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Perspective

# <sup>1</sup> Universal $G' \sim L^{-3}$ Law for the Low-Frequency Shear Modulus of <sup>2</sup> Confined Liquids

3 Alessio Zaccone\* and Laurence Noirez\*



17 water, glycerol, ionic liquids, non-entangled polymer liquids, isotropic liquids crystals). Open questions and potential applications in 18 microfluidics mechanochemistry, energy, and other fields are highlighted.

ur daily life experience and education tell us that when a 19 liquid, e.g., liquid water, is subjected to an infinitesimal 20 21 shear stress, it flows. In physics, the absence of noticeable 22 mechanical resistance can be expressed by saying that the shear 23 elastic modulus G of liquids is exactly zero, where the shear 24 modulus represents the proportionality coefficient between the 25 applied shear stress ( $\sigma$ ) and the resulting deformation of a solid 26 body ( $\gamma$ ), i.e., Hooke's law,  $\sigma = G\gamma$ . Here, a first important 27 distinction comes into play, as the above statement is true for 28 nearly static measurements, i.e., at low frequency or low 29 deformation rate, where the shear modulus is zero, but at high 30 frequency or high rate of deformation the shear modulus of 31 liquids is large (as one would experience by diving into a 32 swimming pool from a considerable height, a possibly painful 33 experience if one is not an experienced diver!). This observation 34 reflects the fact that liquids at high frequency of external drive 35 behave like amorphous solids (e.g., glasses), because the atoms 36 or molecules are made to oscillate at such high speed by the 37 external field that they cannot escape the liquid cage made by 38 their nearest neighbors. This goes along with the ability of 39 liquids to sustain transverse acoustic waves at sufficiently high 40 momenta or high frequencies, as predicted in early work by 41 Yakov Frenkel in the 1940s.

<sup>42</sup>Besides this basic notion, which defines the essential property <sup>43</sup> of a liquid, our understanding of the physics of liquids is <sup>44</sup> incomplete.<sup>2</sup> This is due to the fact that atoms and molecules in a <sup>45</sup> liquid are in a state of disorder and random motions, which is <sup>46</sup> very different from atoms or molecules sitting on a regular lattice in crystalline solids. Therefore, it is difficult to find a 47 mathematical description of the atomic or the molecular 48 dynamics in a liquid, from which quantitative explanations of 49 the macroscopic properties of liquids could be deduced. 50

As a consequence of this state of affairs, it has been impossible 51 so far to rationalize the mechanics of liquids on microscopic 52 scales. Already in 1989–1990, using the newly developed piezo- 53 quartz resonance device, it was experimentally observed that a 54 liquid film confined at a several micrometers scale exhibits an 55 unexpected shear elasticity at low frequency/rate of deforma- 56 tion, a behavior much more akin to solids than to liquids.<sup>3,4</sup> In 57 spite of the fact that these experiments were led by Boris 58 Derjaguin, one of the most prominent Russian physico-chemists 59 of the 20th century, the observation was met with skepticism and 60 abandoned by the scientific community because of the 61 preconceived notion that liquids must have a zero shear 62 modulus. From the early 2000s, many other experiments 63 conducted by different teams across different length scales, 64 however, resulted in the same observation, extending the 65 identification of the "Derjaguin" shear elasticity up to the 66

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**Figure 1.** Dynamic viscoelastic response of liquids. Panel (a) shows a schematic depiction of time-dependent shear stress  $\sigma$  and shear strain  $\gamma$  for solid-like (left) and liquid-like (right) systems and the corresponding frequency-dependent viscoelastic curves. The lower set of figures shows the experimentally measured G' and G'' as a function of oscillation frequency  $\omega$  for glycerol in good (cleaned/thermally regenerated ceramic), panel (b), and poor (non-cleaned ceramic), panel (c), surface-wetting conditions. The measurements are carried out at low gap thickness (e = 0.04 mm) and at room temperature. Panel (b), obtained using a surface state perfectly free of impurities, displays higher viscoelastic moduli and leads to G' > G'', indicating a solid-like response. In panel (c), where the thermal regeneration of the surfaces has not been applied (hence causing a lower energy of adhesion, thus poor wetting and possibly de-wetting), G'' scales exactly linear with  $\omega$ , in line with expectations for purely viscous liquids. Reproduced with permission from ref 19. Copyright 2010 Elsevier.

<sup>67</sup> millimeter scale,<sup>5</sup> using different experimental techniques and <sup>68</sup> different liquids (from liquid water to liquid crystals to ionic <sup>69</sup> liquids and polymer melts).<sup>6–12</sup> A theoretical explanation for <sup>70</sup> these phenomena, however, has remained elusive (despite the <sup>71</sup> keen interest in this problem by Nobel laureate Pierre-Gilles de <sup>72</sup> Gennes, a former colleague of one of us, before his untimely <sup>73</sup> death in 2007).

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In Figure 1 is shown the typical viscoelastic response of a sub-74 75 millimeter confined liquid (glycerol at room temperature), in 76 terms of the storage shear modulus  $G'(\omega)$  and the dissipative loss modulus  $G''(\omega)$ , which represent, respectively, the real and 77 78 imaginary parts of the complex shear modulus,  $G^* = G'(\omega) + G'(\omega)$  $iG''(\omega)$ . Here,  $\omega$  is the frequency of the externally applied 79 <sup>80</sup> mechanical oscillation field (oscillatory strain),  $\gamma = \gamma_0 \sin(\omega t)$ . The shear strain  $\gamma$  is now time-dependent and triggers, under 81 s2 conditions of the linear response, a stress response,  $\sigma = \sigma_0 \sin(\omega t)$  $83 + \delta$ ), where  $\delta$  is a phase which is  $\delta = 0$  for a perfectly elastic <sup>84</sup> response (Hooke's law) and  $\delta = \pi/2$  for a perfectly fluid 85 response. The situation is summarized in Figure 1a. Exper-<sup>86</sup> imentally, the solid-like response displays G' > G'', and a flat  $_{87}$  plateau of G' at low frequency. The hallmark of a liquid-like viscous response, instead, is that  $G' \ll G'' = \eta \omega$  (where  $\eta$  is the 88 se shear viscosity) at low frequency. While bulk liquids have  $\delta = \pi/2$ (perfectly viscous response) for low frequencies of oscillation 90 2 (in the range 0.001–10 Hz), liquids in confined geometry and/ 91 <sup>92</sup> or strongly in interaction with the substrate (wetting conditions) 93 may display  $\delta$  values much smaller than  $\pi/2$ , which indicates a solid-like viscoelastic response. This goes along with a 94 substantial low-frequency plateau value of storage modulus, G' 95 G''. An experimental example which illustrates the solid-like 96 > 97 response of confined liquids is provided in Figure 1b,c, showing 98 the mechanical response of a small-molecule liquid, glycerol, at 99 low frequency and room temperature. Among other simple 100 liquids, very similar mechanical responses at low frequency have

been measured for water,<sup>6</sup> for *o*-terphenyl (OTP),<sup>7</sup> and for ionic 101 liquids.<sup>8</sup> Similar solid-like properties have also been observed 102 with higher-molecular-weight liquids, such as linear alkanes,<sup>9</sup> 103 isotropic liquids crystals,<sup>10</sup> side-chain liquid crystals,<sup>11,12</sup> 104 entangled<sup>7</sup> and non-entangled polymers,<sup>10,13</sup> and supercooled 105 polypropylene glycol.<sup>14</sup> The same phenomena have also been 106 observed at the nanoscale in nano-confined liquids.<sup>15–18</sup> 107

It is important to note the role of boundary conditions, i.e., of 108 the surface energy. For certain solid surfaces in direct contact 109 with the liquid, the liquid molecules are firmly anchored to the 110 solid surface. This is typically associated with high wettability, or 111 good wetting, of the surface, and implies strong attractive 112 interactions between the liquid molecules and the surface. In 113 order to implement these conditions, therefore, it is also 114 necessary to consider the surface energy of the solid, in addition 115 to cleaned, atomically smooth conditions. If that is not the case, 116 e.g., for non-wetting surfaces, the liquid molecules in direct 117 contact with the surface are instead more free in their relative 118 motion with respect to the solid surface. This makes a big 119 difference in order to access the elastic response. It turns out 120 that, in the case of wetting surfaces, the mechanical response of 121 the confined liquid is higher and solid-like, whereas for non- 122 wetting surfaces, the standard purely viscous response is 123 observed. This phenomenon can be mechanistically explained 124 in terms of plane-wave null boundary conditions for the wetting 125 surfaces (or absence thereof, for non-wetting surfaces) in the 126 following theoretical analysis. 127

These experimental findings have recently been rationalized 128 from the point of view of theory. Theories of liquids have 129 normally focused on the high-frequency shear modulus for 130 which atomistic or molecular-level expressions are available, 131 such as the Zwanzig—Mountain formula,<sup>20</sup> precisely because the 132 low-frequency mechanical of bulk liquids was not supposed to 133 exist. In the allied field of amorphous solids, such as glasses, 134

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instead, the mechanical response has been studied across the
entire frequency range, since amorphous solids obviously exhibit
a finite zero-frequency shear modulus. In particular, lattice
dynamics can be extended to deal with disordered systems
where the positions of atoms/molecules are completely random,
to arrive at theoretical expressions for the elastic constants and
for the viscoleastic moduli.<sup>21</sup> The resulting theoretical frameNALD.<sup>22,23</sup> The theory has proved effective in quantitatively
describing elastic, viscoelastic, and plastic responses of systems
as diverse as jammed random packings and random networks,<sup>21</sup>
Furthermore, NALD intrinsically takes into account long-range
correlation phenomena<sup>29,30</sup> that are present also in liquids and
give rise to acoustic wave propagation.

The usual starting point is the equation of motion of a 150 The usual starting point is the equation of motion of a 151 microscopic building block, i.e., an atom or a molecule for 152 atomic liquids or molecular liquids, respectively. In the case of 153 polymers, the building block could be identified with a monomer 154 of the polymer chain.<sup>24</sup> Following previous literature,<sup>21,22</sup> we 155 introduce the Hessian matrix of the system  $\underline{H}_{ij} = -\partial^2 \mathcal{U}/\partial \underline{\mathring{q}}_i \partial \underline{\mathring{q}}_j$ , 156 where  $\mathcal{U}$  is the internal energy, and the affine force field  $\Xi_{i,\kappa\chi} =$ 157  $\partial \underline{f}_i / \partial \eta_{\kappa\chi}$ , where  $\eta_{\kappa\chi}$  is the strain tensor. For example, for simple 158 shear deformation the *xy* entry of tensor  $\eta_{\kappa\chi}$  is given by a scalar  $\gamma$ , 159 i.e., the shear strain introduced above, which coincides with the 160 angle of deformation.

Furthermore,  $\underline{g}_i$  is the coordinate of atom *i* in the initial undeformed frame (denoted with the ring notation), while  $\underline{f}_i = \frac{\partial \mathcal{U}}{\partial \underline{q}_i}$  denotes the force acting on atom *i* in the affine position, i.e., in the initial frame acted upon by the macroscopic deformation, hence the name "affine" force-field. See Figure 2a deformation, hence the name "affine" force-field. See Figure 2a for a visual representation of affine positions in a deformed frame. Greek indices refer to Cartesian components of the macroscopic deformation (e.g.,  $\kappa \chi = xy$  for shear). For liquids (and to a lesser extent, also for glasses<sup>24</sup>), the Hessian  $\underline{H}_{ij}$  is normally evaluated in a reference state obtained from averaging rover MD configurations to include instantaneous normal modes, row INMs (purely imaginary vibrational frequencies).<sup>24</sup> For more

174 As shown in previous works, the equation of motion of atom i175 in a disordered medium subjected to an external small amplitude 176 strain, in mass-rescaled coordinates, can be written as 177 follows:<sup>22,24</sup>

173 details about INMs, see ref 31.

$$\frac{\mathrm{d}^{2}\underline{x}_{i}}{\mathrm{d}t^{2}} + \nu \frac{\mathrm{d}\underline{x}_{i}}{\mathrm{d}t} + \underline{\underline{H}}_{ij}\underline{x}_{j} = \underline{\Xi}_{i,\kappa\chi}\eta_{\kappa\chi} \tag{1}$$

179 where the summation convention is operative for both Latin and 180 Greek indices. Also,  $\eta_{\kappa\chi}$  are the components of the Green–Saint 181 Venant strain tensor, and  $\nu$  is a microscopic friction coefficient 182 which arises from long-range dynamical coupling between atoms 183 mediated by anharmonicity of the pair potential. The term on the right-hand side physically represents the effect of the 184 185 disordered (non-centrosymmetric) environment leading to 186 non-affine motions: a net force acts on atom i in the affine position (i.e., the position prescribed by the external strain 187 188 tensor  $\eta_{\kappa \chi}$ ). The situation is schematically depicted in Figure 2b, 189 which contrasts a centrosymmetric environment (left panel), 190 where all atoms are at mechanical equilibrium, even in the affine 191 position (here for an infinitesimal strain), due to cancellation of 192 nearest-neighbor forces that are mirror-images of each other



Figure 2. Schematic illustration of non-affine displacements in amorphous media. Panel (a) shows the rearrangements or displacements of atoms upon application of an external shear strain. If the deformation were affine, atoms which sit exactly on the dashed lines in the un-deformed frame (left) would still sit exactly on dashed lines also in the deformed frame (right). However, in a disordered environment this does not happen, and in the deformed frame the atoms that were sitting on the dashed lines in the un-deformed frame are no longer sitting on the dashed lines, but are displaced from them. The distance from the actual positions of the atoms to the dashed line corresponds to the non-affine displacements. Panel (b) provides a visual explanation of the origin of non-affine displacements in disordered environments. The left figure shows a perfect lattice where, upon applying a small deformation, the nearest-neighbor forces from surrounding atoms cancel each other out in the affine positions, so there is no need for nonaffine displacements to arise. In the right figure, instead, the tagged atom *i* is not a center of inversion symmetry, which implies that nearestneighbor forces from surrounding atoms do not balance in the affine position, and hence a net force arises which triggers the non-affine displacement in order to maintain mechanical equilibrium.

across the central particle, with the situation in a disordered 193 system (liquid or amorphous solid, right panel). In the latter 194 case, the nearest-neighbor forces do not cancel, thus leading to a 195 net force acting on the central atom. 196

As a consequence, in order to keep mechanical equilibrium on 197 all atoms throughout the deformation, an additional *non-affine* 198 displacement is required in order to relax the force  $f_i$  acting in the 199 affine position. This displacement brings each atom *i* to a new 200 (non-affine) position. A schematic depiction of a non-affine 201 displacement is shown in Figure 2a. 202

The equation of motion eq 1 can also be derived from first- 203 principles, from a model particle-bath Hamiltonian as shown in 204 previous work.<sup>24</sup> Using standard manipulations (Fourier 205 transformation and eigenmode decomposition from time to 206 eigenfrequency<sup>22</sup>), and applying the definition of mechanical 207 stress, we obtain the following expression for the viscoelastic 208 (complex) elastic constants:<sup>22,24</sup> 209

$$C_{\alpha\beta\kappa\chi}(\omega) = C_{\alpha\beta\kappa\chi}^{\text{Born}} - \frac{1}{V} \sum_{n} \frac{\hat{\Xi}_{n,\alpha\beta}\hat{\Xi}_{n,\kappa\chi}}{\omega_{p,n}^2 - \omega^2 + i\omega\nu}$$
(2) 210

where  $C_{\alpha\beta\kappa\chi}^{\text{Born}}$  is the Born or affine part of the elastic constant, i.e., 211 what survives in the high-frequency limit. Here,  $\omega$  represents the 212 oscillation frequency of the external strain field, whereas  $\omega_p$  213 denotes the internal eigenfrequency of the liquid (which results, 214 e.g., from diagonalization of the Hessian matrix<sup>24</sup>). We use the 215 notation  $\omega_p$  to differentiate the eigenfrequency from the external 216 oscillation frequency  $\omega$ . 217



**Figure 3.** Experimental data of low-frequency shear modulus G' versus confinement length L for different systems, where circles represent experimental data while the solid line is the law  $G' \sim L^{-3}$  derived in ref 37. Panel (a) shows the comparison for PAOCH<sub>3</sub>, an isotropic liquid crystal system from ref 10; (b) shows the same comparison for ionic liquids, from ref 8; (c) shows experimental data for short-chain (non-entangled) polystyrene melts from ref 13; and (d) shows experimental data for nanoconfined water from ref 15.

As already mentioned above, an atomistic expression for  $G_{\infty} \equiv$   $C_{\alpha\beta\kappa\chi}^{\text{Born}}$  is provided by the well-known Zwanzig–Mountain 220 formula,<sup>20</sup> in terms of the pair potential V(r) and the radial 221 distribution function g(r). The sum over n in eq 2 runs over all 222 3N degrees of freedom (given by the atomic or molecular 223 building blocks with central-force interactions). Also, we 224 recognize the typical form of a Green's function, with an 225 imaginary part given by damping and poles  $\omega_{p,n}$  that correspond 226 to the eigenfrequencies of the excitations.

At this point we use a key assumption of plane waves; i.e., we 227 assume that, at low values of wavenumber k (i.e., for long 228 wavelengths  $\lambda$ ), liquids can support shear elastic waves. 229 230 Propagation of longitudinal acoustic waves in liquids is, of 231 course, a well-known fact to everyone who does snorkeling or 232 swims under the water level, with firmly established 233 experimental and theoretical evidence of longitudinal acoustic 234 dispersion relations.<sup>32–34</sup> For transverse or shear acoustic waves 235 in liquids, instead, there is no propagation below a characteristic 236 wavenumber. Indeed, there is an onset value of k, that we shall 237 denote  $k_{o}$ , above which these modes can propagate in liquids, 238 known as the k-gap,<sup>2</sup> again a result due originally to Frenkel,<sup>1</sup> 239 and recently demonstrated by Trachenko and co-workers.<sup>35,36</sup> Following the analytical steps presented in ref 37, we arrive at 240 241 the following expression for the complex shear modulus:

$$G^{*}(\omega) = G_{\infty} - B \int_{1/L}^{k_{D}} \frac{\omega_{p,L}^{2}(k)}{\omega_{p,L}^{2}(k) - \omega^{2} + i\omega\nu} k^{2} dk$$
$$- B \int_{k_{\min}}^{k_{D}} \frac{\omega_{p,T}^{2}(k)}{\omega_{p,T}^{2}(k) - \omega^{2} + i\omega\nu} k^{2} dk$$
(3)

243 where the first integral represents the non-affine (negative or 244 softening) contribution due to longitudinal (L) acoustic modes, 245 while the second integral represents the non-affine (also 246 softening) contribution due to the transverse (T) acoustic

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modes. The upper integration limit in the *k* integrals is set, as 247 usual, by the Debye wavenumber  $k_{\rm D}$ . In the above expression, 248  $k_{\rm min} = \max\left(k_{\rm g}, \frac{1}{L}\right)$  is an "infrared" cutoff for the transverse modes, with  $k_{\rm g}$  the onset wavenumber for transverse phonons in 250 liquids (the *k*-gap) and *L* the confinement length along the *z* 251 direction. In the above treatment, dealing with plane waves in 252 3D implies the condition  $\sqrt{k_x^2 + k_y^2 + k_z^2} = k = 2\pi/\lambda$ , which 253 leads to the spherical integrals of eq 3, with the metric factor  $k^2$ . 254

For the longitudinal modes, one can resort to the Hubbard– 255 Beeby theory of collective longitudinal modes in liquids,<sup>32</sup> which 256 has been shown to provide a good description of experimental 257 data, and thus use eq 43 from ref 32 for  $\omega_{p,L}(k)$  inside the 258 integral above. As shown in ref 37, the final result for the low- 259 frequency G' does not depend on the form of the  $\omega_{p,L}(k)$  260 dispersion relation. However, for the mathematical complete- 261 ness of the theory, it is important to specify which analytical 262 forms for the dispersion relations can be used. 263

Anyway, upon taking the real part of  $G^*$ , which gives the 264 storage modulus G', and focusing on low external oscillation 265 frequencies  $\omega \ll \omega_p$  used experimentally, in both integrals the 266 numerator and denominator cancel out, leaving the same 267 expression in both integrals. Therefore, as anticipated above, the 268 final low-frequency result does not depend on the actual form of 269  $\omega_{p,L}(k)$ , nor of  $\omega_{p,T}(k)$ , although the latter, due to the *k*-gap, 270 plays an important role (see the expression for  $k_{\min}$  above) in 271 controlling the "infrared" cutoff of the transverse integral. 272 In the experiments where the size effect of confinement is seen, 273  $k_{\rm g} \ll \frac{1}{r}$ ,  $\frac{38}{r}$  and  $k_{\min} = \frac{1}{r}$ , thus leading to

$$G' = G_{\infty} - \alpha \int_{1/L}^{k_{\rm D}} k^2 \, \mathrm{d}k = G_{\infty} - \frac{\alpha}{3} k_{\rm D}^3 + \frac{\beta}{3} L^{-3} \tag{4}_{275}$$

For the lower limit of the non-affine integral, we used an 276 "infrared" cutoff, 277

$$k_{\min} \equiv |\mathbf{k}_{\min}| = 2\pi \sqrt{(1/L_x)^2 + (1/L_y)^2 + (1/L)^2}$$
(5)

279 by assuming that the liquid is confined along the *z* direction, 280 such that  $L \equiv L_z \ll L_x$ ,  $L_y$ ; the lower limit in the non-affine 281 integral over *k*-space thus reduces to 1/L, as displayed in eq 4. A 282 rigorous estimate of the *k*-integral in eq 4, including exact 283 prefactors, can be found in ref 39.

In eq 4, the only term which depends on the system size is the last term, while  $\alpha$  and  $\beta$  are numerical prefactors. In liquids that last term, while  $\alpha$  and  $\beta$  are numerical prefactors. In liquids that deformations, a different version of the non-affine response last formalism called the stress-fluctuation formalism can be used. It has been rigorously demonstrated that the two versions of the formalism are equivalent in ref 40. Using the stress-fluctuation pron-affine formalism in combination with standard equilibrium statistical mechanics, it has been shown in ref 41 that the affine or high-frequency (Born) term  $G_{\infty}$  and the negative non-affine term (here,  $-\frac{\alpha}{3}k_D^3$ ) cancel each other out exactly, such that  $S^{0}$  under sub-millimeter confinement, only the third term in the protection survives, and we finally obtain

$$_{298} \quad G' \approx \beta' L^{-3} \tag{6}$$

299 where  $\beta' = \beta/3$  is a numerical prefactor. This law was derived for 300 the first time in ref 37. It is worth noticing that  $G_{\infty}$  is 301 independent of *L*. This fact can be seen, e.g., through the 302 Zwanzig-Mountain formula, where the main contribution to 303  $G_{\infty}$  is given as an integral that contains dV(r)/dr as a multiplying 304 factor in the integrand, with dV(r)/dr = 0 after a few molecular 305 diameters. This, in turn, implies that no dependencies on length 306 scales much larger than the nearest-neighbor cage can be present 307 in  $G_{\infty}$ .

The above theory clarifies that the liquid confinement 308 309 between two plates is able to "remove" certain low-frequency 310 normal-mode collective oscillations of molecules, associated 311 with the non-affine motions (i.e., negative contributions to the 312 elasticity), which are responsible for the fluid response of liquids 313 under standard macroscopic ("unconfined") conditions. These motions are responsible for reducing the shear modulus basically 314 315 to zero in macroscopic bulk liquids. Under confinement, instead, 316 the shear modulus is non-zero because these collective 317 oscillations modes are suppressed, and the theory of Zaccone 318 and Trachenko<sup>37</sup> provides the universal law by which the shear 319 modulus grows upon reducing the confinement (gap) size. In 320 particular, the static shear modulus grows with the inverse cubic 321 power of the gap size.

In ref 37, the law  $G' \sim L^{-3}$  was found to provide a perfect description of experimental data measured in an isotropic liquid z4 crystal system (PAOCH<sub>3</sub>) upon varying the confinement length z5 using a conventional rheometer. Here, we show that this law z6 appears to be truly universal, and we present theoretical fittings z27 of several, very different systems in Figure 3.

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In all these systems, i.e., polymer melts of relatively low molecular weight, isotropic liquid crystals, ionic liquids, and weight, isotropic liquid crystals, ionic liquids, and in Figure 3. In the case of nano-confined water,<sup>15</sup> one would expect a crossover from  $\sim L^{-3}$  into an  $L^{-1}$  regime once a 2D monolayer is reached. This is expected because the *k*-space integral in eq 4 contains a metric factor *k* in 2D, instead of  $k^2$  for 35 3D (in fact,  $k^{d-1}$  for generic *d*-dimensional space). This is crossover is not seen in the data of Figure 3d, which calls for integral in equations, both experimentally and theoretically. Experimental data have also been reported for OTP, an organic 338 liquid (data not shown here).<sup>7</sup> Those data also show the  $\sim L^{-3}$  339 behavior, but the data at the shortest confinement length,  $\sim 0.01$  340 mm, suggest the possible existence of a plateau upon going 341 toward lower *L*, while the experimental accuracy is lowered as 342 the confinement increases. The paucity of experimental data 343 does not allow for drawing a definitive conclusion on this effect 344 (i.e., the possible existence of a plateau in *G*' at low *L* in certain 345 systems), which should also be the object of further 346 investigation, both experimentally and in theory. 347

As mentioned earlier, a crucial role is played by the surface 348 anchoring. The solid-like response of confined liquids is indeed 349 observed mainly for atomically smooth surfaces (crystal planes) 350 or for conditions of good wetting between liquid and solid 351 surfaces (high energy surfaces). Instead, a standard, purely 352 viscous response is reported for non-wetting or poorly wetting 353 surfaces; see Figure 4 for a schematic illustration of two different 354 f4



**Figure 4.** Schematic illustration of different levels of surface interaction between liquid molecules and solid substrate. A not-so-strong attractive interaction leads to partial wetting (left panel), whereas a strong attractive interaction with the solid surface leads to total wetting filling in the rough edges (right panel).

surface wetting conditions. The same observation has been 355 made in the case of nanoconfined water, with a substantially 356 higher viscosity measured in the case of good wetting in ref 16. 357

From the theoretical point of view, this fact can be explained 358 by referring to the non-affine lattice dynamic framework 359 summarized above. In particular, wettability connects with the 360 assumption of plane waves and with the implicit null boundary 361 conditions for the displacement field of plane waves. This leads 362 straightforward to the term  $\sim L^{-3}$  in eq 4 above, which is the term 363 responsible for the solid-like elastic response. Without the full 364 wetting boundary conditions between the liquid and the solid 365 surface, the liquid molecules would not be well-anchored to the 366 solid surface, so the null boundary condition for the acoustic 367 waves would not apply, and the very existence of elastic plane 368 waves would then be called into question. Certainly, the above 369 theoretical framework would not be applicable in that case, as it 370 relies on the ability of the liquid to support plane waves. We can 371 speculate that, when molecules are not anchored to the solid 372 surface, the effective "removal" of softening non-affine motions 373 during the displacement and due to confinement is less effective; 374 hence, the rigidifying effect of "cutting" non-affine motions off 375 due to confinement during the deformation, would not be active. 376 A more precise explanation may require a different theoretical 377 approach, possibly working in eigenfrequency domain rather 378 than in k-space, and is an interesting problem for future research. 379

Finally, it is important to point out that the non-zero low- 380 frequency shear elasticity has been measured in various liquids 381 irrespective of their surface tension (alkanes, ionic liquids, and 382

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383 polymer melts typically exhibit low surface tension). Since the 384 interfacial (air/liquid) energy scales as the inverse of the surface,  $_{385}$  significant contributions to G' would be possible if the sample 386 area changes drastically during the oscillatory shear. Bad filling, 387 bad wetting, and large-amplitude oscillator strain (LAOS) 388 conditions could produce a weak "false" torque by breaking the <sup>389</sup> rotational symmetry in the plane.<sup>42</sup> In contrast, the genuine low-390 frequency shear elasticity intrinsic to the confined liquid state is 391 accessible under conditions where the above artifacts are 392 carefully avoided, and by ensuring a good wetting and low 393 strain—that is, as close as possible to the equilibrium conditions. 394 G' then vanishes by increasing the shear strain (at low gap) to 395 make room for the conventional viscous or viscoplastic flow 396 behavior, which could be understood as the non-linear evolution of the finite low-frequency shear elasticity,<sup>3-7</sup> similar to what 397 398 happens in amorphous solids upon crossing the yielding/ plasticity transition.<sup>27,28</sup> 399

A number of outstanding questions remain open for future 400 401 investigations. For example, it could be interesting to explore what kind of connection exists between the above scenario of 402 solid-like elasticity of confined liquids and the glass transition 403 under confinement. The latter is an intensively studied problem, 404 405 especially in the context of thin polymer films.<sup>43,44</sup> Another 406 important issue is the progress that can be made with numerical simulations. In molecular dynamics (MD), the shear modulus of 407 a confined liquid can be measured, but the noise is currently too 408 409 large (since the size of MD-simulated systems is small) to make 410 predictions for most realistic situations.<sup>45</sup> Another issue is 411 represented by the small time step used, especially in atomistic 412 simulations (on the order of femtoseconds), which prevents 413 accessing the low-frequency shear modulus. Quasi-static 414 deformation methods implemented atomistically could be a 415 step forward, but they have not demonstrated quantitative 416 predictive power thus far, in comparison with experiments. Also, 417 the power-law nature of the relation  $G' \sim L^{-3}$  implies the 418 absence of characteristic length scales (as is typical for power-419 laws), which suggests that the same mechanisms apply at  $_{420}$  different length scales; i.e., the same laws apply with L being on 421 the sub-millimeter, micrometer, or nanometer scale. Similarities 422 between trends observed for sub-millimeter<sup>10</sup> and nano-423 confined fluids,<sup>15,16</sup> in terms of both solid-like mechanical <sup>424</sup> properties and role of wetting,<sup>46</sup> suggest that this may indeed be 425 the case, but further research is required to more firmly establish 426 a complete multi-scale framework.

<sup>427</sup> The potential applications of solid-like elasticity of confined <sup>428</sup> liquids, and its tunability via the control and modulation of the <sup>429</sup> interfacial energy and gap size L, are manifold, encompassing <sup>430</sup> fields as diverse as mechanical stress-assisted manipulation of <sup>431</sup> soft and biological matter in microfluidics, <sup>47–49</sup> protective <sup>432</sup> equipment, <sup>50</sup> biomedical flow applications, <sup>51</sup> oil recovery, <sup>52</sup> <sup>433</sup> heat transfer in liquids, <sup>53</sup> and phase transitions under confine-<sup>434</sup> ment. <sup>54</sup> Recently, it has been demonstrated that the thermal <sup>435</sup> response of confined liquids is very similar to that of solids, <sup>55</sup> <sup>436</sup> which opens up new opportunities for exploiting thermo-<sup>437</sup> elasticity of confined fluids, e.g., for energy conversion.

Finally, another potential exciting application of this effect is 439 in the emerging fields of mechanochemistry<sup>56–59</sup> and 440 mechanobiology:<sup>60</sup> a (confined) liquid with a finite low-441 frequency shear modulus G' is able to support mechanical 442 stress-transmission much more efficiently and with much lower 443 dissipative losses compared to standard viscous liquids. This fact 444 may open up new avenues for mechanically induced enhance-445 ment of chemical reaction kinetics, by combining the forcetransmission efficiency typical of elastic solids with all the  $^{446}$  favorable solvation and solubility properties of liquids.  $^{61-64}$   $^{447}$ 

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