



HAL
open science

Inside a Black Hole: the illusion of a Big Bang

Enrique Gaztanaga

► **To cite this version:**

| Enrique Gaztanaga. Inside a Black Hole: the illusion of a Big Bang. 2021. hal-03106344v3

HAL Id: hal-03106344

<https://hal.science/hal-03106344v3>

Preprint submitted on 17 Mar 2021 (v3), last revised 22 Jul 2021 (v7)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Inside a Black Hole: the illusion of a Big Bang

Enrique Gaztañaga

*Institute of Space Sciences (ICE, CSIC), 08193 Barcelona, Spain
Institut d'Estudis Espacials de Catalunya (IEEC), 08034 Barcelona, Spain*

(Dated: March 17, 2021)

What is inside a Black Hole (BH) horizon $r_* = 2GM$? We discuss two non-singular BH solutions, BH.fv and BH.u., within General Relativity, which look like a singular Schwarzschild BH from the outside. BH.fv corresponds to an false vacuum energy $\Delta = 3r_*^{-2}/8\pi G$, while BH.u also has expanding or contracting matter and radiation inside. For the inside comoving observer this BH.u solution looks like an homogeneous and isotropic universe with $\Lambda = 3/r_*^2$. For an outside observer our Universe looks like a regular BH with $M \simeq 5.8 \times 10^{22} M_\odot$. Thus, the Big Bang model is an illusion of the comoving observer and could be much older than previously estimated. Primordial inflation, the measured cosmic acceleration, CMB anomalies and tensions in measurements of cosmological parameters all provide independent observational support for such BH.u solution.

I. INTRODUCTION

A Schwarzschild Black Hole (BH) represents a singular object of mass M . The horizon at $r_* \equiv 2GM$ prevent us from seeing inside. This makes BHs good Dark Matter candidates. But according to Buchdahl [1], BHs are more compact than any form of matter. Here, we look for a physical BH solution defined as a non singular object of size r_* which reproduces the Schwarzschild (SCHW) metric for $r > r_*$ (in empty space). This definition will guide us in our search: we want to find a solution that extends the SCHW coordinate frame inside r_* .

For a perfect fluid the radial pressure inside a BH is negative [2]. Cosmologists are used to this type of fluids, which are called Quintessence, Inflation or Dark Energy (DE). So, could the inside of a BH be DE? Mazur and Mottola [3] argued that the same DE repulsive force that causes cosmic acceleration could also prevent the BH collapse, resulting in the so call gravastar solution. [The idea that the universe might be generated from the inside of a black hole is not new \[4–9\]. The simplest DE is the ground state \$V_0\(\psi\)\$ of a scalar field \$\psi\(x\)\$. In the expanding Big Bang, DE is assumed to be constant as a function of space and time \[10\]. Here we look for a classical BH solution defined by a spatial discontinuity and an event horizon.](#) We revisit these puzzles looking for non-static solutions with radial fluid velocity $u \neq 0$ relative to the outside SCHW observer. The two key questions we want to address are: What are possible metrics for the inside of such a physical BH? What is the meaning of the measured BH mass M ?

II. HOMOGENEOUS SOLUTIONS

We will solve Einstein's field equations [11]:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \equiv -\frac{16\pi G}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}}, \quad (1)$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and \mathcal{L} is the matter Lagrangian. For perfect fluid in spherical coordinates:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \quad (2)$$

where u_ν is the 4-velocity ($u_\nu u^\nu = -1$), ρ , and p are the energy-matter density and pressure. This fluid is in general

made of several components, each with a different equation of state $p = \omega\rho$. For a fluid moving with relative radial velocity u with $u^\nu = (u^0, u, 0, 0)$, we have $u_0^2 = -g_{00}(1 + g_{11}u^2)$ and:

$$\begin{aligned} T_0^0 &= -\rho - u^2(\rho + p)g_{11} & ; & & T_1^1 &= p + u^2(\rho + p)g_{11} \\ T_0^1 &= (\rho + p)u_0 u & ; & & T_2^2 &= T_3^3 = p \end{aligned} \quad (3)$$

For an observer comoving with the fluid $u = 0$. Here we want to consider a global (proper) coordinate frame that is not moving with the fluid so that $T_0^1 \neq 0$. This is necessary to allow for solutions that are not static inside but look like a (static) SCHW BH from outside.

A. Scalar field in curved space-time

Consider a minimally coupled scalar field $\psi = \psi(x_\alpha)$ with:

$$\mathcal{L} \equiv K - V = -\frac{1}{2}\partial_\alpha\psi\partial^\alpha\psi - V(\psi) \quad (4)$$

The Lagrange equations are: $\bar{\nabla}^2\psi = \partial V/\partial\psi$. We can estimate $T_{\mu\nu}(\psi)$ from its definition Eq.1 to find:

$$T_{\mu\nu}(\psi) = \partial_\mu\psi\partial_\nu\psi + g_{\mu\nu}(K - V) \quad (5)$$

comparing to Eq.2 (see Eq.5 in [12] for details):

$$\rho = K + V \quad ; \quad p = |K| - V \quad (6)$$

The stable solution corresponds to $p = -\rho$:

$$\bar{\nabla}^2\psi = \partial V/\partial\psi = 0 \quad ; \quad \rho = -p = V(\psi) = V_i \quad (7)$$

where ψ is trapped in the true minimum V_0 or some false vacuum (FV) state $V_i = V_0 + \Delta$, see Fig.1. The solution to Eq.1 for constant $\rho = -p = V_i$ (without matter or radiation) for a general metric with spherical symmetry in proper coordinates (in Eq.11) is given by deSitter (deS) metric in Eq.13 with $H_\Lambda^2 \equiv 8\pi G\rho_\Lambda/3$ and $\rho_\Lambda = V_i + \Lambda/8\pi G$. This metric is static in proper coordinates which also indicates that the vacuum solution is in equilibrium.

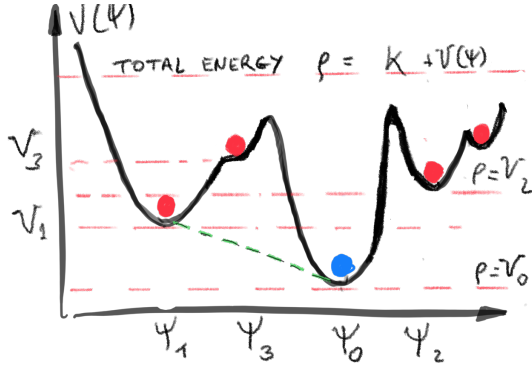


Figure 1. A generic potential $V(\psi)$ for a field $\psi(x)$. A configuration with total energy: $\rho = K + V(\psi)$ (top line) can slowly loose its kinetic energy K (e.g. via Hubble damping) and relax into a (trapped) false ground state $\rho = V_i \equiv V(\psi_i)$, $i = 1, 2, 3$, which we call false vacuum (FV, red circles) with an energy excess $\Delta \equiv V_i - V_0$. The true vacuum is shown as a blue circle at $V_0 = V(\psi_0)$. Note around V_2 how we can have a FV trapped inside other FV. ψ_3 is an unstable (or slow rolling) vacuum, similar to that which generated cosmic inflation. Quantum tunneling (diagonal line) allows particles to escape from a FV.

B. The FLRW metric

The Friedmann-Lemaitre-Robertson-Walker (FLRW) metric in spherical comoving coordinates $\xi^\alpha = (\tau, \chi, \delta, \theta)$, corresponds to an homogeneous and isotropic space-time:

$$ds^2 = f_{\alpha\beta} d\xi^\alpha d\xi^\beta = -d\tau^2 + a(\tau)^2 [d\chi^2 + \chi^2 d\Omega_k^2] \quad (8)$$

where we have introduced: $d\Omega_k^2 \equiv \text{sinc}(\sqrt{k}\chi) d\Omega^2$ with $d\Omega^2 = \cos^2 \delta d\theta^2 + d\delta^2$ and k is the curvature constant $k = \{+1, 0, -1\}$. For the flat case $k = 0$ we have $d\Omega_k^2 = d\Omega^2$. The scale factor, $a(\tau)$, describes the expansion/contraction as a function of time. For a comoving observer, under homogeneity and isotropy:

$$\begin{pmatrix} T_{00} & T_{10} \\ T_{01} & T_{11} \end{pmatrix} = \begin{pmatrix} \rho(\tau) & 0 \\ 0 & p(\tau)a^2 \end{pmatrix} \quad (9)$$

i.e. $u = 0$ in Eq.3. The field equations Eq.1 are:

$$\begin{aligned} 3H^2 &\equiv \left(\frac{\dot{a}}{a}\right)^2 \equiv \left(\frac{\partial_\tau a}{a}\right)^2 = 8\pi G(\rho + \rho_\Lambda) \\ 3\frac{\ddot{a}}{a} &\equiv 3\frac{\partial_\tau^2 a}{a} = -4\pi G(\rho + 3p_R - 2\rho_\Lambda) \\ \rho_\Lambda &\equiv \rho_{\text{vac}} + \frac{\Lambda}{8\pi G}; \quad \rho = \rho_m a^{-3} + \rho_R a^{-4} + \rho_k a^{-2} \end{aligned} \quad (10)$$

Where ρ_m is the matter density today ($a = 1$), ρ_R is the radiation (with pressure $p_R = \rho_R a^{-4}/3$). ρ_{vac} represents vacuum energy: $\rho_{\text{vac}} = -p_{\text{vac}} = V(\psi)$ and $\rho_\Lambda = -p_\Lambda$ is the effective cosmological constant density. Given ρ and p at some time, we can use the above equations to find $a = a(\tau)$ and determine the metric in Eq.8. During inflation, H was dominated by a vacuum (slow rolling) field so that $\rho_\Lambda \approx V(\psi) \equiv 3H_\Lambda^2/8\pi G$, which results in $a \approx e^{\tau H_\Lambda}$. Recent observations show that the expansion rate today is also dominated by ρ_Λ . This indicates that the FLRW metric lives inside a trapped surface

$1/H_\Lambda = (8\pi G\rho_\Lambda/3)^{-1/2}$, which [13] called our causal boundary, and behaves like the interior of a BH.

III. PROPER COORDINATES

The most general shape for a metric with spherical symmetry in proper coordinates (t, r, δ, θ) is [11]:

$$ds^2 = -A(t, r)dt^2 + B(t, r)dr^2 + r^2 d\Omega^2 \quad (11)$$

Empty space $\rho = p = \Lambda = 0$ results in the SCHW metric:

$$ds^2 = -[1 - 2GM/r] dt^2 + \frac{dr^2}{1 - 2GM/r} + r^2 d\Omega^2 \quad (12)$$

where M represents a singular point mass at $r = 0$. This is not a realistic configuration but the horizon at $r_* = 2GM$ prevent us from seeing such naked singularity. Outgoing null radial geodesics cannot leave the interior of r_* , while incoming ones can cross inside r_* . The solution to Eq.1 with the above metric with $\rho = p = M = 0$, but $\Lambda \neq 0$ is the deSitter (deS) metric:

$$ds^2 = -[1 - r^2 H_\Lambda^2] dt^2 + \frac{dr^2}{1 - r^2 H_\Lambda^2} + r^2 d\Omega^2 \quad (13)$$

which is also static and has an trapped surface at $r = 1/H_\Lambda$ where $H_\Lambda^2 \equiv 8\pi G\rho_\Lambda/3$ and $\rho_\Lambda = \Lambda/(8\pi G) + V$. We have included constant V in Eq.7 because it is degenerate with Λ .

Without lost of generality, we rewrite Eq.11 in form that is closer to SCHW and deS metrics:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -[1 - 2\phi]^{1-2\varphi} dt^2 + \frac{dr^2}{1 - 2\phi} + r^2 d\Omega_k^2 \quad (14)$$

where we have used $d\Omega_k$ introduced in Eq.8 to allow for non-flat space. In general $\phi = \phi(t, r)$ and $\varphi = \varphi(t, r)$. The field equations for this case are well known, e.g. see Eq.(7.51) in [11]. The case $p = -\rho$ results in $G_0^0 = G_1^1$, and the solutions are $\varphi = 0$ and:

$$\phi = G/r \int_0^r \rho(r) 4\pi r^2 dr + \Lambda r^2/6 + C_1/r \quad (15)$$

The remaining field equations, $G_2^2 = G_3^3$ provide the time evolution and are equivalent to energy conservation $\nabla_\mu T_\nu^\mu = 0$:

$$\begin{aligned} \partial_t \rho &= -\frac{\rho + p}{1 - 2\phi} \partial_t \phi \\ p' &= (\rho + p) \left[\phi' \frac{1 - 2\phi}{1 - 2\phi} + \varphi' \ln(1 - 2\phi) \right] \end{aligned} \quad (16)$$

for $u = 0$. Note how $\rho = -p$ results in constant ρ and p everywhere, but with a discontinuity at $r = r_*$ corresponding to $2\phi = 1$. This discontinuity is critical to understand a BH solution. One way to address this discontinuity is to [study junction conditions](#) (see §III C).

A. False Vacuum Black Hole (BH.fv)

We take $C_1 = 0$ in Eq.15 to avoid singular solutions. The simplest way to find a physical BH solution is:

$$\rho(r) = -p(r) = \begin{cases} V_0 & \text{for } r > r_* \\ V_0 + \Delta & \text{for } r < r_* \end{cases} \quad (17)$$

where V_0 and $\Delta > 0$. We require the metric to be asymptotically flat [13] because particles should be free at spacial infinity for a finite time of evolution [14]. This requires $\Lambda = -8\pi G V_0$ and the solution (which we called BH.fv) to Eq.15 is:

$$1 - 2\phi = \begin{cases} 1 - r_*/r & \text{for } r > r_* \equiv 2GM \\ 1 - r^2 H_\Lambda^2 & \text{for } r < r_* = 1/H_\Lambda \end{cases} \quad (18)$$

where: $3H_\Lambda^2 \equiv 8\pi G \rho_\Lambda$, $\rho_\Lambda = \rho_M = \Delta$ and $M = \frac{4\pi}{3} r_*^3 \Delta$. This has no singularity at $r = 0$. Coordinates (t, r) don't change sign as we cross inside r_* . In both sides of r_* we have constant values of p and ρ and energy conservation in Eq.16. There is a discontinuity ($\rho' = \infty$ and $p' = \infty$) at $2\phi = 1$ where $r = r_*$, in agreement with Eq.16. But the metric is static and continuous at r_* . This only happens for $r_* = (8\pi G \Delta / 3)^{-1/2}$. The smaller Δ the larger and more massive the BH. In the limit $\Delta \Rightarrow 0$, we have $r_* \Rightarrow \infty$ and we recover Minkowski space.

Is this solution stable [2]? At a fixed location, the field ψ inside the BH is trap in a stable configuration ($\rho = V$) and can not evolve ($K = 0$). The same happens for the field outside (see Fig.1). A FV in Eq.17 with equal Δ but different initial radius $R < r_*$ is subject to a pressure discontinuity at $r = R$ which is not balanced in Eq.16 and results in a bubble growth [15]. [As show in Eq.30 below this junction is stable and asymptotically reaches \$R = r_*\$.](#) The inside of r_* is causally disconnected, so the pressure discontinuity does not act on r_* , which corresponds to a trapped surface. Radial null events ($ds^2 = 0$) connecting $(0, r_0)$ with (t, r) follow:

$$r = r_* \left(\frac{r_* + r_0}{r_* - r_0} e^{2t/r_*} - 1 \right) / \left(\frac{r_* + r_0}{r_* - r_0} e^{2t/r_*} + 1 \right) \quad (19)$$

so that it takes $t = \infty$ to reach $r = r_*$ from any point inside (this also happens from outside, see Eq.31)

The interior deS metric can be transformed into a FLRW metric with constant $H = H_\Lambda$ [16]. This change of coordinates provides a new interpretation for the BH.fv solution in Eq.18. This is not only a solution for a BH inside a universe. The inside comoving observer, sees this solution as an expanding inflationary universe inside a BH, even when the metric is static in proper coordinates and $r = r_*$ is fixed.

Note how we can have FVs inside other FVs (see Fig.1). So we can have BHs inside other BHs. This is a simple solution, but it is not realistic as it has no matter or radiation anywhere.

B. Black Hole Universe (BH.u)

Consider solutions to the metric of Eq.14 where we also have matter $\rho_m = \rho_m(t, r)$ and radiation $\rho_R = \rho_R(t, r)$ inside:

$$\rho(t, r) = \begin{cases} -p = V_0 & \text{for } r > r_* \\ V_0 + \Delta + \rho_m + \rho_R & \text{for } r < r_* \end{cases} \quad (20)$$

with $p = -V_0 - \Delta + \rho_R/3 \neq -\rho$ inside. This means $\partial_t \phi \neq 0$ and $u \neq 0$: the fluid inside has to move relative to proper frame of the outside observer [17]. For $r > r_*$, the solution is the same as Eq.18. For the interior we define: $2\phi \equiv r^2 H^2(t, r)$, so that:

$$1 - 2\phi(t, r) = \begin{cases} 1 - r_*/r & \text{for } r > r_* = 2GM \\ 1 - r^2 H^2(t, r) & \text{for } r < r_* = 1/H_\Lambda \end{cases} \quad (21)$$

where $r_* = 2GM = 1/H_\Lambda$ as before. We can find the interior solution with a change of variables from $x^\mu = [t, r, \delta, \theta]$ to comoving coordinates $\xi^\nu = [\tau, \chi, \delta, \theta]$, where $r = a(\tau)\chi$ and

$$\Lambda_\nu^\mu \equiv \frac{\partial x^\mu}{\partial \xi^\nu} = \begin{pmatrix} \partial_\tau t & \partial_\chi t \\ \partial_\tau r & \partial_\chi r \end{pmatrix} = \begin{pmatrix} (1 - 2\phi)^{\varphi-1} & \frac{a\sqrt{2\phi}}{(1-2\phi)^{1-\varphi}} \\ \sqrt{2\phi} & a \end{pmatrix} \quad (22)$$

where angular variables (δ, θ) are unchanged. This transforms $g_{\mu\nu}$ in Eq.14 into the FLRW metric $f_{\alpha\beta}$ in Eq.8: $f_{\alpha\beta} = \Lambda_\alpha^\mu \Lambda_\beta^\nu g_{\mu\nu}$ with $2\phi = r^2 H^2(\tau)$ and arbitrary φ . So the most general spherically symmetric in-homogeneous metric inside r_* looks homogeneous in comoving coordinates (where r_* is the FLRW trapped surface). Given some $\rho(\tau)$ and $p(\tau)$ in comoving coordinates in the interior of a BH we can use Eq.10 with $\rho_\Lambda = \Delta = 3r_*^{-2}/8\pi G$ to find $H(\tau)$ and $a(\tau)$. The solution in Eq.21 is then just $H(t, r) = H(\tau)$. We call this a BH universe (BH.u).

To find $\tau = \tau(t, r)$ explicitly we need to solve Eq.22 with $2\phi = r^2 H^2(\tau)$. For $H(\tau) = H_\Lambda$ the solution is $\varphi = 0$ with

$$\tau = \tau(t, r) = t + \frac{1}{2H_\Lambda} \ln [1 - H_\Lambda^2 r^2] \quad (23)$$

where $r < r_* = 1/H_\Lambda$. The flat FLRW metric with $H = H_\Lambda$ becomes deS metric in Eq.13 as in the BH.fv solution. In comoving coordinates, (τ, χ) , the metric is expanding: $a(\tau) = e^{\tau H_\Lambda}$, while in proper coordinates, (t, r) , it is static [16].

Given $T_{\mu\nu}$ in Eq.9 we can find $\bar{T}_{\alpha\beta}$ in the proper frame using the inverse matrix of Eq.22: $\bar{T}_{\alpha\beta} = (\Lambda^{-1})_\alpha^\mu (\Lambda^{-1})_\beta^\nu T_{\mu\nu}$:

$$\bar{T}_0^0 = -\frac{\rho + p2\phi}{1 - 2\phi} ; \quad \bar{T}_1^1 = \frac{p + \rho2\phi}{1 - 2\phi} \quad (24)$$

which is independent of φ . Comparing to Eq.3 gives $u^2 = 2\phi = r^2 H^2$.

Solution $H(t, r) = H(\tau)$ in Eq.21 is valid for all $r < r_* = 1/H_\Lambda$ because $H(\tau) > H_\Lambda$. We can see this by considering outgoing radial null geodesic in the FLRW metric of Eq.8:

$$r_{out} = a(\tau) \int_\tau^\infty \frac{d\tau}{a(\tau)} = a \int_a^\infty \frac{da}{a^2 H(a)} < \frac{1}{H_\Lambda} = r_* \quad (25)$$

which shows that signals can not escape from the inside to the outside of the BH.u. But incoming radial null geodesics $a(\tau) \int_0^\tau \frac{d\tau}{a(\tau)}$ can in fact be larger than r_* if we look back in time. This shows that inside observers are trap inside the BH.u but they can nevertheless observe what happened outside.

C. Junction conditions

We can arrive at the same BH.u and BH.fv solutions using junction conditions. Here we follow closely the notation in §12.5 of [11]. We will combine two solutions to the field equations with different energy content, as in Eq.20, on two sides of a hypersurface Σ . The inside g_- is FLRW metric (or deS metric for $H = H_\Lambda$) and the outside g_+ is SCHW metric. This is similar to the case §12.5.1 in [11] with the difference that we use $k = 0$ (instead of $k = 1$) and consider a general FLRW solution $a(\tau)$ with Λ , ρ_m and ρ_R (instead of a pressure-free dust model without Λ). We define Σ to be fixed in comoving coordinates $\chi = \chi_\S$, where χ_\S is the causal boundary in the FLRW metric [13]. For the SCHW coordinate system, Σ is described by $r = R(\tau)$ and $t = T(\tau)$, where τ is the comoving time in the FLRW metric. This is just a particular case of the change of variables in Eq.22. The induced metric h_- on the inside of Σ_- with $y^a = (\tau, \delta, \theta)$ and fixed χ :

$$ds_{\Sigma_-}^2 = h_{-ab} dy^a dy^b = -d\tau^2 + a^2(\tau) \chi_\S^2 d\Omega^2 \quad (26)$$

has to agree with h_+ , the SCHW outside at Σ_+ :

$$-F dt^2 + F^{-1} dr^2 + r^2 d\Omega^2 = -(F\dot{T}^2 - \dot{R}^2/F) d\tau^2 + R^2 d\Omega^2 \quad (27)$$

where $F = 1 - \frac{2GM}{R}$. The matching condition $h_- = h_+$ is:

$$R(\tau) = a(\tau) \chi_\S \quad ; \quad F\dot{T} = \sqrt{\dot{R}^2 + F} \equiv \beta(R, \dot{R}) \quad (28)$$

For a given FLRW solution $a(\tau)$ we know both R and β . The extrinsic curvature K_\pm normal to Σ_\pm from each side is:

$$\begin{aligned} K_{-\tau}^\tau &= 0 \quad ; \quad K_{-\theta}^\theta = K_{-\delta}^\delta = -\frac{1}{a\chi_\S} \\ K_{+\tau}^\tau &= \frac{\dot{\beta}}{R} \quad ; \quad K_{+\theta}^\theta = K_{+\delta}^\delta = -\frac{\beta}{R} \end{aligned} \quad (29)$$

Thus, the second matching condition $K_- = K_+$ requires $\beta = 1$, which using Eq.28 results in:

$$\dot{R}^2 = R^2 H^2 = \frac{R_*}{R} \quad (30)$$

$$T = \int \frac{dR}{HR(1 - H^2 R^2)} \quad (31)$$

where $R_* \equiv 2GM$. Starting from small a , as we increase τ , both R and $\dot{R} = HR$ grow until we reach $HR = c = 1$ which corresponds to the event horizon $R_* = 2GM = 1/H_\Lambda$. It takes $T = \infty$ in the SCHW time of Eq.31 to asymptotically reach R_* . In this limit, Eq.30 reproduces the BH.u solution of Eq.21 and the BH.fv solution of Eq.18 for constant $H = H_\Lambda$.

This proves that the join metric is also a solution to the Einstein's field equations and there are no surface terms in the junction. There is no discontinuity in the two metrics when we join them with the timelike hypersurface of Σ . The Λ term corresponds to a trapped surface $R_* = 1/H_\Lambda$ in the FLRW metric which matches the horizon of a BH in empty space.

IV. DISCUSSION & CONCLUSIONS

The SCHW metric in Eq.12 is well known and studied but the interior solution is not physical because it corresponds to a singular point source of mass M at $r = 0$. Instead of venturing into Quantum-Gravity territory, here we look for classical non-singular solutions for the BH interior. Our motivation is to understand what is the physical meaning of M for BHs that have already been observed. We find two solutions for the BH interior which we label BH.fv and BH.u. BH.fv corresponds to a space-time with a constant density discontinuity in Eq.17 and deS metric inside Eq.18. deS metric has a trapped surface at $r = r_*$ which matches the SCHW event horizon. A constant density corresponds to a centrifugal force, $2\phi = (r/r_*)^2$ that opposes gravity, $2\phi = r_*/r$, i.e. Eq.15. The equilibrium happens when both forces are equal, which fixes r_* , and correspond to stable circular orbits. Similar solutions for BH.fv are known [4, 18–20], such as the gravastar [3] or bubble universes [15]. But they are not very realistic as they don't have matter or radiation.

The second solution is Eq.21, where the BH interior is the FLRW metric. This BH.u solution is new, as far as we know. We can have other BHs, matter and radiation inside a BH within a larger space-time. The inside needs to be expanding or contracting as in the FLRW metric of Eq.8, with the same trapped surface given by ρ_Λ . This holds the expansion and balance gravity at r_* . The join solution Eq.21 is also a solution to Einstein's field equations as the two metrics reduce to the same form on a junction of constant χ in Eq.26, and the extrinsic curvature in Eq.29 is the same in both sides. Non flat $k \neq 0$ junctions were also considered in [7], but only for the pressure-free dust model without Λ . The exterior metric could also be FLRW, as the SCHW metric can be considered a local perturbation within a larger FLRW background.

Both BH type solutions can be interpreted as a BH within our universe or as an expanding universe inside a larger space-time. The idea that the universe might be generated from the inside of a BH is not new [4, 6–8, 18]. Knutsen [21] rightly pointed out the inconsistencies in notation and interpretation in Pathria [6]. For the FLRW metric, the critical density $\rho_c = 3H^2/8\pi G$ corresponds to a BH inside a Hubble radius $R = 1/H$. The FLRW metric is trapped inside $r < r_* = \sqrt{3/\Lambda}$, and is then equivalent to an inhomogeneous spherically symmetric metric of Eq.11, as shown by the change of variables in Eq.22 [22].

The solutions to the field equations are independent of the choice of coordinates but $\tilde{T}_{\mu\nu}(t, r)$ depends on the fluid motion (see Eq.24). We used comoving coordinates (τ, χ) , where the fluid is homogeneous and comoving, to find the interior solution. But we can then transform back to proper (t, r) to find a full BH solution in Eq.21 that is continuous at r_* , like in the BH.fv case. The BH mass density, $\rho_M = M/(4\pi r_*^3/3)$ is $\rho_M = \rho_\Lambda$ which should be interpreted as a (FV) boundary condition $\rho_\Lambda = \langle \rho_m/2 + \rho_R \rangle$ [13]. As it happened in the singular SCHW metric, outgoing radial null geodesics cannot escape the event horizon, but incoming ones can enter (see discussion around Eq.19 and Eq.25).

We can sketch the evolution of our universe with this model. A primordial field ψ settles or fluctuates into a false (or slow

rolling) vacuum which will create a BH.fv with a junction Σ in Eq.26 where the causal boundary is fixed in comoving coordinates and corresponds to the particle horizon during inflation $\chi_{\S} = c/(a_i H_i)$ or the Hubble horizon when inflation begins. The size $R = a(\tau)\chi_{\S}$ of this vacuum grows and asymptotically tends to r_* in Eq.18 following Eq.30 with $H = H_i$. The inside of this BH will be expanding exponentially $a = e^{\tau H_i}$ while the comoving Hubble horizon decreases as $c/(a H_i)$. When this inflation ends [23–26] vacuum energy excess converts into matter and radiation (reheating). This results in BH.u, where an infinitesimal comoving Hubble horizon grows again following the standard Big Bang evolution (see Fig.1 in [13]). The junction condition remains fixed at χ_{\S} throughout and $R = a(\tau)\chi_{\S}$ asymptotically tends to r_* in Eq.21 following Eq.30. In proper coordinates this solution has no Big Bang (or bounce): it is not singular at $r = 0$ or at $\tau = 0$, because we have a non-singular BH.fv before we start the FLRW BH.u phase. The inside comoving observer is trapped inside $r < r_* = 2GM = 1/H_{\Lambda}$ and has the illusion of a Big Bang. The space-time outside (t, r) could be longer and larger than the Big Bang prediction. In a more realistic situation we could have a network of BH.u and there could be matter and radiation in between (e.g. from quantum tunnelling).

We already have some observational evidence that the expanding metric around us is inside a BH.u. We can recover the

Big Bang homogeneous solution in the limit $\Delta \Rightarrow 0$, where we have $r_* \Rightarrow \infty$ and $\rho_{\Lambda} = 0$. But we have measured $\rho_{\Lambda} > 0$ ($\Omega_{\Lambda} \approx 0.7$) which implies $M \approx 5.8 \times 10^{22} M_{\odot}$ and $r_* \approx c/H_0$, as in the BH.u. This model also explains the observed coincidence between ρ_{Λ} and ρ_m today. The BH horizon r_* is what [13] called the primordial causal boundary r_{\S} . If we look back to the CMB times, r_* corresponds to 60 degrees in the sky [13]. The observed anomalies in the CMB temperature maps at larger scales [13, 27] provide additional evidence for the BH.u model. There is also a window to see outside our BH.u using the largest angular scales for $z > 2$ and measurements of cosmological parameters from very different cosmic times. Fosalba and Gaztanaga [27] found variations in cosmological parameters over large CMB regions. Their Fig.30 show that the size of these regions follow the BH.u relation between r_* and ρ_{Λ} . Such observations can therefore provide new evidence for the initial conditions to our universe.

ACKNOWLEDGMENTS

I want to thank Marco Bruni, Robert Caldwell, Alberto Diez-Tejedor and Angela Olinto for their feedback and the anonymous referees for pointing out previous work and the importance of the junction conditions.

-
- [1] H. A. Buchdahl, General relativistic fluid spheres, *Phys. Rev.* **116**, 1027 (1959).
- [2] R. Brustein and A. J. M. Medved, Resisting collapse, *Phys. Rev. D* **99**, 064019 (2019), and references therein
- [3] P. O. Mazur and E. Mottola, Surface tension and negative pressure interior of a non-singular ‘black hole’, *Classical and Quantum Gravity* **32**, 215024 (2015).
- [4] D. A. Easson and R. H. Brandenberger, Universe generation from black hole interiors, *Journal of High Energy Physics* **2001**, 024 (2001), and references therein
- [5] I. Dymnikova, Universes Inside a Black Hole with the de Sitter Interior, *Universe* **5**, 111 (2019).
- [6] R. K. Pathria, The Universe as a Black Hole, *Nature (London)* **240**, 298 (1972).
- [7] W. M. Stuckey, The observable universe inside a black hole, *American Journal of Physics* **62**, 788 (1994).
- [8] N. Popławski, Universe in a Black Hole in Einstein-Cartan Gravity, *Astrophys. J.* **832**, 96 (2016), arXiv:1410.3881 [gr-qc].
- [9] N. Oshita and J. Yokoyama, Creation of an inflationary universe out of a black hole, *Physics Letters B* **785**, 197 (2018).
- [10] A constant vacuum energy does not gravitate [13]. But a false vacuum discontinuity does gravitate, as we will show below.
- [11] T. Padmanabhan, *Gravitation, Cambridge Univ. Press* (2010).
- [12] A. Díez-Tejedor and A. Feinstein, Homogeneous scalar field and the wet dark sides of the universe, *Phys. Rev. D* **74**, 023530 (2006), arXiv:gr-qc/0604031 [gr-qc].
- [13] E. Gaztanaga, The Cosmological Constant as a Zero Action Boundary, *MNRAS*, hal-03015944, darkcosmos.com (2021); The size of our causal Universe, *MNRAS* **494**, 2766 (2020).
- [14] Our FLRW universe has $\rho_{\Lambda} > 0$, the SCHW metric can be considered as a perturbation in a FLRW background.
- [15] A. Aguirre and M. C. Johnson, Dynamics and instability of false vacuum bubbles, *Phys. Rev. D* **72**, 103525 (2005).
- [16] For some additional discussion see A. Mitra, Interpretational conflicts between the static and non-static forms of the de Sitter metric, *Nature Sci. Reports* **2**, 923 (2012).
- [17] This makes physical sense. Specially because the outside observer can not see inside. $T_{\mu\nu}$ could also be anisotropic.
- [18] I. Dymnikova, Vacuum nonsingular black hole, *General Relativity and Gravitation* **24**, 235 (1992).
- [19] R. G. Daghigh, J. I. Kapusta, and Y. Hosotani, False Vacuum Black Holes and Universes, arXiv:gr-qc/0008006 (2000).
- [20] H. Firouzjahi, Primordial Universe Inside the Black Hole and Inflation, arXiv e-prints, arXiv:1610.03767 (2016).
- [21] H. Knutsen, The idea of the universe as a black hole revisited, *Gravitation and Cosmology* **15**, 273 (2009).
- [22] The FLRW metric is only homogeneous in space, but not in space-time. A new frame where comoving time and space are mixed, can break or restore this symmetry [16].
- [23] A. A. Starobinskiĭ, Spectrum of relict gravitational radiation and the early state of the universe, *Soviet J. of Exp. and Th. Physics Letters* **30**, 682 (1979).
- [24] A. H. Guth, Inflationary universe: A possible solution to the horizon and flatness problems, *Phys. Rev. D* **23**, 347 (1981).
- [25] A. D. Linde, A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, *Physics Letters B* **108**, 389 (1982).
- [26] A. Albrecht and P. J. Steinhardt, Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking, *Phys. Rev. Lett.* **48**, 1220 (1982).
- [27] P. Fosalba and E. Gaztanaga, Explaining Cosmological Anisotropy: Evidence for Causal Horizons from CMB data, *MNRAS* submitted (2020), arXiv:2011.00910.