also open the possibility for the emergence of new affordances. Technological affordances are pre-requisites for a trouble-free interaction with Numbas for mathematical learning and assessment purposes. An intuitive and user-friendly interface is thus an important feature of Numbas in line with the primary design goal of the system. In terms of mathematical affordances, the study shows that Numbas has a high level of mathematical content that is correct, sound and congruent with textbook mathematics. It helps to test a great variety of knowledge from primary to secondary mathematics. At the assessment level, Numbas provides several types of feedback to test students' mathematical knowledge, e.g. immediate feedback on correctness of the answer, advice to the solution, summary of the test, and grading.

Third, the paper shows that the immediate feedback of Numbas provides help and hint to test a broad spectrum of mathematical knowledge. As a result, the participating students benefited from Numbas feedback and associated tasks with varied levels of difficulty across grade 1-7, 5-10, and 8-13. The results are in line with the research literature that shows that Numbas improved student experience, increased student engagement and enjoyment (Caroll et. al, 2017; Perfect, 2015). Through the feedback it gave to students, Numbas proved to be a useful formative assessment tool.

Fourth, Numbas in its current form is suited for use as a large question bank to practice basic mathematical skills. Moreover, Numbas is most suitable in a formative mode rather than summative, which occurs at the end of an activity without the possibility for students to change their actions. In a formative mode, students can receive immediate feedback to their answers with additional and useful information, and each question has a "*Try another question like this one*" button, which generates a new question of the same type. Clearly, formative feedback is an essential component of assessment, and it can take many forms, e.g., immediate feedback to students' actions, advices to solutions that combine various information, and solution step by step.

Some students felt nevertheless that Numbas is limited to wrong/right answers, and it should provide more information that fosters conceptual understanding. Highlighting where a student has gone wrong, and not only right or wrong, giving a fully and detailed working solution to a task with additional information on appropriate strategies would provide more powerful feedback that can make students more confident in their mathematical learning (Clark, 2012; Hattie and Timerley, 2007). However, more creative problem-solving tasks and more powerful feedback are much harder to automate. Hence, work remains to be done to provide tasks that foster metacognition, conceptual understanding, and high-order mathematical thinking.

Finally, the role of the teacher cannot be underestimated for an effective use of Numbas. The teacher is a key stakeholder in formulating meaningful tasks and

questions to ensure that students acquire basic skills in mathematics. Clearly, the quality of Numbas feedback depends on the preparatory work and improvements done by the teacher to design, implement and refine purposeful mathematical tasks.

Future work will produce a more elaborated and detailed analysis of the study to ensure more reliability and validity of the results and conclusions in accordance with the theoretical framework. The analysis will include students' responses and comments to open-ended questions to provide a richer empirical basis.

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