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Inside a Black Hole: the illusion of a Big Bang

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We present and interpret two non-singular BH solutions, BH.fv and BH.u., within the laws of classical General Relativity. These solutions look like a singular Schwarzschild BH from the outside. BH.fv corresponds to a false vacuum without matter. BH.u also has matter and radiation inside, which are expanding or contracting. For the inside comoving observer this BH looks like an homogeneous and isotropic universe. The metric of our universe exactly corresponds to such BH.u with $M \simeq 5.8 \times 10^{22} M_\odot$. The Big Bang is just an illusion and our universe could be much older than previously estimated. Primordial inflation, the measured cosmic acceleration, CMB anomalies and tensions in measurements of cosmological parameters all provide independent observational support for such BH.u solution.

I. INTRODUCTION

A Schwarzschild BH represents a singular object of mass M . The horizon at $R_* \equiv 2GM$ prevent us from seeing inside such a BH. But we know that compact relativistic stars have a minimum radius of $9/8R_*$ [1]. Can we find a classical explanation for observed BH and its mass M ? or do we really need to resort to Quantum Gravity to understand them? Here, we look for a physical BH solution defined as a non singular object of size R_* which reproduces the Schwarzschild (SCHW) metric for $r > R_*$ (in empty space). This definition will guide us in our search: we want to find a solution that extends the SCHW coordinate frame inside R_* .

For a perfect fluid the radial pressure inside a BH is negative [2]. Cosmologists are used to this type of fluids, which are called Quintessence, Inflation or Dark Energy (DE). So, could the inside of a BH be DE? Mazur and Mottola [3] argued that the same DE repulsive force that causes cosmic acceleration could also prevent the BH collapse, resulting in the so call gravastar solution. The simplest DE is the ground state $V_0(\psi)$ of some scalar field $\psi(\mathbf{x})$. In the expanding Big Bang, DE is considered to be constant as a function of space and very close to constant as a function of time. But here we seek for a BH solution which represents a spatial discontinuity. We revisit these puzzles by looking for non-static solutions with non zero radial fluid velocity relative to the SCHW observer. The two key questions we want to address are: What are possible metrics for the inside of a physical BH? What is the meaning of M in such a physical BH explanation?

II. HOMOGENEOUS SOLUTIONS

We will solve Einstein's field equations [4]:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \equiv -\frac{16\pi G}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}}, \quad (1)$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$. For a perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} + \pi_{\mu\nu}, \quad (2)$$

where u^μ is the 4-velocity ($u_\mu u^\mu = -1$), ρ and p are the energy-matter density and pressure, and $\pi_{\mu\nu}$ is the anisotropic

stress energy tensor. This fluid is in general made of several components, each with a different equation of state $p = \omega\rho$. For a fluid moving with relative radial velocity v_r with $u^\mu = (u^0, v_r, 0, 0)$, we have $u^0_0 = -g^{00}(1 + g_{11}v_r^2)$ and:

$$\begin{aligned} T_0^0 &= -\rho - v_r^2(\rho + p_r)g_{11} & ; & & T_1^1 &= p_r + v_r^2(\rho + p_r)g_{11} \\ T_0^1 &= (\rho + p_r)u_0 v_r g_{11} & ; & & T_2^2 &= T_3^3 = p_\perp \end{aligned} \quad (3)$$

where p_r is the radial pressure and p_\perp is the perpendicular one, which in general could be different [2]. For an observer comoving with the fluid $v_r = 0$. Here we want to consider a global (proper) coordinate frame that is not moving with the fluid so that $T_0^1 \neq 0$. This is key to allow for solutions that are not static inside but look like a SCHW BH from outside.

A. Scalar field in curved space-time

Consider a minimally coupled scalar field $\psi = \psi(x_\alpha)$ with:

$$\mathcal{L}_m = \frac{1}{2}\bar{\nabla}^2\psi - V(\psi) \quad (4)$$

where we have defined $\bar{\nabla}^2\psi \equiv \partial_\alpha\psi\partial^\alpha\psi$. The Lagrange equations are: $\bar{\nabla}^2\psi = \partial V/\partial\psi$. We estimate $T_{\mu\nu}(\psi)$ from Eq.1.

$$T_{\mu\nu}(\psi) = \partial_\mu\psi\partial_\nu\psi - g_{\mu\nu}\left[\frac{1}{2}\bar{\nabla}^2\psi - V(\psi)\right] \quad (5)$$

Comparing to Eq.2 with $\pi_{\mu\nu} = 0$:

$$\rho = \frac{1}{2}\bar{\nabla}^2\psi + V(\psi) & ; & p = \frac{1}{2}\bar{\nabla}^2\psi - V(\psi) \quad (6)$$

with $u_\mu = \frac{\partial_\mu\psi}{\bar{\nabla}\psi}$. The stable solution corresponds to $p = -\rho$:

$$\bar{\nabla}^2\psi = \partial V/\partial\psi = 0 & ; & \rho = -p = V(\psi) = V_i \quad (7)$$

where ψ is trapped in the true minimum V_0 or some false vacuum (FV) state $V_i = V_0 + \Delta$, see Fig.1. The solution to Eq.1 for constant $\rho = -p = V_i$ (without matter or radiation) for a general metric with spherical symmetry in proper coordinates (in Eq.11) is given by deSitter (deS) metric in Eq.13 with $H_\Lambda^2 \equiv 8\pi G\rho_\Lambda/3$ and $\rho_\Lambda = V_i + \Lambda/8\pi G$. This metric is static in proper coordinates which also indicates that the vacuum solution is stable.

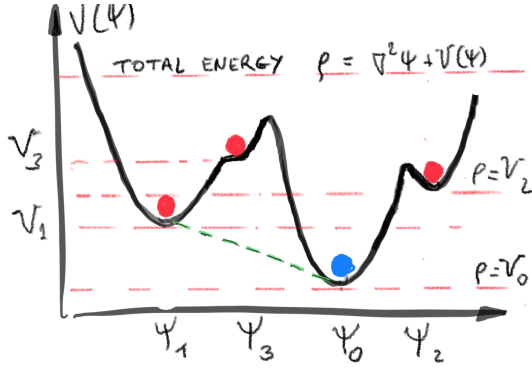


FIG. 1. A generic potential $V(\psi)$ for a field $\psi(\mathbf{x})$. A configuration with total energy: $\rho = \nabla^2\psi + V(\psi)$ (top line) can slowly loose its kinetic energy $\nabla^2\psi$ and relax into a (trapped) false ground state $\rho = V_i \equiv V(\psi_i)$, $i = 1, 2, 3$, which we call false vacuum (FV, red circles) with an energy excess $\Delta \equiv V_i - V_0$. The true vacuum is shown as a blue circle at $V_0 = V(\psi_0)$. Note how there can be FV trapped inside other FV regions. ψ_3 is an unstable (or slow rolling) vacuum, similar to that which generated cosmic inflation. Quantum tunneling (diagonal line) allows particles to escape from a FV.

B. The FLRW metric

The Friedmann-Lemaitre-Robertson-Walker (FLRW) metric in spherical comoving coordinates $\xi^\alpha = (\tau, \chi, \delta, \theta)$, corresponds to an homogeneous and isotropic space-time:

$$ds^2 = h_{\alpha\beta} d\xi^\alpha d\xi^\beta = -d\tau^2 + a(\tau)^2 [d\chi^2 + \chi^2 d\Omega_k^2] \quad (8)$$

where we have introduced: $d\Omega_k^2 \equiv \text{sinc}(\sqrt{k}\chi) d\Omega^2$ with $d\Omega^2 = \cos^2\delta d\theta^2 + d\delta^2$ and k is the curvature constant $k = \{+1, 0, -1\}$. For the flat case $k = 0$ we have $d\Omega_k^2 = d\Omega^2$. The scale factor, $a(\tau)$, describes the expansion/contraction as a function of time. For an isotropic and homogeneous fluid:

$$\begin{pmatrix} T_{00} & T_{10} \\ T_{01} & T_{11} \end{pmatrix} = \begin{pmatrix} \rho(\tau) & 0 \\ 0 & p(\tau)a^2 \end{pmatrix} \quad (9)$$

i.e. $v_r = 0$ and $p_r = p_\perp$ in Eq.3. The field equations Eq.1 are:

$$3H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 \equiv \left(\frac{\partial_\tau a}{a}\right)^2 = 8\pi G(\rho + \rho_\Lambda) \quad (10)$$

$$3\frac{\ddot{a}}{a} \equiv 3\frac{\partial_\tau^2 a}{a} = R_0^0 = -4\pi G(\rho + 3p_R - 2\rho_\Lambda)$$

$$\rho_\Lambda \equiv \rho_{vac} + \frac{\Lambda}{8\pi G}; \quad \rho = \rho_m a^{-3} + \rho_R a^{-4} + \rho_k a^{-2}$$

Where ρ_m is the matter density today ($a = 1$), ρ_R is the radiation (with pressure $p_R = \rho_R a^{-4}/3$). ρ_{vac} represents vacuum energy: $\rho_{vac} = -p_{vac} = V(\psi)$ and $\rho_\Lambda = -p_\Lambda$ is the effective cosmological constant density. Given ρ and p at some time, we can use the above equations to find $a = a(\tau)$ and determine the metric in Eq.8. During inflation, H was dominated by a vacuum field so that $\rho_\Lambda = V(\psi) \equiv 3H_\Lambda^2/8\pi G$, which results in $a = e^{tH_\Lambda}$. Recent observations show that the expansion rate today is also dominated by ρ_Λ [5].

III. PROPER COORDINATES

The most general shape for a metric with spherical symmetry in proper coordinates (t, r, δ, ϕ) is [4]:

$$ds^2 = -A(t, r)dt^2 + B(t, r)dr^2 + r^2 d\Omega^2 \quad (11)$$

Empty space $\rho = p = \Lambda = 0$ results in the SCHW metric:

$$ds^2 = -[1 - 2GM/r] dt^2 + \frac{dr^2}{1 - 2GM/r} + r^2 d\Omega^2 \quad (12)$$

where M represents a singular point mass at $r = 0$. This is not a realistic configuration but the horizon at $R_* = 2GM$ prevent us from seeing such naked singularity. Outgoing null radial geodesics can not leave the interior of R_* , while incoming ones can cross inside R_* . The solution to Eq.11 with $\rho = p = M = 0$, but $\Lambda \neq 0$ is the deSitter (deS) metric:

$$ds^2 = -[1 - r^2 H_\Lambda^2] dt^2 + \frac{dr^2}{1 - r^2 H_\Lambda^2} + r^2 d\Omega^2 \quad (13)$$

which is also static and has an horizon at $r = 1/H_\Lambda$ where $H_\Lambda^2 \equiv 8\pi G\rho_\Lambda/3$ and $\rho_\Lambda = \Lambda/(8\pi G) + V(\psi)$. We have included constant $V(\psi)$ in Eq.7 because it is degenerate with Λ .

Without lost of generality, we rewrite Eq.11 in form that is closer to SCHW and deS metrics:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -[1 - 2\phi]^{1-2\varphi} dt^2 + \frac{dr^2}{1 - 2\phi} + r^2 d\Omega_k^2 \quad (14)$$

where we have used $d\Omega_k$ introduced in Eq.8 to allow for non-flat space. In general $\phi = \phi(t, r)$ and $\varphi = \varphi(t, r)$. The field equations Eq.1 with energy content in Eq.3 are:

$$G_0^1 = \frac{2}{r} \partial_t \phi = -8\pi G \frac{\rho + p_r}{1 - 2\phi} u_0 v_r \quad (15)$$

$$G_0^0 = -2 \frac{(r\phi)'}{r^2} = -8\pi G \rho - \Lambda - v_r^2 \frac{\rho + p_r}{1 - 2\phi} \quad (16)$$

$$G_1^1 = 2 \frac{2\varphi\phi' - f\varphi'}{r} - 2 \frac{(r\phi)'}{r^2} = 8\pi G p_r - \Lambda + v_r^2 \frac{\rho + p_r}{1 - 2\phi}$$

where $f \equiv (1 - 2\phi) \ln(1 - 2\phi)$ and primes correspond to radial derivatives: $\phi' \equiv \partial_r \phi$. The case $p_r = -\rho$ results in $\partial_t \phi = 0$ and $G_0^0 = G_1^1$. The solutions are $\varphi = 0$ and:

$$\phi = G/r \int_0^r \rho(r) 4\pi r^2 dr + \Lambda r^2/6 + C_1/r \quad (17)$$

The remaining non-zero field equations, $G_2^2 = G_3^3$ are equivalent to energy conservation. For $\partial_t \phi = 0$:

$$\nabla_\mu T_\nu^\mu = 0 \Rightarrow \partial_r p_r = -\frac{2}{r}(p_r - p_\perp) \quad (18)$$

A. False Vacuum Black Hole (BH.fv)

We take $C_1 = 0$ in Eq.17 to avoid singular solutions. The simplest way to find a physical BH solution is:

$$\rho(r) = \begin{cases} V_0 & \text{for } r > R_* \\ V_0 + \Delta & \text{for } r < R_* \end{cases} \quad (19)$$

where V_0 and $\Delta > 0$ are just constant values and $p_r = p_\perp = -\rho$. In general, the exterior of this BH is not asymptotically flat because of Λ and V_0 . It is natural that the metric is asymptotically flat because particles should be free at spacial infinity for a finite time [6]. This requires $\Lambda = -8\pi G V_0$. The solution of Eq.17 in then:

$$1 - 2\phi = \begin{cases} 1 - R_*/r & \text{for } r > R_* = 2GM \\ 1 - r^2 H_\Lambda^2 & \text{for } r < R_* = 1/H_\Lambda \end{cases} \quad (20)$$

where: $3H_\Lambda^2 \equiv 8\pi G \rho_\Lambda$, $\rho_\Lambda = \rho_M = \Delta$ and $M = \frac{4\pi}{3} R_*^3 \Delta$. This solution, that we call BH.fv, has no singularity at $r = 0$. Coordinates (t, r) remain time and space as we cross inside R_* [7]. The configuration has a discontinuity in the density (and pressure), which is intrinsic to our physical BH definition. In both sides of R_* we have constant values of p and ρ and energy conservation in Eq.18. The solution is a static metric that is continuous at R_* . This solution requires $R_* = (8\pi G \Delta/3)^{-1/2}$. The smaller the energy difference Δ the larger and more massive the BH. In the limit $\Delta \Rightarrow 0$, we have $R_* \Rightarrow \infty$ and there is nothing, not even ρ_Λ : we just recover Minkowski space.

But is this solution stable under perturbations [2]? When we interpret Δ as a FV in Eq.7, the field ψ_1 inside the BH is trap in a stable configuration and can not evolve. The same happens for the field ψ_0 outside (see Fig.1). So this configuration is a stable solution in both sides of the BH and R_* is fixed. The inside of R_* is causally disconnected: outgoing radial null events ($ds^2 = 0$) follow $r = R_*(e^{2T/R_*} - 1)/(e^{2T/R_*} + 1)$ so that it takes $T = \infty$ to reach $r = R_*$ [8]. So the pressure discontinuity can not exert any force on ψ . There are no classical instabilities in this solution, at least in the (t, r) coordinates [9]. Quantum tunneling (see Fig.1) results in slow BH mass evaporation, similar to the process of Hawking radiation.

The deS metric can be transformed into a FRWM metric with constant $H = H_\Lambda$ [10]. This change of coordinates provides a new interpretation for the BH.fv solution in Eq.20. This is not only a solution for a BH inside a universe. The inside comoving observer, sees this solution as an inflationary universe inside a BH.

Note how we can have FVs inside other FVs (see Fig.1). So we can have BHs inside other BHs. This is a simple solution, but it is not realistic: it has no matter or radiation anywhere.

B. Black Hole Universe (BH.u)

Consider next solutions to Eq.14 where we also have matter $\rho_m = \rho_m(t, r)$ and radiation $\rho_R = \rho_R(t, r)$ inside:

$$\rho(t, r) = \begin{cases} V_0 & \text{for } r > R_* \\ V_0 + \Delta + \rho_m + \rho_R & \text{for } r < R_* \end{cases} \quad (21)$$

$$p_r(t, r) = p_\perp = \begin{cases} -V_0 & \text{for } r > R_* \\ -V_0 - \Delta + \rho_R/3 & \text{for } r < R_* \end{cases}$$

so that $p_r \neq -\rho$ inside. To have a non static solution ($\partial_t \phi \neq 0$) in Eq.15 we also need $v_r \neq 0$: the fluid inside has to move relative to proper frame of the outside observer [11]. For $r > R_*$, the solution is the same as Eq.20. For the interior we

define: $2\phi \equiv r^2 H^2(t, r)$, so that:

$$1 - 2\phi(t, r) = \begin{cases} 1 - R_*/r & \text{for } r > R_* = 2GM \\ 1 - r^2 H^2(t, r) & \text{for } r < R_* = 1/H_\Lambda \end{cases} \quad (22)$$

where $R_* = 2GM = 1/H_\Lambda$ as before. We can find the interior solution with a change of variables from $x^\mu = [t, r, \delta, \theta]$ to comoving coordinates $\xi^\nu = [\tau, \chi, \delta, \theta]$, where $r = a(\tau)\chi$ and

$$\Lambda_\nu^\mu \equiv \frac{\partial x^\mu}{\partial \xi^\nu} = \begin{pmatrix} \partial_\tau t & \partial_\chi t \\ \partial_\tau r & \partial_\chi r \end{pmatrix} = \begin{pmatrix} (1 - 2\phi)^{\varphi-1} & \frac{a\sqrt{2\phi}}{(1-2\phi)^{1-\varphi}} \\ \sqrt{2\phi} & a \end{pmatrix} \quad (23)$$

where angular variables (δ, θ) are unchanged. This transforms $g_{\mu\nu}$ in Eq.14 into the FLRW metric $h_{\alpha\beta}$ in Eq.8: $h_{\alpha\beta} = \Lambda_\alpha^\mu \Lambda_\beta^\nu g_{\mu\nu}$ with $2\phi = r^2 H^2(\tau)$ and arbitrary φ . So the most general spherically symmetric in-homogeneous metric inside the BH looks homogeneous in comoving coordinates. The solution in Eq.22 is then $H(t, r) = H(\tau)$ where $H(\tau)$ is given by Eq.10. We call this a BH universe (BH.u). Given some $\rho(\tau)$ and $p(\tau)$ in comoving coordinates in the interior of a BH we can use Eq.10 to find $H(\tau)$ and $a(\tau)$. To find $\tau = \tau(t, r)$ explicitly we need to solve Eq.23 with $2\phi = r^2 H^2(\tau)$. For example, for a constant $H = H_\Lambda$ the solution is $\varphi = 0$ with

$$\tau = \tau(t, r) = t + \frac{1}{2H_\Lambda} \ln [1 - H_\Lambda^2 r^2] \quad (24)$$

where $r < R_* = 1/H_\Lambda$. The flat FLRW metric with $H = H_\Lambda$ becomes deS metric in Eq.13. In comoving coordinates, (τ, χ) , the metric is expanding: $a(\tau) = e^{tH_\Lambda}$, while in proper coordinates, (t, r) , it is static [10].

Given $T_{\mu\nu}$ in Eq.9 we can find $\bar{T}_{\alpha\beta}$ in the proper frame using the inverse matrix of Eq.23: $\bar{T}_{\alpha\beta} = (\Lambda^{-1})_\alpha^\mu (\Lambda^{-1})_\beta^\nu T_{\mu\nu}$:

$$\bar{T}_0^0 = -\frac{\rho + p2\phi}{1 - 2\phi} ; \quad \bar{T}_1^1 = \frac{p + \rho2\phi}{1 - 2\phi} \quad (25)$$

which is independent of φ . Comparing to Eq.3 gives $v_r^2 = 2\phi = r^2 H^2$. The Lorentz factor is $\gamma = (1 - 2\phi)^{-1/2}$ so that γdr gives the proper length, in agreement with Eq.14.

Solution $H(t, r) = H(\tau)$ in Eq.22 is valid for all $r < R_* = 1/H_\Lambda$ because $H(\tau) > H_\Lambda$. We can see this by considering outgoing radial null geodesic in the FLRW metric of Eq.8:

$$r_{out} = a(\tau) \int_\tau^\infty \frac{d\tau}{a(\tau)} = a \int_a^\infty \frac{da}{aH(a)} < \frac{1}{H_\Lambda} = R_* \quad (26)$$

which shows that signals can not escape from the inside to the outside of the BH.u. But incoming radial null geodesics $a(\tau) \int_0^\tau \frac{d\tau}{a(\tau)}$ can in fact be larger than R_* if we look back in time. This shows that inside observers are trap inside the BH.u but they can nevertheless observe what happened outside.

C. Implications for ρ_Λ

Consider our universe as the interior of a BH.u. In Eq.20 we showed that $\rho_\Lambda = \Delta$. But this assumed that the causal boundary (where the metric becomes asymptotically flat) is at

infinity: $\Sigma = \infty$ (so that $\Lambda = -8\pi G V_0$). For a universe of finite age, the causal boundary Σ could be finite. This requires a boundary term for the action which fixes $\Lambda = 4\pi G \langle \rho + 3p \rangle$ where the average is over the light-cone inside Σ [12]. If the causal boundary is set to $\Sigma = \Sigma_I + \Sigma_O$, where Σ_I and Σ_O are the volumes inside and outside the BH, we find:

$$\frac{\Lambda}{4\pi G} = \langle \rho + 3p \rangle = -2V_0 - 2\Delta \frac{\Sigma_I}{\Sigma} + \langle \rho_m + 2\rho_R \rangle \quad (27)$$

The effective $\rho_\Lambda = V_0 + \Delta + \Lambda/8\pi G$ is then:

$$\rho_M = \rho_\Lambda = \begin{cases} \Delta & \text{for } \Sigma_O \gg \Sigma_I \\ \langle \rho_m/2 + \rho_R \rangle & \text{for } \Sigma_I \gg \Sigma_O \end{cases} \quad (28)$$

The first case corresponds to a small BH inside a larger space where $\langle \rho_m/2 + \rho_R \rangle \simeq 0$ because the BH content is negligible when average over a much larger empty outside volume Σ_O . The second case corresponds to a BH.u that is causally disconnected from the rest of space-time. The observational fact that $\rho_\Lambda \sim \rho_m$ agrees with this second solution [12]. Note that for inflationary models $\langle \rho_m/2 + \rho_R \rangle \simeq \Delta$ because matter and radiation are generated by reheating. DE, inflation and BH interior are different aspects of the same BH.u solution.

IV. DISCUSSION & CONCLUSIONS

The SCHW metric in Eq.12 is well known and studied but the interior solution is not realistic because it corresponds to a singular point source of mass M at $r = 0$. Instead of venturing into Quantum-Gravity territory, here we look for classical non-singular solutions for the BH interior. Our motivation is to understand what is the physical meaning of M for the astrophysical BHs that have already been observed.

We find two solutions for the BH interior which we label BH.fv and BH.u. BH.fv corresponds to deS metric in Eq.20. It corresponds to a space-time with a constant density discontinuity in Eq.19. For this solution to be stable, it should be interpreted as a FV of a classical field (see Eq.7 and Fig.1). This is similar to the gravastar [3] or bubble universes [13]. We have argued, that in proper coordinates (t, r) the BH.fv solution is static and classically stable. The second solution is the FLRW metric in Eq.22 which we call BH.u. The BH.fv is a particular case of the BH.u without matter or radiation. The BH.u solution shows that we can have other BHs, matter and radiation inside a BH within a larger universe. This matter and radiation needs to be expanding or contracting as in the FLRW metric of Eq.8 and requires a ρ_Λ term inside [14].

Both BH type solutions can be interpreted as the inside of a BH within our universe or as an (expanding or contracting) universe inside a larger space-time. For the FLRW solution, the critical density $\rho_c = 3H^2/8\pi G$ corresponds to a BH inside a Hubble radius $R = 1/H$. It is not surprising that several authors [15, 16] have already suggested the idea of a Universe inside a BH. But these were conjectures or analogies, not the solutions that we presented here. For example, Pathria [15] found that R_* for a SCHW-deS metric behaves like R_{max} in

a closed ($k = +1$) FLRW metric. Knutsen [17] argued that these are just similarities, not solutions, and rightly pointed out the inconsistencies in notation and interpretation. Knutsen [17] also argue that p and ρ in the homogeneous FLRW solution is only a function of time (in comoving coordinates) and can not change at $r = R_*$ to become zero in the exterior. But homogeneity is just an illusion of the comoving observer, which is trapped inside $r < R_*$ within the the most general inhomogeneous spherically symmetric metric of Eq.11 [18].

The solutions to the field equations are independent of the choice of coordinates but $\tilde{T}_{\mu\nu}(t, r)$ depends on the fluid motion (see Eq.25). We used comoving coordinates (τ, χ) , where the fluid is homogeneous and comoving, to find the interior solution. But we then transform back to proper (t, r) to find a full BH solution in Eq.22 that is continuous at R_* . The BH mass density, $\rho_M = M/(4\pi R_*^3/3)$ is $\rho_M = \rho_\Lambda$ which should be interpreted as a (FV) boundary condition of Eq.28. We find that, as it happened in the singular SCHW metric, outgoing radial null geodesics can not escape the event horizon, but incoming ones can enter (see discussion around Eq.26).

We can picture the evolution of our universe with Fig.1. A primordial field ψ settles into a false or slow rolling vacuum (ψ_3) which will create a BH.fv. In comoving coordinates the inside of this BH will be expanding exponentially. If the conditions are right [19–22] this inflation ends and the vacuum energy excess Δ converts into matter and radiation (reheating), which results in BH.u where the rest of the standard cosmic evolution can happened. In proper coordinates this solution has no Big Bang (or bounce): it is not singular at $r = 0$ or at $t = 0$, because we have a non-singular BH.fv before we start the FLRW BH.u phase. The inside comoving observer is trapped inside $r < R_* = 2GM = 1/H_\Lambda$ and has the illusion of a Big Bang. The space-time outside could be longer and larger than the Big Bang prediction.

We already have some observational evidence that the expanding metric around us is inside a BH.u. We can recover the Big Bang homogeneous solution in the limit $\Delta \Rightarrow 0$, where we have $R_* \Rightarrow \infty$ and $\rho_\Lambda = 0$. But we have measured $\rho_\Lambda > 0$ ($\Omega_\Lambda \simeq 0.7$) which implies $M \simeq 5.8 \times 10^{22} M_\odot$ and $R_* \simeq c/H_0$. The BH.u model also explains the observed coincidence between ρ_Λ and ρ_m today in Eq.28. The BH horizon R_* is what [12] called the primordial causal boundary r_\S . If we look back to the CMB times, R_* corresponds to 60 degrees in the sky [12]. The observed anomalies in the CMB temperature maps at larger scales [12, 23] provide additional evidence for the BH.u model. There is also a window to see outside our BH.u using the largest angular scales for $z > 2$ and in measurements of cosmological parameters from very different cosmic times.

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- [1] H. A. Buchdahl, General relativistic fluid spheres, *Phys. Rev.* **116**, 1027 (1959).
- [2] R. Brustein and A. J. M. Medved, Resisting collapse, *Phys. Rev. D* **99**, 064019 (2019), and references therein
- [3] P. O. Mazur and E. Mottola, Surface tension and negative pressure interior of a non-singular ‘black hole’, *Classical and Quantum Gravity* **32**, 215024 (2015).
- [4] T. Padmanabhan, *Gravitation, Cambridge Univ. Press* (2010).
- [5] This is no coincidence: the current late time cosmic acceleration is the smoking gun of primordial inflation [12].
- [6] Our universe has $\rho_\Lambda > 0$, but we will argue that this is because it is inside a larger BH. See also [14].
- [7] The metric components don’t change sign as it happens with both the SCHW’s and deS’s metric.
- [8] This only happens when $R_* = (8\pi G\Delta/3)^{-1/2}$. The boundary of a FV with equal Δ but larger or smaller radius R_* is not a solution. It is therefore subject to some dynamics [13] that draws R_* to this solution.
- [9] Using coordinates that are tailored to other purposes, [24] find that deS metric is singular. But these tailored coordinates do not match to the observer looking at the BH. This could relate to the inconsistencies pointed out in [10]. Proper coordinates (t, r) remain valid across R_* and are the only adequate to define a physical BH. In this coordinates, the full metric is static and there are no singularities (other than the horizon at $r = R_*$).
- [10] For some additional discussion see A. Mitra, Interpretational conflicts between the static and non-static forms of the de Sitter metric, *Nature Sci. Reports* **2**, 923 (2012).
- [11] This makes physical sense. Specially because the outside observer can not see inside. $T_{\mu\nu}$ could also be anisotropic.
- [12] E. Gaztanaga, The Cosmological Constant as a Zero Action Boundary, *MNRAS in press hal-03015944* (2021); The size of our causal Universe, *MNRAS* **494**, 2766 (2020).
- [13] A. Aguirre and M. C. Johnson, Dynamics and instability of false vacuum bubbles, *Phys. Rev. D* **72**, 103525 (2005).
- [14] The exterior metric should also be FLRW. But BHs solutions are always given in terms of the SCHW metric because this is a good approximation in most applications. The SCHW metric corresponds to a perturbation within a FLRW background. .
- [15] R. K. Pathria, The Universe as a Black Hole, *Nature (London)* **240**, 298 (1972).
- [16] L. Smolin, Did the universe evolve?, *Classical and Quantum Gravity* **9**, 173 (1992).
- [17] H. Knutsen, The idea of the universe as a black hole revisited, *Gravitation and Cosmology* **15**, 273 (2009).
- [18] These very same paradoxes are present in deS metric [10]. One way to understand this is to notice that the FLRW metric is only homogeneous in space, but not in space-time. A frame where these comoving time and space are mixed, can break or restore this symmetry. Outgoing radial null events never reach R_* . This represents an inhomogeneity or discontinuity in p and ρ (eg. Eq.19 or Eq.21) which characterizes the size of a physical BH.
- [19] A. A. Starobinskiĭ, Spectrum of relict gravitational radiation and the early state of the universe, *Soviet J. of Exp. and Th. Physics Letters* **30**, 682 (1979).
- [20] A. H. Guth, Inflationary universe: A possible solution to the horizon and flatness problems, *Phys. Rev. D* **23**, 347 (1981).
- [21] A. D. Linde, A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, *Physics Letters B* **108**, 389 (1982).
- [22] A. Albrecht and P. J. Steinhardt, Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking, *Phys. Rev. Lett.* **48**, 1220 (1982).
- [23] P. Fosalba and E. Gaztanaga, Explaining Cosmological Anisotropy: Evidence for Causal Horizons from CMB data, *MNRAS submitted* (2020), arXiv:2011.00910.
- [24] E. Farhi and A. H. Guth, An obstacle to creating a universe in the laboratory, *Physics Letters B* **183**, 149 (1987).