

Variational Multiscale error estimates for Navier-Stokes equations. Application to anisotropic mesh adaptation

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ABSTRACT

This work is motivated by the success of the anisotropic adaptive finite element methods in accurately simulating complex physical systems in science and engineering. Anisotropic mesh adaptation is nowadays a mature tool to assist at a low computation cost the high-accuracy resolution. Thanks to error interpolation indicators and metric based adaptation, the mesh adaptation becomes a transparent black-box tool for a large panel of PDEs [2,6,7]. However, error interpolation indicators did not guarantee the robustness of the error estimation which leads to over-estimate the approximation error. This effect is accentuated in the context of the Navier-Stokes equations in particular for turbulent unsteady flows in which different scales interact. Recent works on variational multiscale error estimators for convection diffusion equation are shown the potential of such approaches in particular for dominated convection flows [5]. Indeed, the derived a posteriori error estimator takes into account the solution subscales as well. This approach was extended in [1.3] in order to be used for anisotropic mesh adaptation. The VMS error estimator is combined with the error interpolation error as a scaling function to derive a computable anisotropic error estimator. The latter is used to build variational multiscale metrics for anisotropic mesh adaptation.

The focus of the present paper is on the construction of a VMS error estimator for the Navier-Stokes equations. The latter are discretized with the VMS finite element formulation in which the errors on velocity and pressure are modeled with respect to the equation residuals [4]. As in [1], the VMS error estimator is combined with the interpolation error to derive an anisotropic error estimator to be used in mesh adaptation.

This paper in fine presents a new anisotropic a posteriori error estimator for the Navier-Stokes equations. It controls large and subscale errors in all space directions. From the VMS a posteriori error analysis, we get an optimal metric (optimal mesh) as a minimum of an error estimator function constrained by a given number of elements. The optimal metric obtained is used to build an optimal mesh for the given number of elements. Validation of the proposed approach on 2D and 3D benchmarks will be presented.

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