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# Localization and characterization of buried magnetic structures using a Multi-Scale Tomography

## Application to archaeological structures on Fox-Amphoux site

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**Topic:** Methods and Innovation.

**Key words:** Wavelet transform, Potential field, Remanent magnetization, Cesium magnetometer, Electrical resistivity.

### SUMMARY:

We want to characterize and localize the depth of archaeological structures from the measurement of magnetic potential anomalies above ground to have a non-destructive test of an archaeological site and to suggest a strategy of excavation. A class of tomography methods used in potential field theory are based on cross-correlation technique (Patella), Euler method or generalized Hilbert transform (Nabighian). New class of tomography methods is based on real continuous wavelet transform for characterizing and localizing "homogeneous" buried sources (magnetic, electric and gravity). These last methods use the properties of the Poisson kernel (real by definition), and the wavelet theory. We relax here, the assumption on sources and introduce a method that we call "multi-scale tomography". Our approach is based on the harmonic extension of observed magnetic field to produce a complex source by the use of a complex Poisson kernel solution of the Laplace equation for complex

potential field. A phase and modulus are defined. We show that the phase provides additional information on the total magnetic inclination and the structure of sources, while the modulus allows us to characterize its spatial location, depth and "effective degree". Results of excavation of Roman kilns of Fox Amphoux site (Var, France) are then compared to the estimates.

### INTRODUCTION

The full recovery of magnetic structures from potential anomalies measured at the surface is an ill-conditioned inverse problem (Tarantola, 1983; Parker, 1994; Blakely, 1995). Any practical method has to be limited to the estimation of a few parameters. The purpose of this paper is to introduce a new method of tomography for potential field, that we call Multi-Scale Tomography, where sources are not assumed to be homogeneous, but complex. The method

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is quite general; here it is applied to magnetic fields. The method allows us to obtain estimates of spatial location, depth, inclination and "effective degree" of buried structures from above-ground measurements of the total magnetic field at a fixed height from a cesium vapor magnetometer (Saracco *et al.*, 2007).

Before giving a rough description of the method, we recall a few basic points:

– A stationary magnetic field is the gradient of a scalar magnetic potential. This potential satisfies the Poisson equation, which becomes the Laplace equation in regions without sources (i.e. in the half-space  $z>0$  in which we take measures).

– If we know the potential in any plane with  $z=z_0$  and  $z_0>0$ , we can calculate the potential in the whole half-space  $z>0$ , without making any assumption about the source. This is done by taking the convolution product of the known potential measured at a fixed height  $z_0$  by the dilated Poisson kernel, where the dilation operator acts on the depth. The calculation of the potential field in the whole half-space  $z>0$  is analogous to the continuous wavelet transform of the potential measured at the fixed height  $z_0$ , where the translation operator acts on the horizontal space  $(x,y)$ , while the dilation parameter represents here,  $|z-z_0|$ , the depth or height. The continuous wavelet theory associated with potential theory can be used.

The Multi-Scale Tomography (MST) method developed here, is based on the dilation property of a complex Poisson kernel (solution of the Laplace equation for complex potential fields) (Saracco *et al.*, 2004, 2007) and on the properties of the wavelet theory which analyses local singularities (Grossmann *et al.*, 1987; Saracco *et al.*, 1990; Moreau *et al.*, 1997). The presence of a cone line structure of the potential field anomalies is a necessary condition to estimate the depth and other parameters of arbitrary sources (inclination, degree of structure), that the Poisson kernel be complex or real (Saracco *et al.*, 2004, 2007). If the sources are homogeneous tempered distributions, the "effective degree" is then the "homogeneous degree" defined in Grossmann *et al.* 1987, Moreau *et al.* 1997, 1999 or in Euler Method. The fact that many magnetic sources can be considered in first approximation, as magnetic dipoles, makes it interesting to define an extension of Patella method, the Complex Dipolar Occurrence Tomography (CDOT) (Saracco *et al.*, 2004). This method is based on phase and modulus, as the complex continuous wavelet transform. The CDOT method allows us to obtain rapid information about the position and inclination of buried dipolar archaeological objects.

We recall briefly the geomagnetic field equations, and present the map of magnetic anomalies obtained from the

magnetic survey at Fox-Amphoux (Fig. 1) conducted on cultivated fields. We applied both methods, conjointly to an electrical resistivity tomography. The estimates are then compared to excavation results.

## GEOMAGNETIC POTENTIAL FIELD: SUMMARIZE

In absence of external sources (external current density due to solar wind, solar cycles..., are negligible) and in the quasi-static limit, the equation for magnetic induction  $\mathbf{B}$  of declination  $D$ , and inclination  $I$ , is:  $\nabla \wedge \mathbf{B} = \mathbf{0}$ . It follows that  $\mathbf{B} = -\nabla \Phi$ , where  $\Phi$  is the scalar magnetic potential. Moreover  $\mathbf{B}$  is the sum of the ambient magnetic field  $\mathbf{H}$ , modulated by the diurnal variations, and the total magnetization  $\mathbf{M}$  (induced  $\mathbf{M}_i$ , plus remanent  $\mathbf{M}_r$  magnetization), modulated by ambient noise and topographic effects.  $\mathbf{M}_i$  is parallel and proportional to  $\mathbf{H}$ , while  $\mathbf{M}_r$  depends on the ferromagnetic composition of the material and  $\mathbf{H}$  at the original time. Let  $\varepsilon$  be the set of fluctuations we have:

$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) + \varepsilon = \mu_0(1 + \mathbf{K})\mathbf{H} + \varepsilon$ , where  $\mu_0$  is the vacuum permeability,  $\mathbf{K}$  is the magnetic susceptibility of the ground.  $\mathbf{M}_r$  is here principally due to the natural thermoremanent magnetization acquired during the cooling of

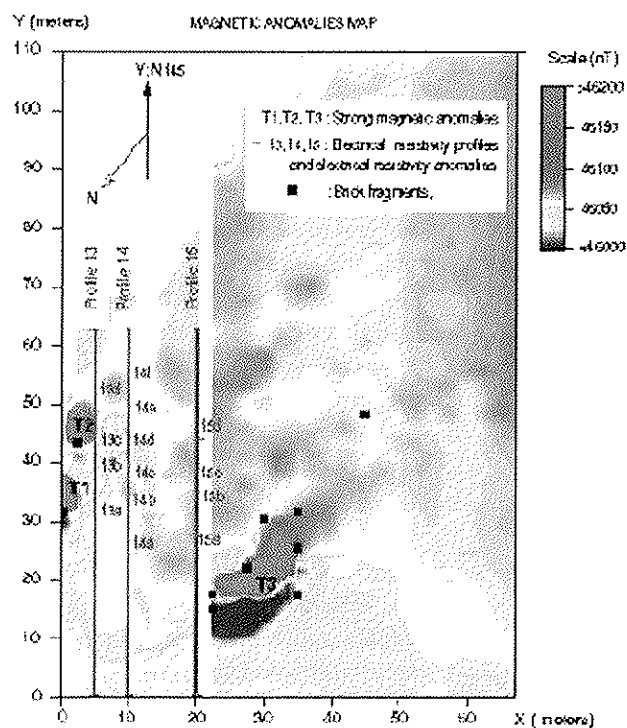


Figure 1 (see color plate): 2D map of magnetic anomalies (Fox-Amphoux, Var, France).

archaeological structures to a temperature below the Curie temperature.

In our experiments, the magnetic data were obtained with a cesium vapor probe with a sensitivity of  $0.01nT$ , where  $M_1$  was 20-50 times higher than  $M_2$ . The present magnetic field given by the IGRF at site coordinates was  $45983nT$  with  $D \approx 0.14^\circ$  and  $I \approx 59^\circ$ . Data acquisition was performed with a magnetic receiver located at 0.8m above ground, every 1m along profiles oriented North  $145^\circ$  at a distance of 2.5m of each other. The temporal drift of the magnetic intensity was  $4nT/hr$ , and the error due to moving position of the receiver during data acquisition was less than  $5nT$ . A linear correction for diurnal variations was made. Three magnetic anomalies corresponding Roman kilns are detected by the probe on a cultivated field of  $70 \times 110m$ . Results are mapped Fig. 1. A kringing extrapolation and a linear correction for diurnal variations were made.

Assuming that the volume of a Roman kiln is a cube of edge  $r_1$ , and the magnetization is induced in the present-day field, the magnetic anomaly  $\Phi$  due to a dipolar source of susceptibility  $K$ , located in an homogeneous medium of susceptibility  $k_0$ , is  $\Phi = (k - k_0)F(r_1/r)^3$ , where  $F$  is the intensity of the total magnetic field, and  $r$  the distance between the probe and the center of the dipole. A rough estimation of the mean depth of structures can be obtain, considering a fixed distance  $r_1$  ( $1 < r_1 < 1.5m$ ),  $F = 45983nT$  and  $K = (k - k_0) = 10^{-2}SI$ . The mean depths corrected of the height of the probe are respectively for the three anomalies 0.54, 0.82 and 1m. This empirical method easily used in first approximation to estimate the depth of simple structures, requires some 'a priori' information on the size of object due to the non-uniqueness of the solution. To reduce these limitations, it is necessary to formulate this problem in terms of an inverse problem in potential theory.

### MULTI-SCALE TOMOGRAPHY OF MAGNETIC POTENTIAL SOURCES

The magnetic field measured at the altitude  $z > 0$  (see Section 2) and generated by a buried source  $\sigma(x, z)$ , ( $x = (x, y)$ ), located in  $z < 0$ , satisfies:  $B(x, z) = -\nabla\Phi(x, z)$ ,  $\Phi$  denoting the scalar potential. We have:

For  $z > 0$ ,  $\Delta\Phi(x, z) = 0$ ; For  $z = 0$ ,  $\Phi(x, 0) = \Phi_{mes}(x)$ ; For  $z < 0$ ,  $\Delta\Phi(x, z) = -\sigma(x, z)$ .

It follows: (1):  $\Phi(x, z) = (D^z P * \Phi_{mes})(x)$ ,

$\Phi$  is called the harmonic extension of  $\Phi_{mes}$  (the measured magnetic potential at a fixed  $z_0$ ) in  $z > 0$ .

\* represents the convolution product;  $D$  the dilation operator acting on the depth  $z$ :  $D^z P(x) = (1/z)P(x/z)$ .

$P$ , the Poisson Kernel, is defined in  $R^2$ ,  $P(x) = C^{st}(1 + |x|^2)^{-5/2}$  (Courant & Hilbert, 1990). The Fourier transform of  $P$  is (2):  $P(u) = \exp(-2\pi|u|)$ , where  $u$  is the dual variable of  $x$ .  $P$  satisfies the crucial equation (3):  $D^z P * D^z P = D^{z+z} P$ , which can be easily verified in Fourier space. This property is the starting point on the method and in (Moreau *et al.*, 1997), on the use of wavelet theory in the study of potential fields.

Since the wavelet transform  $L(b, a)$  of a signal  $F \in R^n$  is defined as the scalar product between  $F$  and the dilated and translated analyzing wavelet we have

(4):  $L(b, a)F = a^{-n} \int g[(x-b)/a] F(x) dx^n$ . The translation parameter has the dimension of the space variable  $x$ , while the dilation parameter ( $a > 0$ ) is dimensionless and plays the role of a zoom in the frequency space ( $k$ ). Small dilations are related to high wave numbers and large dilations to low wave numbers (Saracco, 1994). We have analogous formulas between the calculation (1) of the magnetic potential  $\Phi(x, z)$  in the half-space  $z > 0$ , and the wavelet transform of  $F$  (4), where dilations in (1) are analogous to the depth (Saracco *et al.*, 1990, 2007). The analyzing wavelet  $g$  well localized in space and frequency domain, must verifies some admissibility conditions (Saracco, 1994).

The presence of dilation operator in both formulas allows us to use wavelet theory to characterize and localize buried structures generating magnetic anomalies above ground (Saracco *et al.*, 2007).

A characterization of the local regularity of a signal can be obtained from the modulus of the CMST (complex MST) along the lines of constant phase or from the lines of extrema extracted from the RMST (real MST) (Saracco *et al.*, 2004, 2007). Singularities or structures are generally given by the local Lipschitz exponents (Grossmann, 1986). The exponent, or degree of homogeneity is estimated from the evolution across scales of the wavelet transform in log-log representation and characterizes the local structures of  $s$ . The source depth is estimated from the intersection of lines of extrema (Moreau *et al.*, 1997), or of constant phase (Grossmann *et al.*, 1987; Saracco, 1994). Inclination is estimated from the phase. If  $s(p)$  is a tempered distribution in  $R^n$ ,  $\forall \lambda$  in  $R$ ,  $s$  is homogeneous of degree  $\alpha$ , if  $s$  verifies  $s(\lambda p) = \lambda^{\alpha n} s(p) \Rightarrow L(b, a)s = \lambda^{\alpha n} L(b/a, 1)s$ .

Let  $\beta$  be the degree of the magnetic field  $B$  which differs by one degree from the potential field  $\Phi$ :

$B = -\nabla\Phi$  and  $\Delta\Phi(x, z) = -\sigma(x, z) \Rightarrow \beta = \alpha + 1$ . It follows (Saracco *et al.* 2007):  $L(x, z)\Phi = (a/a+z)^{\gamma-2-\alpha} L[x(a+z)/a, 1]\Phi$ . Accordingly,  $\beta = 1 - \gamma + \alpha$  ( $\gamma$  is the degree of the complex wavelet  $g^\gamma$  defined from  $P(u)$  (2)).  $g^\gamma(u) = P(u) + i TH[P(u)]$ , (TH: Hilbert transform). We obtain  $g^\gamma = u^\gamma^{-1} P(u) (i2\pi^\gamma (u+|u|))$ , ( $\gamma = 2$ ).

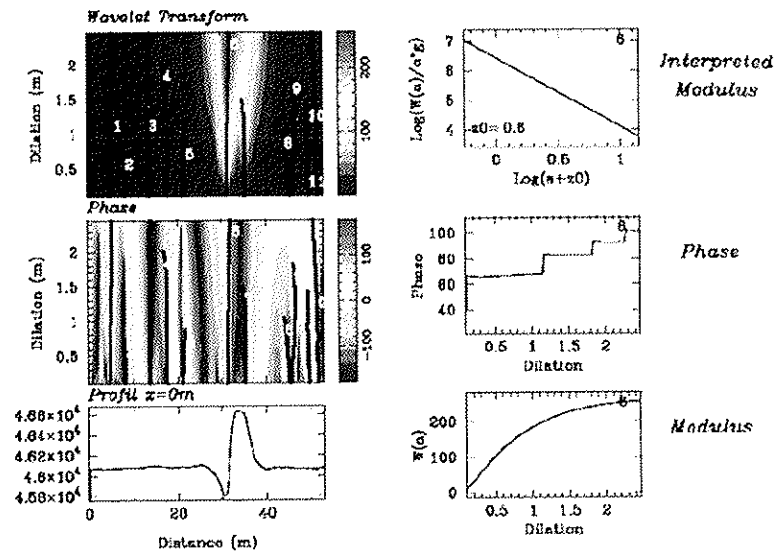


Figure 2 (see color plate): Complex Multi-Scale Tomography (CMST), Characterization and depth anomaly T2.

To obtain a better estimation of depths, magnetic data were interpolated with a sample rate of 0.25m. In order to localize and characterize buried sources  $\sigma$  in the earth ( $z < 0$ ), we use the property of harmonic extension of the solution  $\Phi(x, z)$  (1), and the dilation property (3). It is not necessary to use all the information of the half-plane  $z > 0$ , that is, all wavelet coefficients  $L(x, z)\Phi$ . Only the restriction of the wavelet coefficients along the lines of extrema presenting a cone line structure is necessary. If the lines of extrema do not present a cone-line structure no solution exists for the localization. The convergence of the lines (extrema or constant phase) through the intersection point  $z < 0$  gives the depth of magnetic buried sources. This extension or extrapolation is similar to a contraction  $D^{-z}$  in the half-plane  $z < 0$ . The degree  $\alpha$  of buried structures can be extracted from the slope  $\beta$  of lines of extrema in log-log representation (Fig. 2, top-right). The phase (Fig. 2, middle-right) gives an estimation of the local structure (or inclination). Moreover we obtained information on the localization of lithological structure (limestone substratum), and on archeological fragment objects of the same century (fragments of kiln, tiles, potteries, etc.).

## RESULTS OF ARCHAEOLOGICAL EXCAVATIONS

Excavations were made at Fox-Amphoux (Var) by archeologists (Fig. 3). They found Roman kilns on magnetic anomalies locations (Cf. Fig. 1). The dimensions of kilns were:

2m wide by 2–3m high. The depths found were in agreement with results obtained from MST. Only the top of anomaly T3 was excavated. The strong and wide anomaly observed in this place using both electrical and magnetic surveys is due to the presence of a depot of blows. The kiln (anomaly T1) was partially broken down, and was discovered after the first 0.7m of clearing, corresponding to the estimated depth values. Each structure possesses its own magnetic field, because the overheated earth has a different magnetization from the natural magnetic environment rock. Potteries (overheated earth) contain magnetic minerals, principally iron

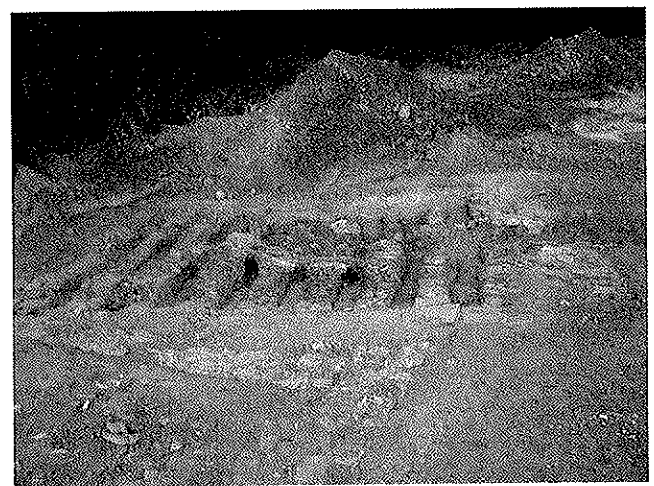


Figure 3: Excavation of anomaly T2.

oxides, responsible for remanent magnetization. When the temperature reaches the Curie or Neel temperature, during the firing this remanent magnetization disappears. During the cooling below this critical temperature, a new remanent magnetization is acquired, guided by the surrounding magnetic field of blows. This new remanent magnetization is then a record of the field during the cooling. T2 and T3 anomalies are in agreement with the depth obtained respectively, by archaeologists after excavation (0.9 and 1.2 m) and (1.3-1.5 m).

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