Exhaustive Modal Analysis of Interconnected European Power System

Mohamed Kouki & Bogdan Marinescu
Ecole Centrale Nantes-LS2N
1, rue de la Noë
44321 Nantes Cedex 3-France

Florent Xavier
RTE-R&D
Immeuble WINDOW-7D, Place du Dôme
92073 Paris La Défense Cedex-France
florent.xavier@rte-france.com

Abstract—The interconnected European power system is evolving due to the new extensions and to the systematic integration of Power Park Modules, i.e., unit or ensemble of units generation electricity (Renewable Energy (RE), Distributed Generation (DG), etc), which is connected to the grid by power electronics and FACTS (STATCOM, HVDC,...). As a consequence, the increase of the size and the emergence of the new oscillatory modes which involve distant dynamic devices, i.e., of coupling modes (modes associated to generator/turbine inertia, and modes associated to Power Park Modules), and of modes related to various regulations. In this paper, a full scan of European power system is given. An exhaustive method that determines all oscillatory modes independently of the system's order, of the topology and without a priori knowledge about the system, is used. The exhaustive method is fully analytic. Indeed, it is based only on the state-space realisation of any given power system and does not require operator manipulation like dynamic simulations. To this end, we quantify and analyse the interaction between the different inputs/outputs of a given system. This leads to the identification of classes of coupled dynamic devices of the power system. Next, starting from a reduced number of classes, a selective modal analysis method is applied to each class to put into evidence a good approximation of all oscillatory modes. Finally, a complete classification of the oscillatory modes found at the previous stage is provided in terms of the types (inter-area, electrical, ...) and their characteristics (oscillation frequency, damping, participation, ...). 2900 oscillatory modes were found for the studied model of the European system.

Index Terms—coupling/inter-area modes, European power system, dynamic reference model, large-scale systems, modal analysis.

I. INTRODUCTION

During the last years, interconnected European power system has evolved. First, its size has increased, due to the new extensions like, e.g, interconnection with the Turkish power system. Next, its structure changed also due to systematic insertion of Power Park Modules (PPMs) for the renewable sources of for the HVDCs systematically used to reinforce border interconnections (e.g., Spain-France or France-Italy). Therefore, the interconnected dynamic devices are not only the power plants with synchronous machines, but they also contain the PPMs. The eigenanalysis of the resulting mathematical model is becoming very difficult, especially when the oscillatory dynamics are expected in an exhaustive manner, i.e., a full scan of the system has to be done and computation of only some preselected modes it is not enough. In fact, an exhaustive analysis should be able to determine all oscillatory modes, not only the inter-area modes, but also the electrical modes (modes related to the electrical parts of distant classic generators and the ones related to the PPMs). For example, In [1], [2], [6] and [7] coupling modes associated to electrical part of the distant classic synchronous generators are highlighted but not of inter-area nature. In [8], and [7] oscillatory modes associated to converters of distant HVDCs, PVs and winds were also highlighted and analysed. Therefore, all these oscillatory modes will be called coupling modes in the sequel.

Several approaches of modal analysis that aim to find all coupling modes of large scale power systems have been developed and discussed (see, e.g., [3]). They are often non exhaustive and provide only a selective modal analysis. This means that, if good initial guesses are given, only few modes can be found in an iterative manner. This supposes that some knowledge about the system is available which is not always the case.

To overcome these difficulties, two exhaustive methodologies were discussed and developed in our previous works [6], [7]. In fact, they provide a way to systematically excite and put into evidence all the coupling modes of a given power system. In addition, they quantify the interactions between the different dynamic devices of any given power system. Indeed, for each device, one can find the other devices to which it is coupled. These coupled devices should be aggregated into a class, and for all devices a set of classes can be constructed. These latter should be used in computing of the coupling modes based on a specific selective modal analysis method [3], [9].

To put into evidence all the European power system modes, the method proposed in [7] is used here with some modifications for large-scale use. Importantly, the analysis of the European power system or of a system of comparable size and complexity in exhaustive way is a novelty in modal analysis and has never been addressed before. Indeed, not only the well-known inter-area modes can be found, but all the coupling modes are put into evidence. In addition, determining the low and high frequencies modes of the network becomes feasible.
As well, aggregating the dynamic devices according to the nature of the coupling modes or according to the oscillation frequencies it is also possible. Preliminary modal analysis results of the European power system were presented in [7]. The analysis is carried out on the Dynamic Reference Model (DRM), which was developed for the IPS/UPS EC study and next maintained [11]. The rest of this paper is organized as follows. Section II provides the mathematical model associated to power system and a brief description of the exhaustive modal analysis methods that were developed and discussed in [6] and [7]. Section III details the specific implementation of the analytic method to large-scale systems. The results of the full modal analysis of the European power system are reported in Section IV. Conclusions are given in Section V.

II. BASIC TOOLS

In this section we present the mathematical model used for power system and a brief description of exhaustive modal analysis methodologies developed in [6] and [7], which will be used subsequently.

A. Power system description

A power system is often modeled by a set of Differential and Algebraic Equations (DAE) called DAE form

\[
\begin{align*}
\dot{x} &= f(x(t), w(t)) \\
0 &= g(x(t), w(t)),
\end{align*}
\]

where, \(x(t)\) and \(w(t)\) are the vectors of differential and algebraic variables, respectively. \(x\) represents the state variables (machines speeds, regulators, active power,...), while \(w\) mainly contains the bus voltages of the power system.

Linearization of (1) around an equilibrium point \((x_0, w_0)\) gives

\[
\dot{x} = Ax, \tag{2}
\]

where \(A \in \mathbb{R}^{n \times n}\).

If a vector of input variables \(u\) and a vector of output variables \(y\) are also considered, (2) leads to an input-output linear model

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]

where, \(B \in \mathbb{R}^{n \times q}\) is the input matrix, and \(C \in \mathbb{R}^{l \times n}\) is the output matrix.

B. New exhaustive modal analysis methodology

[6] and [7] proposed an exhaustive modal analysis methods which hold for any type of coupling modes and power systems, in particular those with high power electronics penetration. They are structured into two main parts.

First, the identification of classes of the coupled dynamic devices (synchronous generators and converters) is done using the simulation technique [6] (i.e., exciting, one by one, each dynamic device with well-chosen events (such as short-circuit)), or by analytic technique [7] (i.e., using the relative sensitivity technique to quantify the interaction between the inputs/outputs), and by analysing the correlation between the obtained outputs/sensitivities of all dynamic devices.

Next, the Selective Modal Analysis (SMA) method [9] is applied to each class in order to determine a good approximation of coupling modes.

SMA is an efficient approach for simplifying and analyzing a selected part of a large-scale linear system (2). It is based on the split of the state vector \(x\) into two parts, a relevant part \(r\) and a less relevant part \(z\)

\[
\begin{bmatrix}
\dot{r} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
A_{rr} & A_{rz} \\
A_{zr} & A_{zz}
\end{bmatrix}
\begin{bmatrix}
r \\
z
\end{bmatrix}. \tag{4}
\]

This partition is illustrated on the Fig. 1.a. Based on this latter, the transfer function \(H(s)\) of the less relevant dynamics could be generated.

\[
H(s) = A_{rz} (sI - A_{zz})^{-1} A_{zr}. \tag{5}
\]

The eigencomputation is run only on the relevant dynamics by taking into account the small effect of the less relevant ones on the eigenvalues of interest. The less relevant dynamics can be exactly introduced in the relevant dynamics by a constant matrix \(M\) that satisfies the linear system of (6).

\[
MV_\pi = [H(\lambda_1)v_1...H(\lambda_r)v_r]\tag{6}
\]

with \(\lambda_r\) are the interest eigenvalues, and \(V_\pi = [v_1...v_r]\) is the matrix of the right eigenvectors associated to \(\lambda_r\). The partition of Fig. 1.a could be represented now as shown in Fig. 1.b. Notice that, in our case, the relevant part is composed by the states of dynamic devices selected in each class.

The flowchart (see Fig. 2) summarizes the main steps of the exhaustive approaches [6], [7].

This methodology is now improved to apply to large-scale power systems such as the European one.

III. EXHAUSTIVE MODAL ANALYSIS OF MODERN LARGE-SCALE POWER SYSTEMS

A. Description of the European case

We use for the European power system, the Dynamic Reference Model (DRM) which was developed for the IPS/UPS EC study and next maintained and currently used for stability studies [11]. It is a detailed nonlinear model including generators along with their regulations (AVRs, PSS and Governors). It consists of 1074 machines, 7652 buses, 10465 lines and 2550 transformers. The order of the resulting linear model, i.e., \(n\) in (2), is 16784.
B. Specific large-scale implementation of the proposed method for exhaustive modal analysis

1) Classes identification: In large-scale systems, the quantification of the interactions and the construction of classes using the simulation technique are numerically fastidious. To overcome this problem, an analytic technique, called Relative Sensitivity Technique (RST) [4], [7], [10] is used here since it is applicable to large-scale systems such as the European one. The RST does not require operator manipulations, such as dynamic simulations, to identify the coupling classes.

In this work, the RST provides a matrix called, in the sequel, Relative Sensitivity Matrix (RSM) [7]. Based on the absolute value of $RSM$ and according to a chosen selection criterion $\delta$, the construction of the classes of coupled dynamic devices of a given power system is done:

- $RSM(i, j) \leq \delta \Rightarrow \text{device } i \text{ and device } j \text{ are not coupled},$
- $RSM(i, j) > \delta \Rightarrow \text{device } i \text{ and device } j \text{ are coupled}.$

This protocol is applied on all the $RSM$ elements. Hence, a quasi-symmetric matrix made up with all coupled devices is obtained. Each row of $RSM$ corresponds to a class.

Notice that, the chosen inputs $u$ and outputs are respectively the references of the voltage regulators, and the speeds of synchronous generators.

2) Modes computing: The use of all obtained classes to put into evidence all coupling modes of large-scale power system gives a high redundancy between the computed modes and it is numerically fastidious. Instead, only some classes can be used. To identify them, a selection threshold $\sigma$ is used as follows:

- if the similarity (i.e., percentage of identic dynamic devices of two classes) between $\text{class}_i$ and $\text{class}_j \geq \sigma$ and $\text{size}(\text{class}_i) > \text{size}(\text{class}_j)$ then $\text{class}_i$ is one of the selected classes.
- if the similarity between $\text{class}_i$ and $\text{class}_j < \sigma$ then $\text{class}_i$ and $\text{class}_j$ are both selected.

Furthermore, in order to improve the computational efficiency of the specific implementation of the exhaustive approach, the stop condition used in [6], [7] is replaced here by a fixed number of iterations $k$.

IV. Analysis Results for the European Power System

A. Classes and modes

As mentioned above, the number of dynamic devices of European power system is equal to 1074. Therefore, using the RST approximation and according to a selection criterion $\delta = 0.001$, 1074 classes are constructed.

As indicated in Table I, according to a selection criterion $\sigma = 50\%$, the number of final classes is 66 and the size of the largest class is 341 machines.

Notice that, the number of coupled machines of the largest class is a little large compared to the total number of machines. This is due to the small value chosen for the selection criterion $\delta$. In general, $\delta$ should be small in order to ensure high redundancy in the construction of classes. Therefore, a
maximum number of modes will be found for each class and, by the end, the eigencalculation is surely exhaustive in the sense that all the coupling modes of the studied power system will be found.

Starting from the final set of classes (66 classes) found for the European power system, SMA is applied on each class in order to put into evidence all coupling modes. The analysis of 66 classes provides real and complex modes. Here, only the complex modes are taken into account. Indeed, only the complex modes are associated to oscillatory phenomena. Furthermore, based on the complex modes, all coupling modes of the European power system can be found. Importantly, this is a challenge and a novelty in modal analysis of large-scale systems.

Table II provides all complex modes (2900 modes) found for the European power system. Their oscillation frequencies are in the low frequency range 0.17Hz to 0.30Hz.

The slowest mode of the European power system is the mode number 1 of Table II. The oscillation frequency of this mode is 0.17Hz. As indicated in columns 2 and 5 of Table IV, mode 1 involves 585 geographically distant synchronous generators with the highest participations associated to the rotors. The generators in Spain, Portugal and France swing against the generators in Turkey and Bulgaria. Therefore, this mode is an inter-area one and is one of the well-known modes identified in the studies of European power system performed by TSOs [5].

An example of electrical coupling mode is reported in Table V. This mode involves 11 geographically distant synchronous generators with significant participation factors. The highest participation parts are not associated to the rotors but to D-axes. Therefore, this mode is not an inter-area mode but an electrical coupling mode. Furthermore, Fig. 4 illustrates the responses (ID current and IQ current) of most participating generators (G1, G2 and G11) to a short-circuit at the connection point of generator G1. This short-circuit is applied at t1 = 60s and eliminated at t2 = 60.1s. From this figure, notice an important variation at t = 60 associated to the ID-current which comes as consequence of electrical coupling.

Table VI illustrates the results for mode number 2198 of Table II. This short-circuit is applied at t1 = 60s and eliminated at t2 = 60.1s. From this figure, notice an important variation at t = 60 associated to the ID-current which comes as consequence of electrical coupling.
II. This latter is another electrical coupling mode type since it involves 3 distant generators with significant participation parts associated to the Q-axes as given in columns 6 and 8 of Table VI. The last type of oscillatory modes are the RM ones. They are local modes in which state variables of the controllers are dominant.

2) Classification according to the participation factor: Participation factors give an interesting quantification of the spread of the modes, i.e., of how many distant devices they involve. Such classification is given in Fig. 5 for the electromechanical modes. Notice that, participations provided here are relative ones, i.e., percentages of participation of the dominant device (which has 100%). As a consequence, the group of 197 modes in Fig. 5 are of local nature, i.e., they mainly concern only one dynamic device. The second and third groups in Fig. 5 contain thus the inter-area modes of the system. Notice first that the inter-area modes are majoritary in the EMM class. The inter-area modes usually studied are the ones geographically spread and which involve a large number of machines, as for example, the East-West modes (see also link with the classification according to frequency).

Other examples from the analysis done here:

- Mode number 5 in Table II has frequency $0.3997 \, \text{Hz}$ and involves 39 generators with a significant participation. These generators form two coherent groups: 6 generators in Denmark and Turkey (group 1) are in phase opposition to 33 generators in France, Poland, Greece, Bulgaria and Romania.
- Mode number 6 in Table II has frequency $0.8529 \, \text{Hz}$ and involves 20 generators: 9 generators in Italy, France,

TABLE IV: Machines with highest participation in mode 1

<table>
<thead>
<tr>
<th>Country</th>
<th>Mac</th>
<th>Rel. Part. (%)</th>
<th>Phase r. evec. δ (deg)</th>
<th>Rot. part</th>
<th>D-axes part.</th>
<th>Q-axes part.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulgaria</td>
<td>G1</td>
<td>95.9</td>
<td>-5.4</td>
<td>0.2628</td>
<td>0.0082</td>
<td></td>
</tr>
<tr>
<td>Turkey</td>
<td>G2</td>
<td>100.0</td>
<td>-1.0</td>
<td>0.0099</td>
<td>0.0024</td>
<td></td>
</tr>
<tr>
<td>Turkey</td>
<td>G3</td>
<td>100.0</td>
<td>-1.0</td>
<td>0.0105</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>Bulgaria</td>
<td>G4</td>
<td>55.3</td>
<td>-8.8</td>
<td>0.0105</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>Bulgaria</td>
<td>G5</td>
<td>94.0</td>
<td>4.0</td>
<td>0.0028</td>
<td>0.0011</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Spain</td>
<td>G581</td>
<td>23.2</td>
<td>164.4</td>
<td>0.0035</td>
<td>0.0002</td>
<td></td>
</tr>
<tr>
<td>Portugal</td>
<td>G582</td>
<td>24.4</td>
<td>165.9</td>
<td>0.0101</td>
<td>0.0019</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>G583</td>
<td>34.4</td>
<td>175.6</td>
<td>0.0077</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>G584</td>
<td>44.2</td>
<td>177.3</td>
<td>0.0127</td>
<td>0.0007</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>G585</td>
<td>36.8</td>
<td>178.9</td>
<td>0.0064</td>
<td>0.0040</td>
<td></td>
</tr>
</tbody>
</table>

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TABLE V: Machines with highest participation in mode 2308

<table>
<thead>
<tr>
<th>Country</th>
<th>Mac</th>
<th>Rel. Part. (%)</th>
<th>Phase r. evec. δ (deg)</th>
<th>Rot. part</th>
<th>D-axes part.</th>
<th>Q-axes part.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td>G1</td>
<td>100</td>
<td>0.0</td>
<td>0.7633</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G2</td>
<td>15.4</td>
<td>69.6</td>
<td>0.6263</td>
<td>0.1239</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G3</td>
<td>3.5</td>
<td>42.5</td>
<td>0.0093</td>
<td>0.2520</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>G9</td>
<td>1.8</td>
<td>162.5</td>
<td>0.1456</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G10</td>
<td>13.4</td>
<td>103.0</td>
<td>0.4318</td>
<td>0.1085</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G11</td>
<td>20.2</td>
<td>103.0</td>
<td>0.6115</td>
<td>0.1582</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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of the European power system are computed using a fully analytic method. The latter is exhaustive and does not require any a priori knowledge neither about the nature of modes nor about the participating devices. It was extended here to deal with large-scale systems. For a given system, this kind of analysis allows one first to check the model: are any unstable modes? any abnormal frequencies of oscillations? Next, ranges of the dynamics of the system (slowest to fastest modes) are put into evidence. This is useful for control and model reduction actions. Finally, the couplings between the dynamic elements of the system are important to settle protection and control actions. The important novelty is to provide all this information in an exhaustive manner, i.e., to be able to make a general and full scan of the system. For the European system it is provided for the first time such exhaustive information. Dynamics are classified in accordance to their frequencies, spread of the participating devices, damping and nature of the modes. This improved knowledge of the European power system will be exploited for any further dynamic correction actions: control, tuning of protection, defense plans,...

**V. CONCLUSION**

In this paper, the results of full modal analysis of the European power system are presented. All coupling modes range ($\leq 0.2Hz$) in which these modes are considered). These modes are less spread (see also classification 3) and can thus be damped with more local control solutions as, for example, POD control of HVDC links.

4) **Classification according to the damping**: Classification according to the damping ratio is given in Fig. 8. Based on this classification more than half of the coupling modes are damped at more than 40% and thus do not have significant impact on system dynamics. Most of them are regulator and electrical modes but not electromechanical. Indeed, the critical (with damping less than 5%) ones are of electromechanical nature.

**REFERENCES**


