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From an analysis of content to an analysis of ordinary lecturing practices: A case-study in mathematical analysis

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This paper bears on teaching practices in lecture courses on analysis in the first year of tertiary education. This case-study shows how the knowledge accumulated by the didactical community on the challenges in the teaching of a specific notion – the formal definition of limits of sequences – allows for a fine-grained analysis of teachers' practices. On the basis of this knowledge, three lecture courses were analysed and compared. Post-teaching interviews were used to test our hypotheses as to the didactical knowledge, choices and repertoire of the lecturers. This progress reports aims to sketch the research rationale and to discuss a small sample of results.

Keywords: Teachers' and students' practices at university level, teaching and learning of analysis and calculus.

INTRODUCTION

This paper reports on a research project developed in the framework of activity theory (Hache and Robert, 2013). It aims to contribute to its adaptation to the study of the teaching practices in lecture-courses (Bridoux, Grenier-Boley, Hache and Robert, 2016), with a specific emphasis on the challenges of higher education (as in (Grenier-Boley, Bridoux and Hache, 2016)). In this context, the focus is on "teaching", regarded as a professional activity. We hypothesize that teaching practices can be described, analysed, (to some extent) accounted for, and (possibly) altered.

Within this larger context, this paper is of a methodological nature. We aim to study to what extent the analysis of the *relief* of the mathematical content at stake in a given teaching context – i.e. an analysis combining mathematical, epistemological, didactical, and institutional aspects (Bridoux et al., 2016) – allows the researcher to carry out an analysis of the empirical data based on the identification of *observables*. These observables will be denoted as "control points" in this paper. In particular, this approach provides means to overcome two common difficulties in the analysis of lecture-courses: The lack of information on actual student activity, on the one hand, and the difficulty to objectively identify what is *not* done, on the other hand. Moreover, the list of control points supports the construction of questionnaires which enable to researchers to test their hypotheses trough post-lectures interviews with the lecturer.

Beyond its contribution to this research program on teaching practices in higher education (with a focus on lecture-courses), this case studies suggests avenues for the professional development of teachers in higher education (Lison, 2013), (Rogalski & Robert, 2015). This aspect can only be touched upon in this short progress report.

THE *RELIEF* OF THE FORMAL DEFINITION OF LIMITS IN THE FIRST YEAR OF TERTIARY EDUCATION

The challenges of the transition from an intuitive notion of limit and the associated techniques (algebra of limits, connections with inequalities) to a formal definition used to prove theorems – in the larger context of the proof system of analysis – has been a continuous focus of didactical investigation since the 1970s (Artigue, 2016). Numerous works have identified the epistemological, didactical and cognitive challenges students face in this transition. On this basis, several didactical interventions have sought – occasionally with some success – to identify and put to the trial teaching paths which can prove conducive to the formal definition (see (Chorlay, 2019) for a recent survey). Moreover, this vast literature forms a fairly coherent whole, since, in spite of the variety of theoretical frameworks, the same key-phenomena are generally identified.

Consequently, we drew on this solid body of didactical knowledge to devise a large list of *control points* on the basis of which actual teaching practices can be described. The fact that we chose to derive our grid of analysis from a part of the didactical literature that is subject-specific – i.e. which bears on the formal definition of limits of sequences – comes at a price. Indeed, the results could be too subject-specific, thus making transfer to other contexts more difficult. Even so, we deemed the price worth paying, for two reasons. First, we wanted to see to what extent a body of knowledge bearing of mainly on students (their misconceptions, their documented behaviour in specific milieus whose didactic variables can be finely-tuned) could provide tools for the analysis of "ordinary" teaching practices, i.e. teaching practices which are not based on this research literature, by lecturers whose professional identity (de Hosson & al., 2018) and trajectory are independent from the didactical community. Second, in terms of professional development (see (Lison, 2013), (Rogalski & Robert, 2015)), studies suggest that professionals are keener to engage in a reflective practice when the focus is on very specific issues rather than on fairly general challenges.

In the two tables below, we list a series of control points which we identified in the literature on the teaching of limits. This is only a sample – albeit a significant sample – and, for lack of space, each of the points is described only sketchily and without systematic references to the literature. A wide-ranging list of references can be found in (Chorlay, 2019). The first table lists negative control points, bearing mainly on cognitive difficulties and misconceptions. The second table mentions more positive control points, since the literature allows for the identification of teaching strategies and moves which can prove conducive to the concept of limit, or, more generally, to advanced mathematical concepts.

The concept image of limits usually encompasses "primitive" models which are non-congruent with the formal definition. In particular, primitive models are usually "*x*-first"or "covariant", and resort to temporal, dynamic and causal imageries (the values of the variable *n goes* to infinity, and, as a consequence, the corresponding values of the sequence does this or that), whereas the

definition is "*y*-first" or "contravariant" (the condition on the values of the function or sequence determines conditions on the values of the variable) and string of quantifiers does not reflect or capture any notion of temporal evolution or causality.
The concept image of limits usually encompasses misconceptions, erroneous beliefs, or in-act-theorems such as: • Every convergent sequence is monotonic (at least as from a certain rank). • Every sequence which tends to $+\infty$ is monotonic increasing (at least from a certain rank). • If a sequence converges, then the distance to the limit is monotonic decreasing. • If the distance between a sequence and a given number L is monotonic decreasing, then the sequence tends to L.
The nested quantifiers can be interpreted incorrectly in several ways: • The order of quantifiers *does* matter, and a $\forall\exists$ sentence is not equivalent to the corresponding $\exists\forall$ sentence. • In a $\forall\exists$ sentence, the second variable depends on the first, but this dependence is not of a functional nature (more explicitly: in "$\forall\varepsilon>0\ \exists\ N \in \mathbb{N}$, N can be written N_ε or $N(\varepsilon)$, but N is not actually determined uniquely by the value of ε). • The presence of the third quantifier is typically not regarded as necessary by students, who, for instance, tend to regard "$\forall A>0\ \exists\ N \in \mathbb{N}\ \ u_N>A$" as an acceptable definition of the infinite limit. This incomplete definition echoes standard informal formulations such as "the values become arbitrarily large". • More generally, in the absence of the third quantifier, distinct mathematical concepts conflate: finite limit and that of subsequential limit; positive infinite limit and non-boundedness.
The transition between several formulations which are mathematically equivalent can prove challenging for students. For instance: • For finite limits: $\lvert u_n - L\rvert < \varepsilon \quad \Leftrightarrow \quad L-\varepsilon < u_n < L+\varepsilon \quad \Leftrightarrow \quad u_n \in]L-\varepsilon, L+\varepsilon[$ • For infinite limits: $\forall\, M \in \mathbb{R}\ \ \exists\, n_M \in \mathbb{N}\ \ \ \forall\, n \in \mathbb{N}\ \ \ n \geq n_M \implies u_n \geq M$ $\Leftrightarrow$ $\forall\, M \in \mathbb{R}\ \ \ u_n \geq M$ except for (at most) a finite number of terms.
The definition of a convergent sequence can be used in two different contexts, and a non-trivial shift of viewpoints is necessary to use it in a relevant way.

• Context 1 (C1): In order to *prove* that a given sequence tends to a given number, any positive ε has to be taken into account, and the existence of at least one corresponding N value has to be proved. • Context 2 (C2): If some property is to be derived for sequences which are *known* to converge, then one is at liberty to select and use one specific value for ε, and the existence of N is warranted.

Table 1: Challenges in the teaching of the formal definition of limits of sequences

There are diagrams, such as the ε-strip diagram, which can be conducive from a change of perspective, from an *x*-first to a *y*-first perspective.
Discussing and comparing several *correct* definitions can have a positive effect in terms of conceptual understanding.
Some non-standard yet correct definitions can more likely be in the ZPD of students that the standard definition. This has been documented for the "non-implicative" form of the definition (here for infinite limits): $\forall M \in \mathbb{R} \ \exists n_M \in \mathbb{N} \quad \forall n \in \mathbb{N} \qquad u_{n_M+n} \geq M$
Discussing and assessing *incorrect* definitions can have a positive effect in terms of conceptual understanding, in particular to help student distinguish between neighbouring concepts (e.g. +∞ limit and non-boundedness).

Examples can come in several types, and can be used to serve a variety of purposes, all of which with a positive effect in terms of conceptual understanding:
• In addition to *examples* (of a given concept) and *counter-examples* (which invalidate incorrect universal statements), *non-examples* can help students get a better grasp of the scope of a concept.
• *Boundary-examples*, i.e. examples of a given concept which are not typically part of the students' concept image, can help students get a better grasp of the scope of a concept. Standard instances are: Constant sequences are convergent sequences; straight line graphs are curves which coincide with their tangents etc.
• In *situations of definition construction*, examples can be used in several ways, among which:
  ○ The situation can rest on an *example space*
  ○ Examples can be used to put a candidate-definition to the test. In these situations, it is made clear from the start that the sought-for definition should be such that some objects should be examples, while some others should be non-examples.

Table 2: Affordances for the teaching of the limit concept and other advanced concepts

TERRAIN AND METHOD

Three lectures were video-recorded in the first semester of the 2018-2019 academic year at Paris Diderot University. The three lectures were delivered by three experienced lecturers (C, F, and B), two research mathematicians, and a secondary-school teacher with a university teaching position. The lectures were part of the same, 1rst year, introductory course to mathematical analysis, covering, in this order: functions, continuity (without proofs), properties of real numbers; a final period was dedicated to the formal notion of limits (finite and infinite) for sequences and to proving in analysis. All the recorded sessions covered: formal definition of converging sequences, proof of the uniqueness of the limit for a converging sequence, proof that converging sequences are bounded. Lecturer B also proved that a subsequence of a converging sequence does converge to the same limit. Lecturer F also covered infinite limits and a few properties (such as: tends to $+\infty \Rightarrow$ not bounded above, while the converse is invalid).

We hypothesize that the lecture-courses under study are "ordinary", or "standard", for several reasons: They took place in standard institutional contexts; they were delivered by experienced lecturers with stable practices, with no involvement with didactical research, and with no claim to pedagogical innovation (as was checked in preliminary interviews); they were not designed specifically for this study, and we have no indication that they might have been affected by the fact that they would be recorded for research purposes.

The videos were analysed by the researchers. Both the comparison among lecturers and the comparison between what was done and what could have been done (as identified in the list of control points) provides a wealth of information. Table 3 below shows a sketchy sample of results ("X" stands for "Yes"):

	C	F	B
Uses informal formulations before the formal definition	X	X	X
Comments on the two contexts of use for the definition (C1, C2)			
Mentions alternative correct definitions			
Mentions boundary examples			
Uses an example space to assess a candidate-definition			
Use of *x*-first, dynamic, informal formulations		X	
Explicitly warns students against common misconceptions		X	X
Mentions non-examples		X	X
Uses diagrams to introduce or illustrate the definition of a converging sequence	X		X
Uses the ε-strip diagram for this purpose			X

Mentions incorrect definitions			X

Table 3: Overview of the three lectures (partial)

As mentioned above, the use of the control points allows us to identify things which did *not* happen, even in cases when the comparison among the three lectures would not point to them (lines 2 to 5 of table 3).

The purpose of this paper is not to analyse these data in detail, but to account for how they were used to further the analysis of lecturing practices. The long post-lecture interviews (about 50' each) were not carried out right after the teaching sessions. The videos were first analysed by the researchers. On this basis, the list of topics and the specific questions for the interviews were chosen to allow the researcher to study to what extent the observed practices reflected choices, and on what form of knowledge (*lato sensu*) these choices rested. The topics addressed in the three interviews were the same, but the wording of the questions differed. To promote reflective analysis, the questions were first based, for each lecturer, on her/his own practice, as in a explicitation interview (Vermersch, 1994). Then, alternative elements of practice – preferably based on the other two lectures – were mentioned, for the lecturer to discuss and assess.

More precisely:

- When one of the control points was positive in table 3 ("X"), we attempted to investigate to what extent this reflected a choice (for instance, did the lecturer mentioned alternative moves?) and, if, so, how it was justified: What item of knowledge or what belief did the lecturer but forth – if any – to account for it?
- When one of the control points was negative in table 3 (" "), we also attempted to investigate to what extent this reflected a choice (i.e. a choice *not to* do something).
    - If so, how was it justified? What alternatives had been considered and rejected? What item of knowledge or what belief did the lecturer but forth – if any – to account for this choice?
    - If not, we mentioned alternatives so as to study how lecturers made sense of them and assessed them. When their assessment was globally positive, we attempted to spot signs indicating that it could be a lever for professional development: Was the reaction very positive? Did the lecturer readily imagine ways to include this new element of practice in her/his own lecture (extension of the repertoire[1])?

[1] We use "repertoire" to denote the "stock of skills or types of behaviour that a person habitually uses" (Lexico dictionary, https://www.lexico.com/, accessed Jan. 24th, 2020.)

SAMPLE OF RESULTS

Use of *x*-first and/or dynamical informal formulations

In the interviews, the lecturers were first asked to comment on their informal introduction to the definition. In the lecture courses, F was the only one to use contravariant, dynamical formulations; a fact with which he felt completely comfortable in the post-lecture interview, arguing that this formulation carries an intuitive representation of the concept of limit. He mentioned no possible effects in terms of misconceptions:

Int.: All three of you made a similar choice, in so far as you introduced the notion of limit informally, in everyday language, before diving into the formal definition. Let me quote you: "a convergent sequence, a sequence which has a limit, is a sequence which, as it grows, has values which come closer to the limit value". What is the role of this sentence for you? How would you describe this sentence?

F: What matters is for students to retain the <u>meaning</u> [oral emphasis] of this definition. (…) It's a difficult notion, so it seems to me it's important to emphasize the intuitive side of the notion of limit.

By contrast, C et B used only contravariant rhetorical formulations such as "If (u_n) is a complex sequence and L a complex number. One says that sequence (u_n) converges to L when u_n is "as close to L as one might want provided *n* is large enough" " (lecturer C, sentence projected). The selection of this formulation reflects a choice for C and B. In C's case, little justification is provided:

Int.: Maybe you've come across other informal formulations, some which you would rather <u>not</u> use? Some which you find less acceptable?

C: Yes

Int.: Can you think of any one in particular, for which you said to yourself "hmmm, I definitely should not say that"?

C: No, nothing comes to mind straight away. You should not do too much hand-waving. If you go "*bla, bla, bla* comes closer" … it's meaningless!

Relevance of the ε-strip diagram to introduces or illustrate convergence

Neither C nor F used the ε-strip diagram, whereas B used an on-line applet which not only displayed the diagram but allowed the user to change the ε-value and visualize the effect on the horizontal strip and on the associated value of N(ε) (see figure 1 below). B explained that he chose this applet after watching several of them, and accounted for his choice by mentioning students' concept image of the limit in a way which could reflect the *x*-first/*y*-first distinction:

Int. : This on-line applet […] did you sometimes use something else, is that something you chose from among others ?

B : Yes, I chose it, I watched several. What is not intuitive in the definition, compared with the students' representation … students tend to think: there, it's coming closer; while we say that it enters the interval, an interval which becomes ever smaller. It's not exactly the same idea.

Figure 1: The ε-strip applet used in B's lecture[2]

When commenting on the diagram, B mentioned another didactic variable in the milieu provided by the applet to the class: the fact that the distance between the sequence and the limit is not decreasing. But the fact that N is not necessarily the rank of the first value of u which enters the strip was not mentioned, neither to the class, neither during the interview

It turns out that *not* using this type of diagrams was *not* a choice for C or F: They readily admitted they had never come across it, nor thought about it. By contrast, both reacted very positively, which suggests avenues for professional development.

Int.: (…) Some of your colleagues use this diagram to introduce the formal definition. Would you find it interesting, or dangerous?

F: Yes, yes, I find it … I should do it! I love this kind of diagrams. I'm in favour of as many diagrams as possible.

Int.: What qualities or advantages do you think this one has?

F: If we start with a definition in everyday language, and a hard [i.e. formal] definition, with epsilons, this gives a 3rd definition, a visual one. The more definitions we have the better. It's a good training for students, it trains them to reflect on drawings. (…)

In spite of his attempts, the interviewer did not manage to elicit any specific analysis of the qualities (or drawbacks) of this diagram. Lecturer C was equally enthusiast, and, when pressed to comment on the choice of the displayed sequence, managed to spot one of the didactic variables in the milieu provided by the applet: The specific sequence on display is not monotonic, "what's not bad is that is goes above then below, so we

[2] Retrieved from http://gilles.dubois10.free.fr/analyse_reelle/suitesconvergentes.html (accessed oct.9, 2019).

don't have the problem I mentioned earlier" (she had mentioned the belief that converging sequences are monotonic). Other didactic variables went undetected.

Scaffolding the shift of viewpoints when switching between C1 and C2 contexts

As can be seen in the interview below, it took C a little time to identify the issue at stake, but she eventually formulated it in the clearest of ways:

Int.:	Maybe it didn't happen in your lecture this time, but we prove the "convergent ⇒ bounded" property, it is not uncommon for students to ask questions. The proof typically begins with "The sequence being convergent to L, I want to show it is bounded. So let's take a neighbourhood of size 1, let's say ε = 1" [C nods, approvingly]. Students sometime say "I don't get it, ε has to be indeterminate, that's the definition!" Did you ever get such questions? How would you respond to "You can't do that, ε has to be indeterminate".
C:	[Merely rephrases the definition]
Int:	So, later on, if in exercises students study the convergence of sequences by saying: "I say ε = 1" …
C:	[laughs] Oh no! Of course not!
Int.:	How come?
C:	No, but, there's a difference of … of … in one case we know the sequence converges (…) but when I want to prove that it does (…).

The reaction was roughly the same for the other two lecturers. It shows that not mentioning this difficulty in their lectures was not a choice. Also, it suggests that this question is a lever for professional development, since it triggers the realization of a difficulty. However, the reactions of the lecturers were not as enthusiastic as for the ε-strip, and none of them spontaneously mentioned taking this issue into account in their future teaching.

DISCUSSION AND PERSPECTIVES

Our first analyses show that for two of the three teachers, most of the elements observed, then discussed in the interviews, can be justified in some way, but do not reflect choices insofar as no alternatives are clearly mentioned. This conclusion needs to be qualified for the case of B, who mentioned more alternatives and justified choices on the basis of his experience of students' behaviour. The interviews also show a potential for teacher training, as all three teachers adopt a reflective position that leads them to consider changes in their practices. Further analysis is needed to understand why discussing control points and alternatives elicits different reactions.

The point of this admittedly sketchy progress report was to describe the goal and the method of an on-going project. We could not present any results in detail, nor discuss the connections with several theoretical frameworks and issues. From a theoretical viewpoint, this research aims to contribute to collective work of adaptation to the

specificities of tertiary education of all-purpose theoretical tools such as the MKT framework or the ergonomic and didactic approach to teaching practices (Hache and Robert, 2013). It should also prove instrumental in the adaptation of tools for the assessment of the quality of instruction (LMTP, 2011), in a context where the correctness of the mathematical content taught is not an issue.

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