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To link to this article : DOI:10.1016/j.matcom.2015.11.001
URL : http://dx.doi.org/10.1016/j.matcom.2015.11.001

Regression methods for improved lifespan modeling of low voltage machine insulation

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Abstract

This paper deals with the modeling of insulation material lifespan in a partial discharge regime under certain accelerated electrical stresses (voltage, frequency and temperature). An original model, relating the logarithm of the insulation lifespan, the logarithm of the electrical stress and an exponential form of the temperature, is considered. An estimation of the model parameters is performed using three methods: the design of experiments (DoE) method, the response surface method (RSM) and the multiple linear regression (MLR) method. The estimation is obtained on learning sets determined according to each method specification. The performance, in terms of estimation, of each of the three methods is evaluated on a test set composed of additional experiments. For economic reasons and flexibility, the learning and test sets are composed of experiments carried out on twisted pairs of wires covered by an insulator varnish. The ability of the DoE and the RSM methods to organize and to limit the number of experiments is confirmed. The MLR method, however, shows more flexibility with regard to the studied configurations. Thus, it offers an efficient solution when organization is not required or not possible. Moreover, the flexibility of MLR allows specific ranges for the factors to be explored. A local analysis of the estimation performance shows that very short and long lifespans cannot be simultaneously represented by the same model.

Keywords: Lifespan; Twisted pairs; Design of experiments; Response surface; Regression

1. Introduction and problem formulation

New applications in aeronautics, especially in more electric aircrafts, should widen the use of low voltage (under 1 kV) electrical rotating machines. Consequently, the lifespan of these machines becomes a key issue for aircraft reliability assessment. Approximately 40% of the failures in low voltage rotating machines originate from the stator-winding insulation materials [29]. For these reasons, this paper focuses on insulation lifespan statistical modeling.

The objective is to provide a reliable lifespan model under extreme conditions for further lifespan prediction under nominal conditions. For economic reasons and flexibility, experiments were conducted on twisted pairs (entwined copper cords, coated by an insulator varnish) as they are expected to behave in a similar manner to the stator-winding

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http://dx.doi.org/10.1016/j.matcom.2015.11.001

insulator with respect to thermal and electrical stresses. Obviously, the study of mechanical stress influence would require different devices to be tested. However, a mechanical stress study is out of the scope of this paper.

Considering the average device lifespan (several thousands of hours under nominal stress), carrying out full aging tests would be too expensive. Accelerated Life Testing (ALT) allows us to tackle this problem [15,24,30]. The principle of ALT is to test the components under combinations of higher-than-usual levels of the stress lifespan variables (e.g. rate of use, temperature, voltage or pressure) in order to bring the lifespan below an acceptable level. The purpose is to obtain information about the failure-time distribution at specified levels of these variables. Then, data from these tests are extrapolated, through a physically reasonable statistical model, to obtain estimates of lifespan or long-term performance at lower, nominal levels of the stress variables [2]. This kind of test has been used for different objects in electrical engineering such as insulated gate bipolar transistor (IGBT) modules in high temperature power cycling [27] or nano-structured enamels on twisted pairs [8]. In this paper, ALT is applied to the lifespan modeling of insulating materials.

The lifespan of components used in electrical or electronic engineering has been modeled as a function of a given factor according to the Arrhenius law (temperature) [6,9], according to the inverse power law (voltage or pressure) [6,25,32], or according to the Coffin–Manson law (temperature cycling) [2], for instance. Lifespan models involving several factors have also been proposed such as the Crine model (electrothermal stress) [6,17], the Hallberg–Peck (temperature and humidity) [2,26] or the Eyring (electrothermal stress) [2] models. Unfortunately, these models are limited to two factors only. Moreover, they generally require the prior knowledge of some physical properties of the studied material or of particular constants (for instance, the activation energy in the case of Arrhenius law and Eyring model or some thresholds regarding the applied stresses in the case of the inverse power law, etc.). The estimation of these quantities depends on the studied material [2]; it generally requires complex experimental setups and is only valid for a given stress range [30].

Empirical models which relate multi-stress levels to insulation lifespan have also been developed. For instance, the Simoni model [21] and the Ramu model [6], derived from the general Eyring model, the Fallou exponential model [6] and the Montanari probabilistic model [20] allow us to describe insulation aging processes occurring in particular stress ranges. However, these so-called multi-factor models are in fact limited to two factors: the electrical and the thermal stresses.

Consequently, the existing lifespan models have been applied on a variety of electrical components and materials. However, they often remain too simplistic since they are restricted to a single factor or two factors and are only valid for particular factor ranges. Moreover, they do not provide an explicit term for the interaction between factors. In real-life operations, materials are subjected to a multitude of operational and environmental stressors acting simultaneously. Thus the contribution of each stressor to material lifespan reduction cannot be studied independently from the others. The synergetic effects, including interaction or coupling terms, should be taken into account for a more realistic modeling of real operation conditions. Finally, these state-of-the-art lifespan models require a specific experimental setup and procedure for each studied material. To our knowledge, there has been no unification approach for the development of these models before now.

In this paper, we introduce a new statistical approach for the modeling of lifespan that can be applied to different types of insulating materials [15,24,30], with no prior knowledge of their physical properties. Contrary to the existing models, the proposed lifespan models include all possible interactions between the factors and they consider wide variation ranges. Experiments are based on ALT and involve three main aging factors (voltage, frequency and temperature) as they have been identified as the predominant aging factors. The number of experiments and their configuration are fully specified according to a design method under the constraint of a reduced number of experiments.

The first model studied in this paper includes the interactions between factors using the Design of Experiments (DoE) method [3,15,24,30]. The second method, the Response Surface Method (RSM) [12,22,24,30], is applied to improve model accuracy by adding the quadratic forms of the factors. The proposed methodology, based on DoE and RSM, has already been applied in [15,24] and validated for the lifespan modeling of insulating Poly-Ether-Imide (PEI) films of thermal class 180 °C. This paper shows that the same methodology can be applied to different material: the twisted pairs of wires covered with a double layer of insulating varnish of Poly-Ether-Imide (PEI) and Poly-Amide-Imide (PAI) of thermal class 200 °C.

In addition to the DoE and RSM methods, this paper considers lifespan modeling as a Multiple Linear Regression (MLR) problem [30,31] where no specific experimental design is required. The results obtained using these three different methods are analyzed and compared. The predictability of the models is studied with respect to the insulator lifespan ranges by performing a detailed analysis of the model-relative errors. Finally, the flexibility of MLR allows appropriate models for specific lifespan ranges to be identified.

2. Proposed methods

2.1. Design of experiments

The Design of Experiments method (DoE) organizes the experiments in order to optimize a process or to estimate a model. It was first introduced by Fisher in 1935 [3] and made popular by Taguchi [28]. The main objective is to maximize accuracy whilst minimizing the number of experiments.

DoE can be applied to several domains such as agronomics, mechanics, chemistry, electronics, or even computer simulations [13]. It is particularly recommended when a large number of parameters are involved as in chemistry or mechanics, or when experiments are time consuming as in agronomics. The advantages of this method are the reduction of the number of experiments, improved accuracy, the ability to deal with several parameters and their coupling effects, and to determine the most influential factors according to their effect values.

The basic idea in DoE is to assign N_e levels to each investigated factor and to organize the experiments in such a way that each configuration involves a combination of these levels. The number of required experiments is then $n = N_e^k$ where N_e is the number of levels and k is the number of factors. DoE provides a specific procedure to choose the levels of each factor in each experiment. A very basic example with only 2 factors and 2 levels for each is presented in Table 1. Centered and reduced variables are used, i.e. (-1) represents the low level and $(+1)$ the high level of each factor. Each line is devoted to an experiment and each column to a factor. For each experiment i, the response value y_i is measured.

In the case of two factors, DoE considers the following mathematical model relating response Y to its average value M, to the two factors F_1 and F_2, and to their interaction I_{12} (1):

$$Y \sim M + E_1 \cdot F_1 + E_2 \cdot F_2 + E_{12} \cdot I_{12}. \tag{1}$$

In our case, response Y represents the insulation lifespan, M is the average lifespan derived from all aging test responses, F_i is the ith factor level, I_{ij} is the level of interaction between factors F_i and F_j. Finally, E_i is the effect of the ith factor and E_{ij} is the effect of the interaction I_{ij} on global response Y.

According to DoE [15], the effect E_1 of factor F_1 is obtained from (2):

$$E_1 = \frac{-y_1 + y_2 - y_3 + y_4}{4}. \tag{2}$$

For example, if $E_1 = +0.12$, it means that factor F_1 at high level $(+1)$ has an effect of $+0.12$ on the response with respect to its average value denoted as M.

Moreover, one key point of the DoE method is that the effect of interaction between factors 1 and 2 can also be determined. Eq. (2) presents the average effect of interaction E_{12} between factors 1 and 2 on the desired response.

$$E_{12} = \frac{y_1 - y_2 - y_3 + y_4}{4}. \tag{3}$$

The estimated effect vector $\hat{E}$ can be derived using the following relationship (4).

$$\hat{E} = X^{-1} \cdot Y \quad \text{with } \hat{E} = \begin{pmatrix} M \\ E_1 \\ E_2 \\ E_{12}. \end{pmatrix} \tag{4}$$

$\hat{E}$ is the vector of the different factor and interaction effects, X is the square matrix of levels composed of the columns 2 to 5 of Table 1 and Y is the vector of the experimental values presented in the last column of Table 1. A graphical representation of the experimental DoE full factorial plan is displayed as black dots in Fig. 1 for three factors F_1, F_2 and F_3.

Table 1
Experimental plan for DoE method—simple case of 2 factors with 2 levels each.

Experiment number		Factor 1	Factor 2	Interaction 1–2	Response y_i
1	1	−1	−1	+1	y_1
2	1	+1	−1	−1	y_2
3	1	−1	+1	−1	y_3
4	1	+1	+1	+1	y_4

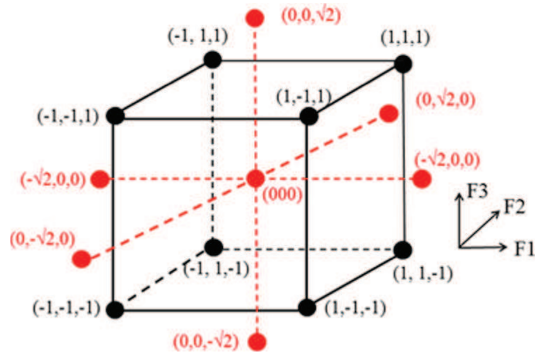

Fig. 1. Graphic representation of the experimental points in a DoE (black) and in a CCD (in red and black) with $\mu = \sqrt{2}$ for 3 factors (2 levels each in a DoE and 5 levels each in a RSM). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2.2. *Response surface method*

DoE provides an appropriate model estimation if the hypothesis of a linear relationship between the response, the factors and their interactions is valid. However, a model involving only the main effects and their interactions is not always sufficient. The response surface method (RSM) extends the DoE model by considering the quadratic effects of the main factors [12,16,24]. Consequently, k additional terms are added to the model (where k denotes the number of factors).

RSM still organizes the experiments and minimizes their number but also improves the DoE model accuracy. However, it requires some central experiments (all levels equal to 0 when considering centered, reduced variables) to be tested too. Consequently, the RSM design will have at least three levels, increasing the number of levels compared to DoE. Fortunately, experiments used in DoE can be reused in RSM.

The most popular design of RSM is the Central Composite Design (CCD) which considers an extra level μ for each factor and is designed as follows [22]:

- A complete $n_f = 2^k$ DoE design,
- n_0 repetitions at the design center,
- Two points on the axis of each parameter situated at a distance μ from the design center.

The total number of required experiments in RSM is therefore $n = n_f + n_0 + 2k$. The values of μ and n_0 are chosen to achieve particular properties [22] such as orthogonality, rotability, uniform precision, optimality criteria, etc. Orthogonal designs are of particular interest since they lead to uncorrelated model coefficients with minimum variances, thus improving the accuracy of the obtained model. The design is orthogonal if μ and n_0 are chosen such that [12]:

$$(n_f + 2\mu^2)^2 = n_f n. \tag{5}$$

In this paper, to ensure the orthogonality property with a number of four experiments in the center of the experimental domain, and according to (5), μ must be equal to $\sqrt{2}$ as displayed in Fig. 1. Consequently, five levels $[-\sqrt{2}; -1; 0; +1; +\sqrt{2}]$ are chosen for each factor.

The effect vector $\hat{E}$ can be computed as in the classical DoE methodology, but since X matrix is no longer square (the number of experiments is larger than the number of variables), $\hat{E}$ is estimated by the Ordinary Least Square

(OLS) method according to expression (6):

$$\hat{E} = \left(X^t \cdot X\right)^{-1} \cdot X^t \cdot Y. \tag{6}$$

More details about the least square estimator are given in the next section.

2.3. Multiple linear regression

DoE or RSM can be seen as regression methods taking into account nonlinear relationships between the stress variables. However, these two methods impose a design of experiments with a fixed number of variables and experimental data points. More generally, regression methods offer more flexibility by imposing no constraints on the experiment organization or on the studied variables.

The most common regression method is linear regression. Nevertheless, as we are dealing with a multi-stress modeling problem, the Multiple Linear Regression method (MLR) [31] is used instead of simple linear regression. In general, MLR method is used to model the linear relationship between response Y with n observations and p independent variables ($n > p$). The model expresses the value of the predicted variable as a linear function of the predictor variables and an error term (7):

$$Y = \beta_0 + \sum_{i=1}^{p} \beta_i \cdot X_i + \varepsilon \tag{7}$$

where X_i is a $(n \times 1)$ vector representing the ith column of the variable matrix X $(i = 1 \ldots p)$, β_i $(i = 0 \ldots p)$ are the model coefficients to be estimated, ε is the $(n \times 1)$ error vector and Y is the $(n \times 1)$ vector of the experimental measures.

By adding a $(n \times 1)$ vector of ones to the first column of matrix X, a more compact expression can be written in the form of (8):

$$Y = \sum_{i=0}^{p} \beta_i \cdot X_i + \varepsilon. \tag{8}$$

Calculations are similar to those of DoE or RSM but, in this case, the elements of matrix X are the real values of each factor instead of normalized levels. Moreover, there are no prior constraints on the choice of the experimental points, except that their number must be greater than the number of variables considered in the model ($n > p$).

MLR model estimation is based on the OLS method: the sum of the squared differences between observed and predicted values is minimized. However, the MLR model obtained by OLS estimation is based on several assumptions [5]:

- Full rank X: the p columns of matrix X are linearly independent,
- Non stochastic X: errors are uncorrelated with the predictors,
- Uncorrelated error: errors are random and uncorrelated,
- Zero mean error: the expected value of the error term is zero,
- Homoscedasticity: the error variance is constant,
- Normality: the error is normally distributed. This assumption must be satisfied for conventional tests of significance of coefficients and other statistics of the regression equation to be valid.

Provided these assumptions are verified, the OLS regression estimators are optimal in the sense that they are unbiased, efficient, and consistent.

MLR performs the derivation of the OLS estimation of the model coefficients as in (9) [5,18]:

$$\hat{\beta} = \left(X^t \cdot X\right)^{-1} \cdot X^t \cdot Y. \tag{9}$$

MLR model performance is evaluated through statistical criteria [4,11,31]:

1. **Standard error** σ:

   The residual Mean Square Error (MSE) is the estimate of the variance of the regression residuals and is computed as follows (10):

$$\sigma^2 = \frac{1}{n-p-1}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2 \tag{10}$$

   where $\hat{y}_i$ is the predicted response value and y_i is the corresponding measured value.

2. **Coefficient of determination** R^2:

   The explanatory ability of the regression model is summarized by its "R-squared" value which represents the proportion of variance explained by the regression model and is computed from the sums of squared terms as in (11):

$$R^2 = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{11}$$

   where $\bar{y}$ is the average of all measured values y_i. However, the R^2 value can increase when some predictors are added to the model. The adjusted R^2 (12) is then introduced to compensate for this artificial increase by taking into account the number of predictors p and of data points n.

$$R^2 adj = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2/(n-p)}{\sum_{i=1}^{n}(y_i - \bar{y})^2/(n-1)}. \tag{12}$$

3. **Confidence intervals of the estimated coefficients**:

   If the regression assumptions on the residuals are satisfied, including the normality assumption, then the sampling distribution of the estimated regression coefficients is normal with variances proportional to the residual MSE. The coefficient variances are the diagonal elements of the matrix obtained by expression (13):

$$\mathrm{var}(\hat{\beta}) = \sigma^2 (X^T \cdot X)^{-1}. \tag{13}$$

   Consequently, a $(1-\alpha)$ Confidence Interval (C.I.) of a coefficient β_j can be expressed as:

$$C.I._{1-\alpha}(\beta_j) = \left[\hat{\beta}_j - t_{(1-\alpha/2)}\sqrt{\sigma^2(X^T \cdot X)^{-1}};\, \hat{\beta}_j + t_{(1-\alpha/2)}\sqrt{\sigma^2(X^T \cdot X)^{-1}}\right] \tag{14}$$

   where $t_{(1-\alpha/2)}$ is the $(1-\alpha/2)$ percentile of Student $(n-p-1)$ distribution.

4. **Residual graphics**:

   It is often necessary to check the normality assumption of the regression residues. This can be achieved by QQ-plots. A QQ-plot ("Q" stands for quantile) compares two probability distributions by plotting their quantiles against each other. It is also useful for comparing the experimental distribution of a data set to a theoretical distribution. In OLS regression models, normal distribution must be verified, thus, residue quantiles are compared to the corresponding theoretical normal quantiles. If the two distributions are identical, the QQ plot follows the 45° line $y = x$.

3. Description of system

3.1. Materials

This work aims to estimate the lifespan of motor wirings through experiments on twisted pairs which are composed of wires coated with a double electrical insulator varnish. Electrical insulator varnish degrades with time when submitted to electrical and thermal constraints.

The insulator is designed to last for several thousand hours but, if the nominal constraints are exceeded, the lifespan decreases dramatically. Partial discharges (PD) appear in the insulation of rotating machines fed by inverters under

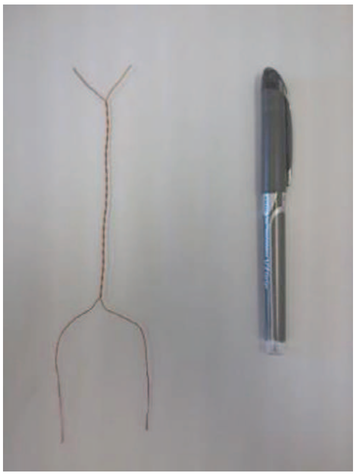

Fig. 2. Twisted pairs EDERFIL C200 as the test samples.

Table 2
Extreme values of the three stress factors.

Factors	Minimum value	Maximum value
Voltage (kV)	1	3
Frequency (kHz)	5	15
Temperature (°C)	−55	180

high voltage stress [7,10,19]. The discharge is said to be partial since it does not immediately bridge the space between two conductors. The insulation varnish is the first element in contact with PD and is therefore the most affected part of the insulation.

In order to establish a lifespan model of the rotating electrical machine wiring insulation, the tested materials are twisted pairs covered with a double layer of Poly-Ether-Imide (PEI) and Poly-Amide-Imide (PAI) varnish with a thermal class of 200 °C (Ederfil C200 with a diameter of 0.5 mm), as shown in Fig. 2. Twisted pairs are manufactured according to the American National Standards Institute [1].

3.2. *Stresses*

The first step in our lifespan modeling methodology consists in defining the stress factors and their range of variation. Experiments are based on ALT, i.e. materials are submitted to high constraints to test their degradation without waiting until normal failure.

Voltage and frequency are the principal causes of PD into the insulator [10,19,23]. In addition, several other factors can affect the insulation such as temperature, depression, humidity, chemical or mechanical stresses. Furthermore, the cyclic recurrence of some of these stresses can also impact lifespan and should therefore be considered as a degradation factor [14,17].

A review of the existing literature highlights four major factors that affect lifespan: voltage (periodic square wave with amplitude V), frequency (F), temperature (T) and temperature cycling [14,15]. However, for simplicity and as a first modeling attempt, only the first three stresses are studied in a domain where the experimental aging conditions must ensure that the degradation is principally due to PD.

The extreme stress values are chosen either to accelerate the degradation or to represent extreme conditions. They are given in Table 2.

3.3. *Experimental setup*

Experiments are carried out with the help of a climatic chamber that maintains a constant temperature which can be set between −55 and 180 °C. A power electronic system generates a square voltage controlled in amplitude and frequency. The experimental setup is depicted in Fig. 3.

Fig. 3. Climatic chamber and power electronics as a test bench for the insulation materials.

Table 3
Number of experiments for the three methods.

DoE	8 exp.
RSM	DoE + 6 exp. RSM + 4 central points
Extra experiments	12 exp.

Table 4
Normalized levels of the three stress factors for centered and reduced variables.

Factors	Level ($-\sqrt{2}$)	Level (-1)	Level (0)	Level ($+1$)	Level ($+\sqrt{2}$)
Log(10 $*$ Voltage (kV))	Log($10 * 1$)	Log($10 * 1.174$)	Log($10 * 1.73$)	Log($10 * 2.554$)	Log($10 * 3$)
Log(Frequency (kHz))	Log(5)	Log(5.872)	Log(8.7)	Log(12.77)	Log(15)
Exp($-b *$ Temperature (°C))	Exp($55b$)	Exp($34.82b$)	Exp($-26.12b$)	Exp($-119.74b$)	Exp($-180b$)

Finally, the lifespan of each sample is measured, with an average error of two seconds. Six twisted pairs were simultaneously tested under each stress configuration.

4. Models

This study compares the models obtained with DoE, RSM and MLR. The experiments were chosen in order to be used in the three methods and are organized as explained in Table 3: thirty-two experiments were carried out with six repetitions for each. Some of these experiments are used to estimate the models and compose the so-called learning set. The other experiments are used to test the validity of the models and compose the so-called test set.

In order to identify the form of the stress factors that will be used in these three models, the next step in our methodology is to identify the form of variation of the lifespan with respect to each of the three stress factors.

According to [15], in the case of PEI films under PD regime, the insulation lifespan logarithm follows an inverse power model depending on $\mathrm{Log}(V),\ \mathrm{Log}(F)$ and $\exp(-bT)$. Constant $b = 5.64 \times 10^{-3}$ has been derived as explained in [15] from lifespan experimental measures under various temperature constraints. In the case of twisted pairs, three different tests were carried out to model the variation of the lifespan as a function of each of the three factors taken separately. In each case, only one factor varies whilst the other two are fixed. The results are shown in Fig. 4.

As expected from previous work on PEI films [15], lifespan logarithm of the twisted pairs varies approximately linearly as a function of $\mathrm{Log}(V),\ \mathrm{Log}(F)$ and $\exp(-bT),\ b = 4.825 \times 10^{-3}$ respectively. Hence, in the following, $\mathrm{Log}(L)$ will be modeled as a linear combination of $\mathrm{Log}(V),\ \mathrm{Log}(F),\ \exp(-bT)$ and their interactions.

The five levels of the stress factors used in the following DoE and RSM models are given in Table 4.

After defining the constraint ranges, forms and levels, the experimental design is now fully specified. Table 5 displays the constraint values in each experiment and the corresponding measured lifespan. Experiments 1–8

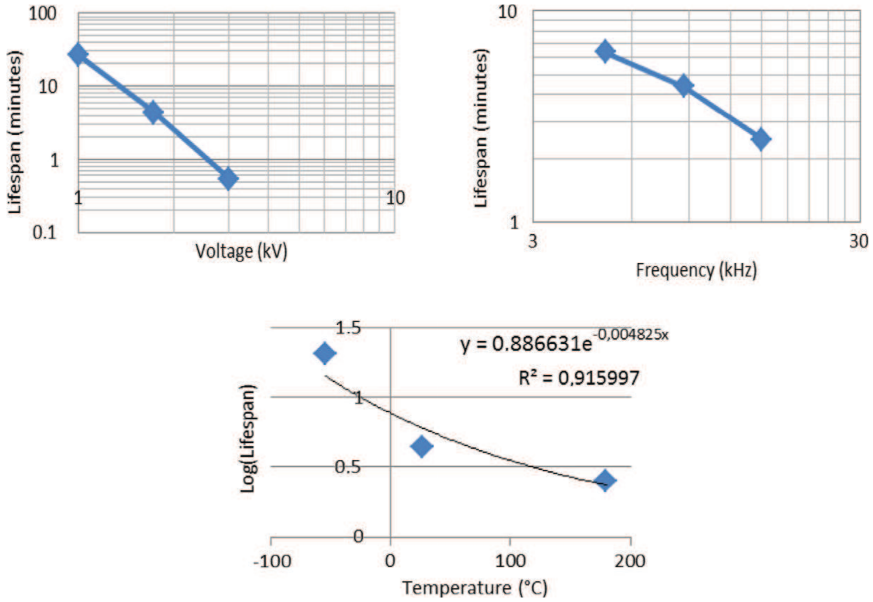

Fig. 4. Variations of Log(L) with respect to Log(V), Log(F) and exp($-bT$) studied separately.

correspond to DoE and are represented by the black dots in Fig. 1, whereas the red dots correspond to experiments 9–20 and are used in the RSM model. Finally, experiments 21–32 have random positions with respect to black and red dots and serve as extra experiments either to validate DoE and RSM models, or to generate regression models with randomly selected data points.

Note that in DoE and RSM models, the X matrix consists of factor levels, whereas in regression models, the real values of factors are used as in Table 5.

4.1. DoE and RSM

In this section, models are estimated according to DoE and RSM methods. Thanks to DoE, the lifespan model M_1 can be expressed as follows:

$$\begin{aligned}\mathrm{Log}(L) \approx\ & M + E_V \cdot \mathrm{Log}(10V) + E_F \cdot \mathrm{Log}(F) + E_T \cdot \mathrm{Exp}(-b \cdot T) + I_{VF} \cdot \mathrm{Log}(10V) \cdot \mathrm{Log}(F) \\ & + I_{VT} \cdot \mathrm{Log}(10V) \cdot \mathrm{Exp}(-b \cdot T) + I_{FT} \cdot \mathrm{Log}(F) \cdot \mathrm{Exp}(-b \times T) \\ & + I_{VFT} \cdot \mathrm{Log}(10V) \cdot \mathrm{Log}(F) \cdot \mathrm{Exp}(-b \cdot T). \qquad (15)\end{aligned}$$

In Table 5, 8 experiments are used to estimate the DoE model coefficients; the model is then tested on the remaining 24 experiments. The estimated values of $M, E_V, E_F, \ldots, I_{VFT}$ are shown in Fig. 5. The most influential factors and/or interactions are those having the highest effects i.e. the voltage, the temperature and their interaction.

The average, maximum and minimum relative errors between predicted and measured lifespan are shown in Table 6. From this table, it can be assumed that this model does not provide a very accurate lifespan prediction on the 24 extra experiments composing the test set. It is due to the fact that only 8 experiments are used to compute this model. The maximum error derived from the test set is far too high, thus, there is a need to extend the model using RSM.

In the case of RSM, the lifespan model M_2 is the same as for DoE with 3 extra quadratic terms (16):

$$\mathrm{Log}(L) \approx Model_{DoE} + I_{VV} \cdot \mathrm{Log}^2(10V) + I_{FF} \cdot \mathrm{Log}^2(F) + I_{TT} \cdot \mathrm{Exp}(-2b \cdot T). \qquad (16)$$

In this case, the model parameters are estimated from the 8 DoE experiments plus the 10 additional experiments needed for RSM and then tested on the remaining 14 experiments. The effects of the stress factors are shown in Fig. 6 which also points out that the most influential contributions on the lifespan model are the voltage, temperature and their interaction, plus the quadratic term of the temperature. The average, maximum and minimum relative errors between predicted and measured lifespan are shown in Table 7.

Table 5
Values of the stress constraints in the 32 experiments for DoE, RSM and MLR and of the 6 corresponding measured lifespans (in minutes).

		V (kV)	F (kHz)	T (°C)	Exp 1	Exp 2	Exp 3	Exp 4	Exp 5	Exp 6
1	DoE	1.174	5.872	−34.82	60	75	81	40.05	55.5	64.5
2	DoE	1.174	5.872	119.74	12.3	14.283	16.05	10.5	14.83	16
3	DoE	1.174	12.77	−34.82	10	19	25.5	21.4	36	44
4	DoE	1.174	12.77	119.74	7	7.66	7.66	4.26	4	4.91
5	DoE	2.554	5.872	−34.82	2.11	4.26	5.45	5.4	5.48	6
6	DoE	2.554	5.872	119.74	1.98	2.316	2.26	1.75	2.25	2.26
7	DoE	2.554	12.77	−34.82	1.05	1.5	1.66	1.3	1.6	1.85
8	DoE	2.554	12.77	119.74	0.21	0.75	0.9	0.2	0.88	0.95
9	RSM	1	8.66	26.12	19.62	20.73	25.23	26.02	26.72	30.23
10	RSM	3	8.66	26.12	0.12	0.27	0.43	0.58	0.7	0.8
11	RSM	1.732	5	26.12	2.17	3.05	5.02	7	8.23	8.55
12	RSM	1.732	15	26.12	1.78	2.18	2.18	2.37	2.65	2.78
13	RSM	1.732	8.66	180	2.367	2.43	2.65	2.44	2.35	2.47
14	RSM	1.732	8.66	−55	13	26.5	28.5	11.5	14.5	16.6
15	Central point	1.732	8.66	26.12	4.35	1.78	5.05	4.62	2.87	5.03
16	Central point	1.732	8.66	26.12	3.35	4.25	6.2	3.92	2.88	3.53
17	Central point	1.732	8.66	26.12	5.15	1.7	5.22	4.58	5.03	4.93
18	Central point	1.732	8.66	26.12	4.3	5.72	4.65	4.42	4.33	4.77
19	Central point	1.732	8.66	26.12	4.17	3.27	4.75	5.05	6.13	4.35
20	Central point	1.732	8.66	26.12	4.83	3.82	3.52	4.42	5.72	3.8
21	Rndm selection	2.5	14.52	−35	0.57	1.3	1.5	1.18	1.27	1.47
22	Rndm selection	1.5	10	−15	13.7	16.17	17.67	17.17	18.93	21.1
23	Rndm selection	2.81	14	−10	0.32	0.52	0.7	0.33	0.7	0.87
24	Rndm selection	2.53	13.79	2	0.55	0.85	1.27	0.98	1	1.28
25	Rndm selection	2.56	5.4	10	2.82	3.53	4.28	1.66	4.77	4.92
26	Rndm selection	2	10.97	36	2.9	3.55	4.58	2.17	2.85	4.03
27	Rndm selection	2.3	7.48	40	1.43	2.27	3.97	3.05	4.08	4.55
28	Rndm selection	1.01	11.54	61	12	13.58	17.58	16.5	17.83	19.83
29	Rndm selection	1.54	10.58	76	1.83	4.56	4.95	4.33	5.5	5.8
30	Rndm selection	1.92	9.68	90	2.5	2.65	3.4	2.8	3.05	3.28
31	Rndm selection	1.1	7.98	164	9.75	9.91	10.13	9.2	9.3	9.65
32	Rndm selection	1.31	7.79	180	5.5	5.83	5.95	4.9	5	5.43

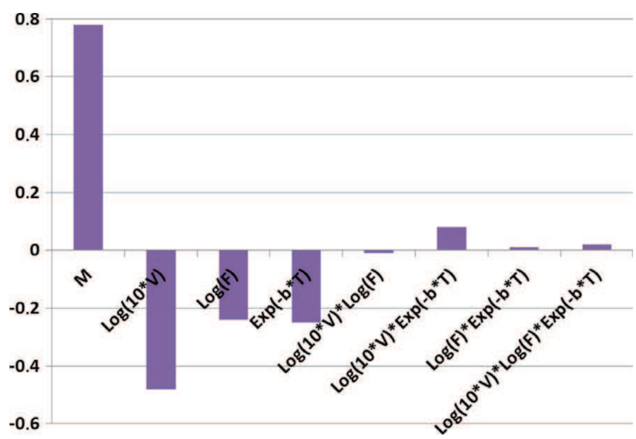

Fig. 5. Average effects of the stress factors and their interactions with DoE.

It would seem that the mean and maximum errors remain high with the RSM model. However, there is an aberrant value with a relative error of 1096% corresponding to a lifespan measurement of 1 min (median lifespan of experiment 24 in Table 5). This is actually the same data point that also led to the maximum error of 1692% in the DoE model M_1. Since it belongs to both DoE and RSM test sets, it can be ignored in error analysis. Indeed, this point acts as an outlier and may distort the results in these two models.

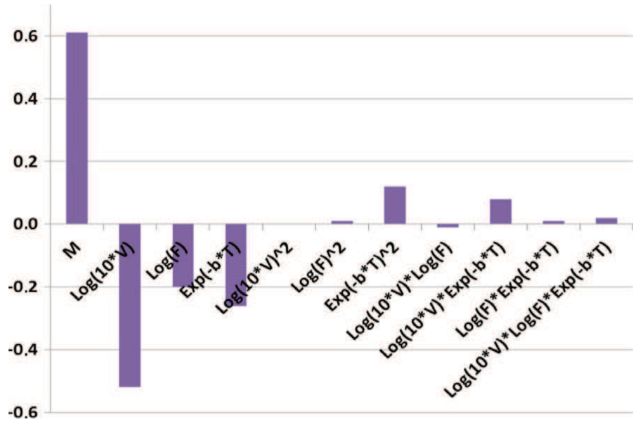

Fig. 6. Average effects of the stress factors and their interactions with RSM.

Table 6
Relative errors between predicted and measured lifespan (Model M_1).

Average error	70%
Maximum error	1692%
Minimum error	0.00%

Table 7
Relative errors between predicted and measured lifespan (Model M_2).

Average error	52%
Maximum error	1096%
Minimum error	0.45%

By removing this point from error calculations, the mean errors in DoE and RSM models decrease to 18% and 17% respectively, and their maximum errors decrease to 136% and 60%, respectively. This significant decrease suggests a further analysis of the model errors with respect to the measured lifespan is required; this will be studied in Section 4.3. After removing this "outlier" data point from both DoE and RSM test sets, the RSM model shows a better performance with regard to the computed relative errors compared to the DoE model.

On the other hand, it is important to point out that the presented methodology based on DoE and RSM methods is efficient regardless of the studied insulating material. As in [15,24], the DoE method establishes a simple lifespan model with only 8 experiments. However, while the DoE model of PEI films was validated only on 8 extra points [15], we showed in this paper that, with twisted pairs, the validity of the DoE model (Model M_1) is confirmed on a larger test set composed of 23 experiments (outlier point excluded). The extended RSM model improved model accuracy in both cases (PEI films and twisted pairs), again with a larger test set in the case of twisted pairs for the validity assessment. Note, however, that in the case of twisted pairs, the lifespan measurements have a larger dispersion compared to PEI film measurements, leading to higher relative errors in both DoE and RSM models with respect to [15,24] DoE and RSM models.

4.2. MLR

In this section, we introduce a third method for the lifespan modeling that has not been applied on PEI films [15,24]. In this method, the lifespan model is considered as a Multiple Linear Regression (MLR) problem. The same parameters used in the RSM model (main factors, interactions and quadratic effects) are included and the regression model has the form of (17):

$$Y \approx \sum_{i=0}^{10} \hat{\beta}_i \cdot X_i . \qquad (17)$$

Table 8
Relative errors between predicted and measured lifespan (Model M_3).

Average error	20%
Maximum error	174%
Minimum error	0.00%

As defined previously, the $\hat{\beta}_i$'s are the model parameters and the X_i's are the real values of the so-called explanatory variables 1, $\text{Log}(10V)$, $\text{Log}(F)$, $\text{Exp}(-bT)$, $\text{Log}^2(10V)$, etc. in the same order as in the RSM model for comparison purposes.

In an attempt to compare the different methodologies, our regression model will be estimated from a minimum number of experiments. Since 10 parameters are considered (expression (17)), at least 11 experiments are required to estimate our regression model.

The 11 experiments are randomly chosen from the 32 experiments of Table 5 (more precisely, they are selected among the 27 different configurations since we have six repeated center point experiments), leading to the so-called M_3 model, which will also be tested on the remaining 21 experiments. The random selection of the learning set experiments was performed several times in order to obtain a set of experiments that covers the largest possible range of variation of the three main factors, similarly to DoE and RSM learning sets. After several random selections, the best combination of 11 experiments for the learning set of Model M_3 is composed of experiments 1, 5, 6, 8, 9, 11, 21, 24, 29, 30 and 31. This learning set covers a large range of variation of the three main factors and of the measured lifespans.

The average, maximum and minimum relative errors between predicted and measured lifespan are shown in Table 8.

However, in this case also, the maximum error of 174% corresponds to a lifespan of less than 1 min (median lifespan of experiment 10 in Table 5). By removing this outlier point from error analysis, maximum and average errors decrease down to 74% and 15%. Thus, as in DoE and RSM models, some data points in the MLR model behave as outliers with respect to the other points, and lead to significantly higher errors in the models. This behavior is probably due to the short lifespans. This will be confirmed in the next section.

The comparison of average relative errors obtained after removing outlier points from the test sets of M_1, M_2 and M_3 reveals that the MLR model M_3 has a similar performance to that of the DoE and RSM models. However, in order to obtain such a good performance with a MLR model constructed from a minimum number of randomly selected experiments, the learning set must provide a wide range of variation of the constraints and this is why the learning set of model M_3 was chosen after several random selections of 11 experiments. Generally, such models, defined over a large range of variation of constraints and of lifespans, provide more accurate predictions on their test sets than those estimated from restricted learning sets. Nevertheless, with regression, there is no need for a prior design specification in order to determine the constraint levels for each experiment as in DoE or RSM. Therefore, when no organization is required or possible, it can be interesting to benefit from the flexibility of the MLR method to establish a lifespan model. Good results such as those obtained with DoE and RSM models can be obtained with MLR models as well, provided that their learning set constraints and lifespans have a wide range of variation.

4.3. *Discussion*

Studying the relative errors obtained with models M_1, M_2 and M_3 (learning and test sets) with respect to measured lifespan reveals that the highest errors are always for very short lifespans, as shown in Fig. 7.

This fact seems to indicate a particular degradation regime leading to short lifespan and therefore the studied models are not suitable to evaluate long and short lifespans simultaneously. For instance, even in M_2's learning set, the model was unsuitable for two data points of short lifespan (experiments 7 and 8), the corresponding errors are higher than 100%, taking into consideration that in M_1 and M_3, the number of learning set experiments is equal to the number of model parameters, therefore, all the errors in their learning sets are zero, even for short lifespans.

From this error analysis, it can be assumed that very short lifespans do not follow the same model as longer lifespans: a single model with a reasonable number of parameters is not sufficient to simultaneously represent short and long lifespans. Thus, to improve model performance and to focus on the most pertinent part, the experiments

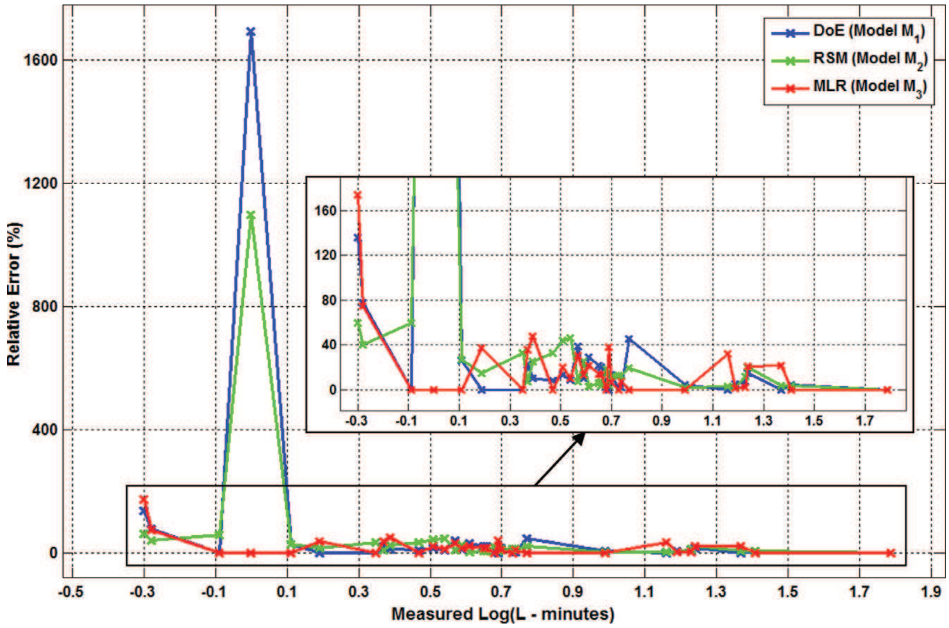

Fig. 7. Model (DoE, RSM and MLR) relative error distribution according to measured lifespan.

corresponding to very short lifespan must be removed from the models learning and test sets. Indeed, short lifespan can be seen as an outlier in the need for long lifespan models. Fortunately, we are more interested in modeling long lifespans.

However, DoE and RSM methods require a fixed design of the learning set that may contain long and short lifespans, as seen in Table 5. From this point of view, MLR shows more flexibility. Some ranges of interest from our data set (Table 5) can be chosen to generate MLR models, particularly, in the long lifespan range.

Since the errors in the learning set of M_1 and M_3 are equal to zero regardless of the lifespan range, we will focus on the variation of errors in model M_2 with respect to the measured lifespan (green curve). Model M_2 errors tend to significantly decrease for $\text{Log}(L) > 0.5$, this threshold corresponds to a lifespan value $L \cong 3$ min. As a first attempt, this value will be used in the following sections as a threshold to define short and long lifespan intervals.

5. Regression analysis

In the previous section, it was shown that the shortest lifespans lead to the highest errors and thus a single model cannot represent all the insulator lifespan ranges. To confirm this hypothesis, we will perform a regression analysis for different models derived from different lifespan ranges and using all experimental data in the considered lifespan range as a learning set. These models are compared based on relative errors and the criteria defined in Section 2.3.

Note that for these model estimations the 6 repetitions will be used in the Y vector instead of using only one median value per experiment. This procedure has the advantage of enriching the learning set while reducing errors that may be caused by taking the median as a representative value for the 6 repetitions. Consequently, the X matrix must also be reshaped. In fact, for each line in Table 5, the 6 measured lifespans are subjected to the same constraints (V, F and T). And since the 6 lifespan repetitions are now considered in the Y vector, each line of the original X matrix must be repeated 6 times (6 identical rows) to represent the same constraints applied to the 6 measured lifespans of a given configuration.

5.1. Model (a) with all experimental data points

First, all data points ($32 * 6 = 192$) are used to construct a MLR model (Model (a)) regardless of the lifespan range. The regression coefficient vector $\hat{\beta}$ is then estimated by OLS as previously described:

$$\hat{\beta} = \left(X^t \cdot X\right)^{-1} \cdot X^t \cdot Y \tag{18}$$

where X and Y are respectively the new matrix and response vector just defined.

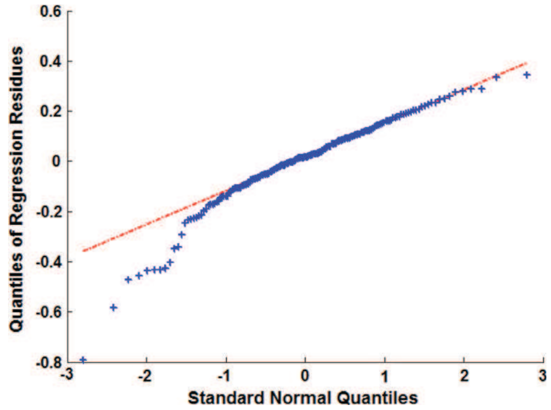

Fig. 8. QQ-plot of model (a) residues.

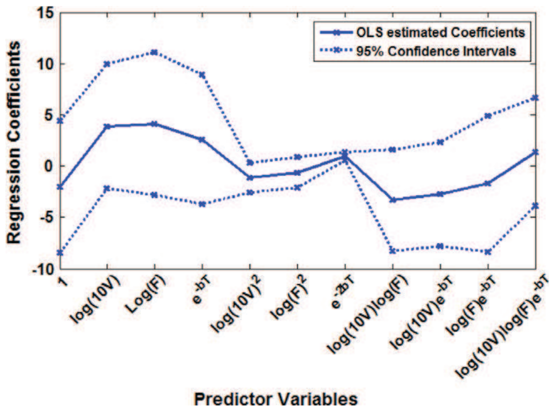

Fig. 9. Model (a) regression coefficients with 95% C.I.

Fig. 8 shows the QQ-plot of the regression residues in order to verify their normality. Since only 5% of the total numbers of observations (192) do not fit in this normality plot (left side of Fig. 8), we will consider that residue normality is globally verified. After this verification, the upper and lower confidence interval bounds of the model coefficients can be calculated (Fig. 9).

This figure points out that most of these coefficients have large confidence intervals which means that their variances are too high and therefore they cannot be used to represent a unique model over the entire lifespan domain.

On the other hand, the average relative error between the estimated and the measured values is 47%. By computing the average relative error only on high lifespans (defined by $L > 3$ min), a value of 14% is obtained, meaning that the highest errors are mostly located in the short lifespan range.

It is also important to notice that the points on the left side of Fig. 8 that do not fit to normal distribution all belong to the short lifespan range ($L < 3$ min).

5.2. Model (b) only with long lifespan data points ($L > 3$ min)

In this model, only long lifespans ($L > 3$ min) are taken into account to generate the regression model, the number of data points is therefore reduced to 121 instead of 192.

In Fig. 10, and after removing the short lifespans, the residues show a better fitting to normal distribution than in Model (a). The verification of residue normality allows us to compute the coefficients confidence intervals shown in Fig. 11. In this case, smaller confidence intervals are obtained for the model coefficients, which provide a better confidence level for the global model in this lifespan range ($L > 3$ min) with respect to Model (a).

Finally, the average relative error between the estimated and measured values in this model is 10%, which is significantly lower than the average error in Model (a) (47%).

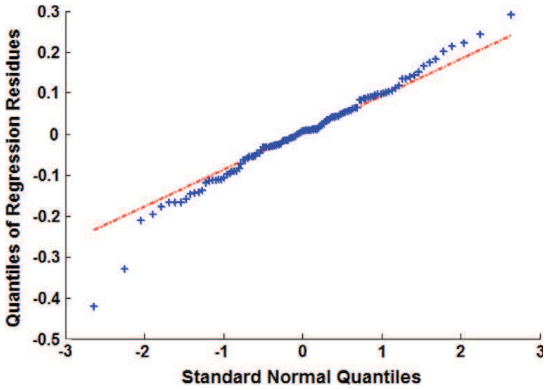

Fig. 10. QQ-plot of model (b) residues.

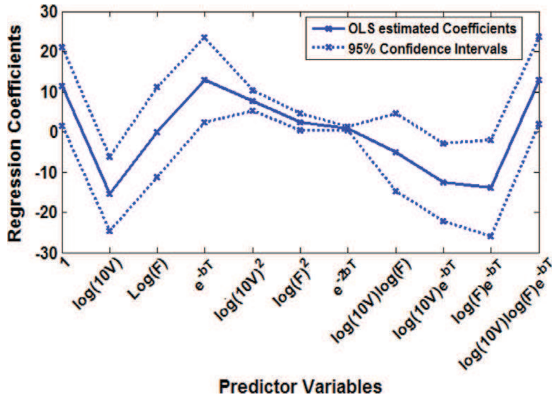

Fig. 11. Model (b) regression coefficients with 95% C.I.

5.3. Comparison of model (a) and model (b) with radar plots

In order to illustrate and compare the performance of these two models, a radar plot is used. A radar plot is a simple graphic tool used to display performance metrics defined by m number of criteria. It is composed of m axes having equal lengths, starting from the same point and graduated according to each criterion value. Finally, a line is drawn connecting the values on each axis and thus describing the global performance.

We will define 8 criteria to perform our comparison: the standard error σ, the adjusted determination coefficient R^2, the maximum relative error, the maximum relative error between the estimated values and the mean value of the 6 repeated measurements on each experiment, the mean relative error, the mean relative error between the estimated values and the mean value of the 6 repeated measurements, the proportion of data points having a relative error higher than 20% and finally the same when the error is computed between the estimated values and the mean value of the 6 repeated measurements. Note that mean measurements are considered in order to analyze the average model performance, regardless of the dispersion of the measurements corresponding to the same experiment.

Since a model has a higher performance if it presents lower errors, and since all our criteria represent different forms of errors except for adjusted determination coefficient R^2 (a appropriateness criterion varying between 0 and 1), we will consider $1 - R^2$ instead. The two corresponding plots are shown in Fig. 12.

This analysis shows a significantly better global performance in the case where only the long lifespans are accounted for in the regression model. It confirms, once again, our hypothesis that very short and long lifespans cannot be modeled simultaneously.

In conclusion, a better performance is achieved by including only long lifespans in our model. Fortunately, it is also our domain of interest since our prospective objective is to validate a lifespan model that can be applied to nominal constraints leading to long (or even very long) lifespans.

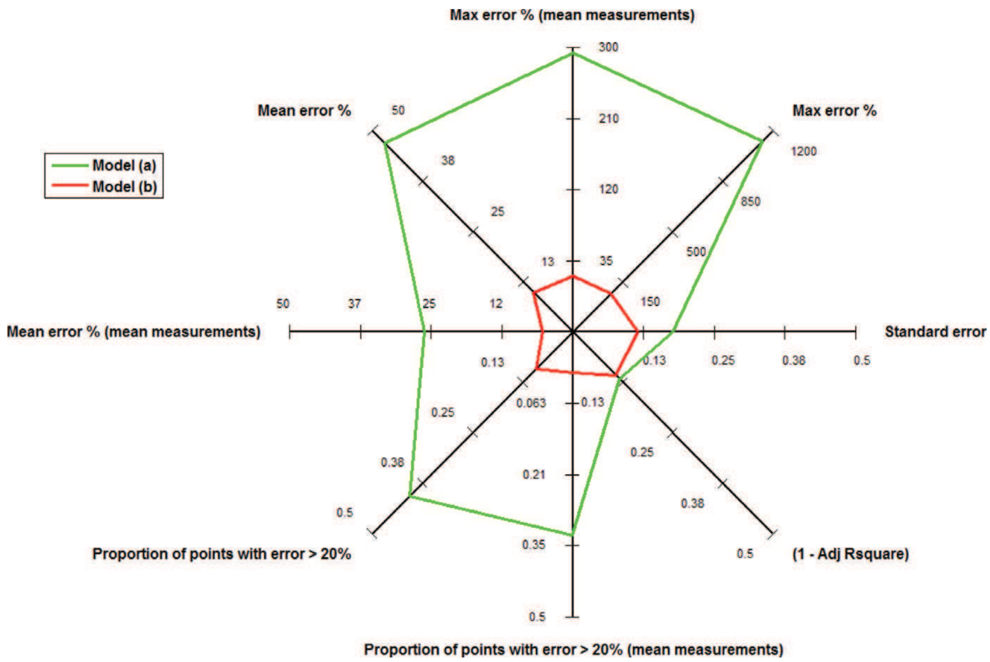

Fig. 12. Radar plot of Models (a) (all lifespans) and (b) (lifespans $L > 3$ min) versus 8 quality criteria.

6. Conclusion

Insulation lifespan is modeled through three statistical methods: DoE, RSM and MLR. After DoE and RSM models were validated in previous works on PEI films, we showed in this paper that the same methodology can be applied to a different kind of insulating material: twisted pairs covered with a double layer of insulating varnish of PEI and PAI. The ability of DoE and RSM to organize and to limit the number of experiments is confirmed. However, these models impose a fixed configuration and a fixed number of the experiments in their learning sets. From this point of view, the MLR method shows more flexibility. MLR was introduced in this paper as an alternative to DoE and RSM models with no particular design requirements. Therefore, MLR offers an efficient solution for lifespan modeling when no organization is possible.

After a first analysis of DoE, RSM and MLR models, the studied relative errors reveal that these models have a better predictability in the long lifespan range. Moreover, a detailed regression analysis was performed to confirm that short and long lifespans cannot be modeled simultaneously with a unique model involving the considered number of parameters (10 parameters). Hence, it was shown that regression methods can help to analyze different regions inside the studied domain and to eliminate outliers that might be due to a different aging mechanism.

Future work will investigate the ability of MLR to refine the model obtained with DoE and RSM methods in terms of stress variation range and to elaborate a more sophisticated method to detect the threshold between short and long lifespan models. Moreover, MLR provides a theoretical framework for the analysis of the significance of the global model and of each predictor variable independently. This analysis may be used for the model validation and eventually to simplify the model by keeping only the most significant predictor variables. In order to provide a more realistic lifespan model, other aging factors will be considered such as pressure, humidity, mechanical vibrations, etc. Finally, more realistic devices will be tested such as small coils inserted into machine slots.

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