



HAL
open science

An Overview of a Break Assignment Problem Considering Area Coverage

Marin Lujak, Álvaro Garcia Sánchez, Miguel Ortega Mier, Holger Billhardt

► **To cite this version:**

Marin Lujak, Álvaro Garcia Sánchez, Miguel Ortega Mier, Holger Billhardt. An Overview of a Break Assignment Problem Considering Area Coverage. 2021. hal-03104345

HAL Id: hal-03104345

<https://hal.science/hal-03104345>

Preprint submitted on 8 Jan 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

An Overview of a Break Assignment Problem Considering Area Coverage

Marin Lujak¹, Álvaro Garcia Sánchez², Miguel Ortega Mier², and Holger Billhardt³

¹ IMT Lille Douai, Univ. Lille, Unité de Recherche Informatique Automatique,
F-59000 Lille, France, marin.lujak@imt-lille-douai.fr,

² Department of Organization Engineering, Business Administration and Statistics,
Universidad Politécnica de Madrid (UPM), 28006 Madrid, Spain,
alvaro.garcia@upm.es, miguel.ortega.mier@upm.es

³ University Rey Juan Carlos, calle Tulipán, s/n, Móstoles, Madrid, Spain,
holger.billhardt@urjc.es

Abstract. Prolonged focused work periods decrease efficiency with related decline of attention and performance. Therefore, emergency fleet break scheduling should consider both area coverage by idle vehicles (related to the fleet's target arrival time to incidents) and vehicle crews' service requirements for breaks to avoid fatigue. In this paper, we propose a break assignment problem considering area coverage (BAPCAC) addressing this issue. Moreover, we formulate a mathematical model for the BAPCAC problem. Based on available historical spatio-temporal incident data and service requirements, the BAPCAC model can be used not only to dimension the size of the fleet at the tactical level, but also to decide upon the strategies related with break scheduling. Moreover, the model could be used to compute (suboptimal) locations for idle vehicles in each time period and arrange vehicles' crews' work breaks considering given break and coverage constraints.

Keywords: fleet scheduling, break assignment, emergency fleets

1 Introduction

Usually, in emergency fleets (such as police and ambulance fleets), the relation between the dynamic spatio-temporal area coverage requirements and the number of breaks and their duration is not taken into account for break assignment and the breaks are regulated by (static) labor rules. Thus, it may happen that the fleet is undersized and recurrently falls behind its performance expectations.

Even though a vehicle crew having a break can be assigned to assist an incident, its arrival time is delayed due to the preparation time necessary to stop the break and start the mission. This additional delay (often in terms of minutes) can mean the difference between life and death both in the case of urgent out-of-hospital patients and rapidly evolving disasters.

To guarantee efficient and timely emergency assistance, a minimum number of stand-by vehicles with in-vehicle emergency crews must always be available to

cover each area of interest, i.e., to assist a probable incident within a predefined maximum delay in the arrival. This can be done by dynamically transferring stand-by vehicles to (time- and area-dependent) locations that maximize the coverage of a region of interest in each time period. These locations can be a set of strategically positioned depots, parking lots, terminals, garages, and similar.

In the emergency fleet context with temporarily and geographically varying workload, if the effective number and duration of previous work breaks has not been taken into account in the assignment of vehicles to incidents, it may also happen that a vehicle crew has worked during extended periods of time without any break or with interrupted breaks that were insufficiently long for effective crew’s rest. Such a disparity between a too high workload and insufficient breaks leads to increased fatigue that can have detrimental effects on emergency assistance. The consequence is the feeling of being drained with related deterioration of attention and performance that may be hazardous for emergency assistance. Hence, strategic fleet dimensioning performed before the fleet’s deployment should consider expected workload dynamics to guarantee frequent and sufficiently long time for rest for the vehicles’ crews.

In this paper, we formulate this problem under the name of the break assignment problem considering area coverage (BAPCAC). Based on the historical data, the BAPCAC problem consists in determining the vehicles with in-vehicle crews necessary for optimal fleet’s deployment in each time period of a workshift, and their breaks’ type, number, and durations based on the legal constraints, and the forecasted workload dynamics in terms of the area coverage by the fleet. By the area coverage, we consider that incidents appearing in the region of interest may be potentially assisted within a predefined target delay in the arrival. Moreover, the BAPCAC model relocates vehicles at each time period to the locations that best cover the forecasted incident demand. The objective of the BAPCAC model is an efficient dimensioning and an effective choice of break coordination strategy in emergency vehicle fleets based on historical data while increasing both the quality of service of emergency fleets as well as the well-being of the vehicles’ crew members while reducing their absenteeism due to fatigue.

The paper is organized as follows. In Section 2, we describe the state-of-the-art practice in break scheduling and introduce the break assignment problem considering area coverage (BAPCAC) and its mathematical model. We conclude the paper with a discussion of the model in Section 3.

2 BAPCAC: Preliminaries and the model

Breaks in emergency fleets are usually considered preemptive and in the case of an incident, the closest available vehicle having the required equipment is called to assist the incident independently of the past crew’s break and workload dynamics and if it is momentarily in a break or not. This approach may induce a delay in arrival of up to several minutes, which in critical cases can be hazardous.

A review of the literature on personnel scheduling problems can be found in [2]. Beer et al. [1] address a complex real-world break-scheduling problem for

supervisory personnel and present a scheduling system that can help professional planners create high-quality shift plans. Similarly, Di Gaspero et al. [4] also consider the problem of scheduling breaks that fulfill different constraints about their placement and lengths.

Mesquita et al. [6] present different models and algorithms for the integrated vehicle and crew scheduling problem, where crews can be assigned to different vehicles. In their algorithms, they consider several complicating constraints corresponding to workload regulations for crews. Defraeye and Van Nieuwenhuysse [3] provide a state-of-the-art overview of research in the period 1991-2013 on personnel staffing and scheduling approaches for systems with a stochastic demand with a time-varying rate. Rekik et al. in [7] consider a break scheduling problem that includes different forms of flexibility in terms of shift starting times, break lengths and break placement. In the BAPCAC problem, we consider the $F|V|W$ break model. Here, F stands for the fractionable breaks with an upper and lower bound on the number of breaks that can be assigned to each vehicle crew; V stands for variable break lengths with an aggregated break length given, which may be split into sub-breaks for each vehicle crew. W stands for the workstretch duration that defines a lower and upper bound on the number of consecutive periods of uninterrupted work before and after each sub-break can start, see, e.g., [5]. We also assume that the plan of *vehicle crew shifts* that make a *work-day shift* is given a priori.

We assume that incidents appear, w.l.o.g., in a 2D square environment $Env = [0, l]^2 \subset \mathbf{R}^2$ of side length $l > 0$. Environment Env is divided in mutually exclusive and nonempty rectangular unit areas $e \subset Env$ of equal size such that $e_i \cap e_j = \emptyset$ for $i \neq j$ and $\bigcup_{e \in Env} e = Env$. Vehicle agents are positioned and may move only within the subset $\mathcal{J} \subseteq Env$ of unit areas.

Let \mathcal{T} be a set of consecutive time periods $\{1, \dots, |\mathcal{T}|\}$, representing a vehicle crew shift, where $|\mathcal{T}|$ is the cardinality of \mathcal{T} . For each time period $\tau \in \mathcal{T}$, $D_{e\tau}$ represents incident density or demand at area $e \in Env$. $\mathcal{I} \subseteq Env$ is a subset of unit areas $i \in \mathcal{I}$ with positive incident density. Then, each vehicle $a \in A$ can have one of mutually exclusive vehicle states: $\{idle, on - break, occupied\}$: *idle* - a vehicle is waiting for a new incident assistance; *on-break* - currently on a break. An idle vehicle is considered to be covering unit area $i \in \mathcal{I}$ if it is located in any unit area from which it can arrive to unit area i within a predefined and given target arrival time Δt , while for the vehicles on-break, the arrival time is increased by an additional preparation delay.

Let $B = \{b_s, b_l\}$ be a set of short and long breaks respectively. The minimum number of short and long breaks is related to the duration of the shift, labor rules, and legal requirements. We track accumulated workload of each vehicle crew since the last break at each time period $\tau \in \mathcal{T}$ to comply with a given maximum allowed work time MAX_b^w before the start of a break $b \in B$. Each break has a minimal duration Δ_b^{MIN} and a maximal duration Δ_b^{MAX} , that specify the required amount of break time based on legal requirements.

Since the problem of area coverage is interconnected with the break assignment problem in the emergency fleet context, we approach it by a unique math-

ematical programming model in which the decisions on the number of vehicles necessary for the coverage, their locations, identities, and the break assignments are decided all at once for the whole workshift.

Sets and indices

- \mathcal{T} time horizon; a set of time periods in a work shift; $\tau \in \mathcal{T}$
- A set of agents $a \in A$ representing $|A|$ capacitated vehicles with vehicle crews
- B set of break types for example, $\{b_s, b_l\}$, where b_s stands for short breaks, whereas b_l stands for long breaks
- \mathcal{I} set of unit areas $i \in \mathcal{I}$ with positive incident density
- \mathcal{J} set of unit areas $j \in \mathcal{J}$, apt for vehicle locations

Parameters

- $D_{i\tau}$ estimated demand and, thus, required minimum proportion of vehicles for unit area $i \in \mathcal{I}$ at time $\tau \in \mathcal{T}$
- Δ_b^{MIN} minimal duration of break $b \in B$
- Δ_b^{MAX} maximal duration of break $b \in B$
- $IM_{jj'}$ valued 1 if an agent can move from $j \in \mathcal{J}$ to $j' \in \mathcal{J}$ based on the maximum transport time between 2 consecutive periods; 0 otherwise
- N_{ij} 1 if an idle vehicle in $j \in \mathcal{J}$ can assist an incident at $i \in \mathcal{I}$ within a given target arrival time, 0 otherwise
- \bar{N}_{ij} 1 if a vehicle on a break located at $j \in \mathcal{J}$ can assist an incident at $i \in \mathcal{I}$ within a predefined target arrival time, 0 otherwise
- MAX_b^w maximum allowed work time before assigning break $b \in B$ usual this time will be 3 hours for short and 4-6 hours for long breaks

Decision variables

- $x_{a\tau j}$ valued 1 if agent a at time $\tau \in \mathcal{T}$ is located at unit area $j \in \mathcal{J}$; 0 otherwise
- $y_{a\tau}$ binary break assignment variable valued 1 if agent $a \in A$ is assigned a break at time $\tau \in \mathcal{T}$; 0 otherwise
- $z_{a\tau i}$ real nonnegative coverage assignment variable representing the part of density at incident unit area $i \in \mathcal{I}$ assigned to *idle* agent $a \in A$ at time $\tau \in \mathcal{T}$, where $0 \leq z_{a\tau i} \leq 1$
- $\bar{z}_{a\tau i}$ real nonnegative coverage assignment variable representing the part of density at incident unit area $i \in \mathcal{I}$ assigned to *on-break* agent $a \in A$ at time $\tau \in \mathcal{T}$, where $0 \leq \bar{z}_{a\tau i} \leq 1$
- $\delta_{i\tau}$ real nonnegative density of area $i \in \mathcal{I}$ uncovered at time $\tau \in \mathcal{T}$
- $\alpha_{ab\tau}$ binary variable valued 1 if agent $a \in \mathcal{A}$ starts a break of type $b \in B$ at $\tau \in \mathcal{T}$; 0 otherwise

(BAPCAC):

$$\min w \cdot \sum_{i \in \mathcal{I}, \tau \in \mathcal{T}} \delta_{i\tau} + (1 - w) \cdot \sum_{a \in A, \tau \in \mathcal{T}} (1 - y_{a\tau}), \quad (1)$$

such that:

$$\sum_{a \in A} (z_{a\tau i} + \bar{z}_{a\tau i}) = D_{i\tau} - \delta_{i\tau}, \forall i \in \mathcal{I}, \tau \in \mathcal{T} \quad (2)$$

$$z_{a\tau i} \leq \sum_{j \in \mathcal{J} | N_{ij}=1} D_{i\tau} x_{a\tau j}, \forall a \in A, \tau \in \mathcal{T}, i \in \mathcal{I} \quad (3)$$

$$\bar{z}_{a\tau i} \leq \sum_{j \in \mathcal{J} | \bar{N}_{ij}=1} D_{i\tau} x_{a\tau j}, \forall a \in A, \tau \in \mathcal{T}, i \in \mathcal{I} \quad (4)$$

$$z_{a\tau i} \leq D_{i\tau}(1 - y_{a,\tau}), \forall a \in A, \tau \in \mathcal{T}, i \in \mathcal{I} \quad (5)$$

$$\bar{z}_{a\tau i} \leq D_{i\tau} y_{a,\tau}, \forall a \in A, \tau \in \mathcal{T}, i \in \mathcal{I} \quad (6)$$

$$\sum_{i \in \mathcal{I}} z_{a\tau i} \leq 1, \forall \tau \in \mathcal{T}, a \in A \quad (7)$$

$$\sum_{i \in \mathcal{I}} \bar{z}_{a\tau i} \leq 1, \forall \tau \in \mathcal{T}, a \in A \quad (8)$$

$$\sum_{j \in \mathcal{J}} x_{a\tau j} = 1, \forall \tau \in \mathcal{T}, a \in A \quad (9)$$

$$x_{a\tau j} + x_{a(\tau+1)j'} \leq 1 + IM_{jj'}, \forall a \in A, \tau = \{1, |\mathcal{T}| - 1\}, j, j' \in \mathcal{J} \quad (10)$$

$$\sum_{\tau'=\tau}^{\tau+\Delta_b^{MIN}} y_{a\tau'} \geq \Delta_b^{MIN} \alpha_{ab\tau}, \forall a \in A, b \in B, \tau = \{1, |\mathcal{T}| - \Delta_b^{MIN}\} \quad (11)$$

$$y_a(\tau+\Delta_b^{MAX+1}) + \alpha_{ab\tau} \leq 1, \forall a \in A, b \in B, \tau = \{1, |\mathcal{T}| - \Delta_b^{MAX} - 1\} \quad (12)$$

$$\sum_{\tau'=\tau}^{\tau+MAX_b^w} \alpha_{ab\tau'} = 1, \forall a \in A, b \in B, \tau \in \{1, \mathcal{T} - MAX_b^w\} \quad (13)$$

$$\sum_{\tau'=\tau}^{\tau+MAX_{b_s}^w} \alpha_{ab_s\tau'} + \sum_{\tau'=\tau}^{\tau+MAX_{b_l}^w} \alpha_{ab_l\tau'} \leq 2, \forall a \in A, \tau \in \{1, \mathcal{T} - MAX_{b_l}^w\} \quad (14)$$

$$z_{a\tau i}, \bar{z}_{a\tau i}, \delta_{i\tau} \geq 0, x_{a\tau j}, y_{a\tau}, \alpha_{ab\tau} \in \{0, 1\}, \forall i \in \mathcal{I}, j \in \mathcal{J}, \tau \in \mathcal{T}, a \in A \quad (15)$$

3 Discussion

In (1), w is the weight assigned to the coverage mean such that $0 \leq w \leq 1$. Constraints (2) relate to the coverage of each unit area i by *idle* vehicles and vehicles *on-break*. Here, minimizing the arrival time to an incident has priority over the performance of a break in its totality. If we exclude $\bar{z}_{a\tau i}$ from the above and other constraints, our model turns into a non-preemptive break model.

Constraints (3) and (4) limit the coverage of each idle agent and an agent on-break to at most density $D_{i\tau}$ of unit area i if it is positioned within the travel time defined by adjacency matrix N_{ij} and \bar{N}_{ij} , respectively. These constraints ensure that part $z_{a\tau i}$ and $\bar{z}_{a\tau i}$ of the coverage of area i by an idle agent and an agent on-break a respectively, is at most the sum of the densities of the areas $j \in \mathcal{J}$ within its reach at time $\tau \in \mathcal{T}$. Furthermore, constraints (5) and (6) limit idle coverage and the coverage on-break only to idle agents and the agents at break, respectively. Moreover, constraints (7) guarantee that the incident density covered by each idle agent sums up to at most 1. Similarly, constraints (8) capacitate the coverage for each agent on-break.

Constraints (9) assign to each agent a at each time period τ a unique location $j \in \mathcal{J}$, by means of $x_{a\tau j}$, while constraints (10) relate agents' positions $x_{a\tau j}$ in two consecutive time periods $t, t + 1$ based on the maximum distance travelled established in $IM_{jj'}$. Moreover, minimal and maximal break durations are imposed by constraints (11) and (12). Constraints (13) assure that short and long breaks start at latest after a maximum uninterrupted work time MAX_b^w of consecutive time periods without a break. Additionally, (14) specifies that long and short breaks are coordinated within the maximum allowed working times. Finally, constraints (15) are non-negativity constraints on the decision variables.

Acknowledgements This work has been partially supported by an STSM Grant from COST Action TD1409 “Mathematics for Industry Network (MI-NET)”.

References

1. Beer, A., Gartner, J., Musliu, N., Schafhauser, W., Slany, W.: An AI-based break-scheduling system for supervisory personnel. *IEEE Intel. Sys.* 25(2), 60–73 (2010)
2. Van den Bergh, J., Beliën, J., De Bruecker, P., Demeulemeester, E., De Boeck, L.: Personnel scheduling: A literature review. *European Journal of Operational Research* 226(3), 367–385 (2013)
3. Defraeye, M., Van Nieuwenhuysse, I.: Staffing and scheduling under nonstationary demand for service: A literature review. *Omega* 58, 4–25 (2016)
4. Di Gaspero, L., Gärtner, J., Musliu, N., et al.: Automated shift design and break scheduling. In: *Automated sched. and plann.* pp. 109–127. Springer (2013)
5. Kiermaier, F., Frey, M., Bard, J.: The flexible break assignment problem for hierarchical tour scheduling problems. In: *INFORMS 2014 Annual Meeting* (Nov 2014)
6. Mesquita, M., Moz, M., Paias, A., Paixão, J., Pato, M., Respício, A.: A new model for the integrated vehicle-crew-rostering problem and a computational study on rosters. *Journal of Scheduling* 14(4), 319–334 (2011)
7. Rekik, M., Cordeau, J.F., Soumis, F.: Implicit shift scheduling with multiple breaks and work stretch duration restrictions. *Journal of Scheduling* 13(1), 49–75 (2010)