



HAL
open science

Inside a Black Hole: the illusion of a Big Bang

Enrique Gaztanaga

► **To cite this version:**

| Enrique Gaztanaga. Inside a Black Hole: the illusion of a Big Bang. 2021. hal-03106344v2

HAL Id: hal-03106344

<https://hal.science/hal-03106344v2>

Preprint submitted on 10 Feb 2021 (v2), last revised 22 Jul 2021 (v7)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Inside a Black Hole: the illusion of a Big Bang

Enrique Gaztañaga

Institute of Space Sciences (ICE, CSIC), 08193 Barcelona, Spain
Institut d'Estudis Espacials de Catalunya (IEEC), 08034 Barcelona, Spain

(Dated: February 10, 2021)

What is inside a Black Hole (BH) horizon $R_* = 2GM$? We discuss two non-singular BH solutions, BH.fv and BH.u., within General Relativity, which look like a singular Schwarzschild BH from the outside. BH.fv corresponds to an false vacuum $\Delta = 3R_*^{-2}/8\pi G$, while BH.u also has expanding or contracting matter and radiation inside. For the inside comoving observer this BH.u solution looks like an homogeneous and isotropic universe with $\Lambda = 3/R_*^2$. For an outside observer our Universe looks like a regular BH with $M \simeq 5.8 \times 10^{22} M_\odot$. Thus, the Big Bang model is an illusion of the comoving observer and could be much older than previously estimated. Primordial inflation, the measured cosmic acceleration, CMB anomalies and tensions in measurements of cosmological parameters all provide independent observational support for such BH.u solution.

I. INTRODUCTION

A Schwarzschild Black Hole (BH) represents a singular object of mass M . The horizon at $R_* \equiv 2GM$ prevent us from seeing inside. This makes BHs good Dark Matter candidates. But according to Buchdahl [1], BHs are more compact than any form of matter. Here, we look for a physical BH solution defined as a non singular object of size R_* which reproduces the Schwarzschild (SCHW) metric for $r > R_*$ (in empty space). This definition will guide us in our search: we want to find a solution that extends the SCHW coordinate frame inside R_* .

For a perfect fluid the radial pressure inside a BH is negative [2]. Cosmologists are used to this type of fluids, which are called Quintessence, Inflation or Dark Energy (DE). So, could the inside of a BH be DE? Mazur and Mottola [3] argued that the same DE repulsive force that causes cosmic acceleration could also prevent the BH collapse, resulting in the so call gravastar solution. The idea that the universe might be generated from the inside of a black hole is not new [4, 5], but such models don't contain matter and radiation, only scalar fields. The simplest DE is the ground state $V_0(\psi)$ of a scalar field $\psi(x)$. In the expanding Big Bang, DE seems to be constant as a function of space and time[6]. But here we look for a BH solution which is not homogeneous but is defined by a spatial discontinuity. We revisit these puzzles looking for non-static solutions with radial fluid velocity $u \neq 0$ relative to the outside SCHW observer. The two key questions we want to address are: What are possible metrics for the inside of such a physical BH? What is the meaning of the measured BH mass M ?

II. HOMOGENEOUS SOLUTIONS

We will solve Einstein's field equations [7]:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \equiv -\frac{16\pi G}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}}, \quad (1)$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and \mathcal{L} is the matter Lagrangian. For perfect fluid in spherical coordinates:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \quad (2)$$

where u_ν is the 4-velocity ($u_\nu u^\nu = -1$), ρ , and p are the energy-matter density and pressure. This fluid is in general made of several components, each with a different equation of state $p = \omega\rho$. For a fluid moving with relative radial velocity u with $u^\nu = (u^0, u, 0, 0)$, we have $u_0^2 = -g_{00}(1 + g_{11}u^2)$ and:

$$\begin{aligned} T_0^0 &= -\rho - u^2(\rho + p)g_{11} & ; & \quad T_1^1 = p + u^2(\rho + p)g_{11} \\ T_0^1 &= (\rho + p)u_0 u & ; & \quad T_2^2 = T_3^3 = p \end{aligned} \quad (3)$$

For an observer comoving with the fluid $u = 0$. Here we want to consider a global (proper) coordinate frame that is not moving with the fluid so that $T_0^1 \neq 0$. This is necessary to allow for solutions that are not static inside but look like a (static) SCHW BH from outside.

A. Scalar field in curved space-time

Consider a minimally coupled scalar field $\psi = \psi(x_\alpha)$ with:

$$\mathcal{L} \equiv K - V = -\frac{1}{2}\partial_\alpha\psi\partial^\alpha\psi - V(\psi) \quad (4)$$

The Lagrange equations are: $\bar{\nabla}^2\psi = \partial V/\partial\psi$. We can estimate $T_{\mu\nu}(\psi)$ from its definition Eq.1 to find:

$$T_{\mu\nu}(\psi) = \partial_\mu\psi\partial_\nu\psi + g_{\mu\nu}(K - V) \quad (5)$$

comparing to Eq.2 (see Eq.5 in [8] for details):

$$\rho = K + V \quad ; \quad p = |K| - V \quad (6)$$

The stable solution corresponds to $p = -\rho$:

$$\bar{\nabla}^2\psi = \partial V/\partial\psi = 0 \quad ; \quad \rho = -p = V(\psi) = V_i \quad (7)$$

where ψ is trapped in the true minimum V_0 or some false vacuum (FV) state $V_i = V_0 + \Delta$, see Fig.1. The solution to Eq.1 for constant $\rho = -p = V_i$ (without matter or radiation) for a general metric with spherical symmetry in proper coordinates (in Eq.11) is given by deSitter (deS) metric in Eq.13 with $H_\Lambda^2 \equiv 8\pi G\rho_\Lambda/3$ and $\rho_\Lambda = V_i + \Lambda/8\pi G$. This metric is static in proper coordinates which also indicates that the vacuum solution is in equilibrium.

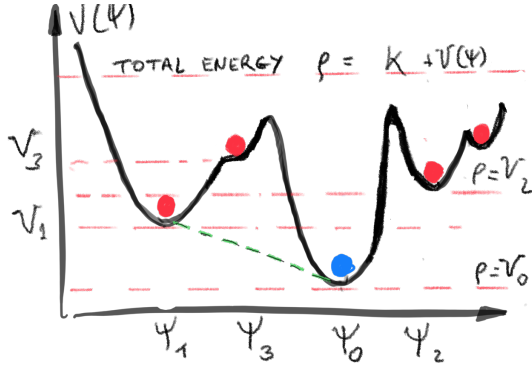


FIG. 1. A generic potential $V(\psi)$ for a field $\psi(x)$. A configuration with total energy: $\rho = K + V(\psi)$ (top line) can slowly lose its kinetic energy K (e.g. via Hubble damping) and relax into a (trapped) false ground state $\rho = V_i \equiv V(\psi_i)$, $i = 1, 2, 3$, which we call false vacuum (FV, red circles) with an energy excess $\Delta \equiv V_i - V_0$. The true vacuum is shown as a blue circle at $V_0 = V(\psi_0)$. Note around V_2 how we can have a FV trapped inside other FV. ψ_3 is an unstable (or slow rolling) vacuum, similar to that which generated cosmic inflation. Quantum tunneling (diagonal line) allows particles to escape from a FV.

B. The FLRW metric

The Friedmann-Lemaitre-Robertson-Walker (FLRW) metric in spherical comoving coordinates $\xi^\alpha = (\tau, \chi, \delta, \theta)$, corresponds to an homogeneous and isotropic space-time:

$$ds^2 = h_{\alpha\beta} d\xi^\alpha d\xi^\beta = -d\tau^2 + a(\tau)^2 [d\chi^2 + \chi^2 d\Omega_k^2] \quad (8)$$

where we have introduced: $d\Omega_k^2 \equiv \text{sinc}(\sqrt{k}\chi) d\Omega^2$ with $d\Omega^2 = \cos^2 \delta d\theta^2 + d\delta^2$ and k is the curvature constant $k = \{+1, 0, -1\}$. For the flat case $k = 0$ we have $d\Omega_k^2 = d\Omega^2$. The scale factor, $a(\tau)$, describes the expansion/contraction as a function of time. For a comoving observer, under homogeneity and isotropy:

$$\begin{pmatrix} T_{00} & T_{10} \\ T_{01} & T_{11} \end{pmatrix} = \begin{pmatrix} \rho(\tau) & 0 \\ 0 & p(\tau)a^2 \end{pmatrix} \quad (9)$$

i.e. $u = 0$ in Eq.3. The field equations Eq.1 are:

$$\begin{aligned} 3H^2 &\equiv \left(\frac{\dot{a}}{a}\right)^2 \equiv \left(\frac{\partial_\tau a}{a}\right)^2 = 8\pi G(\rho + \rho_\Lambda) \\ 3\frac{\ddot{a}}{a} &\equiv 3\frac{\partial_\tau^2 a}{a} = -4\pi G(\rho + 3p_R - 2\rho_\Lambda) \\ \rho_\Lambda &\equiv \rho_{\text{vac}} + \frac{\Lambda}{8\pi G}; \quad \rho = \rho_m a^{-3} + \rho_R a^{-4} + \rho_k a^{-2} \end{aligned} \quad (10)$$

Where ρ_m is the matter density today ($a = 1$), ρ_R is the radiation (with pressure $p_R = \rho_R a^{-4}/3$). ρ_{vac} represents vacuum energy: $\rho_{\text{vac}} = -p_{\text{vac}} = V(\psi)$ and $\rho_\Lambda = -p_\Lambda$ is the effective cosmological constant density. Given ρ and p at some time, we can use the above equations to find $a = a(\tau)$ and determine the metric in Eq.8. During inflation, H was dominated by a vacuum (slow rolling) field so that $\rho_\Lambda \approx V(\psi) \equiv 3H_\Lambda^2/8\pi G$, which results in $a \approx e^{\tau H_\Lambda}$. Recent observations show that the expansion rate today is also dominated by ρ_Λ [9]. This

indicates that the FLRW metric lives inside a trapped surface $1/H_\Lambda = (8\pi G\rho_\Lambda/3)^{-1/2}$, which [10] called our causal boundary, and behaves like the interior of a BH.

III. PROPER COORDINATES

The most general shape for a metric with spherical symmetry in proper coordinates (t, r, δ, θ) is [7]:

$$ds^2 = -A(t, r)dt^2 + B(t, r)dr^2 + r^2 d\Omega^2 \quad (11)$$

Empty space $\rho = p = \Lambda = 0$ results in the SCHW metric:

$$ds^2 = -[1 - 2GM/r] dt^2 + \frac{dr^2}{1 - 2GM/r} + r^2 d\Omega^2 \quad (12)$$

where M represents a singular point mass at $r = 0$. This is not a realistic configuration but the horizon at $R_* = 2GM$ prevent us from seeing such naked singularity. Outgoing null radial geodesics cannot leave the interior of R_* , while incoming ones can cross inside R_* . The solution to Eq.1 with the above metric with $\rho = p = M = 0$, but $\Lambda \neq 0$ is the deSitter (deS) metric:

$$ds^2 = -[1 - r^2 H_\Lambda^2] dt^2 + \frac{dr^2}{1 - r^2 H_\Lambda^2} + r^2 d\Omega^2 \quad (13)$$

which is also static and has an trapped surface at $r = 1/H_\Lambda$ where $H_\Lambda^2 \equiv 8\pi G\rho_\Lambda/3$ and $\rho_\Lambda = \Lambda/(8\pi G) + V$. We have included constant V in Eq.7 because it is degenerate with Λ .

Without lost of generality, we rewrite Eq.11 in form that is closer to SCHW and deS metrics:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -[1 - 2\phi]^{1-2\varphi} dt^2 + \frac{dr^2}{1 - 2\phi} + r^2 d\Omega_k^2 \quad (14)$$

where we have used $d\Omega_k$ introduced in Eq.8 to allow for non-flat space. In general $\phi = \phi(t, r)$ and $\varphi = \varphi(t, r)$. The field equations for this case are well known, e.g. see Eq.(7.51) in [7]. The case $p = -\rho$ results in $G_0^0 = G_1^1$, and the solutions are $\varphi = 0$ and:

$$\phi = G/r \int_0^r \rho(r) 4\pi r^2 dr + \Lambda r^2/6 + C_1/r \quad (15)$$

The remaining field equations, $G_2^2 = G_3^3$ provide the time evolution and are equivalent to energy conservation $\nabla_\mu T_\nu^\mu = 0$:

$$\begin{aligned} \partial_t \rho &= -\frac{\rho + p}{1 - 2\phi} \partial_t \phi \\ p' &= (\rho + p) \left[\phi' \frac{1 - 2\varphi}{1 - 2\phi} + \varphi' \ln(1 - 2\phi) \right] \end{aligned} \quad (16)$$

for $u = 0$. Note how $\rho = -p$ results in constant ρ and p everywhere, but with a discontinuity at $r = R_*$ corresponding to $2\phi = 1$. This discontinuity is critical to understand a BH solution. One way to address this discontinuity is to add junction conditions [11].

A. False Vacuum Black Hole (BH.fv)

We take $C_1 = 0$ in Eq.15 to avoid singular solutions. The simplest way to find a physical BH solution is:

$$\rho(r) = -p(r) = \begin{cases} V_0 & \text{for } r > R_* \\ V_0 + \Delta & \text{for } r < R_* \end{cases} \quad (17)$$

where V_0 and $\Delta > 0$. We require the metric to be asymptotically flat [10] because particles should be free at spacial infinity for a finite time of evolution [12]. This requires $\Lambda = -8\pi G V_0$ and the solution (which we called BH.fv) to Eq.15 is:

$$1 - 2\phi = \begin{cases} 1 - R_*/r & \text{for } r > R_* \equiv 2GM \\ 1 - r^2 H_\Lambda^2 & \text{for } r < R_* = 1/H_\Lambda \end{cases} \quad (18)$$

where: $3H_\Lambda^2 \equiv 8\pi G \rho_\Lambda$, $\rho_\Lambda = \rho_M = \Delta$ and $M = \frac{4\pi}{3} R_*^3 \Delta$. This has no singularity at $r = 0$. Coordinates (t, r) remain time and space as we cross inside R_* [13]. In both sides of R_* we have constant values of p and ρ and energy conservation in Eq.16. There is a discontinuity ($\rho' = \infty$ and $p' = \infty$) at $2\phi = 1$ where $r = R_*$, in agreement with Eq.16. But the metric is static and continuous at R_* . This only happens for $R_* = (8\pi G \Delta / 3)^{-1/2}$. The smaller Δ the larger and more massive the BH. In the limit $\Delta \Rightarrow 0$, we have $R_* \Rightarrow \infty$ and we recover Minkowski space.

Is this solution stable [2]? At a fixed location, the field ψ inside the BH is trap in a stable configuration ($\rho = V$) and can not evolve ($K = 0$). The same happens for the field outside (see Fig.1). A FV in Eq.17 with equal Δ but different initial radius $R < R_*$ is subject to a pressure discontinuity at $r = R$ which is not balanced in Eq.16 and results in a bubble growth [14]. Additional junction conditions [11] could be added in such case, as in the gravastar [3] solution. But the configuration should be stable when R is fixed to R_* [4, 15]. This is because the inside of R_* is causally disconnected, so the pressure discontinuity does not act on R_* , which corresponds to a trapped surface. Radial null events ($ds^2 = 0$) connecting $(0, r_0)$ with (t, r) follow:

$$r = R_* \left(\frac{R_* + r_0}{R_* - r_0} e^{2t/R_*} - 1 \right) / \left(\frac{R_* + r_0}{R_* - r_0} e^{2t/R_*} + 1 \right) \quad (19)$$

so that it takes $t = \infty$ to reach $r = R_*$ from any point inside. Quantum tunneling (see Fig.1) results in slow BH evaporation, similar to the process of Hawking radiation.

The interior deS metric can be transformed into a FLRW metric with constant $H = H_\Lambda$ [16]. This change of coordinates provides a new interpretation for the BH.fv solution in Eq.18. This is not only a solution for a BH inside a universe. The inside comoving observer, sees this solution as an expanding inflationary universe inside a BH, even when the metric is static in proper coordinates and $r = R_*$ is fixed.

Note how we can have FVs inside other FVs (see Fig.1). So we can have BHs inside other BHs. This is a simple solution, but it is not realistic as it has no matter or radiation anywhere.

B. Black Hole Universe (BH.u)

Consider next solutions to Eq.14 where we also have matter $\rho_m = \rho_m(t, r)$ and radiation $\rho_R = \rho_R(t, r)$ inside:

$$\begin{aligned} \rho(t, r) &= \begin{cases} V_0 & \text{for } r > R_* \\ V_0 + \Delta + \rho_m + \rho_R & \text{for } r < R_* \end{cases} \quad (20) \\ p(t, r) &= \begin{cases} -V_0 & \text{for } r > R_* \\ -V_0 - \Delta + \rho_R/3 & \text{for } r < R_* \end{cases} \end{aligned}$$

so that $p \neq -\rho$ inside. This means $\partial_t \phi \neq 0$ and $u \neq 0$: the fluid inside has to move relative to proper frame of the outside observer [17]. For $r > R_*$, the solution is the same as Eq.18. For the interior we define: $2\phi \equiv r^2 H^2(t, r)$, so that:

$$1 - 2\phi(t, r) = \begin{cases} 1 - R_*/r & \text{for } r > R_* = 2GM \\ 1 - r^2 H^2(t, r) & \text{for } r < R_* = 1/H_\Lambda \end{cases} \quad (21)$$

where $R_* = 2GM = 1/H_\Lambda$ as before. We can find the interior solution with a change of variables from $x^\mu = [t, r, \delta, \theta]$ to comoving coordinates $\xi^\nu = [\tau, \chi, \delta, \theta]$, where $r = a(\tau)\chi$ and

$$\Lambda_\nu^\mu \equiv \frac{\partial x^\mu}{\partial \xi^\nu} = \begin{pmatrix} \partial_\tau t & \partial_\chi t \\ \partial_\tau r & \partial_\chi r \end{pmatrix} = \begin{pmatrix} (1 - 2\phi)^{\varphi-1} & \frac{a\sqrt{2\phi}}{(1-2\phi)^{1-\varphi}} \\ \sqrt{2\phi} & a \end{pmatrix} \quad (22)$$

where angular variables (δ, θ) are unchanged. This transforms $g_{\mu\nu}$ in Eq.14 into the FLRW metric $h_{\alpha\beta}$ in Eq.8: $h_{\alpha\beta} = \Lambda_\alpha^\mu \Lambda_\beta^\nu g_{\mu\nu}$ with $2\phi = r^2 H^2(\tau)$ and arbitrary φ . So the most general spherically symmetric in-homogeneous metric inside R_* looks homogeneous in comoving coordinates (where R_* is the FLRW trapped surface). Given some $\rho(\tau)$ and $p(\tau)$ in comoving coordinates in the interior of a BH we can use Eq.10 with $\rho_\Lambda = \Delta = 3R_*^{-2}/8\pi G$ to find $H(\tau)$ and $a(\tau)$. The solution in Eq.21 is then just $H(t, r) = H(\tau)$. We call this a BH universe (BH.u).

To find $\tau = \tau(t, r)$ explicitly we need to solve Eq.22 with $2\phi = r^2 H^2(\tau)$. For $H(\tau) = H_\Lambda$ the solution is $\varphi = 0$ with

$$\tau = \tau(t, r) = t + \frac{1}{2H_\Lambda} \ln [1 - H_\Lambda^2 r^2] \quad (23)$$

where $r < R_* = 1/H_\Lambda$. The flat FLRW metric with $H = H_\Lambda$ becomes deS metric in Eq.13 as in the BH.fv solution. In comoving coordinates, (τ, χ) , the metric is expanding: $a(\tau) = e^{\tau H_\Lambda}$, while in proper coordinates, (t, r) , it is static [16].

Given $T_{\mu\nu}$ in Eq.9 we can find $\bar{T}_{\alpha\beta}$ in the proper frame using the inverse matrix of Eq.22: $\bar{T}_{\alpha\beta} = (\Lambda^{-1})_\alpha^\mu (\Lambda^{-1})_\beta^\nu T_{\mu\nu}$:

$$\bar{T}_0^0 = -\frac{\rho + p2\phi}{1 - 2\phi} ; \bar{T}_1^1 = \frac{p + \rho2\phi}{1 - 2\phi} \quad (24)$$

which is independent of φ . Comparing to Eq.3 gives $u^2 = 2\phi = r^2 H^2$. The Lorentz factor is $\gamma = (1 - 2\phi)^{-1/2}$ so that γdr gives the proper length, in agreement with Eq.14.

Solution $H(t, r) = H(\tau)$ in Eq.21 is valid for all $r < R_* = 1/H_\Lambda$ because $H(\tau) > H_\Lambda$. We can see this by considering outgoing radial null geodesic in the FLRW metric of Eq.8:

$$r_{out} = a(\tau) \int_\tau^\infty \frac{d\tau}{a(\tau)} = a \int_a^\infty \frac{da}{aH(a)} < \frac{1}{H_\Lambda} = R_* \quad (25)$$

which shows that signals can not escape from the inside to the outside of the BH.u. But incoming radial null geodesics $a(\tau) \int_0^\tau \frac{d\tau}{a(\tau)}$ can in fact be larger than R_* if we look back in time. This shows that inside observers are trap inside the BH.u but they can nevertheless observe what happened outside.

C. Implications for ρ_Λ

Consider our universe as the interior of a BH.u. In Eq.18 we showed that $\rho_\Lambda = \Delta$. But this assumed that the causal boundary (where the metric becomes asymptotically flat) is at infinity: $\Sigma = \infty$ (so that $\Lambda = -8\pi G V_0$ [10]). For a universe of finite age, the causal boundary Σ could be finite. This requires a boundary term for the action which fixes $\Lambda = 4\pi G < \rho + 3p >$ where the average is over the light-cone inside Σ [10]. If the causal boundary is set to $\Sigma = \Sigma_I + \Sigma_O$, where Σ_I and Σ_O are the volumes inside and outside the BH, we find:

$$\frac{\Lambda}{4\pi G} = < \rho + 3p > = -2V_0 - 2\Delta \frac{\Sigma_I}{\Sigma} + < \rho_m + 2\rho_R > \quad (26)$$

The effective $\rho_\Lambda = V_0 + \Delta + \Lambda/8\pi G$ is then:

$$\rho_M = \rho_\Lambda = \begin{cases} \Delta & \text{for } \Sigma_O \gg \Sigma_I \\ < \rho_m/2 + \rho_R > & \text{for } \Sigma_I \gg \Sigma_O \end{cases} \quad (27)$$

The first case corresponds to a small BH inside a larger space where $< \rho_m/2 + \rho_R > \approx 0$ because the BH content is negligible when average over a much larger (empty?) outside volume Σ_O . The second case corresponds to a BH.u that is causally disconnected from the rest of space-time. The observational fact that $\rho_\Lambda \sim \rho_m$ agrees with this second solution [10]. This agreement (the coincidence problem) is therefore telling us that the light-cone volume outside our BH.u is not very large. Note that for inflationary models $< \rho_m/2 + \rho_R > \approx \Delta$ because matter and radiation are generated by reheating. DE, inflation and BH interior are different aspects of the same BH.u solution.

IV. DISCUSSION & CONCLUSIONS

The SCHW metric in Eq.12 is well known and studied but the interior solution is not realistic because it corresponds to a singular point source of mass M at $r = 0$. Instead of venturing into Quantum-Gravity territory, here we look for classical non-singular solutions for the BH interior. Our motivation is to understand what is the physical meaning of M for the astrophysical BHs that have already been observed. We find two solutions for the BH interior which we label BH.fv and BH.u. BH.fv corresponds to a space-time with a constant density discontinuity in Eq.17 and deS metric inside Eq.18. deS metric has a trapped surface at $r = R_*$ which matches the SCHW event horizon. A constant density corresponds to a centrifugal force, $2\phi = (r/R_*)^2$ that opposes gravity, $2\phi = R_*/r$, i.e. Eq.15. The equilibrium happens when both forces are equal, which fixes R_* , and correspond to stable circular orbits. Similar solutions for BH.fv are known [4, 5, 15, 18], such as the gravastar [3] or bubble universes [14]. But they are not very realistic as they don't have matter or radiation.

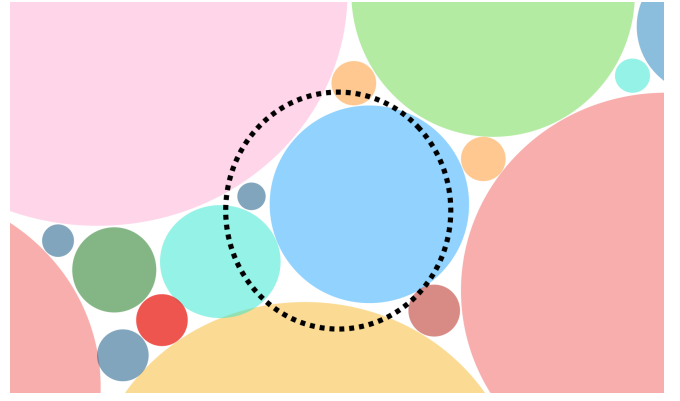


FIG. 2. The Apollonian Universe: spatial distribution of False Vacuums (FV). Each color circle represents a region where the field ψ has settled into a FV with energy excess Δ . The BH solution requires $R_* = (8\pi G\Delta/3)^{-1/2}$. So the smaller the difference the larger and more massive the BH. Our galaxy is depicted at the center of the dashed circle which represents our observable universe.

The second solution is Eq.21, where the BH interior is the FLRW metric. This BH.u solution is new, as far as we know. We can have other BHs, matter and radiation inside a BH within a larger space-time. The inside needs to be expanding or contracting as in the FLRW metric of Eq.8, with the same trapped surface given by ρ_Λ . This holds the expansion and balance gravity at R_* . These solutions also work for non flat $k \neq 0$ geometries: note $d\Omega_k$ in Eq.14. The exterior metric could also be FLRW, as the SCHW metric can be considered a perturbation within a FLRW background.

Both BH type solutions can be interpreted as a BH within our universe or as an expanding universe inside a larger space-time. For the FLRW metric, the critical density $\rho_c = 3H^2/8\pi G$ corresponds to a BH inside a Hubble radius $R = 1/H$. Pathria [19] noted that R_* for a SCHW-deS metric behaves like R_{max} in a closed ($k = +1$) FLRW metric. Knutsen [20] argued that these are just similarities, not solutions, and rightly pointed out the inconsistencies in notation and interpretation. Knutsen [20] also argue that p and ρ in the homogeneous FLRW solution is only a function of time (in comoving coordinates) and can not change at $r = R_*$ to become zero in the exterior. This is an important point which is broken by the presence of ρ_Λ . Homogeneity is then an illusion of the comoving observer. The FLRW metric is trapped inside $r < R_* = \sqrt{3/\Lambda}$, and is then equivalent to an inhomogeneous spherically symmetric metric of Eq.11, as shown in Eq.22 [21].

The solutions to the field equations are independent of the choice of coordinates but $\tilde{T}_{\mu\nu}(t, r)$ depends on the fluid motion (see Eq.24). We used comoving coordinates (τ, χ) , where the fluid is homogeneous and comoving, to find the interior solution. But we can then transform back to proper (t, r) to find a full BH solution in Eq.21 that is continuous at R_* , like in the BH.fv case. The BH mass density, $\rho_M = M/(4\pi R_*^3/3)$ is $\rho_M = \rho_\Lambda$ which should be interpreted as a (FV) boundary condition of Eq.27. As it happened in the singular SCHW metric, outgoing radial null geodesics cannot escape the event horizon, but incoming ones can enter (see discussion around

Eq.19 and Eq.25).

We can picture the evolution of our universe with Fig.1. A primordial field ψ settles into a false (or slow rolling vacuum) which will create a BH.fv. The size R of this vacuum has to grow to match R_* in Eq.15. In comoving coordinates the inside of this BH will be expanding exponentially. When this inflation ends vacuum energy excess converts into matter and radiation (reheating), which results in BH.u, where the rest of the standard cosmic evolution can happen. As noted by [4], the picture of standard inflation [22–25] is not necessary to achieve such solution. In proper coordinates this solution has no Big Bang (or bounce): it is not singular at $r = 0$ or at $t = 0$, because we have a non-singular BH.fv before we start the FLRW BH.u phase. The inside comoving observer is trapped inside $r < R_* = 2GM = 1/H_\Lambda$ and has the illusion of a Big Bang. The space-time outside could be longer and larger than the Big Bang prediction. In a more realistic situation we could have a network of BH.u and there could be matter and radiation in between (e.g. from quantum tunnelling). Fig.2 shows an illustration for such Apollonian universe.

We already have some observational evidence that the expanding metric around us is inside a BH.u. We can recover the Big Bang homogeneous solution in the limit $\Delta \Rightarrow 0$, where we have $R_* \Rightarrow \infty$ and $\rho_\Lambda = 0$. But we have measured $\rho_\Lambda > 0$

($\Omega_\Lambda \simeq 0.7$) which implies $M \simeq 5.8 \times 10^{22} M_\odot$ and $R_* \simeq c/H_0$, as in the BH.u. This model also explains the observed coincidence between ρ_Λ and ρ_m today in Eq.27. The BH horizon R_* is what [10] called the primordial causal boundary r_\S . If we look back to the CMB times, R_* corresponds to 60 degrees in the sky [10]. The observed anomalies in the CMB temperature maps at larger scales [10, 26] provide additional evidence for the BH.u model. There is also a window to see outside our BH.u using the largest angular scales for $z > 2$ and measurements of cosmological parameters from very different cosmic times. Fosalba and Gaztanaga [26] found variations in cosmological parameters over large CMB regions. Their Fig.30 show that the size of these regions follow the BH.u relation between R_* and ρ_Λ . Such observations can therefore provide new evidence for the initial conditions to our universe.

ACKNOWLEDGMENTS

I want to thank Marco Bruni, Robert Caldwell, Alberto Diez-Tejedor and Angela Olinto for their feedback. This work has been supported by Spanish MINECO grants PGC2018-102021-B-I00 and EU grants LACEGAL 734374 and EWC 776247 with ERDF funds. IEEC is funded by the CERCA program of the Generalitat de Catalunya.

-
- [1] H. A. Buchdahl, General relativistic fluid spheres, *Phys. Rev.* **116**, 1027 (1959).
 - [2] R. Brustein and A. J. M. Medved, Resisting collapse, *Phys. Rev. D* **99**, 064019 (2019), and references therein
 - [3] P. O. Mazur and E. Mottola, Surface tension and negative pressure interior of a non-singular ‘black hole’, *Classical and Quantum Gravity* **32**, 215024 (2015).
 - [4] D. A. Easson and R. H. Brandenberger, Universe generation from black hole interiors, *Journal of High Energy Physics* **2001**, 024 (2001), and references therein
 - [5] I. Dymnikova, Vacuum nonsingular black hole, *General Relativity and Gravitation* **24**, 235 (1992).
 - [6] A constant vacuum energy does not gravitate [10]. But a false vacuum discontinuity does gravitate, as we will show below.
 - [7] T. Padmanabhan, *Gravitation, Cambridge Univ. Press* (2010).
 - [8] A. Díez-Tejedor and A. Feinstein, Homogeneous scalar field and the wet dark sides of the universe, *Phys. Rev. D* **74**, 023530 (2006), arXiv:gr-qc/0604031 [gr-qc].
 - [9] This is no coincidence: the current late time cosmic acceleration is the smoking gun of primordial inflation [10].
 - [10] E. Gaztanaga, The Cosmological Constant as a Zero Action Boundary, *MNRAS*, hal-03015944, darkcosmos.com (2021); The size of our causal Universe, *MNRAS* **494**, 2766 (2020).
 - [11] A. Huber, Junction conditions and local spacetimes in general relativity, *European Physical Journal C* **80**, 1132 (2020), arXiv:1908.08735 [gr-qc].
 - [12] Our FLRW universe has $\rho_\Lambda > 0$, but we will argue that this is because it is inside a larger BH. In general, the SCHW metric can be considered as a perturbation in a FLRW background.
 - [13] The metric components don’t change sign, as it happens with both the SCHW’s and deS’s metric.
 - [14] A. Aguirre and M. C. Johnson, Dynamics and instability of false vacuum bubbles, *Phys. Rev. D* **72**, 103525 (2005).
 - [15] R. G. Daghigh, J. I. Kapusta, and Y. Hosotani, False Vacuum Black Holes and Universes, arXiv:gr-qc/0008006 (2000).
 - [16] For some additional discussion see A. Mitra, Interpretational conflicts between the static and non-static forms of the de Sitter metric, *Nature Sci. Reports* **2**, 923 (2012).
 - [17] This makes physical sense. Specially because the outside observer can not see inside. $T_{\mu\nu}$ could also be anisotropic.
 - [18] H. Firouzjahi, Primordial Universe Inside the Black Hole and Inflation, arXiv e-prints, arXiv:1610.03767 (2016).
 - [19] R. K. Pathria, The Universe as a Black Hole, *Nature (London)* **240**, 298 (1972).
 - [20] H. Knutsen, The idea of the universe as a black hole revisited, *Gravitation and Cosmology* **15**, 273 (2009).
 - [21] One way to understand this is to notice that the FLRW metric is only homogeneous in space, but not in space-time. A new frame where comoving time and space are mixed, can break or restore this symmetry [16].
 - [22] A. A. Starobinskiĭ, Spectrum of relict gravitational radiation and the early state of the universe, *Soviet J. of Exp. and Th. Physics Letters* **30**, 682 (1979).
 - [23] A. H. Guth, Inflationary universe: A possible solution to the horizon and flatness problems, *Phys. Rev. D* **23**, 347 (1981).
 - [24] A. D. Linde, A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, *Physics Letters B* **108**, 389 (1982).
 - [25] A. Albrecht and P. J. Steinhardt, Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking, *Phys. Rev. Lett.* **48**, 1220 (1982).
 - [26] P. Fosalba and E. Gaztanaga, Explaining Cosmological Anisotropy: Evidence for Causal Horizons from CMB data, *MNRAS* submitted (2020), arXiv:2011.00910.