

A damage energy criterion for cohesive zone model André Chrysochoos, Loïc Daridon, Mathieu Renouf

▶ To cite this version:

André Chrysochoos, Loïc Daridon, Mathieu Renouf. A damage energy criterion for cohesive zone model. 2021. hal-03098095v1

HAL Id: hal-03098095 https://hal.science/hal-03098095v1

Preprint submitted on 5 Jan 2021 (v1), last revised 11 Mar 2022 (v5)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A damage energy criterion for cohesive zone model

A. Chrysochoos^{a,b}, L. Daridon^{a,b,*}, M. Renouf^{a,b}

^aLMGC, Université de Montpellier, CNRS, Montpellier, France ^bMIST, Université de Montpellier, IRSN, CNRS, France

5 Abstract

1

2

3

4

The objective of this paper is to present an energy damage criterion for cohe-6 sive zone models (CZM) within the framework of the non-linear thermodyna-7 mics of irreversible processes (TIP). An isotropic elastic damageable material 8 is considered for isothermal transformations. Damage is then the only irre-9 versible effect accompanying the deformation process and this mechanism is 10 supposed to be fully dissipative. Once a separation law and a damage state 11 variable have been chosen, the paper shows that the damage criterion can 12 be automatically derived from the energy balance. From this observation, 13 a CZM is derived for a given choice of traction-separation law and damage 14 state variable and the quality of its numerical predictions is analyzed using 15 an experimental benchmark bending test extracted from literature. Finally, 16 damage, elastic and dissipated energy fields around the crack path are shown 17 during this rupture test. 18

Keywords: cohesive zone, damage, fracture, thermodynamics of irreversible
processes, energy balance, Finite element analysis

^{*.} Corresponding author

Email addresses: andre.chrysochoos@umontpellier.fr (A. Chrysochoos), loic.daridon@umontpellier.fr (L. Daridon), mathieu.renouf@umontpellier.fr (M. Renouf)

²¹ 1. Introduction

In many engineering applications, the fracture behavior of the structure 22 is crucial, which is why damage mechanisms have been studied over the last 23 decades, from a theoretical, numerical and experimental point of view, using 24 different frameworks [5; 46]. Since the pioneering work of L. M. Kachanov [39], 25 continuum damage mechanics has become a scientific discipline focusing on 26 the effects of various microdefects on the macroscopic behavior of materials 27 and structures. Macroscopic damage descriptions often use a damage variable 28 which is linked to loss of stiffness [15; 43]. For a given elastic material, the 29 stiffness tensor $\mathbf{E}_{\mathbf{d}}$ at a certain level of isotropic damage, denoted by d, is very 30 often related to the stiffness tensor $\mathbf{E}_{\mathbf{0}}$ of the undamaged material as follows : 31 $\mathbf{E}_{\mathbf{d}} = (1 - d)\mathbf{E}_{\mathbf{0}}$. In the particular case of the Thermodynamics irreversible 32 Processes (TIP) framework [6], the elastic free energy of damageable material, 33 ψ , is also defined in the same way $\psi = (1 - d)\psi_0$, where ψ_0 is the elastic 34 free energy of the non damaged material. The conjugate variable associated 35 with the damage state variable, d, is by definition $Y_d = \frac{\partial \psi}{\partial d}$, leading to a 36 thermodynamic force X_d which is given by $X_d = -Y_d = \psi_0$, the elastic 37 energy of the undamaged material. Classically, for material behavior using 38 threshold criterion, the thermodynamic force X_d is then used to define the 39 damage rate d, through a yield function F and its associated flow rule [21; 46]. 40 An alternative numerical strategy to predict the evolution of damage and 41 crack propagation is to introduce a 2D cohesive zone model (CZM) bet-42 ween two elastic layers. The CZM model is considered as a zero-thickness 43 medium where the traction-separation law can be derived from a surface's 44 free energy. This numerical approach has been widely used in many areas of 45

computational mechanics [4; 22; 23; 25; 53; 54; 69]. Indeed the fracture of 46 concrete and the delamination of composites are some important examples 47 of application areas of damage mechanics. To model elastic damageable ma-48 terials, the main assumption is that all the damage that occurs in the bulk 49 are gathered in a cohesive zone. In a finite element approach the cohesive 50 zone is then located between two elements where elastic behavior remains 51 linear [9]. Since the pioneering works carried out by Dugdale and Baren-52 blatt [7; 29], many cohesive-zone models were proposed in the literature 53 [17; 22; 23; 33; 34; 72; 73]. Cohesive-zone models taking fatigue into account 54 were developed to simulate crack propagation under cyclic loading conditions 55 [55; 66] while others were developed to combine two irreversible phenomena 56 such as damage and plasticity [42]. The identification of CZM requires the 57 coupling of experimental data and numerical studies to accurately define the 58 traction separation law [10; 20; 35; 40; 41; 64]. 59

60

A classic criticism of the cohesive zone methods is their relative depen-61 dency on mesh size. For example, intrinsic CZM approach for crack propa-62 gation has a deficiency of introducing artificial compliance to the model and 63 crack path dependency because the cohesive elements are inserted between 64 every 2D or 3D elements [38; 67; 74]. To remedy this mesh dependency, as-65 sociated with the vanishing of stiffness, a new class of so-called "non-local 66 methods" has appeared in the domains of damage and fracture mechanics. 67 Two main regularization techniques exist to avoid pathological localization, 68 namely the integral [63] or the gradient [5; 47; 61] damage approaches. Both 69 consist in introducing non-local terms in the formulae of the cohesive model 70

associated with a characteristic length. For example, in the Thick Level Set 71 method, which is an integral damage approach, the undamaged zone is se-72 parated from the totally damaged zone by a level set [50; 51]. The damage 73 variable is then an explicit function of the level set. The damage growth in 74 solids is based on the movement of a layer of finite thickness l_c within which 75 the damage varies continuously. Then, the damage rate is directly linked to 76 the set propagation. The non-local aspect of this method is essentially due to 77 the fact that the configurational force driving the damage front is an average 78 value over the thickness of the level set in the wake of the front [8; 45]. As 79 in bulk damage mechanics, this approach allows the cracks' initiation and 80 propagation within the same framework. 81

82

Another way to regularize the damage progress is to add a gradient-83 dependent term and to derive the problem of damage evolution from a va-84 riational approach based on an energy formulation [48]. This approach has 85 also been used to couple the models for gradient-damage and those for plas-86 ticity [2; 3] or to develop a cohesive zone model suitable for fatigue fracture 87 [1; 13; 62]. The macroscopic behavior can be seen as brittle fracture with 88 a Griffith-like criterion associated with cohesive fracture of the Barenblatt 89 or Dugdale types [24; 62]. The variational formulation of fracture mechanics 90 framework has also been used to develop the Eigen-erosion scheme [58; 59]. 91 In this finite element approximation scheme, the crack tracking problem is 92 done by successively eroding elements when the attendant elastic energy re-93 leaseable exceeds the critical fracture energy. 94

95

The objective of the following sections is to construct an energy damage 96 criterion for an isotropic elastic damageable material within the TIP fra-97 mework. The construction of the CZM is performed under the small strain 98 hypothesis. The damage law is based on the premise that the damage pro-99 gress is linked to a prescribed evolution in the maximum elastic energy that 100 can be stored within the material for a given damage state. The damage 101 mechanisms are the only microstructural irreversible effects accompanying 102 the deformation processes and these mechanisms are fully dissipative (no 103 energy storage is induced by the material degradation). Naturally, damage 104 dissipation may induce self heating leading to non-isothermal deformation 105 processes that are consequently irreversible due to heat diffusion. However 106 for sake of simplicity, only isothermal transformations are considered and the 107 chosen state variables are the displacement jump \mathbf{u} and a scalar damage va-108 riable, denoted by u_d . The damage variable u_d can be related to the effective 109 displacement as used in [9; 25; 26; 60]. The 2D cohesive zone model proposed 110 in this paper is constructed this way. 111

112

The layout of the paper is as follows. The energy criterion of the dama-113 geable elastic cohesive zone model is presented in Section 2 through a 1D 114 scenario within the TIP framework. In Section 3, a vectorial extension of the 115 cohesive zone law is proposed for an isotropic damage evolution. In Section 4, 116 the capability of the model is investigated using an experimental benchmark 117 test (i.e. a single-edge notch-bending specimen for fracture toughness testing) 118 [31; 50; 51; 70]. Mechanical and energy responses are shown and discussed. 119 In particular, several damage, elastic and dissipated energy fields around the 120

fracture paths are plotted during the crack propagation. The computation of the dissipated energy fields is of special interest inasmuch as they can be compared with the ones derived from quantitative IR techniques [18].

124 2. 1D scenario

The objective of the following section is to briefly review the mechanical concepts classically introduced with CZM in the case of a 1D monotonic traction and to embed them into the TIP framework to derive, through an energy criterion, a damage evolution law.

129 2.1. Mechanical aspects

In the literature [12], the mechanical response of the cohesive zone is 130 described by the correspondence between the " normal traction" force f sup-131 ported by the interface and its normal opening displacement often called 132 "separation" during a monotonic opening. Depending on the chosen form 133 of the traction-separation diagram, the relationships are called bilinear, po-134 lynomial or exponential cohesive laws. In Figure 1 a polynomial form has 135 been chosen to illustrate the most commonly characteristics of these curves. 136 We find the cohesive strength f_0 corresponding to the maximum of the 137 traction-separation curve or its associated opening displacement u_0 , the 138 maximum value of separation u_c corresponding to the crack opening. An 139 energy parameter is also often mentioned [57]: this is the fracture energy 140 $A_c = \int_0^{u_c} f(u) du$ (work of separation), which is the area below the traction-141 separation curve. 142

This traction-separation curve is considered as a threshold over which the damage develops irreversibly. This threshold is an intrinsic characteristic of the cohesive zone behavior. When unloading is considered, it is supposed to be purely elastic, assuming that the damage progress stops as soon as the loading point is below the threshold curve. For convenience, the elastic unloading paths are often directed towards the origin of the traction-separation diagram (see Figure 1). This implies that the elasticity remains linear and that there is no residual opening at the end of the unloading.



FIGURE 1: Traction-separation diagram. Monotonic envelope (continuous line), elastic unload or reload (dashed line). An arbitrary polynomial cohesive law has been chosen.

The progress of the damage can be depicted by a continuous decrease of the secant stiffness $K = \frac{f}{u}$ towards zero until rupture at u_c . A classical scalar definition of the damage variable can then be given by :

$$D_k = \frac{K_0 - K}{K_0} , (1)$$

where K_0 is the initial stiffness of the cohesive zone [11]. The variable D_k

progressively increases from 0 to 1 when the opening displacement increases from 0 to u_c (or from u_e to u_c when a pure elastic domain, $[0, u_e]$, is introduced in the traction-separation curve (see Figure 1)).

A second possibility is to consider a normalized deformation energy definition of the damage [57] :

$$D_A = \frac{A}{A_c}$$
, where $A = \int_0^u f(v) dv$. (2)

Here again, this last definition slightly changes when an elastic domain limited by the point (u_e, f_e) is introduced. In such a case, Eq.(2) requires a renormalization :

$$D_A^* = \frac{A^*}{A_c^*}$$
, where $A^* = \int_{u_e}^u f(v) dv$ and $A_c^* = \int_{u_e}^{u_c} f(v) dv$. (3)

Then by construction D_A and D_A^* belong to [0,1]. In fact, there are 163 many ways to define damage. The damage process being assumed irreversible, 164 the damage variable rate is often chosen to be non-negative whatever the 165 loading history, to depict its monotonic evolution. Damage develops when 166 the mechanical state (u, f) corresponds to a point of the cohesive threshold 167 curve. In what follows we have chosen a kinematic definition of the damage 168 variable. Like previously done by numerous authors (e.q. [68]), we have chosen 169 the maximum value of the separation u_d ever reached by the cohesive zone 170 until instant t. This damage variable is then defined at instant t by : 171

$$u_d = \max\left\{u(\tau), \forall \tau \leq t\right\} . \tag{4}$$

This variable monotonically increases during the damage progress from 0 to u_c whatever the loading path (see Fig.1)).

174 2.2. Energy aspects

Usually during a load cycle, the deformation energy w_{def} , which corres-175 ponds to the area surrounded by the loading curve Eq.(7) is transformed into 176 dissipated energy, denoted by w_d , and stored energy, denoted by w_s , due to 177 the irreversible microstructural transformations accompanying the deforma-178 tion process. Part of w_{def} can also involve strong thermomechanical coupling 179 energy (heat) w_{thm} [19]. An illustrative example of the coupling effects on 180 the mechanical response can be given by the famous thermoelastic damping 181 presented by Zener in [71]. The general form of the energy balance over a 182 loading cycle can then be written as : 183

$$w_{def} = w_d + w_s + w_{thm}$$
 (5)

184 For any other loading the elastic energy, w_e , has to be added so that :

$$w_{def} = w_e + w_d + w_s + w_{thm} , (6)$$

 w_e vanishing, by construction, over a loading cycle. In the present situation, we only consider isothermal transformations with no thermomechanical coupling. Moreover, we assume that damage is a pure dissipative mechanism and that, consequently, no energy storage or release of stored energy, due to microstructural changes, occurs during the loading. These assumptions imply $w_s = 0$ and $w_{thm} = 0$.

For any kind of separation-controlled loading $\{u(\tau), \forall \tau \leq t\}$, the deformation energy at instant t is here defined by :

$$w_{def}(t) = \int_0^t f(\tau) \dot{u}(\tau) d\tau .$$
(7)

For monotonic loadings, the mechanical state follows the traction-separation curve. The deformation energy then represents the mechanical energy required to reach the damage state $u_d = u(t)$. This cost in deformation energy can be defined by :

$$w_{def}^d(u_d) = \int_0^{u_d} f(\upsilon) d\upsilon \ . \tag{8}$$

Another important mechanical energy term is the elastic energy, $w_e(u, u_d)$, in the cohesive zone at a given state of damage u_d . It is defined by :

$$w_e(u, u_d) = \frac{1}{2} K(u_d) u^2$$
 . (9)

¹⁹⁹ Note that this energy is mechanically recoverable during the unloading. ²⁰⁰ This is the reason why it did not appeare in the general form of the energy ²⁰¹ balance proposed in Eq.(5) for a complete loading cycle.

As previously done for the deformation energy during monotonic loading, we can define the elastic energy $w_e^d(u_d)$ by :

$$w_e^d(u_d) = \frac{1}{2} K(u_d) u_d^2 = w_e(u_d, u_d) , \qquad (10)$$

which represents the maximum elastic energy mechanically recoverable for a given damage state, defined by u_d .

As previously supposed (no thermomechanical coupling energy, no energy storage) the difference between $w_{def}^d(u_d)$ and $w_e^d(u_d)$ is attributed to the energy dissipation accompanying the irreversibility of damage mechanisms. We can then define the dissipated energy by $w_d^d(u_d)$:

$$w_d^d(u_d) = w_{def}^d(u_d) - w_e^d(u_d) , \qquad (11)$$

 $w_d^d(u_d), w_e^d(u_d) \text{ and } w_{def}^d(u_d) \text{ are illustrated in Figure 2.}$



FIGURE 2: Energy illustration of the traction-separation diagram. Monotonic envelope (continuous blue line), elastic unload (dashed blue line).

Based on the mechanical response chosen in Figure 1, the evolutions of 211 the three different energies associated with a loading-unloading tensile testing 212 are shown in Figure 3. The deformation, elastic, and dissipated energies are 213 plotted in green, blue, and red respectively. The deformation energy w_{def} is 214 naturally the sum of the dissipated w_d and elastic w_e energies Eq.(6) since 215 the damage is supposed to be the only microstructural transformation which 216 is fully dissipative during loading, see Eq.(11) (no energy storage is induced 217 by the microstructural transformations). Figure 3a illustrates that during 218 the elastic unloading w_e^d remains constant (no evolution of damage) while w_e 219 returns to zero. In parallel, the deformation energy w_{def} also decreases and 220 tends towards the energy previously dissipated during the first loading cycle, 221 w_d^d . In Figure 3b the elastic reloading while $u \leq u_d$ is shown (dashed lines) 222 and extended by a monotonic loading until rupture for $u_d = u_c$ (full lines). 223



(a) Loading up to $u = u_d$ and unloading. (b) Reloading until rupture.

FIGURE 3: Energy balance evolution during a load-unload-reload process - Continuous lines are associated with the monotonic envelope, dashed lines correspond to the elastic unload and reload.

Under these restrictive assumptions, the area under the traction-separation 224 curve, Figure 1 (equivalent to a monotonic traction rupture) is completely 225 dissipated when the cohesive zone vanishes. In the next sub-section once the 226 thermodynamic working framework has been specified, this important pro-227 perty is discussed. Then, another point to underline is that if the traction-228 separation curve is classically considered as the constituent element of the 229 behavior of the cohesion zone, it is thus the same for the evolutions of $w_{def}^d(u_d)$ 230 and $w_e^d(u_d)$. Therefore, instead of using the tension-separation curve to des-231 cribe the damage progress, associated with the loss of stiffness, it is also 232 possible to use the evolution of the allowable maximum elastic energy w_e^d to 233 define the threshold function associated with the damage rate. 234

235 2.3. Thermodynamics aspects

In this sub-section, we propose to integrate the above results and comments into the TIP framework.

238 2.3.1. Cohesive zone potential and state laws

In the case of isothermal transformations, the chosen state variables are (u, u_d) . A first gambling of the thermomechanical approach is to assume the existence of a potential $\psi(u, u_d)$ capable of gathering all the state laws. Here we identify this potential to the elastic energy w_e defined in Eq.(10):

$$\psi(u, u_d) = \frac{1}{2} K(u_d) u^2 .$$
(12)

The state laws are by construction the partial derivatives of the potential with respect to the state variables. We then define the conjugate variable f^r , associated with u which represents the reversible part of the traction force, and A_d associated u_d respectively :

$$\begin{cases} f^r = \frac{\partial \psi}{\partial u} = K(u_d)u\\ A_d = \frac{\partial \psi}{\partial u_d} = \frac{1}{2}K'(u_d)u^2 \end{cases}, \tag{13}$$
where $K'(u_d) = \frac{\mathrm{d}K(u_d)}{\mathrm{d}u_d}.$

243

244 2.3.2. Clausius-Duhem inequality

The irreversibility of the mechanisms accompanying the opening of the cohesive zone is depicted by the Clausius-Duhem inequality which enables the definition of the intrinsic dissipation w_d^o . In the present framework, it can be written as :

$$w_d^o = w_{def}^o - \dot{\psi} = f\dot{u} - \frac{\partial\psi}{\partial u}\dot{u} - \frac{\partial\psi}{\partial u_d}\dot{u}_d = f^{ir}\dot{u} + X_d\dot{u}_d \ge 0.$$
(14)

The terms w_d^o and w_{def}^o determine the dissipated and deformation energy 245 rates, respectively. The symbol $(-)^{o}$ is introduced to underline that w_{d} and 246 w_{def} are not a priori state functions and are then path dependent. Eq.(14) 247 also introduce the irreversible part of the traction force, $f^{ir} = f - f^r$, and the 248 thermodynamic force X_d associated with \dot{u}_d . Note that during an irreversible 249 transformation $\dot{u}_d > 0$ we get $X_d = -A_d$. If damage is the only irreversible 250 process, no dissipation has to be associated with \dot{u} . In such a case the 251 irreversible traction force vanishes $f^{ir} = 0$. The traction force f can then be 252 directly defined via the state law : 253

$$f = f^r = K(u_d)u . (15)$$

Moreover, the intrinsic dissipation becomes with Eqs (13) and (14):

$$w_d^o = X_d \dot{u_d} = -\frac{1}{2} K'(u_d) u^2 \dot{u_d} \ge 0 .$$
(16)

The fact that $\dot{u}_d \geq 0$ implies $K'(u_d) \leq 0$ what is physically consistent. The irreversible nature of damage leads to a degradation of the secant stiffness.

258 2.3.3. Threshold function and damage evolution law

In the TIP framework the thermodynamic forces are supposed to be function of the state variable rates. In the case of the linear TIP proposed by Onsager [56], the correspondence between thermodynamic forces and state variable fluxes is linear. The Onsager's matrix is supposed to be symmetric positive definite in order to verify the Clausius-Duhem inequality (positive dissipation) whatever the thermodynamic process. Extension to non-linear

theory exists as for example the formalism of Generalized Standard Materials 265 [6]. Based on the hypothesis of normal dissipation, the thermodynamic forces 266 derive from a convex dissipation potential or equivalently, state variables 267 rates derived from a dual dissipation potential, function of the thermodyna-268 mic forces. This dissipation potential can also involve the state variables of 269 the model as parameters. This last option is referred to as non-associated 270 behavior [46]. For threshold laws (e.g. in plasticity), these options are often 271 chosen. A common approach is then : 272

to define a threshold function depending on the thermodynamic forces
(and possibly state variables)

to write that irreversibility occurs and develops if the thermodynamic
state is on the threshold and remains on it during a time increment.
Note that once the state laws (derived from the thermodynamic potential)
and complementary laws (derived from the dissipation potential) have been
written, it is then possible to deduce the evolution of the energy balance
associated with the transformation.

In what follows in as much as our approach is directly based on the energy balance form imposing by construction non-negative dissipation, the existence of the threshold function will not be associated with the normal dissipation hypothesis. Indeed, the current elastic domain is characterized by $w_e^d(u_d)$ the maximum elastic energy available for a given damage state which also corresponds to the energy required to further damage the material. Then we have :

$$w_e(u, u_d) \le w_e^d(u_d) . \tag{17}$$

The evolution law for u_d is then derived from the fact that for the damage to occur the maximum elastic energy allowable in the material has to be and remain on the threshold during the loading step, i.e.

$$\begin{cases} w_e(u, u_d) = w_e^d(u_d) & (a) \\ \dot{w}_e(u, u_d) = \dot{w}_e^d(u_d) & (b) \end{cases}.$$
 (18)

The first equality gives naturally $u_d = u$. The second equality leads to a proposal of evolution equation for the damage :

$$\dot{u}_d = \begin{cases} \dot{u} & \text{if } u = u_d \quad \text{and} \quad \dot{u} \ge 0\\ 0 & \text{if } u < u_d \quad \text{or} \quad \dot{u} \le 0 \end{cases},$$
(19)

what is consistent if we remind the definition of the damage state variable Eq.(4) and the fact that the damage increases irreversibly, $\dot{u}_d \ge 0$.

To be fully compatible with non-linear TIP framework, the final step is to propose a threshold function that takes the thermodynamic force X_d into account. As previously stated, we consider a derivative form of the energy balance to get this threshold function Eq.(18)b. By using Eq.(13) and Eq.(16), we get :

$$\frac{d w_e}{dt} = -X_d \dot{u_d} + K(u_d) u \dot{u} .$$
⁽²⁰⁾

On the threshold, Eq.(20) becomes :

$$\frac{d w_e^d}{d u_d} \dot{u}_d = (-X_d + K(u_d)u_d) \ \dot{u}_d \ . \tag{21}$$

Then a threshold function F involving the thermodynamic force X_d and the state variables can be taken under the form :

$$F(X_d; u, u_d) = K(u_d)u - X_d - \frac{d w_e^d}{d u_d} .$$
(22)

To be consistent with the incremental form of the energy balance, the equality $F(X_d; u, u_d) = 0$, gives once again $u_d = u$ while the consistency condition dF = 0 leads to $du = du_d$, or equivalently to Eq.(19).

To be precise, the full calculation of dF = 0 at $u = u_d$ leads to :

$$(K(u_d) + 2K'(u_d)u_d)(du - du_d) = 0, \qquad (23)$$

then $du = du_d$, except possibly when $u_d = -\frac{K(u_d)}{2K'(u_d)}$.

301 2.3.4. Some comments about the damage evolution equations

To depict the evolution of damage, in addition to the traction-separation curve data, the literature often proposes a specific evolution equation in the form of $\dot{D} = \dot{D}(f, D, \dot{u})$ whatever the definition of the damage variable D[11; 44; 65].

In the foregoing, because of the hypotheses explicitly made on the energy balance (i.e. damage is the only dissipative mechanism and it is totally dissipative), the damage evolution law is fixed by the definition of the damage variable itself and by the explicit form of the energy balance. With the chosen definition of the damage variable given in Eq.(4), we derive an evolution equation given in Eq.(18), which is an extremely simple form of the general equation proposed by [65].

Another evolution equation could lead, in our case, to no longer respect the properties of the energy balance as first assumed. The consequences could be the appearance of energy storage mechanisms, i.e. $\dot{w}_d^d < \dot{w}_{def}^d - \dot{w}_e^d$, or internal energy transformation into dissipated energy (release of stored energy), i.e. $\dot{w}_d^d > \dot{w}_{def}^d - \dot{w}_e^d$. Taking this stored energy variations should lead to the introduction of new internal state variables and/or to a change of the deformation energy rate definition [30].

320 3. 2D cohesive zone model

In this paragraph, we propose an extension to a vectorial version of the CZM where the isotropic damage is controlled by the evolution of the maximum storable elastic energy. Isotropic damage means here that a scalar state variable is solely used to describe the damage evolution. This generalization has been made by following the same approach as the one previously proposed.

327 3.1. Mechanical variables

Regarding the mechanical description of the cohesive zone, the traction 328 force and the separation become now vectors. Let us introduce a frame of 329 reference where directions 1 and 2 correspond to the tangent plane of the 330 cohesive zone while direction 3, is the normal direction. The traction vector, 331 **f**, whose components are (f_{t_1}, f_{t_2}, f_n) and the separation vector, **u**, which 332 has 3 components denoted by (u_{t_1}, u_{t_2}, u_n) are introduced. As is conventio-333 nally admitted in CZM, the normal move jump denoted by u_n is positive or 334 null. For a long time, this unilateral condition has been taken into account 335 by a Signori relation in numerical simulations performed with the software 336 program, LMGc90 [27]. 337

338 3.2. Cohesive zone potential and state equations

Before defining the cohesive zone potential that is a state function, a set of state variables has to be chosen. Here we selected the components (u_{t_1}, u_{t_2}, u_n) of the separation vector and a scalar damage variable denoted by u_d . Then, to generalize the form of the cohesive zone potential proposed in Eq.(12), the following form, inspired by [11], is adopted :

$$\psi(\mathbf{u}, u_d) = w_e(\mathbf{u}, u_d)$$

$$= \frac{1}{2} \left(K_n(u_d) u_n^2 + K_t(u_d) u_{t_1}^2 + K_t(u_d) u_{t_2}^2 \right)$$

$$= \frac{1}{2} K_n(u_d) (u_n^2 + \alpha u_{t_1}^2 + \alpha u_{t_2}^2)$$

$$= \frac{1}{2} K_n(u_d) u_{eq}^2$$
(24)

where :

$$u_{eq} = (u_n^2 + \alpha u_{t_1}^2 + \alpha u_{t_2}^2)^{\frac{1}{2}} .$$
(25)

The variable α is the ratio between $K_t(u_d)$, the tangential and $K_n(u_d)$, the normal secant stiffnesses at a given u_d . In the case of isotropic damage α is a constant.

In Eq.(26), a 3D formulation of the scalar depicting the isotropic damage is given. By construction, u_d takes a the 3D aspect of the separation vector **u** into account and then \dot{u}_d is non-negative and de facto respects the irreversibility of the damage progress.

$$u_d = \max \left\{ u_{eq}(\tau), \forall \tau \leq t \right\} .$$
(26)

By definition, the state laws are the partial derivatives of the cohesive zone potential Eq.(24). They introduce the components of the reversible traction vector \mathbf{f}^r and the conjugate variable A_d associated with (u_{t_1}, u_{t_2}, u_n) and u_d respectively :

$$\begin{cases}
f_n^r = \frac{\partial \psi}{\partial u_n} = K_n(u_d)u_n \\
f_{t_1}^r = \frac{\partial \psi}{\partial u_{t_1}} = \alpha K_n(u_d)u_{t_1} = K_t(u_d)u_{t_1} \\
f_{t_2}^r = \frac{\partial \psi}{\partial u_{t_2}} = \alpha K_n(u_d)u_{t_2} = K_t(u_d)u_{t_2} \\
A_d = \frac{\partial \psi}{\partial u_d} = \frac{1}{2}K'(u_d)u_{eq}^2
\end{cases}$$
(27)

Because only damage induces irreversibility, no dissipation has to be associated with the component of the separation vector. The reversible part \mathbf{f}^r of the separation can therefore be identified with \mathbf{f} , then $\mathbf{f} = \mathbf{f}^r$.

353 3.2.1. Energy definition of the damage threshold

To extend our approach to an isotropic 3D CZM the evolution of the chosen damage variable, u_d , is directly linked to the evolution of the elastic energy w_e^d . This elastic energy, for a given damage state u_d , describes in the displacement space a half spheroid of radii $r_n = \left(\frac{2w_e^d}{k_n(ud)}\right)$ and $r_t = \left(\frac{2w_e^d}{\alpha k_n(ud)}\right) = \frac{r_n}{\alpha}$ as shown in Figure 4. As the normal jump denoted u_n is by definition positive or null only half of the spheroid is reachable for any separation states.

As long as the further separation states, \mathbf{u} , remain within the corresponding spheroid (i.e. $w_e(\mathbf{u}, u_d) \leq w_e^d(u_d)$), the behavior remains elastic. Then for a given opening such that $u_{eq} = u_d$, the elastic energy reaches the maximal value associated with this damage state (i.e. $w_e(\mathbf{u}, u_d) = w_e^d(u_d)$). Once the surface of the spheroid is reached :

 $_{366}$ — either the separation increment $\delta \mathbf{u}$ is directed towards the inside of the spheroid, and an elastic unloading at constant damage can be observed,

369

- or $\delta \mathbf{u}$ is directed towards the outside of the spheroid, and then the



FIGURE 4: A 3D representation of the elastic energy allowable, $w_e^d(u_d)$, for a given damage state u_d . The color variation represents only the height U_n .

damage develops defining a new elastic limit surface.

370

For isotropic damage, a single evolution equation for u_d is required. We have 371 already underlined that for threshold behavior law, the yield function de-372 pends on the thermodynamic forces and possibly on the states variables 373 themselves, acting as parameters. In the present case, the thermodynamic 374 force of the model , associated with the damage variable rate, is X_d . A ge-375 neralized form of the yield criterion proposed in Eq.(22) is chosen where the 376 role of u used in the 1D scenario is played by u_{eq} . So, the proposed yield 377 criterion, derived of $\frac{d w_e^1}{dt}$, may be written as : 378

$$F(X_d; u_{eq}, u_d) = K_n(u_d) u_{eq} - X_d - \frac{d w_e^d}{d u_d} .$$
(28)

Damage develops if the threshold is reached, $F(X_d; u_{eq}, u_d) = 0$ and if the consistency condition is verified, $\dot{F}(X_d; u_{eq}, u_d) = 0$. For the same reasons as the ones shown for the 1D model, the evolution law of the parameter u_d is written as :

$$\dot{u}_d = \dot{u}_{eq} \text{ if } u_d = u_{eq} \text{ and } \dot{u}_{eq} \ge 0 , \qquad (29)$$

results which, moreover, are imposed by the very definition of the damagevariable.

An illustration of the energy criteria is given in Figure 5. Following a monotonic loading (i.e. remaining on the $w_e^d(u_d)$ curve), $K_n(u_d)u_d$ is the slope of the deformation energy w_{def}^d , X_d is the slope of the dissipated energy w_d^d and $(w_e^d)'$ is naturally the slope of the maximal allowable elastic energy w_e^d .



FIGURE 5: Illustration of the damage energy criterion - $u_{eq} = u_d$

To conclude section 3, we would like to stress once more the fact that

the damage threshold of is not here a matter of choice. It is imposed by the chosen form of the energy balance and by the definition of the damage state variable.

The state equations Eqs(27) and the evolution equation Eq.(29) will be, in what follows, implemented in a home-made finite element code. The different material parameter of the constitutive equations will be specified. In order to show the capabilities of such a CZM, simulations will be made and compared with some experimental results extracted from the literature.

399 4. Numerical Implementation

To illustrate the potentiality of the proposed model, simulations reprodu-400 cing a common benchmark extracted from the literature [32][49] were carried 401 out. It is important to notice that the objective of this practical compari-402 son is simply to show the operability of the model and not to optimize its 403 parameters in order to fit the benchmark. The numerical implementation of 404 the previous model is then done in the code L^MGc90 based on Non-Smooth 405 Contact Dynamics (NSCD) [36; 37; 52]. The NSCD method is dedicated to 406 solving problems related to dynamic systems with unilateral constraints. It 407 is therefore particularly suitable for contact friction problems. It proposes 408 a non-smooth treatment (no compliance, no penalty) of the conditions of 409 contact [36], which is explicit in definition of u_n . The way which adhesion is 410 taken into account in this method makes it possible to consider each point 411 of contact as a cohesive zone. Then the mechanical behavior of the cohesive 412 zones may vary at any point of the spatial discretization of the problem. 413 This relevant modeling framework was then adopted to numerically simulate 414

⁴¹⁵ crack propagation with cohesive zone [16].

416

417 4.1. Benchmark

To compare the proposed model with a benchmark found in literature 418 [32][49], the form of the maximum storable elastic energy, which we remember 419 is $w_e^d(u_d)$, must be specified in order to be able to implement it in LMGc90 420 , the open source platform 1 used to carry out the simulations [28]. This 421 benchmark, illustrated by Figure 6, traces the evolution of a crack in mixed 422 mode to be followed. In the context of this feasibility study, a simple quadratic 423 form of $w_e^d(u_d)$ is proposed. In what follows, we also assumed the existence of 424 a pure elastic domain and thus the existence of a threshold equivalent elastic 425 deplacement u_{eq}^{e} , simply denoted by u_{e} . The maximum storable elastic energy 426 as a function of the damage parameter u_d simply reads : 427

$$w_e^d(u_d) = A (u_d - u_c)^2 + B (u_d - u_c), \text{ if } u_e \le u_d \le u_c$$
, (30)

where u_c is the critical equivalent displacement corresponding to the crack onset. Parameters A and B are two constants chosen to ensure the C_1 continuity of the maximum storable elastic energy, $w_e^d(u_d)$, at the threshold equivalent elastic deplacement, $w_e^d(u_e) = \frac{1}{2} K_n^0 u_e^2$. They are defined by :

$$\begin{cases} A = -\frac{1}{2} K_n^0 u_e \frac{(2u_e - u_e)}{(u_e - u_e)^2} \\ B = -K_n^0 \frac{u_e u_e}{u_e - u_e} \end{cases}, \tag{31}$$

where K_n^0 is the initial normal stiffness of the CZM.

^{1.} https://git-xen.lmgc.univ-montp2.fr/lmgc90/



FIGURE 6: Characteristics of the benchmark issued from [32] used for simulation

The thickness of the sample, denoted by H, is equal to 0.3m while its 429 length is equal to 1.2m. A 0.15m pre-crack is located in the middle on the 430 lower edge. The point B is fixed in both x and y directions whereas the point 431 A is only fixed in the y direction. A displacement is imposed on the point 432 A to load the structure. The mesh is composed of 3 parts : Two continuous 433 meshes (the left and the right parts of the structure) composed respectively 434 of 958 and 2063 T3 elements and 6723 meshes composed of a single T3 435 element assuring the continuity of the structure (domain Ω_1 on Figure 6). The 436 interactions between elements of Ω_1 are governed by the proposed cohesive 437 zone model where the initial secant elastic stiffness, K_n^0 and K_t^0 , are chosen 438 to satisfy the criterion proposed in [9] to limit the reduction of stiffness due 439 to the presence of CZM. It is important to underline that the objective of 440 this practical comparison is simply to show the operability of the model and 441 not to optimize its parameters in order to fit the benchmark. The values of 442 the CZM parameters are summarized in Table 1. 443

444

Figure 7 shows the evolution, for different simulation times, of different

$K_n^0~{ m (N/m)}$	α	$u_e (\mathrm{m})$	u_c (m)
$2.48 10^9$	0.5	$0.5 \ 10^{-6}$	$1.5 \ 10^{-6}$

TABLE 1: Parameter values of the CZM

characteristic quantities associated with the model : the damage variable, the elastic energy w_e and the dissipated energy w_d . In order to present a quantity varying from 0 to 1 the damage ratio, as a function the damage variable, is introduced and defined by $\frac{\langle u_d - u_e \rangle^+}{u_c - u_e}$. To improve the visibility of these different quantities supported by the interfaces, they are projected on adjacent elements.

Figure 7(a), corresponding to a pre-cracking state, shows a concentration 451 of the elastic energy at the outset of the crack tip. However, the damage 452 criterion has not been reached within the cohesive zone so that no damage 453 or dissipation has yet occurred (see Eq.(17)). The corresponding map to 454 $\frac{\langle u_d - u_e \rangle^+}{u_c - u_e}$ and w_d are then uniformally egal to 0. As expected, Figure 7 (b) 455 and (c), corresponding to two post-cracking steps, highlight the correlation 456 between the evolution of the dissipated energy and the damage ratio. The 457 elastic energy is still concentrated ahead of the crack tip, then returns to zero 458 along the crack lips. In contrast, the dissipated energy related to the damage 459 evolution can be exhibited all along the crack path. Similarly, the damage 460 field allows the cracking path to be tracked. 461

To exhibit the capability of our CZM where only the shape of the cohesive energy associated with a simple energy balance is needed (cf. Eq.(30)), different quantities, numerically obtained, are compared with experimental



FIGURE 7: Visualization of the damage ratio (top row), the elastic energy (center) and the dissipated energy (bottom row) during the crack propagation

measurements present in the literature [14]. For such comparisons, Figure 8
presents both the classical crack path monitoring and the load vs. CMOD
curve (Crack Mouth Opening Displacement).



FIGURE 8: Comparison of numerical macroscopic measurements associated to the crack evolution with experimental results [14] : a) The crack path and b) the load vs. CMOD curve.

In Figure 8(a) and Figure 8(b), the red dot line corresponds to the simulation result while the black dashed lines represent the crack envelop obtained experimentally [14]. In Figure 8a, the crack obtained numerically corresponds closely to the experimental envelope. The starting angle is strongly related to the discretization around the initiation point, explaining the slight difference at the beginning of the initiation. Then, the path is corrected and repositioned in the experimental envelope until the end of the simulation.

475 Concerning the force vs. CMOD curves, they fit perfectly in the section

corresponding to the linear increase. This highlight that the introduction of 476 a 2D interface element, where the values of K_n^0 and K_t^0 satisfy the criterion 477 proposed in [9] between each elements of Ω_1 do not affect the global stiffness 478 of the sample. The maximum force obtained is also in good agreement with 479 that obtained in the experiment, as well as the beginning of the non-linear 480 decreasing part of the CMOD curve occurring at the initiation of cracking. In 481 the last part, the curves diverge. This difference is partly explained by the fact 482 that the numerical simulation is two-dimensional while the experiments are 483 three-dimensional. Indeed, not all deformation modes are taken into account 484 (especially out-of-plane modes), which explains this different behavior at the 485 end of the simulation. Moreover, we have arbitrarily chosen a 2nd degree 486 polynomial to characterize the damage of the cohesive zone model, Eq.(30). 487 This choice could be fine-tuned in order to better account for experiences by 488 taking a Needleman-type damage, [26; 54]. 489

490 4.2. Sensitivity study

Finally, in order to see the impact of a variation in the parameters u_e 491 and u_c on the overall behavior of the system and more particularly on the 492 evolution of the force vs. CMOD curves, a sensitivity study is proposed. The 493 influence of these parameters on the crack path is not presented because it is 494 not very significant. The influence of these parameters on the energy available 495 to be dissipated in the model is pointed out in Figure 9. The parametric 496 study is carried out relative to the reference point (0,0) corresponding to the 497 results presented on the Figure 8(b) with the parameters define in table 1. 498 With the chosen law, a variation of u_c has almost the same consequence as 499 a variation of u_e in terms of the energy available to be dissipated. Then the 500

map presented on Figure 9 is symmetric in the regard of the circle-triangle diagonal. During the different parametric studies, the color code used for the curves will refer to the one defined in the Figure 9.



FIGURE 9: Map of the normalized dissipated energy variations as a function of the variations of u_e and u_c . The symbols used at the four corners of the map identify the curves shown in Figure 10.

The Figure 10 presents the normalized plots of $w_e^d(u_d)$ for different values 504 of u_e and u_c (Figure 10(a)) and the corresponding force vs. CMOD curves 505 (Figure 10(b)). The normalization parameters are $u_{e,0} = 0.5 \, 10^{-2}$ and $w_{e,0} =$ 506 $w_{e,0}^d(u_{e,0})$ using the values of parameters in Tab. 1. Even if the shape of the 507 curves is significantly different in Figure 10 (a), the energies available to be 508 dissipated for the case represented by a cross and the one represented by 509 a square are of same order of magnitude. The maxima order observed at 510 the scale of the CZM models (Figure 10(a)) is conserved at the scale of the 511 structure (Figure 10(b)). 512



FIGURE 10: (a) Normalized plots of $w_e^d(u_d)$ for different values of u_e and u_c . The normalization parameters are $u_{e,0} = 0.5 \, 10^{-2}$ and $w_{e,0} = w_{e,0}^d(u_{e,0})$ using the values of parameters in Tab.1. (b) Corresponding force vs. CMOD curves.



FIGURE 11: Comparison between the reference Load vs. CMOD curve (dash line) and the ones related to the variation of u_c of $\pm 20\%$ while u_e constant.



FIGURE 12: Comparison between the reference Load vs. CMOD curve (dash line) and the ones related to the variation of u_e of $\pm 20\%$ while u_c constant.

On figures Figure 11 to Figure 13, we observe respectively the influence of 513 u_c and u_e on the force vs. CMOD curves. These figures show that the influence 514 of the variation of u_c is less than that of u_e . Indeed, where we observe for a 515 variation of u_c a variation of less than 10% on the critical values of the curve 516 $(F_{max} \text{ and } CMOD_{max})$, while a variation of more than 20% for an equivalent 517 variation of u_e is observed. Nevertheless, in both cases, an increase of the 518 damage energy w_d induces an increase of the CMOD and loading maxima in 519 the Load vs. CMOD curve. In this model where an elastic domain is assumed, 520 u_e is the threshold where the damage begins to occur. This value determines 521 the outset of the non-linear response of the structure. This is exhibited in 522 Figure 12 where an increase of u_e at the local scale induces an increase of the 523 maximal force at the macroscopic scale and a delay of the occurrence of the 524 nonlinear response of the curves. 525



FIGURE 13: Comparison between the reference Load vs. CMOD curve (dash line) and the ones related to the variation of u_c and u_e of $\pm 20\%$ W constant.

Figure 13 demonstrates that the non-linear region of the curve is also 526 governed by the shape of the energy curve (cf. Figure 10). Although the 527 dissipative energy in this parametric study is almost constant, we observe 528 a variation of about 10% on the characteristic values of the response curve. 529 So by combining the effects of u_e , u_c and the shape of the local curve (cf. 530 Figure 10) it is possible to obtain a better optimal result to fit experiments. 531 Thus the experimental characterization of this type of local curve depicting 532 the micro-structural phenomenon linked to fracture is relevant and is still an 533 ongoing problem. 534

535 5. Conclusion

In this paper we present an energy criterion for cohesive zone models where the damage progress is assessed together with the ability of the ma-

terial to store energy elastically. The damage parameter used is $u_d(t)$ = 538 $Sup \{u_{eq}(\tau), \tau \leq t\}$ where u_{eq} is an equivalent strain compatible with the 539 isotropic evolution of the damage progress. The cohesive zone law is then 540 completely defined through the function $w_e^d(u_d)$ which represents the maxi-541 mum elastic energy which can be stored in the material for a given da-542 mage state associated with a simple energy balance. The identification of the 543 function, $w_e^d(u_d)$ will be based on the analysis of the experimentally derived 544 energy balance associated with the material transformations, in addition to 545 the kinematic aspects near the crack tip, the dissipative effects accompanying 546 the damage progress, inducing temperature variations at the crack tip. The 547 thermodynamic coherence of the proposed model effectively makes it pos-548 sible to directly relate damage and temperature. We have implemented the 549 chosen model in the code L^MGc90 based on Non-Smooth Contact Dynamics 550 (NSCD) and performed numerical simulations in the case of a bending test. 551 The results obtained for this 2-D modeling are encouraging. Using a simple 552 quadratic function $w_e^d(u_d)$ for the interface, we obtained a close correlation 553 between the simulations and the experimental observations of the crack path. 554 A first parametric study of the macroscopic response of the structure natu-555 rally demonstrates the importance of the shape of the function $w_e^d(u_d)$ which 556 characterize the interface behavior between two elements. In subsequent de-557 velopments, we will consider high cycle fatigue in cohesive zone models while 558 preserving this framework to define and quantify damage. An extension to 559 investigate the non isotropic degradation of a material to store elastic energy 560 could also be envisaged. 561

562 Références

- [1] Abdelmoula, R., Marigo, J.-J., Weller, T., 2009. Construction des lois
 de fatigue à partir de modèleles de forces cohésivesives : cas de fissures
 en mode i. Comptes Rendus Mécanique 337 (3), 166 172.
- 566 URL http://www.sciencedirect.com/science/article/pii/ 567 S1631072109000382
- [2] Alessi, R., Jul. 2013. Variational approach to fracture mechanics with
 plasticity. Theses, Ecole Polytechnique X.
- URL https://pastel.archives-ouvertes.fr/pastel-00847970
- [3] Alessi, R., Marigo, J.-J., Vidoli, S., 2015. Gradient damage models
 coupled with plasticity : Variational formulation and main properties.
 Mechanics of Materials 80 (Part B), 351 367, materials and Interfaces.
 URL http://www.sciencedirect.com/science/article/pii/
 S0167663614000039
- [4] Allix, O., Ladevèze, P., Corigliano, A., 1995. Damage analysis of
 interlaminar fracture specimens. Composite Structures 31 (1), 61 74.
- 578 URL http://www.sciencedirect.com/science/article/pii/ 579 026382239500002X
- [5] Amor, H., Marigo, J.-J., Maurini, C., 2009. Regularized formulation
 of the variational brittle fracture with unilateral contact : Numerical
 experiments. Journal of the Mechanics and Physics of Solids 57 (8),
 1209 1229.

⁵⁸⁴ URL http://www.sciencedirect.com/science/article/pii/ ⁵⁸⁵ S0022509609000659

- [6] B., H., Q.S., N., 1975. Sur les matériaux standards generalisés. Journal
 de mécanique 14, 39,63.
- ⁵⁸⁸ [7] Barrenblatt, G., 1962. The mathematical theory of equilibrium of cracks
 ⁵⁸⁹ in brittle fracture. Adv. Appl. Mech. 7, 55–129.
- [8] Bernard, P., Moës, N., Chevaugeon, N., 2012. Damage growth modeling
 using the thick level set (tls) approach : Efficient discretization for
 quasi-static loadings. Computer Methods in Applied Mechanics and
 Engineering 233-236, 11 27.
- ⁵⁹⁴ URL http://www.sciencedirect.com/science/article/pii/
 ⁵⁹⁵ S004578251200062X
- [9] Blal, N., Daridon, L., Monerie, Y., Pagano, S., 2011. Criteria on the
 artificial compliance inherent to the intrinsic cohesive zone. Comptes
 Rendus Mécanique 339 (12), 789 795.
- 599 URL http://www.sciencedirect.com/science/article/pii/ 500 S1631072111001677
- [10] Blaysat, B., Hoefnagels, J., Lubineau, G., Alfano, M., Geers, M., 2015.
 Interface debonding characterization by image correlation integrated
 with double cantilever beam kinematics. International Journal of Solids
 and Structures 55 (Supplement C), 79 91, special Issue Computational
 and Experimental Mechanics of Advanced Materials A workshop held at
 King Abdullah University of Science and Technology Jeddah, Kingdom

of Saudi Arabia July 1-3, 2013.

608 URL http://www.sciencedirect.com/science/article/pii/
 609 S0020768314002443

- [11] Bouvard, J., Chaboche, J., Feyel, F., Gallerneau, F., 2009. A cohesive
 zone model for fatigue and creep-fatigue crack growth in single crystal
 superalloys. International Journal of Fatigue 31 (5), 868 879.
- ⁶¹³ URL http://www.sciencedirect.com/science/article/pii/
 ⁶¹⁴ S0142112308002521
- [12] Brocks, W., Cornec, A., Scheider, I., july 2003. Computational aspects of nonlinear fracture mechanics. Technical Report GKSSForschungszentrum Geesthacht GmbH (Germany) 3, 127–209.
- [13] Cazes, F., Moës, N., July 2015. Comparison of a phase-field model and
 of a thick level set model for brittle and and quasi-brittle fracture. Numerical Methods in Engineering 103 (2), 114–143.
- [14] Cendón, D., Gálvez, J., Elices, M., Planas, J., Jun 2000. Modelling the
 fracture of concrete under mixed loading. International Journal of Fracture 103 (3), 293–310.
- ⁶²⁴ URL https://doi.org/10.1023/A:1007687025575
- [15] Chaboche, J., Feyel, F., Monerie, Y., 2001. Interface debonding models :
 a viscous regularization with a limited rate dependency. International
 Journal of Solids and Structures 38 (18), 3127 3160.
- ⁶²⁸ URL http://www.sciencedirect.com/science/article/pii/
 ⁶²⁹ S0020768300000536

- [16] Champagne, M., Renouf, M., Berthier, Y., 01 2014. Modeling Wear for
 Heterogeneous Bi-Phasic Materials Using Discrete Elements Approach.
 Journal of Tribology 136 (2), 021603.
- ⁶³³ URL https://doi.org/10.1115/1.4026053
- [17] Chen, Z., Bunger, A., Zhang, X., Jeffrey, R. G., 2009. Cohesive zone
 finite element-based modeling of hydraulic fractures. Acta Mechanica
 Solida Sinica 22 (5), 443 452.

⁶³⁷ URL http://www.sciencedirect.com/science/article/pii/
 ⁶³⁸ S0894916609602950

[18] Chrysochoos, A., 2012. Infrared thermography applied to the analysis
of material behavior : a brief overview. Quantitative InfraRed Thermography Journal 9 (2), 193–208.

⁶⁴² URL https://doi.org/10.1080/17686733.2012.746069

[19] Chrysochoos, A., 2012. Thermomechanical analysis of the cyclic behavior of materials. Procedia IUTAM 4, 15 – 26, iUTAM Symposium on
Full-field Measurements and Identification in Solid Mechanics.

⁶⁴⁶ URL http://www.sciencedirect.com/science/article/pii/
 ⁶⁴⁷ S2210983812000272

[20] Chrysochoos, A., Daridon, L., Wattrisse, B., Jul. 2014. Prediction of damage evolution in bonded material using cohesive zone model. In : 11th
World Congress on Computational Mechanics - 5th European Conference on Computational Mechanics. Barcelone, Spain.

652 URL https://hal.archives-ouvertes.fr/hal-01103157

- [21] Corigliano, A., 1993. Formulation, identification and use of interface
 models in the numerical analysis of composite delamination. International Journal of Solids and Structures 30 (20), 2779 2811.
- ⁶⁵⁶ URL http://www.sciencedirect.com/science/article/pii/
 ⁶⁵⁷ 002076839390154Y
- [22] Corigliano, A., Allix, O., 2000. Some aspects of interlaminar degradation in composites. Computer Methods in Applied Mechanics and
 Engineering 185 (2), 203 224.
- ⁶⁶¹ URL http://www.sciencedirect.com/science/article/pii/
 ⁶⁶² S0045782599002601
- [23] Corigliano, A., Ricci, M., 2001. Rate-dependent interface models :
 formulation and numerical applications. International Journal of Solids
 and Structures 38 (4), 547 576.
- ⁶⁶⁶ URL http://www.sciencedirect.com/science/article/pii/
 ⁶⁶⁷ S0020768300000883
- ⁶⁶⁸ [24] Daridon, L., Cochelin, B., Ferry, M. P., 1997. Delamination and fiber
 ⁶⁶⁹ bridging modelling in composite samples. Journal of Composite Mate⁶⁷⁰ rials 31 (9), 874–888.
- ⁶⁷¹ URL https://doi.org/10.1177/002199839703100902
- ⁶⁷² [25] Daridon, L., Wattrisse, B., Chrysochoos, A., Potier-Ferry, M., 2011.
 ⁶⁷³ Solving fracture problems using an asymptotic numerical method.
 ⁶⁷⁴ Computers & Structures 89 (5), 476 484.
- ⁶⁷⁵ URL http://www.sciencedirect.com/science/article/pii/
 ⁶⁷⁶ S0045794910002841

- ⁶⁷⁷ [26] den Bosch, M. V., Schreurs, P., Geers, M., 2006. An improved des⁶⁷⁸ cription of the exponential xu and needleman cohesive zone law for
 ⁶⁷⁹ mixed-mode decohesion. Engineering Fracture Mechanics 73 (9), 1220
 ⁶⁸⁰ 1234.
- ⁶⁸¹ URL http://www.sciencedirect.com/science/article/pii/
 ⁶⁸² S0013794406000026
- [27] Dubois, F., Jean, M., 2006. The non smooth contact dynamic method :
 recent LMGC90 software developments and application. Springer Berlin
 Heidelberg, Berlin, Heidelberg, pp. 375–378.
- ⁶⁸⁶ URL https://doi.org/10.1007/3-540-31761-9_44
- [28] Dubois, F., Jean, M., Renouf, M., Mozul, R., Martin, A., Bagnéris, M.,
 May 2011. LMGC90. In : 10e colloque national en calcul des structures.
 Giens, France, p. Clé USB.
- ⁶⁹⁰ URL https://hal.archives-ouvertes.fr/hal-00596875
- [29] Dugdale, D., 1960. Yielding of steel sheets containing slits. Journal of
 the Mechanics and Physics of Solids 8 (2), 100 104.
- ⁶⁹³ URL http://www.sciencedirect.com/science/article/pii/
 ⁶⁹⁴ 0022509660900132
- [30] Fremond, M., 2002. Non-smooth Thermomechanics. Springer-Verlag
 Berlin Heidelberg.
- [31] Gálvez, J., Elices, M., Guinea, G., Planas, J., december 1998. Mixed
 mode fracture of concrete under proportional and nonproportional loading. International Journal of Fracture 94 (3), 267–284.

- [32] Galvez, J., Elices, M., Guinea, G. V., Planas, J., Jun 1996. Crack trajectories under mixed mode and non-proportional loading. International
 Journal of Fracture 81 (2), 171–193.
- ⁷⁰³ URL https://doi.org/10.1007/BF00033181
- [33] Giulio, A., May 2006. On the influence of the shape of the interface law
 on the application of cohesive-zone models. Composites Science and
 Technology 66 (6), 723–730.

 707
 URL
 http://www.sciencedirect.com/science/article/

 708
 B6TWT-4FF8WMX-2/2/ee8b268696a03502b511be734dd46694

- [34] Giulio, A., Elio, S., 10 2006. Combining interface damage and friction
 in a cohesive, zone model. International Journal for Numerical Methods
 in Engineering 68 (5), 542–582.
- ⁷¹² URL http:https://doi.org/10.1002/nme.1728
- ⁷¹³ [35] Huon, V., Richefeu, V., Shuang, W., Chrysochoos, A., Monerie, Y., Wat-
- trisse, B., 2010. Experimental characterisation of a cohesive zone model

⁷¹⁵ using digital image correlation. EPJ Web of Conferences 6, 43004.

⁷¹⁶ URL https://doi.org/10.1051/epjconf/20100643004

- ⁷¹⁷ [36] Jean, M., 1999. The non-smooth contact dynamics method. Computer ⁷¹⁸ Methods in Applied Mechanics and Engineering 177 (3), 235 – 257.
- URL http://www.sciencedirect.com/science/article/pii/
 S0045782598003831
- [37] Jean, M., Acary, V., Monerie, Y., 2001. Non-smooth contact dynamics
 approach of cohesive materials. Philosophical Transactions of the Royal

- Society of London A : Mathematical, Physical and Engineering Sciences
 359 (1789), 2497–2518.
- URL http://rsta.royalsocietypublishing.org/content/359/
 1789/2497
- ⁷²⁷ [38] Julian Rimoli, J. R., Khemani, F., 2012. On the mesh dependency
 ⁷²⁸ of cohesive zone models for crack propagation analysis. In : 53rd
 ⁷²⁹ AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and
 ⁷³⁰ Materials Conference Honolulu, Hawaii.
- [39] Kachanov, L., 1986. Introduction to continuum damage mechanics.
 springer.
- [40] Kolluri, M., Hoefnagels, J. P. M., Samimi, M., van Dommelen, H.,
 van der Sluis, O., Geers, M. G. D., 2012. An in situ experimentalnumerical approach for characterization and prediction of interface delamination : Application to culf-mce systems. Advanced Engineering Materials 14 (11), 1034–1041.
- ⁷³⁸ URL http://dx.doi.org/10.1002/adem.201200110
- [41] Kolluri, M., Hoefnagels, J. P. M., van Dommelen, J. A. W., Geers, M.
 G. D., Sep 2013. A practical approach for the separation of interfacial toughness and structural plasticity in a delamination growth experiment.
 International Journal of Fracture 183 (1), 1–18.
- ⁷⁴³ URL https://doi.org/10.1007/s10704-013-9871-y
- [42] Kolluri, M., Hoefnagels, J. P. M., van Dommelen, J. A. W., Geers, M.
 G. D., Jan 2014. Irreversible mixed mode interface delamination using

- a combined damage-plasticity cohesive zone enabling unloading. Inter-
- national Journal of Fracture 185 (1), 77–95.
- ⁷⁴⁸ URL https://doi.org/10.1007/s10704-013-9899-z
- [43] Kondo, D., Welemane, H., Cormery, F., 2007. Basic concepts and models in continuum damage mechanics. Revue Européenne de Génie Civil
 11 (7-8), 927–943.
- ⁷⁵² URL https://doi.org/10.1080/17747120.2007.9692970
- [44] Kuna, M., Roth, S., 2015. General remarks on cyclic cohesive zone models. International Journal of Fracture 196 (1), 147–167.

⁷⁵⁵ URL https://doi.org/10.1007/s10704-015-0053-y

- [45] Lé, B., Moës, N., Legrain, G., 2018. Coupling damage and cohesive
 zone models with the thick level set approach to fracture. Engineering
 Fracture Mechanics 193, 214 247.
- ⁷⁵⁹ URL http://www.sciencedirect.com/science/article/pii/
 ⁷⁶⁰ S0013794417312523
- ⁷⁶¹ [46] Lemaitre, J., 1996. A course on dammage mechanics. springer.
- [47] Lorentz, E., Andrieux, S., 1999. A variational formulation for nonlocal
 damage models. International Journal of Plasticity 15 (2), 119 138.
- ⁷⁶⁴ URL http://www.sciencedirect.com/science/article/pii/
 ⁷⁶⁵ S0749641998000576
- [48] Marigo, J.-J., Maurini, C., Pham, K., Dec 2016. An overview of the
 modelling of fracture by gradient damage models. Meccanica 51 (12),

768 3107-3128.

⁷⁶⁹ URL https://doi.org/10.1007/s11012-016-0538-4

- [49] Moës, N., Belytschko, T., 2002. Extended finite element method for
 cohesive crack growth. Engineering Fracture Mechanics 69 (7), 813 –
 833.
- URL http://www.sciencedirect.com/science/article/pii/
 S001379440100128X
- ⁷⁷⁵ [50] Moës, N., Stolz, C., Bernard, P.-E., Chevaugeon., N., 4 2011. A level set
 ⁷⁷⁶ based model for damage growth : The thick level set approach. Interna⁷⁷⁷ tional Journal for Numerical Methods in Engineering 86 (3), 358–380.
 ⁷⁷⁸ URL http:https://doi.org/10.1002/nme.3069
- [51] Moës, N., Stolz, C., Parrilla-Gomez, A., 2015. Passage du modèle d'endommagement thick level set au modèle de zone cohésive et vice versa :
 cas unidimensionnel. In : 12e Colloque national en calcul des structures.
- [52] Moreau, J. J., 1988. Unilateral Contact and Dry Friction in Finite Free dom Dynamics. Springer Vienna, Vienna, pp. 1–82.
- ⁷⁸⁴ URL https://doi.org/10.1007/978-3-7091-2624-0_1
- [53] Needleman, A., 1987. A continuum model for void nucleation by inclusion debonding. Journal of applied mechanics 54, 525–531.
- ⁷⁸⁷ [54] Needleman, A., 1990. An analysis of tensile decohesion along an
 ⁷⁸⁸ interface. Journal of the Mechanics and Physics of Solids 38 (3), 289 –
 ⁷⁸⁹ 324.

⁷⁹⁰ URL http://www.sciencedirect.com/science/article/pii/ ⁷⁹¹ 002250969090001K

- ⁷⁹² [55] Nguyen, O., Repetto, E., Ortiz, M., Radovitzky, R., Aug 2001. A co⁷⁹³ hesive model of fatigue crack growth. International Journal of Fracture
 ⁷⁹⁴ 110 (4), 351–369.
- ⁷⁹⁵ URL https://doi.org/10.1023/A:1010839522926
- ⁷⁹⁶ [56] Onsager, L., Feb 1931. Reciprocal relations in irreversible processes. i.
 ⁷⁹⁷ Phys. Rev. 37, 405–426.
- ⁷⁹⁸ URL https://link.aps.org/doi/10.1103/PhysRev.37.405
- ⁷⁹⁹ [57] Ortiz, M., Pandolfi, A., 1999. Finite-deformation irreversible cohesive
 ⁸⁰⁰ elements for three-dimensional crack-propagation analysis. International
 ⁸⁰¹ journal of numerical method in engineering. 44 (9), 1267–1282.
- ⁸⁰² [58] Pandolfi, A., Ortiz, M., 2003. A cohesive model for fatigue crack. In :
 ⁸⁰³ Acta Fracturae, Atti convegni Nazionali IGF ISSN : 2281-1443.
- ⁸⁰⁴ [59] Pandolfi, A., Ortiz, M., novembre 2013. Modeling fracture by material⁸⁰⁵ point erosion. International Journal of Fracture 184 (1-2), 3–16.
- [60] Park, K., Paulino, G. H., Roesler, J. R., 2009. A unified potential-based
 cohesive model of mixed-mode fracture. Journal of the Mechanics and
 Physics of Solids 57 (6), 891 908.
- ⁸⁰⁹ URL http://www.sciencedirect.com/science/article/pii/
 ⁸¹⁰ S0022509608001713
- [61] Pham, K., Marigo, J.-J., 2010. Approche variationnelle de l'endommagement : Ii. les modèles à gradient. Comptes Rendus Mécanique

338 (4), 199 - 206.

814 URL http://www.sciencedirect.com/science/article/pii/ 815 S1631072110000446

- ⁸¹⁶ [62] Pham, K., Marigo, J.-J., 2012. Damage localization and rupture with
 ⁸¹⁷ gradient damage models. Fracture and structural integrity 19, 5–19.
- [63] Pijaudier-Cabot, G., Bazant, Z. P., 1987. Nonlocal damage theory. Journal of Engineering Mechanics 113 (10), 1512–1533.
- ⁸²⁰ [64] Richefeu, V., Chrysochoos, A., Huon, V., Monerie, Y., Peyroux, R.,
 ⁸²¹ Wattrisse, B., 2012. Toward local identification of cohesive zone models
 ⁸²² using digital image correlation. European Journal of Mechanics ⁸²³ A/Solids 34 (Supplement C), 38 51.
- ⁸²⁴ URL http://www.sciencedirect.com/science/article/pii/
 ⁸²⁵ S0997753811001690
- [65] Roe, K., Siegmund, T., 2003. An irreversible cohesive zone model
 for interface fatigue crack growth simulation. Engineering Fracture
 Mechanics 70 (2), 209 232.
- ⁸²⁹ URL http://www.sciencedirect.com/science/article/pii/
 ⁸³⁰ S0013794402000346
- [66] Roth, S., Hütter, G., Kuna, M., Jul 2014. Simulation of fatigue crack
 growth with a cyclic cohesive zone model. International Journal of Fracture 188 (1), 23–45.
- URL https://doi.org/10.1007/s10704-014-9942-8

- ⁸³⁵ [67] Roy, Y. A., Dodds, R. H., 2001. Simulation of ductile crack growth in
 thin aluminum panels using 3-d surface cohesive elements. International
 Journal of Fracture 110 (1), 21–45.
- URL https://doi.org/10.1023/A:1010816201891
- [68] Serpieri, R., Sacco, E., Alfano, G., 2015. A thermodynamically
 consistent derivation of a frictional-damage cohesive-zone model with
 different mode i and mode ii fracture energies. European Journal of
 Mechanics A/Solids 49, 13 25.
- ⁸⁴³ URL http://www.sciencedirect.com/science/article/pii/
 ⁸⁴⁴ S0997753814000837
- ⁸⁴⁵ [69] Suo, Z., Hutchinson, J. W., May 1990. Interface crack between two elastic layers. International Journal of Fracture 43 (1), 1–18.

⁸⁴⁷ URL https://doi.org/10.1007/BF00018123

- [70] Wojtacki, K., Daridon, L., Dubois, F., Moës, N. N., Monerie, Y., May
 2015. Analyse comparative de trois méthodes performantes de simulation numérique de la fissuration. 13e colloque national en calcul des
 structures CSMA.
- [71] Zener, C., Jan 1938. Internal friction in solids ii. general theory of thermoelastic internal friction. Phys. Rev. 53, 90–99.
- URL https://link.aps.org/doi/10.1103/PhysRev.53.90
- [72] Zhang, X., Liu, H.-Y., Mai, Y.-W., 2003. Rate-dependent bridging law
 and its application to dynamic crack growth in brittle-matrix composite
 materials. Composites Part A : Applied Science and Manufacturing

858 34 (11), 1053 - 1063.

⁸⁵⁹ URL http://www.sciencedirect.com/science/article/pii/ ⁸⁶⁰ S1359835X03002355

[73] Zhang, X., Mai, Y.-W., Jeffrey, R. G., 2003. A cohesive plastic and
damage zone model for dynamic crack growth in rate-dependent
materials. International Journal of Solids and Structures 40 (21), 5819
- 5837.

⁸⁶⁵ URL http://www.sciencedirect.com/science/article/pii/ ⁸⁶⁶ S0020768303003706

- ⁸⁶⁷ [74] Zhou, F., Molinari, J. F., 2004. Dynamic crack propagation with
 ⁸⁶⁸ cohesive elements : a methodology to address mesh dependency.
 ⁸⁶⁹ International Journal for Numerical Methods in Engineering 59 (1),
 ⁸⁷⁰ 1–24.
- URL https://onlinelibrary.wiley.com/doi/abs/10.1002/nme.
 857