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# Workspace Analysis in the Design Parameter Space of a 2-DOF Spherical Parallel Mechanism for a Prescribed Workspace: Application to the Otologic Surgery

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## Abstract

*During Otologic surgery, and more broadly during microsurgery, the surgeon encounters several difficulties due to the confined spaces and micro-manipulations. The purpose of the paper is to design a robot with a prescribed regular workspace shape to handle an endoscope to assist the Otologic surgery. A spherical parallel mechanism with two degrees of freedom is analysed in its design parameter space. This mechanism is composed of three legs (2USP-U) to connect the base to a moving platform connected to a double parallelogram to create a remote center of motion (RCM). Its kinematic properties, i.e. the singularity locus and the number of direct kinematic solutions, are investigated. For some design parameters, non-singular assembly modes changing trajectories may exist and have to be investigated inside the prescribed regular workspace shape. Two sets of design parameters are presented with their advantages and disadvantages.*

*Keywords:* Spherical Mechanism, Otologic Surgery, Cusp points, Parallel robot, Singularities

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## Highlights

- Design of mechanism for Otologic Surgery;
- Kinematic structure of the proposed mechanism;
- Workspace and Joint space analysis of the proposed 2-DOF Spherical mechanism;
- Influence of the design parameter on the workspace properties;
- Workspace without non-singular assembly modes changing trajectories.

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## 1 Nomenclature

<i>SIROPA</i>	<i>Library for manipulator singularities analysis</i>
<i>CAD</i>	<i>Cylindrical Algebraic Decomposition</i>
<i>IKP</i>	<i>Inverse Kinematic problem</i>
<i>DKP</i>	<i>Direct Kinematic problem</i>
<i>det</i>	<i>Determinant of Jacobian matrix</i>
<i>R</i>	<i>Revolute Joint</i>
<i>P</i>	<i>Prismatic Joint</i>
<i>S</i>	<i>Spherical Joint</i>
$\rho$	<i>Actuated Joint Variables</i>
<i>X</i>	<i>Pose Variables</i>
<b>A</b>	<i>Direct parallel Jacobian matrices</i>
<b>B</b>	<i>Inverse serial Jacobian matrices</i>
$S_\alpha$	$\sin(\alpha)$
$S_\beta$	$\sin(\beta)$
$C_\alpha$	$\cos(\alpha)$
$C_\beta$	$\cos(\beta)$

## 2 1. Introduction

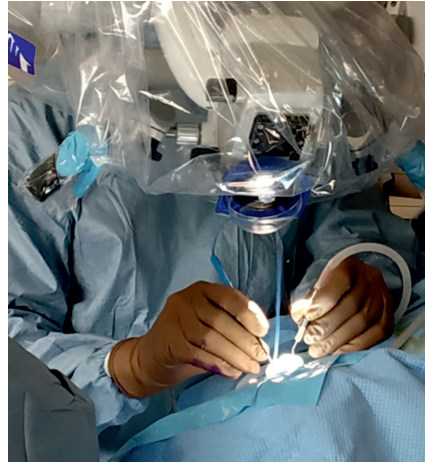
Traditionally, the ear surgeries are performed under binocular Loupes, which allows the surgeon to use both hands during the surgery. Presently, the development of Otologic surgery by endoscopy allows a better view of areas difficult to access [1, 2]. However, the use of an endoscope restricts the surgeon to use both the hands while doing surgery. Therefore, there is a strong expectation on the part of the otologic surgeon for a robotic assist system to perform the surgery comfortably.

There are many advantages for using an endoscope instead of the conventional microscope in otologic surgery. Some of them include a better view with easy access to the operation area and proximity to the patient during the operation. Endoscopic surgeries result in less bone sacrifice and thus improve the patient's recovery period. Despite the many advantages, endoscopic surgeries pose different challenges for the surgeon. Currently, the surgeon can only use one tool at a time in endoscopic surgery, as opposed to two tools with the use of a microscope, as shown in Figure 1. This makes endoscopic surgeries cumbersome because the surgeon must switch from one tool to the other to operate and manage bleeding in the ear. At the time of patient set-up, several factors can change the anatomical position of the head and thus the workspace in which the robot must operate. This leads to changes in the robot's approach angle. It must be able to compensate for this by working amplitudes that are greater than the working space inside the ear alone.

This paper is organized as follows. The first part comprises of the application of robot for the otologic surgery in order to determine the required workspace. A two DOF RCM spherical mechanism is proposed in the second part with detailed kinematic equations. The third part reports the singularity and workspace analysis of the proposed robot in the design parameter space with two case studies to fulfil the design requirements. In the later parts, discussions and conclusions are presented.



(a) The use of single hand to hold endoscope limits the number of instruments



(b) The surgeon can use two instruments while using a microscope

Figure 1: The comparison of the number of instruments possible to use simultaneously while using an endoscope and a microscope (Dr. Guillaume Michel, CHU Nantes).

## 31 2. Application of Robotics for otologic surgery

### 32 2.1. Robotics for otologic surgery

33 Currently, several robotic systems are developed [3], reflecting the interest for  
 34 robotic assistance in microscopic surgery. In the context of designing a robot to  
 35 assist the surgeon in otologic surgery, a spherical robot with a parallel structure  
 36 associated with a double parallelogram was developed [4]. This robot can handle  
 37 an endoscope to increase the efficiency of the surgeon when compared to the  
 38 classical binocular performed surgery. The parallel structure should increase the  
 39 rigidity compared to the existing solution [5]. Many spherical mechanisms exist  
 40 [6, 7, 8, 9, 10]. They can be divided into two main families (i) those with a virtual  
 41 center of rotation and (ii) those constrained by a spherical joint or a universal  
 42 joint (three or two DOF). To form a remote center of motion (RCM), one  
 43 solution is to use a universal joint associated with two parallelograms. Actuation  
 44 can be done with prismatic or revolute actuators. The advantage of prismatic  
 45 actuators is that there is only one solution to the inverse geometric model or  
 46 one working mode [11, 12]. In order to give the surgeon more mobility, it is  
 47 necessary to have the largest working space without singularity. By adding an  
 48 offset in the classical design, we are able to increase this workspace but the  
 49 properties of the robot change [13]. In [13] the parameter  $h$  is added between  
 50 the universal joint and the mobile platform but the offset on the both legs are  
 51 removed.

52 For most parallel or serial manipulators, pose variables and joints variables  
 53 are linked by algebraic equations and form a so called algebraic variety (some-  
 54 times after a straight forward change of variables). The two-kinematic problems  
 55 (the direct kinematic problem (DKP) and the inverse kinematic problem (IKP))  
 56 consist in studying the pre-image of the projection of this algebraic variety onto  
 57 a subset of unknowns. Solving the DKP remains to computing the possible  
 58 poses for a given set of joint variables values while solving the IKP remains

59 to computing the possible joints variables values for a given pose. Algebraic  
60 methods have been deeply used in several situations for studying parallel and  
61 serial mechanisms, but finally their use is quite confidential in the design pro-  
62 cess. The number of aspects changes and the number of solutions to the DKP  
63 increases. This means that the presence of cusp points or the uniqueness do-  
64 mains in the workspace must be investigated [14]. In this paper we will present  
65 the mechanism properties for a given offset that allow non-singular assembly  
66 mode changing trajectories [15]. The SIROPA library written in Maple will be  
67 used to compute the singularity equations using Groebner bases, do the cylin-  
68 drical algebraic decomposition (CAD) as well as the trajectories [16, 17]. The  
69 paper presents the design of a robotic system, which assist the surgeon in visu-  
70 alizing the inner part of the ear while performing surgery using an endoscope.  
71 This would facilitate the use of the endoscope in microsurgery or even replace  
72 any use of the operating microscope. The study of workspace, joint space and  
73 singularities together assists the engineers and researchers in the efficient task  
74 planning and the selection of the particular configuration of the manipulator for  
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84 of workspace, joint space and singularities together assists the engineers and  
85 researchers in the efficient task planning and the selection of the particular  
86 configuration of the manipulator for a desired task.

## 87 *2.2. Specifications for the otological surgeries*

88 Figure 2 shows the ear in two orientations. The optimal position, for most  
89 of the otological surgeries, is when the mastoid is horizontal. Unfortunately,  
90 the patient's anatomy does not always allow the ear to be in this position. In  
91 the figure on the right, the axis of the ear can therefore be tilted by about  
92  $54^\circ$ . Mobility inside the ear has been studied in [18] with a scan analysis on a  
93 population of variable age and sex ( $n=16$ , patients from 2 to 79 years old). It  
94 can be hypothesized that the center of rotation of the endoscope will be placed  
95 in the middle of the ear canal as shown in Figure 3.

96 In this study, we can isolate the dimensional parameters giving maximum  
97 angular variation. Figure 4 represents these variations with a zero diameter  
98 for the endoscope in the centre and on the right and left for an endoscope of  
99 3 and 4 mm diameter where the tilt angles are equal to  $\pm 13^\circ$  and  $\pm 11^\circ$ . To  
100 ensure that the robot can be used for all patients as well as having the necessary  
101 mobility for all ears. Figure 5(left) depicts the nominal cone and its inclination  
102 of  $54^\circ$  to reach the most inclined position of Figure 3. The total angle becomes  
103  $67^\circ$ . However we must slightly increase this value because we will define the  
104 limit of the workspace by the singularities, where the robot behavior will be less  
105 accurate. We choose to search for a tilt angle close to  $75^\circ$ . Workspace can be  
106 defined using azimuth, tilt and torsion representation [19] where azimuth angle  
107 is  $[0...360^\circ]$ , tilt angle is  $[0...75^\circ]$  and no torsion. For the studied mechanism, we

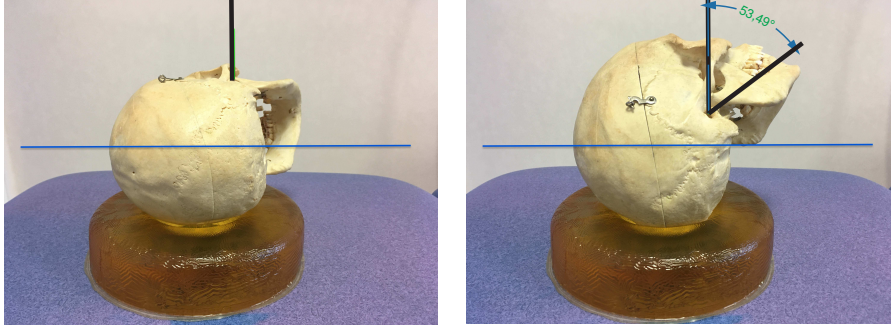


Figure 2: Placement of the ear in relation to the operating table in the optimal (right) and unfavourable (left) position

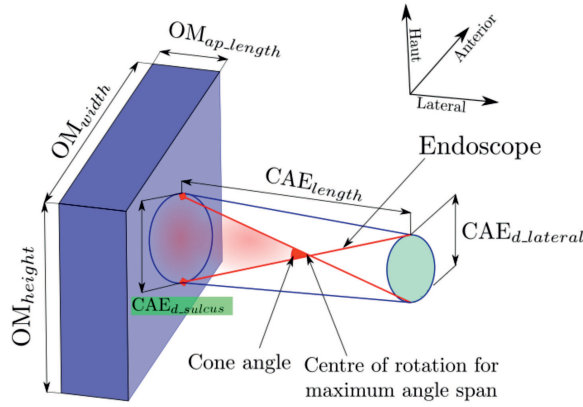


Figure 3: Schematic workspace of the external (cylinder) and middle ear

108 will use a universal joint based on two orthogonal revolute joints  $\alpha$  and  $\beta$ . Figure  
 109 5(right) depicts in red the image of the regular workspace shape [20] requested  
 110 to have the tilt motion in the Euler coordinates. It is possible to include this  
 111 space either by a square defined by  $\alpha = \beta = [-75^\circ \dots 75^\circ]$  or a centered circle  
 112 with a radius equal to  $75^\circ$ .

113 Another constraint is to limit the possible interference between the mecha-  
 114 nism and the surgeon. Therefore, we can offset the center of rotation by using  
 115 a RCM made by parallelograms.

### 116 3. A remote center of motion mechanism to carry an endoscope

117 Figure 6 shows an RCM mechanism carrying an endoscope for operations in  
 118 the ears made by coupling two DOF spherical mechanism with double parallel-  
 119 ograms. This mechanism is coupled to a translation mechanism for positioning  
 120 in the middle ear centre. Another mechanism, not shown in the figure, allows  
 121 translation of the endoscope for insertion, cleaning and ejection in case the

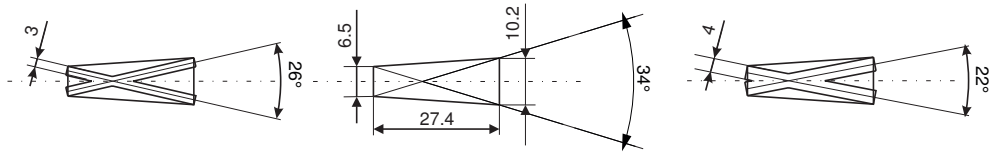


Figure 4: Mobility of the endoscope inside the middle ear

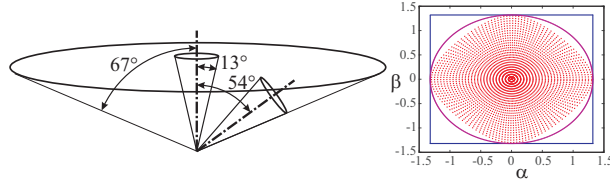


Figure 5: Prescribed regular workspace shape for Otologic surgery

122 patient wakes up (patent pending) [21]. The spherical parallel mechanism is  
 123 composed of three limbs and one moving platform. The two first legs, UPS,  
 124 are composed of a universal joint, a prismatic joint and a spherical joint and the  
 125 last one is made by a single universal joint and constrains its mobility. The two  
 126 prismatic joints are actuated. The double parallelogram is attached to the two  
 127 axes of this joint. Usually, the end points of the UPS legs are in the same plane  
 128 as the axes of rotation of the universal joint.

129 In order to obtain the desired workspace, it is possible to vary several design  
 130 parameters. In this study, one parameter is normalized, the structure remains  
 symmetrical and three length parameters are varied.

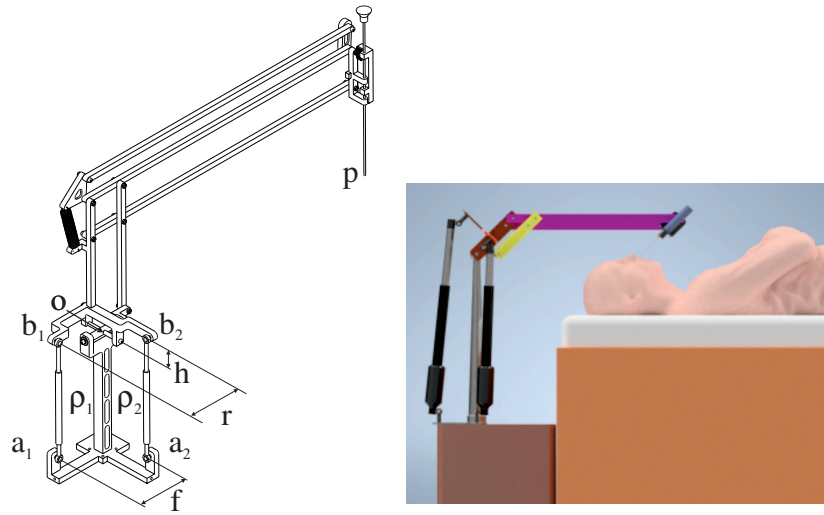


Figure 6: RCM Mechanism with spherical parallel mechanism in its home pose and its location close to the patient

131

### 132 3.1. Kinematic equations

Let  $\mathbf{a}_1$  and  $\mathbf{a}_2$  be attached to the base,  $O$  the center of the universal joint and  $\mathbf{b}_1$  and  $\mathbf{b}_2$  be attached to the mobile platform in the moving reference frame.

The coordinates are given by

$$\mathbf{o} = [0, 0, 0], \mathbf{a}_1 = [f, 0, -1]^T, \mathbf{a}_2 = [0, f, -1]^T, \mathbf{b}_1 = [r, 0, h]^T, \mathbf{b}_2 = [0, r, h]^T \quad (1)$$

The orientation space of the moving platform is fully represented with the variables  $(\alpha, \beta)$ . The rotation matrix  $\mathbf{R}$  from the base frame to the moving frame is expressed as follows:

$$\mathbf{R} = \mathbf{R}_\alpha \mathbf{R}_\beta = \begin{bmatrix} C_\beta & 0 & S_\beta \\ S_\alpha S_\beta & C_\alpha & -S_\alpha C_\beta \\ -C_\alpha S_\beta & S_\alpha & C_\alpha C_\beta \end{bmatrix} \quad (2)$$

The orientation angles are defined in such a way that  $\alpha = \beta = 0$ , which represents the “home” pose as depicted in Figure 6. The coordinates of  $\mathbf{b}_1$  and  $\mathbf{b}_2$  can be written in the base frame as

$$\mathbf{c}_1 = \mathbf{R}\mathbf{b}_1 \quad \mathbf{c}_2 = \mathbf{R}\mathbf{b}_2 \quad (3)$$

The distance constraints from the two prismatic joints yields

$$\|\mathbf{a}_i \mathbf{c}_i\| = \rho_i \quad \text{with } i = 1, 2 \quad (4)$$

133 This leads to the two constraint equations:

$$-2(fh + C_\alpha r)S_\beta + 2(hC_\alpha - fr)C_\beta + f^2 + h^2 + r^2 + 1 = \rho_1^2 \quad (5)$$

$$2h(fS_\alpha + C_\alpha)C_\beta - 2fC_\alpha r + f^2 + h^2 + r^2 + 2S_\alpha r + 1 = \rho_2^2 \quad (6)$$

#### 134 4. Singularity and workspace analysis

The singularity analysis is done by differentiating the two constraint equations with respect to time that leads the velocity model:

$$\mathbf{A}\omega + \mathbf{B}\dot{\rho} = 0 \quad (7)$$

135 where  $\mathbf{A}$  and  $\mathbf{B}$  are the parallel and serial Jacobian matrices, respectively,  $\omega$   
136 is the angular velocity and  $\dot{\rho} = [\dot{\rho}_1 \ \dot{\rho}_2]^T$  joint velocities [11]. The singularity  
137 locus in the workspace can be written as follows without specifying the design  
138 parameters  $f, r$  and  $h$  [22]:

$$\begin{aligned} & 4(C_\beta r + h(f + S_\beta))r(C_\alpha)^2 + (4f^2h^2(C_\beta)^2 - 4f(-S_\alpha r + h(fS_\beta - 1))rC_\beta + \\ & 4(h(fS_\beta - 1)S_\alpha - fS_\beta r)r)C_\alpha + 4f((-h^2S_\alpha + rh)(C_\beta)^2 + \\ & h(r(f + S_\beta)S_\alpha + S_\beta h)C_\beta - r(frS_\beta S_\alpha + h)) = 0 \end{aligned} \quad (8)$$

139 There are three conditions for simplifying and factorizing the singularity equa-  
140 tion.

- 141 •  $h = 0$ :  $(\sin(\alpha) + \cos(\alpha))(+\sin(\beta) - \cos(\alpha)\cos(\beta)) = 0$
- 142 •  $f = 0$ :  $\cos(\alpha)r(h\sin(\alpha) - \cos(\alpha)\cos(\beta)r - \cos(\alpha)\sin(\beta)h) = 0$
- 143 •  $r = 0$ :  $\cos(\beta)fh^2(\cos(\alpha)\cos(\beta)f - \sin(\alpha)\cos(\beta) + \sin(\beta)) = 0$



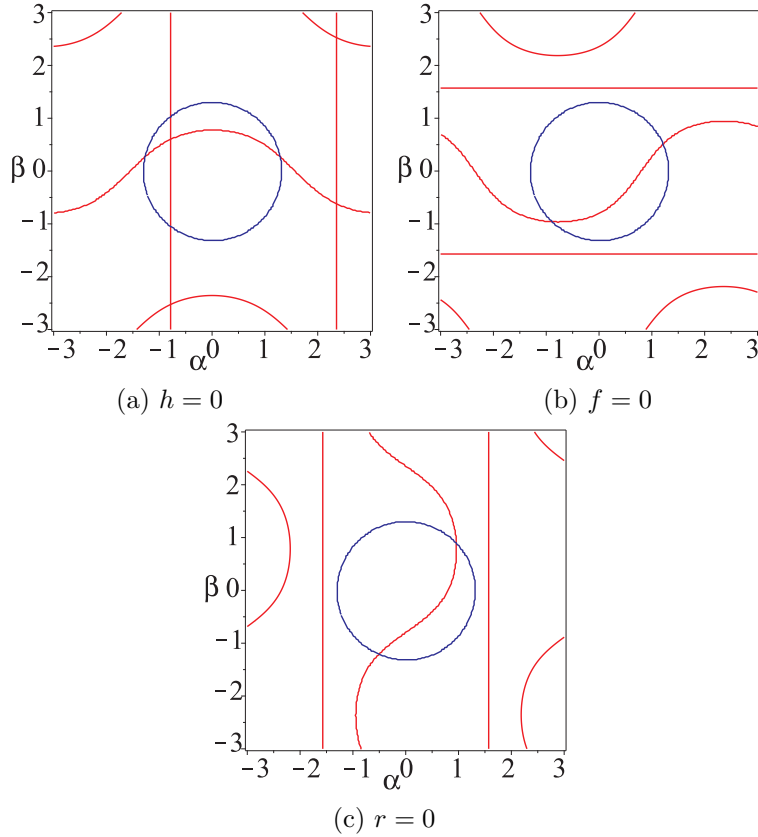


Figure 7: Singularity locus in the workspace of the spherical joint with four aspects where red represents the singularity locus and blue represents the border of the regular workspace shape

144        When there is simplification of the singularity equations, the DKP admits  
 145 two of four solutions and each one is located in a given aspect which refers to the  
 146 maximum singularity free regions in the workspace. These properties are not  
 147 stable because when a small offset is added, the locus of the singularities changes.  
 148 The aspects, i.e. the maximum singularity free regions in the workspace are  
 149 shown in Figure 8 where there are four aspects as shown in Figure 8(a) and (c)  
 150 and only two in Figure 8(b) [11].

151        When there is less aspect than solutions to DKP, it means that non-singular  
 152 assembly mode change trajectories exist, in other words that the mechanism is  
 153 cuspidal. In order to guarantee the safety of the movements, it is necessary to  
 154 analyze the uniqueness domains and the behavior around the cusp points.

#### 155 4.1. Sets of design parameters for the desired workspace

156        The search for design parameters allowing the inclusion of the desired workspace  
 157 can be done by optimization or by analyzing the number of intersections be-  
 158 tween the singularities and the workspace boundary. This is the second solution  
 159 which is used in this study. It consists in using the CAD of a system formed by  
 160 the parametric polynomial system of singularities and the equation of the circle,  
 161 border of the workspace. This system is a function of three design parameters

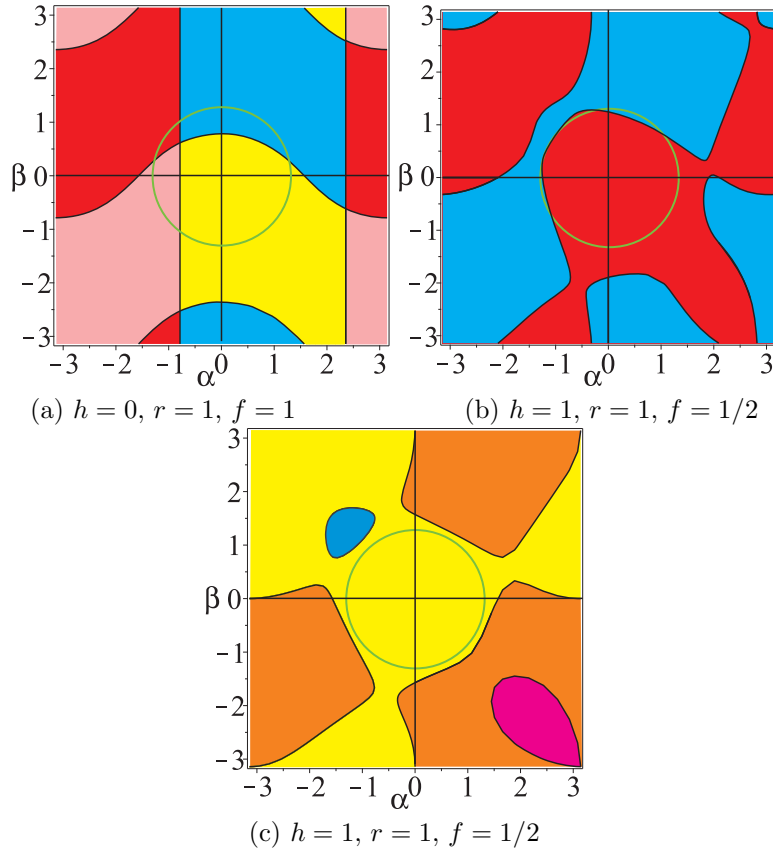


Figure 8: Singularity locus and aspects in the workspace for three sets of parameters where each color represents a given aspect

162 and  $\alpha$  and  $\beta$  angles. The CAD decomposes the parameter space of the system  
 163 into cells in which the original system has a constant number of solutions and  
 164 we retain the components with zero solutions. For each cell obtained, the CAD  
 165 returns the values of a set of parameters. The following list is obtained by vary-  
 166 ing by discretization the parameter  $f$  to have a 2D view of the other parameters  
 167  $h$  and  $r$ . Table 1 is a list of solutions by a set of parameters solutions of each cell  
 168 for  $r = 1$  and Figure 9 depicts the singularity locus of each set of parameters.

169 If the design parameters make it possible to obtain the desired workspace,  
 170 there are two singularity typologies. Two examples are now presented, one  
 171 where the equation of factor singularities factorizes and the other where we  
 172 cannot, to divide the workspace into four aspects. Figure 10(a) is the set of  
 173 design parameters for  $f = 0$ . This set include  $h = 0$  but also small value of  $h$   
 174 with respect to  $r$ . Figure 10(b) and (c) are the set of solutions for  $f = 1$  and  
 175  $f = 2$ , respectively.

176 *4.2. Example 1:  $h = 0, r = 1$  and  $f = 1/10$*

For  $h = 0, r = 1$  and  $f = 1/10$ , the singularity locus is defined as

$$(10 \cos(\alpha) \cos(\beta) - \sin(\beta))(\sin(\alpha) + 10 \cos(\alpha)) = 0 \quad (9)$$

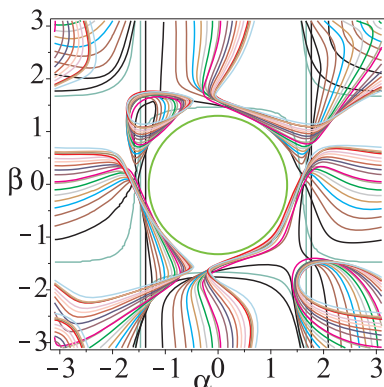


Figure 9: Singularity locus from the parameters defined in Table 1 where the blue curve is the boundary of the prescribed workspace

$h$	0	0	0.233	0.408	0.549	0.648	0.735	0.818	0.898	1.038
$f$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$h$	1	1.204	1.341	1.478	1.616	1.753	2	2	2	2.308
$f$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2

Table 1: Set of parameters solutions of each cell for  $r = 1$  obtained by the CAD

177 Figure 11 depicts the workspace and the joint space as well as the boundary  
 178 of the prescribed workspace. As there is only two (area in green) or four real  
 179 solutions (area in red) to the DKP and the singularity can be factorized, there is  
 180 no cusp point in the joint space. As the boundary of the prescribed workspace  
 181 is on the boundary of the aspect, its image in the joint space is also on the  
 182 boundary. This solution does not yield a safety problem during the trajectory  
 183 planning.

#### 184 4.3. Example 2: $h = r = f = 1$

185 For  $h = r = f = 1$ , the singularity locus is defined as

$$\begin{aligned}
 & 2(C_\beta + S_\beta + 1)C_\alpha^2 + (2C_\beta^2 + (-2S_\beta + 2S_\alpha + 2)C_\beta + (-2S_\alpha + 2)C_\beta^2 \\
 & + (2S_\beta - 2)S_\alpha - 2S_\beta)C_\alpha + ((2S_\beta + 2)S_\alpha + 2S_\beta)C_\beta - 2S_\alpha S_\beta = 2 \quad (10)
 \end{aligned}$$

186 Figure 12(b) depicts the joint space where the DKP admits either two real  
 187 solutions, in green, four real solutions, in red and six real solutions, in yellow.  
 188 Eight cusps exist (i)  $C_1, C_2$  and  $C_3$  are between the two- and four-solution  
 189 regions solutions to the DKP, and (ii)  $C_4, C_5, C_6, C_7$  and  $C_8$  are between the  
 190 four- and six-solution regions. The workspace divided into four aspects (regions  
 191 in orange, pink, blue and yellow).

192 The analysis of the image of the boundary of the prescribed workspace (Fig-  
 193 ure 12(a)) in the joint space has been studied in [22]. This curve surrounds 3  
 194 cusp points  $C_4, C_5$  and  $C_6$  (Figure 12(b)). When a prescribed workspace is in-  
 195 scribed in a single aspect, it does not lead to the conclusion that the trajectories  
 196 that the robot will make will be in a single domain of uniqueness, i.e. there is  
 197 no non-singular assembly mode changing.

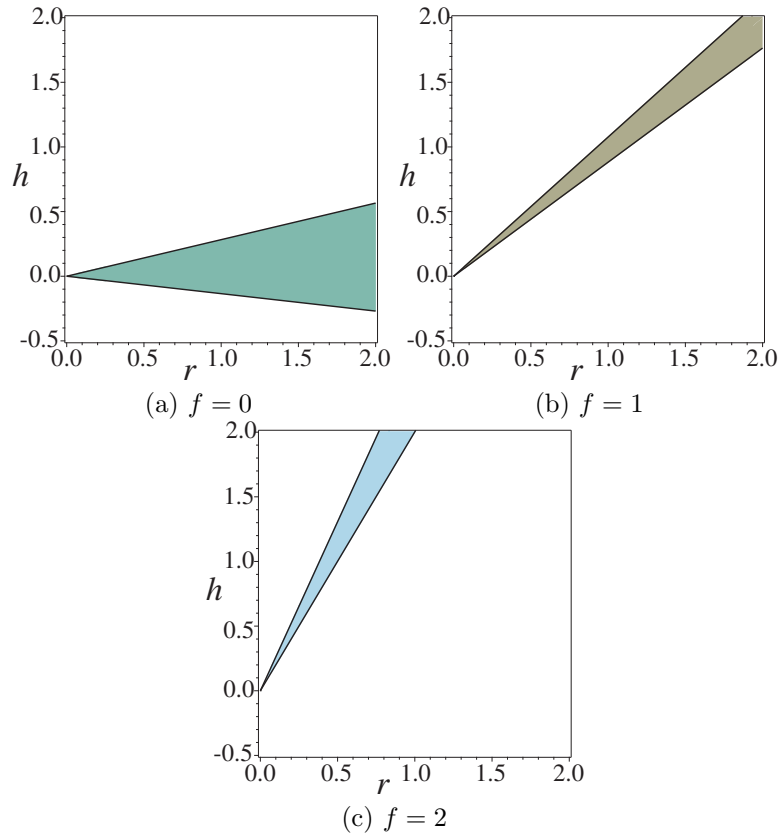


Figure 10: Set of solutions obtained by CAD

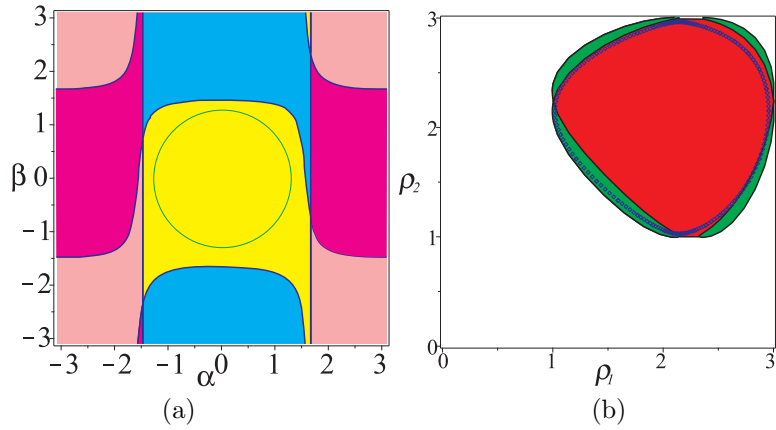


Figure 11: Workspace (a) and joint space (b) without any cusps points with the boundary of the prescribed workspace and its image in the joint space for  $h = 0$ ,  $r = 1$  and  $f = 1/10$

198     Either point  $Q$  belonging to the workspace boundary whose image in the  
 199     workspace is located in a region where DKP admits six solutions, called  $P_i$ . We  
 200     realize a loop that includes the cusp point  $C_6$  (Figure 13(a)).

201     The images of this trajectory shown in Figure 13(b) in the workspace are lo-

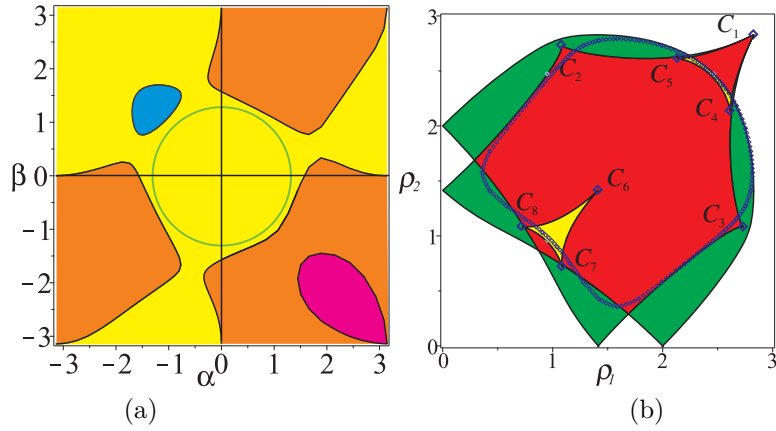


Figure 12: Workspace (a) and joint space (b) with height cusps points with the boundary of the prescribed workspace and its image in the joint space for  $h = r = f = 1$

202 cated in basic regions depicted in yellow and blue regions where  $\det(\mathbf{A}) > 0$  and  
 203 in basic region depicted in red and green where  $\det(\mathbf{A}) < 0$ . In Figure 13(b), we  
 204 can observe from any stating point  $P_i$  (i) two singular trajectories between two  
 205 aspects ( $P_4 - P_6$  and  $P_5 - P_6$ ) and meet singular positions in  $S_1$  and  $S_2$ , respectively,  
 206 (ii) one non-singular changing trajectory in the same aspect ( $P_4 - P_5$ ),  
 207 and (iii) three loops in the workspace located in the same aspect ( $P_1 - P_1$ ,  
 208  $P_2 - P_2$ ,  $P_3 - P_3$ ). Only the trajectory ( $P_1 - P_1$ ) is located in the regular  
 209 workspace shape. We can therefore conclude that even if this trajectory sur-  
 210 rounds a cusp point in the joint space, it is not a non-singular trajectory in a  
 211 changing assembly mode.

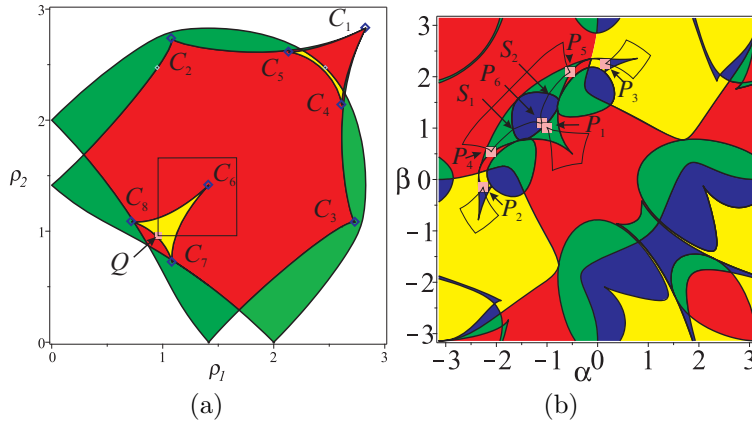


Figure 13: Trajectory that encircles a cusp point in the joint space and its image in the workspace

#### 212 4.4. Discussions

213 The study of mechanism in the design parameter space allows a better un-  
 214 derstanding of the architecture and the working of the 2-PUS-U robot. There  
 215 are three design parameters (namely  $f$ ,  $r$  and  $h$ ) associated with the proposed

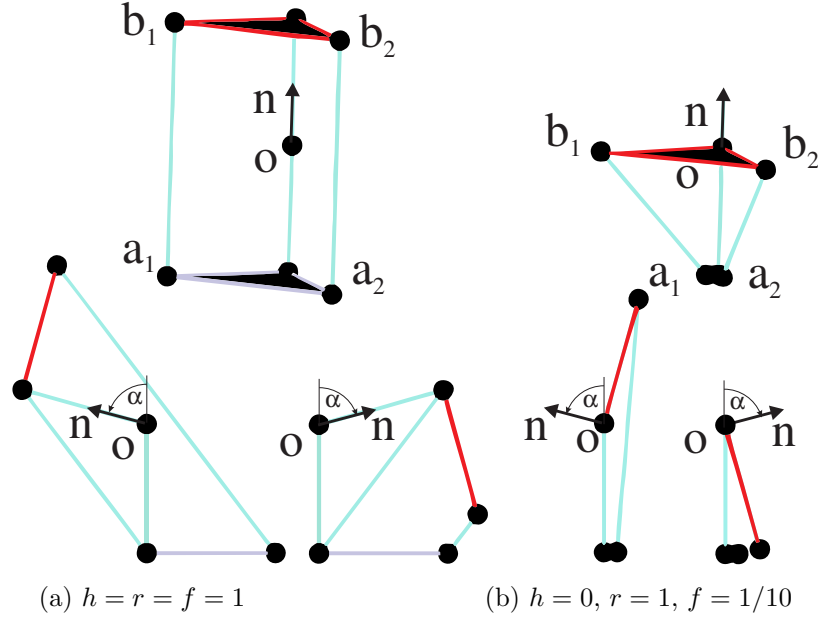


Figure 14: Graphic rendering of the two cases under study in the home pose and for  $\alpha = \pm 75^\circ$  with the mobile platform in red and  $\mathbf{n}$  the normal to the mobile platform

216 robot. The singularity equation is analysed, based on the three conditions  
 217 ( $f = 0, r = 0$  and  $h = 0$ ), which further helps to simplify and factorize the  
 218 singularity equations which is shown in Figure 7. It is shown that robot will  
 219 have non-singular assembly mode changing trajectories i.e. it will be cuspidal  
 220 if it has less aspect than the solutions to the DKP. The analysis of uniqueness  
 221 domains and the behavior around the cusp points infers the safe movement of  
 222 the robot. The selection of design parameters for the desired workspace is ob-  
 223 tained by analyzing the number of intersections between the singularities and  
 224 the workspace boundary. The CAD algorithm is used to compute and visualize  
 225 the two dimensional surface in  $h - r$  plane by varying the design parameter  $f$   
 226 value. More homogeneous sets of parameters exist but make the robot cuspidal  
 227 with possible dangers when generating trajectories. It has been shown that in  
 228 reported case, the part of the workspace studied does not lead to non-singular  
 229 assembly mode changes. However, this does not allow us to infer this result for  
 230 all design parameters.

231 Figure 14 depicts the graphic rendering of the robot with the same scaling  
 232 for the two examples studied. In the “home” position, for the first example, the  
 233 design is more compact than the second example as the width of the first case  
 234 is twice the width of the second case while its height is half the height of the  
 235 second case. However, this selection of design parameters is not generic and it is  
 236 impossible to design the robot as one of the design parameter equal to zero. At  
 237 the maximum tilt rotations, it is necessary to take into account the interference  
 238 problems between the UPS leg actuators and the central link formed by the  
 239 universal joint.

## 240 5. Conclusions

241 Based on the analysis of the surgeon's needs of otologic operations, the  
242 definition of a regular workspace shape was defined. This property allow the  
243 robot to be adapted to the human anatomy of a large part of the population. A  
244 2UPS-U spherical parallel robot was presented and its kinematic equations were  
245 formulated. The properties of this robot were studied in the design parameter  
246 space. Several simplifications have been proposed which allow a factorization of  
247 the singularity locus when one of the parameters is equal to zero. The workspace  
248 can be divided into two or four aspects. A CAD is used to characterize the  
249 sets of design parameters where there is no intersection between the singularity  
250 curves and the regular workspace shape. From this set of possible solutions,  
251 two examples are presented, a non cuspidal robot and a cuspidal robot. Their  
252 properties were illustrated in the workspace and in the joint space. In both cases,  
253 the prescribed workspace is safe because the robot does not change the assembly  
254 mode. Further research will be conducted in order to verify the collision between  
255 the legs and the joint limits of the passive joints and to use a kinetostatic criteria  
256 to select the optimized design parameters of the mechanism for the surgery  
257 application.

## 258 References

- 259 [1] Cohen, M.S., Basonbul, R.A., Barber, S.R., Kozin, E.D., Rivas, A.C. and  
260 Lee, D.J. Development and validation of an endoscopic ear surgery classi-  
261 fication system. *The Laryngoscope*, 128(4), pp.967-970 (2018).
- 262 [2] Iannella, G., Marcotullio, D., Re, M., Manno, A., Pasquariello, B., An-  
263 geletti, D., Falasca, V. and Magliulo, G.. Endoscopic vs microscopic ap-  
264 proach in stapes surgery: advantages in the middle ear structures visual-  
265 ization and trainee's point of view. *The journal of international advanced*  
266 *otology*, 13(1), p.14 (2017).
- 267 [3] Dahroug, B., Tamadazte, B., Weber, S., Tavernier, L. and Andreff,  
268 N. Review on otological robotic systems: Toward microrobot-assisted  
269 cholesteatoma surgery. *IEEE reviews in biomedical engineering*, 11,  
270 pp.125-142 (2018).
- 271 [4] Schena, B., Robotic manipulator with remote center of motion and com-  
272 pact drive, Patent WO 2008/157225 (2007).
- 273 [5] Rosen, J., Brown J.D., Chang, L., Barreca, M., Sinanan, M., Hannaford,  
274 B., The Blue-DRAGON - a system for measuring the kinematics and dy-  
275 namics of minimally invasive surgical tools in-vivo, In: *Proc. IEEE Inter-*  
276 *national Conference on Robotics and Automation* (2002).
- 277 [6] Gosselin, C., Hamel, J.-F., The agile eye: a high-performance three-degree-  
278 of-freedom camera-orienting device. In: *Proc. IEEE international confer-*  
279 *ence on robotics and automation*, pp. 781-786 (1994).
- 280 [7] Cheng H.H., Real-time manipulation of a hybrid serial-and-parallel driven  
281 redundant industrial manipulator. *ASME J. of Dynamic Systems, Mea-*  
282 *surement and Control*, 116(4), pp. 687-701 (1994).

- 283 [8] Agrawal S.K., Desmier G., Li S., Fabrication and analysis of a novel 3 dof  
284 parallel wrist mechanism, *ASME J. of Mechanical Design*, 117(2), pp. 343–  
285 345 (1995).
- 286 [9] Caron, F., Analyse et développement d’un manipulateur parallèle  
287 sphérique à deux degrés de liberté pour l’orientation d’une caméra, M.Sc.,  
288 Université Laval, Québec, August (1997).
- 289 [10] Karouia M. and Hervé J.M. A three-dof tripod for generating spherical  
290 motion, In *ARK*, pp.395–402, Piran, 25-29 June (2000).
- 291 [11] Chablat, D., and Wenger P., Working modes and aspects in fully par-  
292 allel manipulators; *Proceedings, 1998 IEEE International Conference on*  
293 *Robotics and Automation*, Vol. 3 (1998).
- 294 [12] Bonev, I. A., Chablat D., and Wenger P., Working and assembly modes  
295 of the Agile Eye, *Proceedings 2006 IEEE International Conference on*  
296 *Robotics and Automation* (2006).
- 297 [13] Kumar, S., Nayak, A., Peters, H., Schulz, C., Müller, A., Kinematic analy-  
298 sis of a novel parallel 2SPRR+ 1U ankle mechanism in humanoid robot, *In-*  
299 *ternational Symposium on Advances in Robot Kinematics*. Springer, Cham  
300 (2018).
- 301 [14] Chablat D., Wenger P., Séparation des solutions aux modèles géométriques  
302 direct et inverse pour les manipulateurs pleinement parallèles, *Mechanism*  
303 *and Machine Theory*, Vol 36/6, pp. 763–783, (2001).
- 304 [15] Wenger Ph., Chablat D., Definition Sets for the Direct Kinematics of Par-  
305 allel Manipulators, *8th International Conference in Advanced Robotics*,  
306 pp. 859–864 (1997).
- 307 [16] Jha, R., Chablat, D., Baron, L., Rouillier, F., Moroz, G., Workspace,  
308 joint space and singularities of a family of delta-like robot, *Mechanism*  
309 *and Machine Theory*, 127, pp. 73–95, (2018).
- 310 [17] Chablat, D., Moroz, G., Rouillier, F., Wenger, P., Using Maple to analyse  
311 parallel robots. In *Maple Conference 2019*, October, (2019).
- 312 [18] Michel, G., Haribhau Salunkhe, D., Chablat, D. and Bordure, P., A New  
313 RCM Mechanism for an Ear and Facial Surgical Application, *Proceed-*  
314 *ings Advances in Service and Industrial Robotics*, Springer International  
315 Publishing, Cham, pp. 408–418, 2020.
- 316 [19] Bonev, I. A., D. Zlatanov, and C. M. Gosselin. ”Advantages of the modified  
317 Euler angles in the design and control of PKMs.” *2002 Parallel Kinematic*  
318 *Machines International Conference* (2002).
- 319 [20] Chablat, D., Wenger, P., Majou, F., and Merlet, J. P.”An interval analysis  
320 based study for the design and the comparison of three-degrees-of-freedom  
321 parallel kinematic machines”. *The International Journal of Robotics Re-*  
322 *search*, 23(6), 615-624, 2004.
- 323 [21] Chablat, D., Michel, G., Bordure, P. (2019), Dispositif d’aide à la chirurgie,  
324 PCT/EP2020/076353, European Patent.



- 325 [22] Chablat, D., Michel, G., Bordure, P., Jha, R., & Venkateswaran, S. (2020,  
326 September). Joint space and workspace analysis of a 2-DOF Spherical Par-  
327 allel Mechanism. In European Conference on Mechanism Science (pp. 181-  
328 188). Springer, Cham.