



HAL
open science

Data-driven stability analysis and enforcement for Loewner Data-Driven Control

Pauline Kergus

► **To cite this version:**

Pauline Kergus. Data-driven stability analysis and enforcement for Loewner Data-Driven Control. Workshop Intersections between Learning, Control and Optimization (IPAM, UCLA), 2020, Los Angeles, United States. hal-03095975

HAL Id: hal-03095975

<https://hal.science/hal-03095975>

Submitted on 4 Jan 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

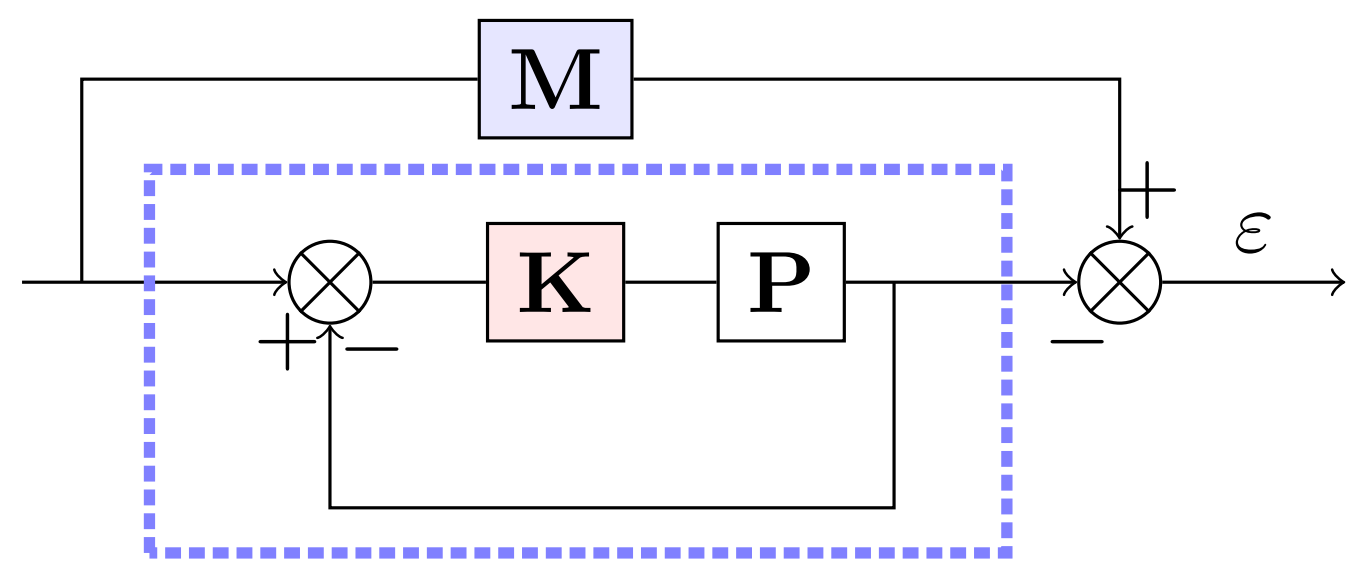
L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

DATA-DRIVEN STABILITY ANALYSIS AND ENFORCEMENT FOR LOEWNER DATA-DRIVEN CONTROL

PAULINE KERIGUS
LUND UNIVERSITY
DEPARTMENT OF AUTOMATIC CONTROL



LOEWNER DATA-DRIVEN CONTROL: GENERAL FORMULATION



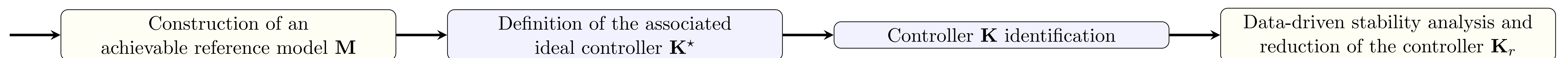
Proposed methodology

1. Computation of the ideal controller K^* frequency-response:
$$K^*(\omega_i) = \Phi_i^{-1} M(\omega_i) (I - M(\omega_i))^{-1}.$$
2. Interpolation and reduction of the ideal controller K^* through the Loewner framework.

Input data

- Frequency-domain data from the plant P : $\{\omega_i, \Phi_i\}, i = 1 \dots N.$
- Reference model $M.$

- Data $\{\omega_i, \Phi_i\}_{i=1}^N$
- Performance specifications



A simple example

$$P(s) = \frac{0.03616(s-140.5)(s-40)^3}{(s^2+1.071s+157.9)(s^2+3.172s+1936)} \quad M(s) = \frac{1}{0.01s^2+0.25s+1}$$

$$K^*(s) = k \frac{(s^2 + 1.071s + 157.9)(s^2 + 3.172s + 1936)}{s(s+10)(s-140.5)(s-40)^3}$$

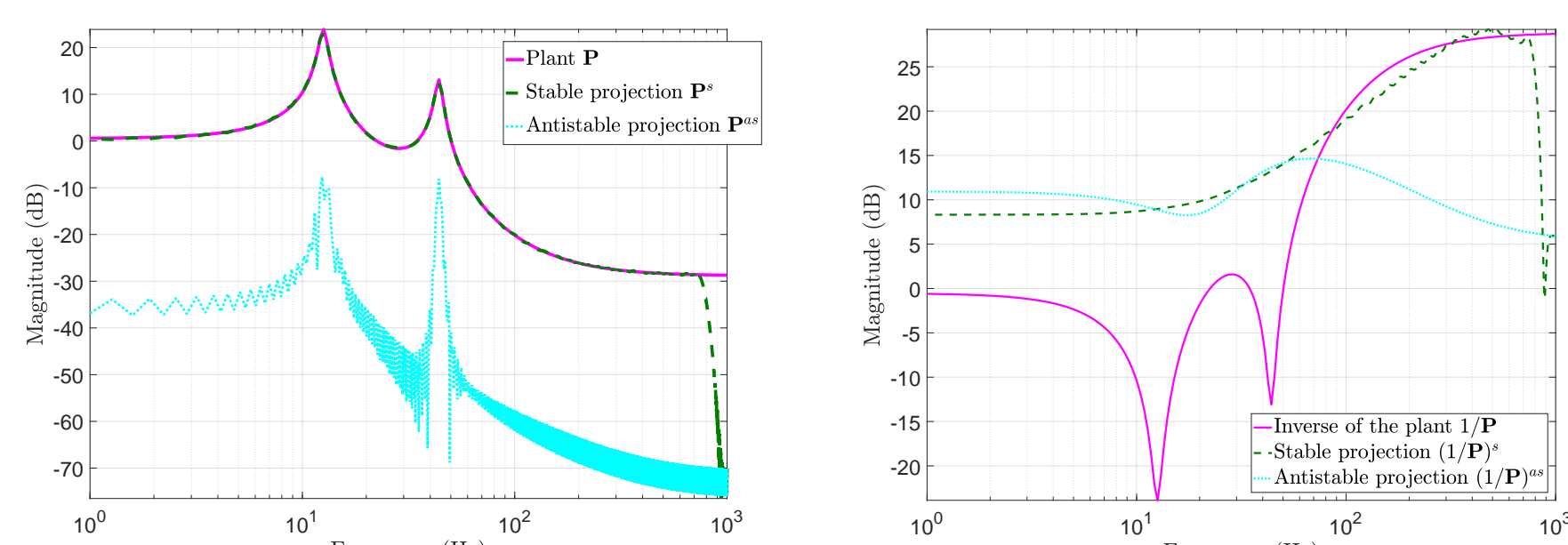
→ The reference model should be achievable by the plant.

$$\begin{cases} y_{z_i}^T P(z_i) = 0 \\ y_{p_j}^T P(p_j) = \infty \end{cases} \Rightarrow \begin{cases} y_{z_i}^T M(z_i) = 0 \\ M(p_j) y_{p_j} = y_{p_j} \end{cases}$$

→ A data-driven closed-loop stability analysis is needed.

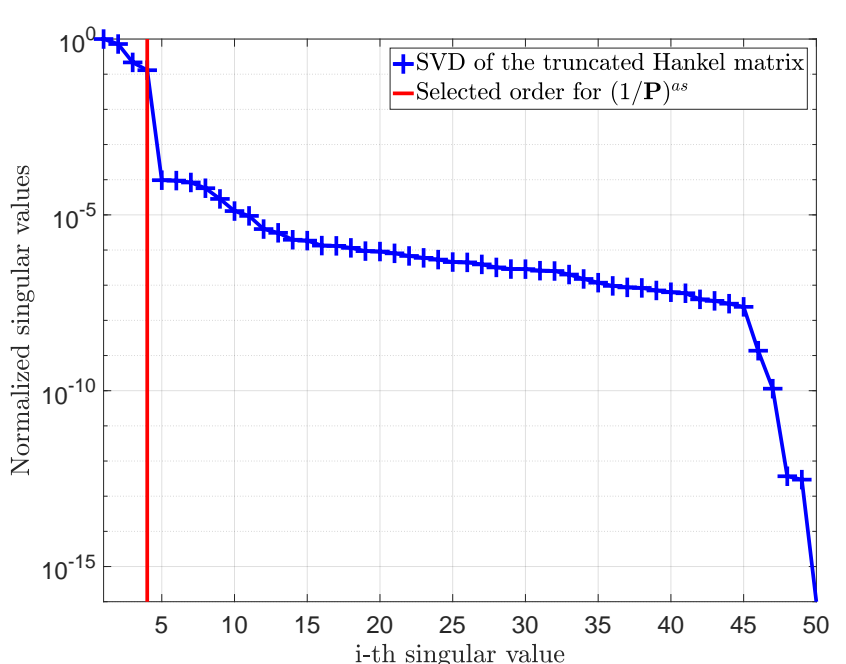
CHOICE OF THE REFERENCE MODEL

1) Projection of the available data to determine the nature of P :



→ The system is stable but Non-Minimum Phase (NMP).

2) Principal Hankel Components technique to determine the number of NMP zeros and obtain an estimate of the instabilities.



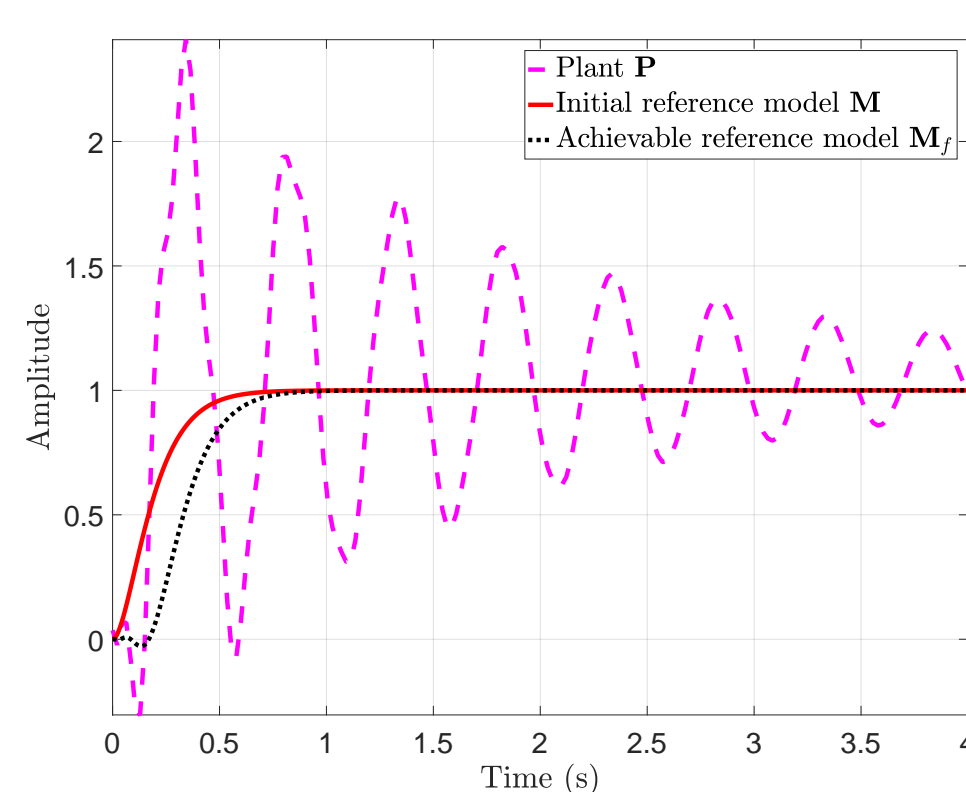
True z_i	Estimated \tilde{z}_i
140.5	140.58
40	41.3+2i
40	41.3-2i
40	37.4

3) Construction of an achievable reference model M_f .

$$M_f = M B_z$$

$$B_z(s) = \prod_{i=1}^{n_z} \frac{s - \tilde{z}_i}{s + \tilde{z}_i}$$

$$|B_z(\omega)| = 1$$



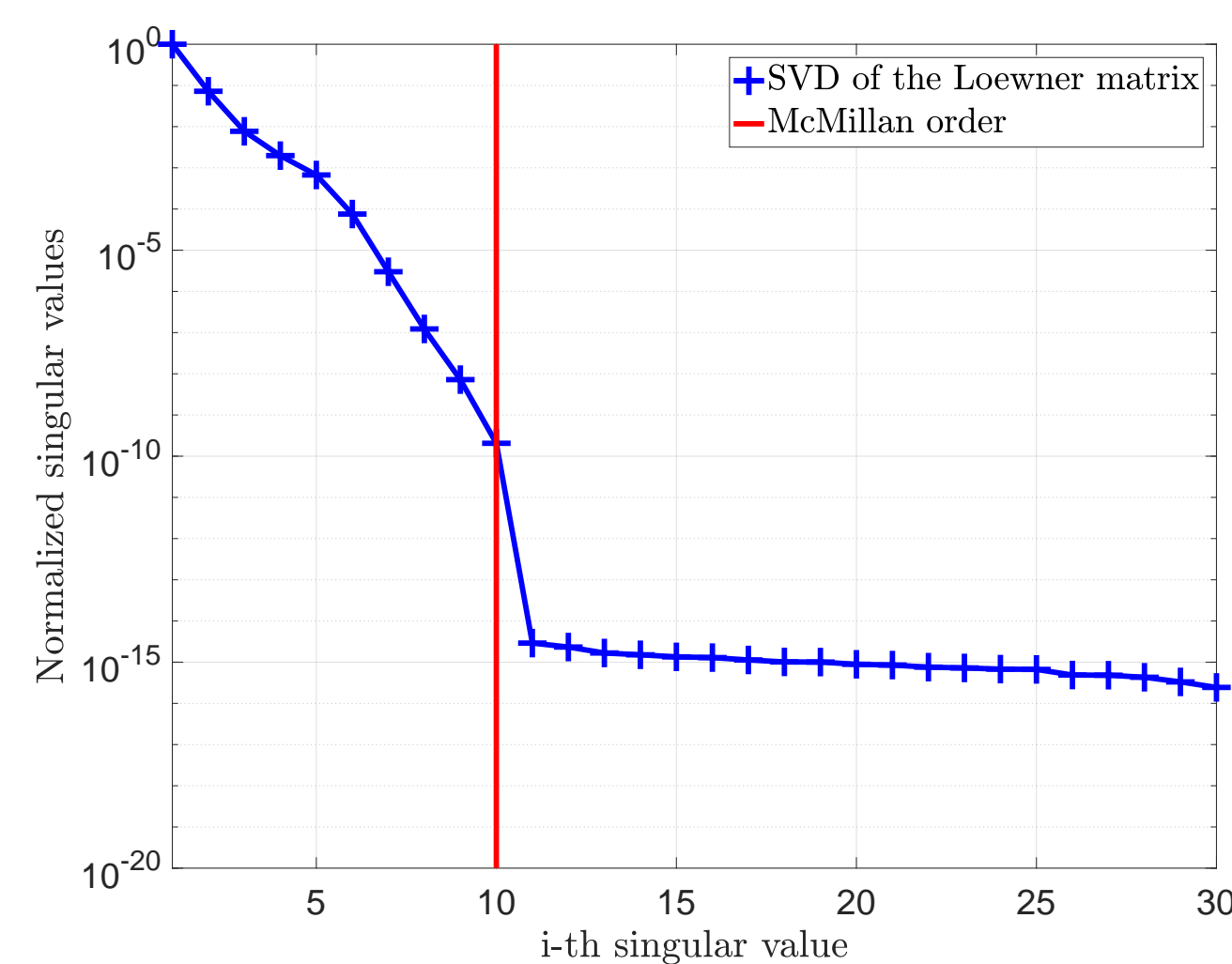
CONTROLLER IDENTIFICATION

The objective is to obtain a rational model $K = (E, A, B, C, D)$ such that:

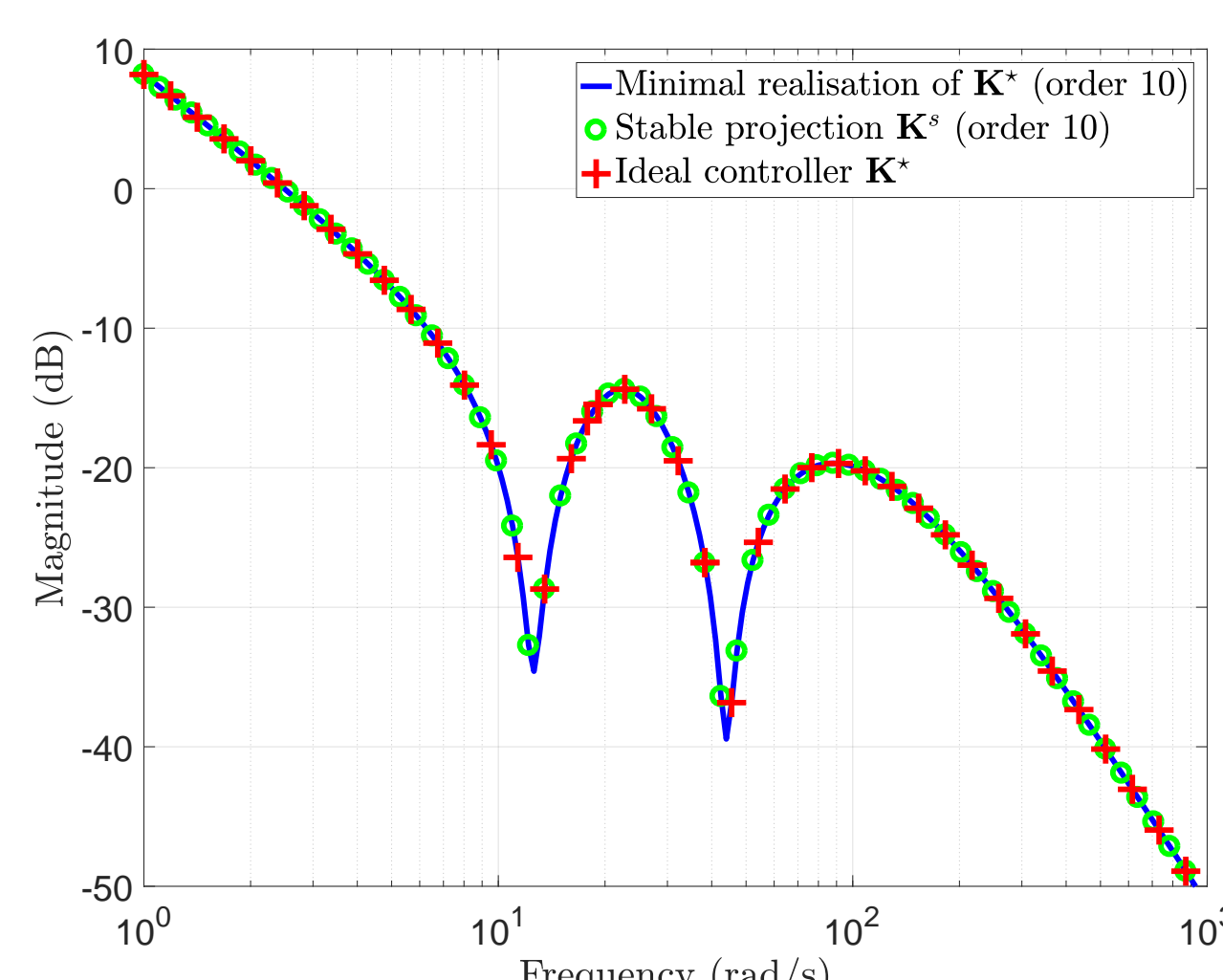
$$\forall i = 1 \dots N, K(\omega_i) = K^*(\omega_i).$$

→ Use of the Loewner pencil $[\mathbb{L}, \mathbb{L}_\sigma]$

1. Embedded order reduction

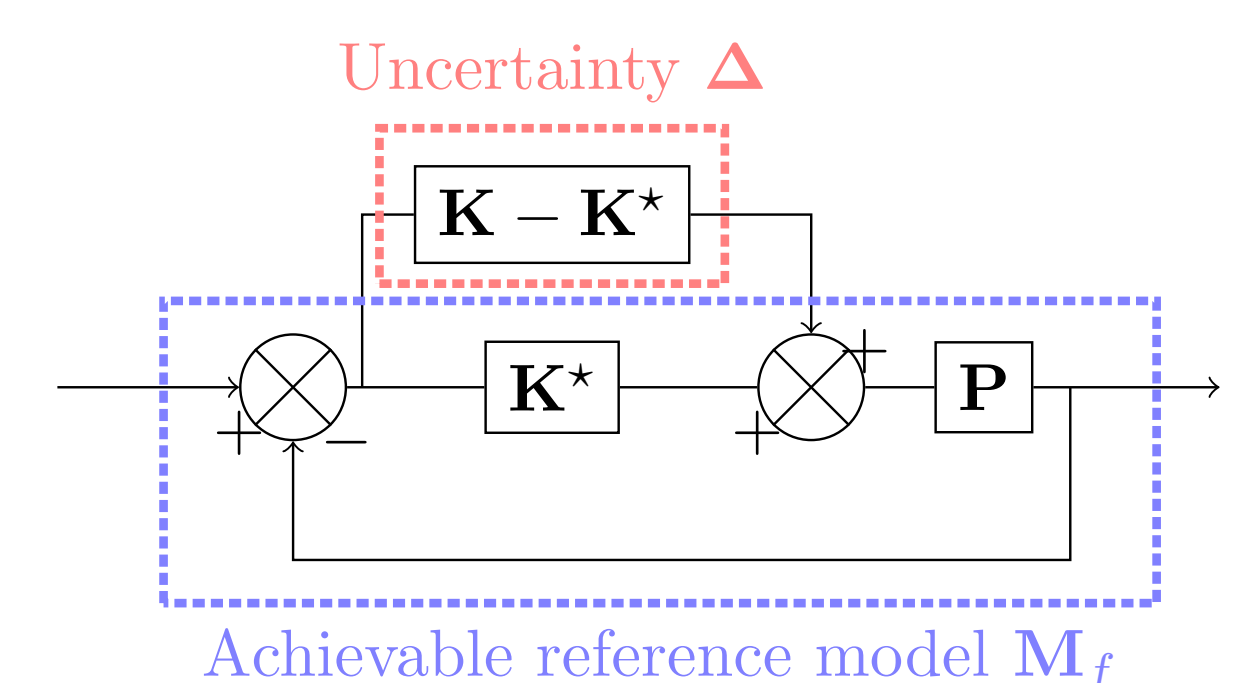


2. Stability of the identified model K



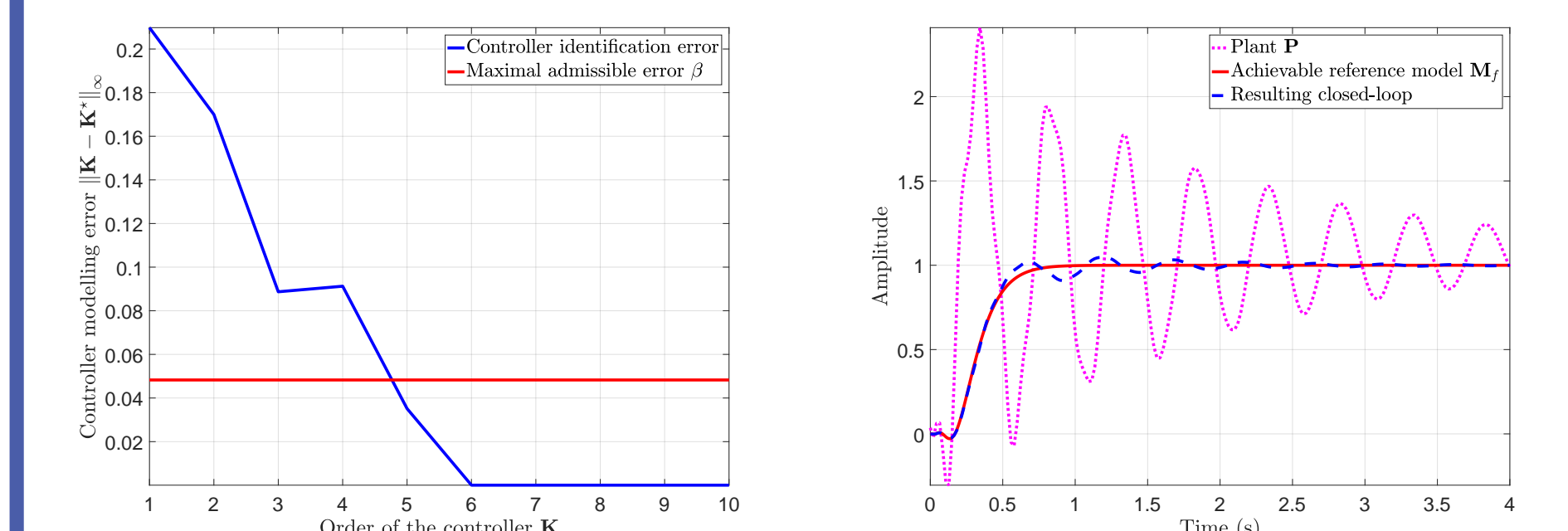
Proposed choice of M → no more compensation of instabilities in the open-loop!

CONTROLLER REDUCTION



The resulting closed-loop is well-posed and internally stable for all stable Δ such that $\|\Delta\|_\infty \leq \beta$ if and only if $\|(1 - M_f)P\|_\infty < \frac{1}{\beta}$.

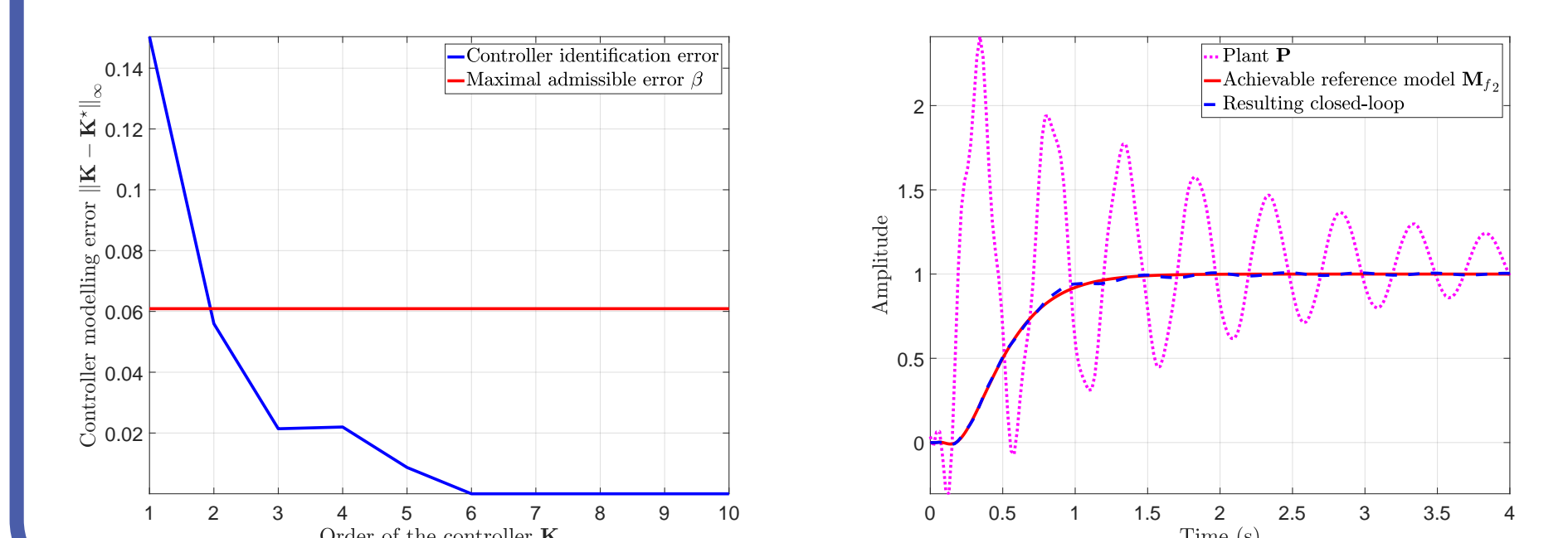
→ Limiting the controller modelling error allows to ensure closed-loop internal stability!



→ Conservatism of the small-gain theorem and importance of the choice of the initial specifications

$$M_2(s) = \frac{1}{0.04s^2 + 0.4s + 1}$$

$$M_{f2} = M_2 B_z$$



STRUCTURED LDDC DESIGN

General structuration of the controller:

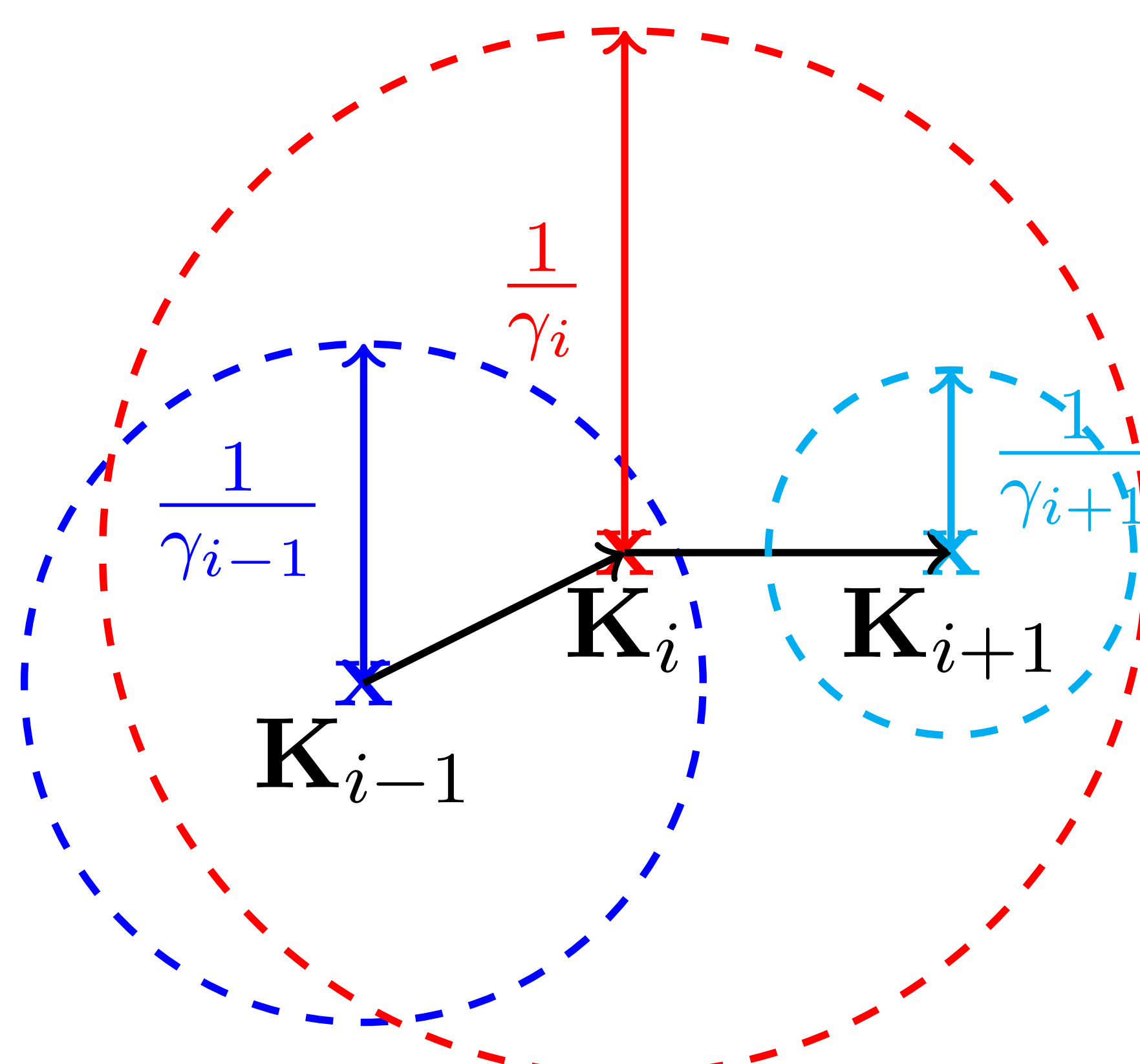
$$K(s, \theta) = \frac{1}{D(s, \theta)} N(s, \theta)$$

At iteration k , problem \mathcal{P}_k is solved:

$$\begin{aligned} \min_{\theta_k} & \|\mathbf{M}(\omega_i) - \mathbf{H}(\theta_k, \omega_i)\|_F^2 \\ \text{s.t.} & \|\mathbf{K}(\theta_k) - \mathbf{K}(\theta_{k-1})\|_\infty < \frac{1}{\beta_{k-1}} \\ & P\theta < 0 \end{aligned}$$

where

$$\mathbf{H}(\theta_k, \omega_i) = (I + \Phi_i \mathbf{K}(\theta_k, \omega_i))^{-1} \Phi_i \mathbf{K}(\theta_k, \omega_i)$$



REFERENCES

1. Kergus, P., Olivi, M., Poussot-Vassal, C., Demourant, F. (2019). *From reference model selection to controller validation: Application to Loewner Data-Driven Control*. IEEE L-CSS.
2. Cooman, A., Seyfert, F., Olivi, M., Chevillard, S., Baratchart, L. (2017). *Model-free closed-loop stability analysis: A linear functional approach*. IEEE TMTT.
3. Cooman, A., Seyfert, F., Amari, S. (2018). *Estimating unstable poles in simulations of microwave circuits*. IEEE IMS.
4. Van Heusden, K., Karimi, A., Bonvin, D. (2009). *Data-driven controller validation*. IFAC Symposium on System Identification.