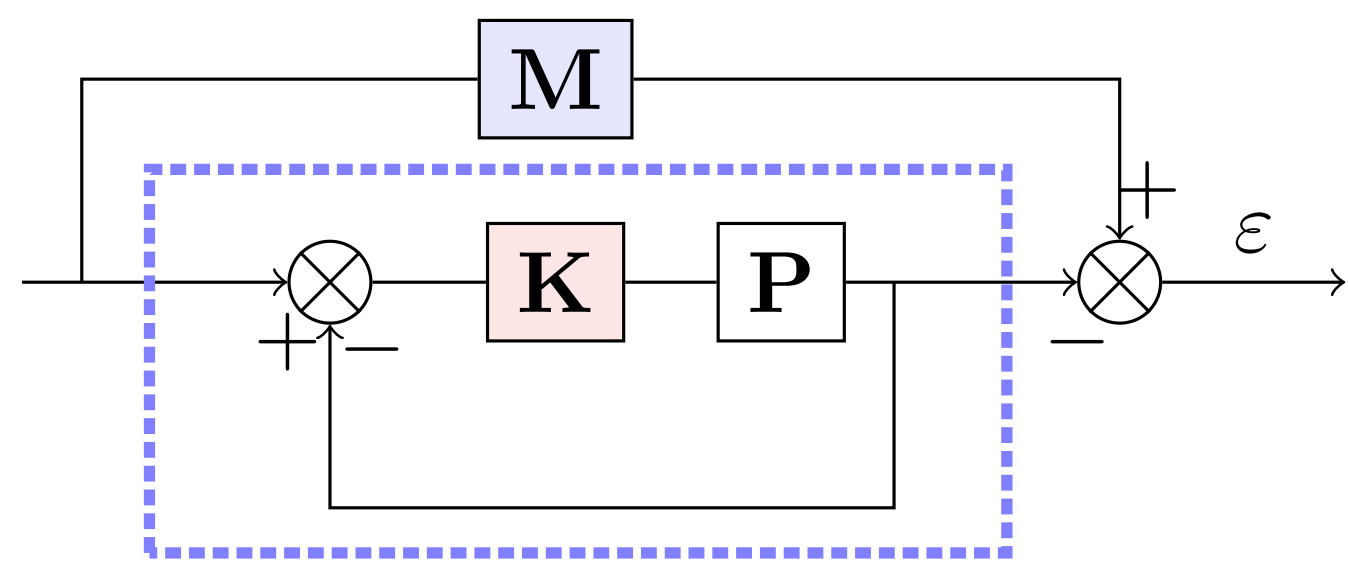


DATA-DRIVEN STABILITY ANALYSIS AND ENFORCEMENT FOR LOEWNER DATA-DRIVEN CONTROL

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LOEWNER DATA-DRIVEN CONTROL: GENERAL FORMULATION

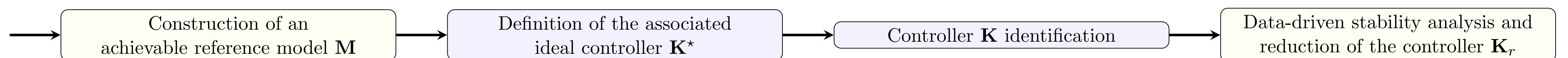


Proposed methodology

1. Computation of the ideal controller K^* frequency-response:
$$K^*(\omega_i) = \Phi_i^{-1} M(\omega_i) (I - M(\omega_i))^{-1}.$$
2. Interpolation and reduction of the ideal controller K^* through the Loewner framework.

Input data

- Frequency-domain data from the plant P : $\{\omega_i, \Phi_i\}, i = 1 \dots N.$
- Reference model $M.$
- Data $\{\omega_i, \Phi_i\}_{i=1}^N$
- Performance specifications



A simple example

$$P(s) = \frac{0.03616(s-140.5)(s-40)^3}{(s^2+1.071s+157.9)(s^2+3.172s+1936)} \quad M(s) = \frac{1}{0.01s^2+0.25s+1}$$

$$K^*(s) = k \frac{(s^2 + 1.071s + 157.9)(s^2 + 3.172s + 1936)}{s(s+10)(s-140.5)(s-40)^3}$$

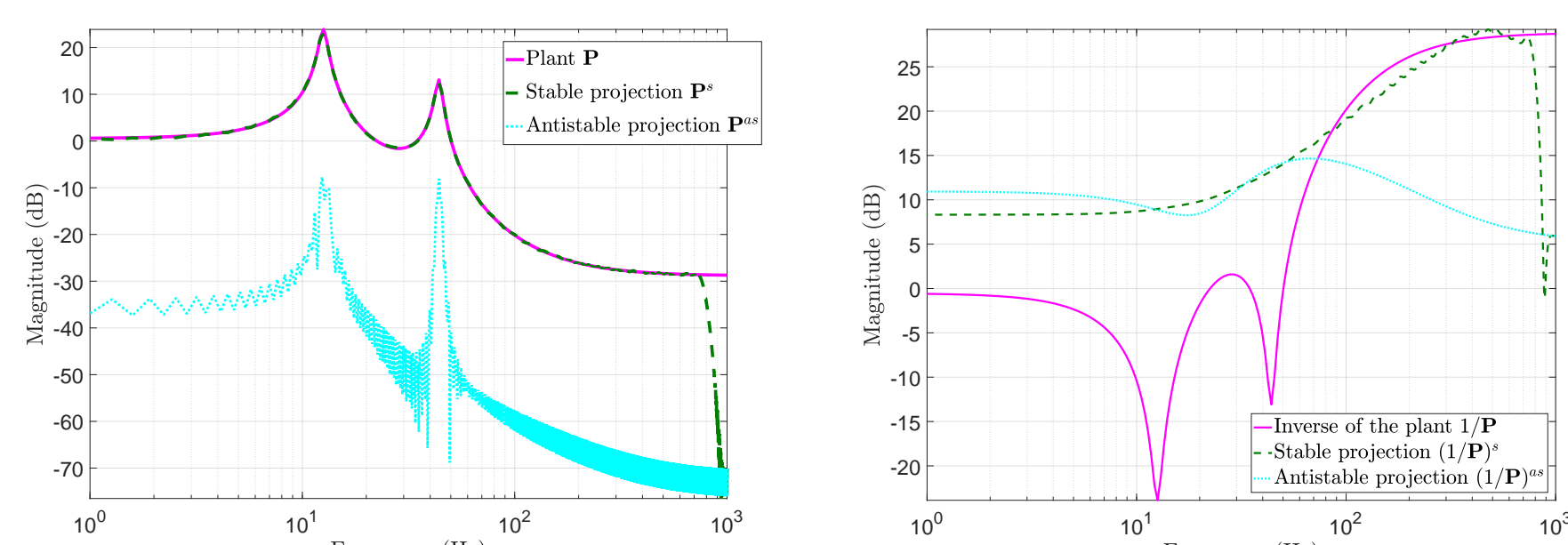
→ The reference model should be achievable by the plant.

$$\begin{cases} y_{z_i}^T P(z_i) = 0 \\ y_{p_j}^T P(p_j) = \infty \end{cases} \Rightarrow \begin{cases} y_{z_i}^T M(z_i) = 0 \\ M(p_j) y_{p_j} = y_{p_j} \end{cases}$$

→ A data-driven closed-loop stability analysis is needed.

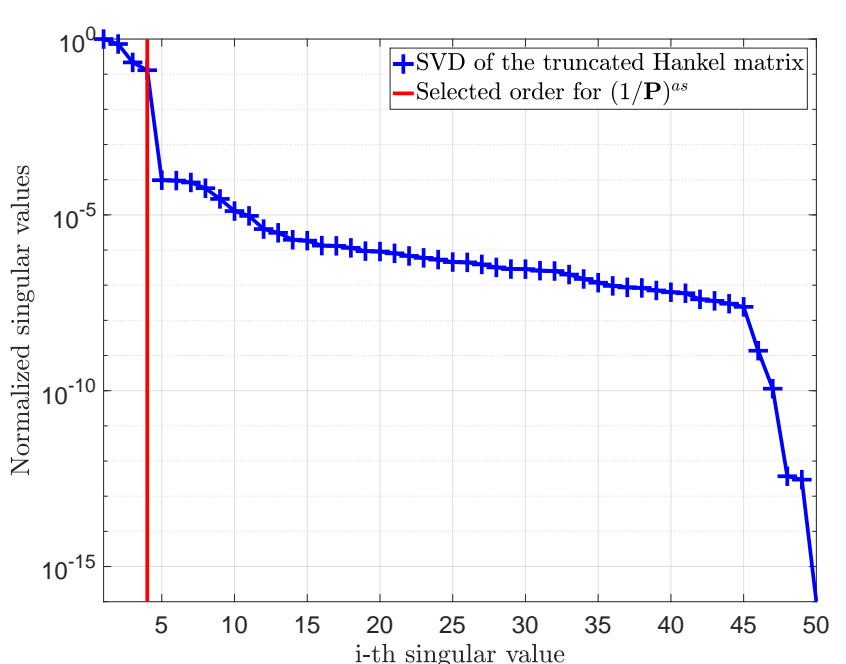
CHOICE OF THE REFERENCE MODEL

1) Projection of the available data to determine the nature of P :



→ The system is stable but Non-Minimum Phase (NMP).

2) Principal Hankel Components technique to determine the number of NMP zeros and obtain an estimate of the instabilities.



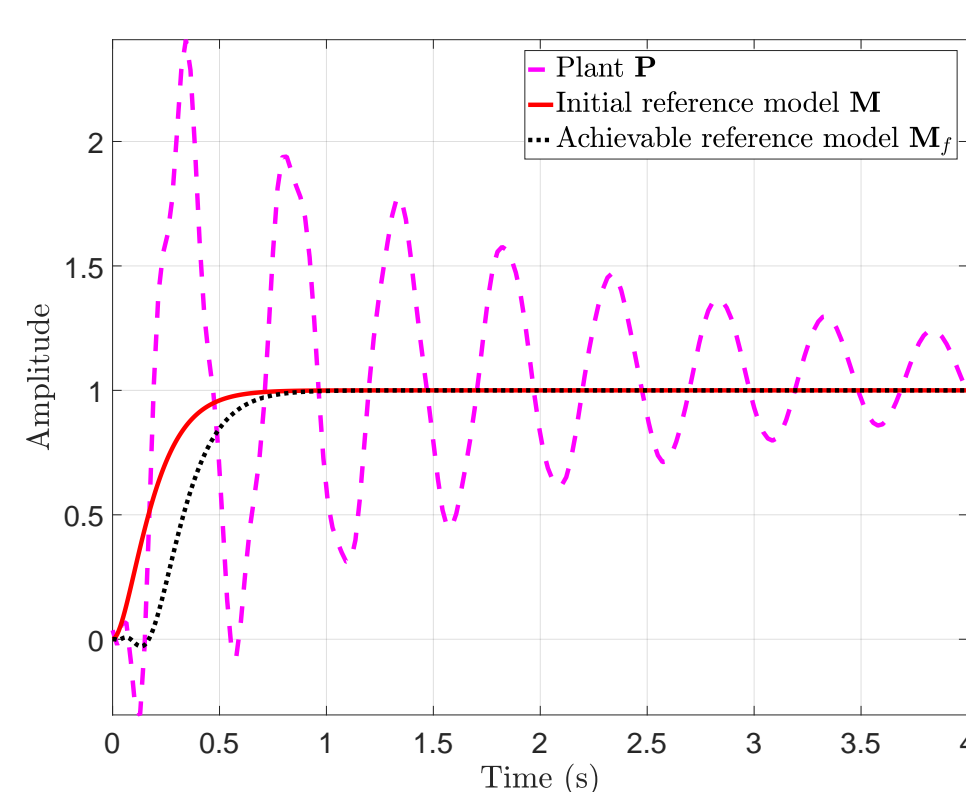
True z_i	Estimated \tilde{z}_i
140.5	140.58
40	41.3+2i
40	41.3+2i
40	37.4

3) Construction of an achievable reference model M_f .

$$M_f = M B_z$$

$$B_z(s) = \prod_{i=1}^{n_z} \frac{s - \tilde{z}_i}{s + \tilde{z}_i}$$

$$|B_z(\omega)| = 1$$



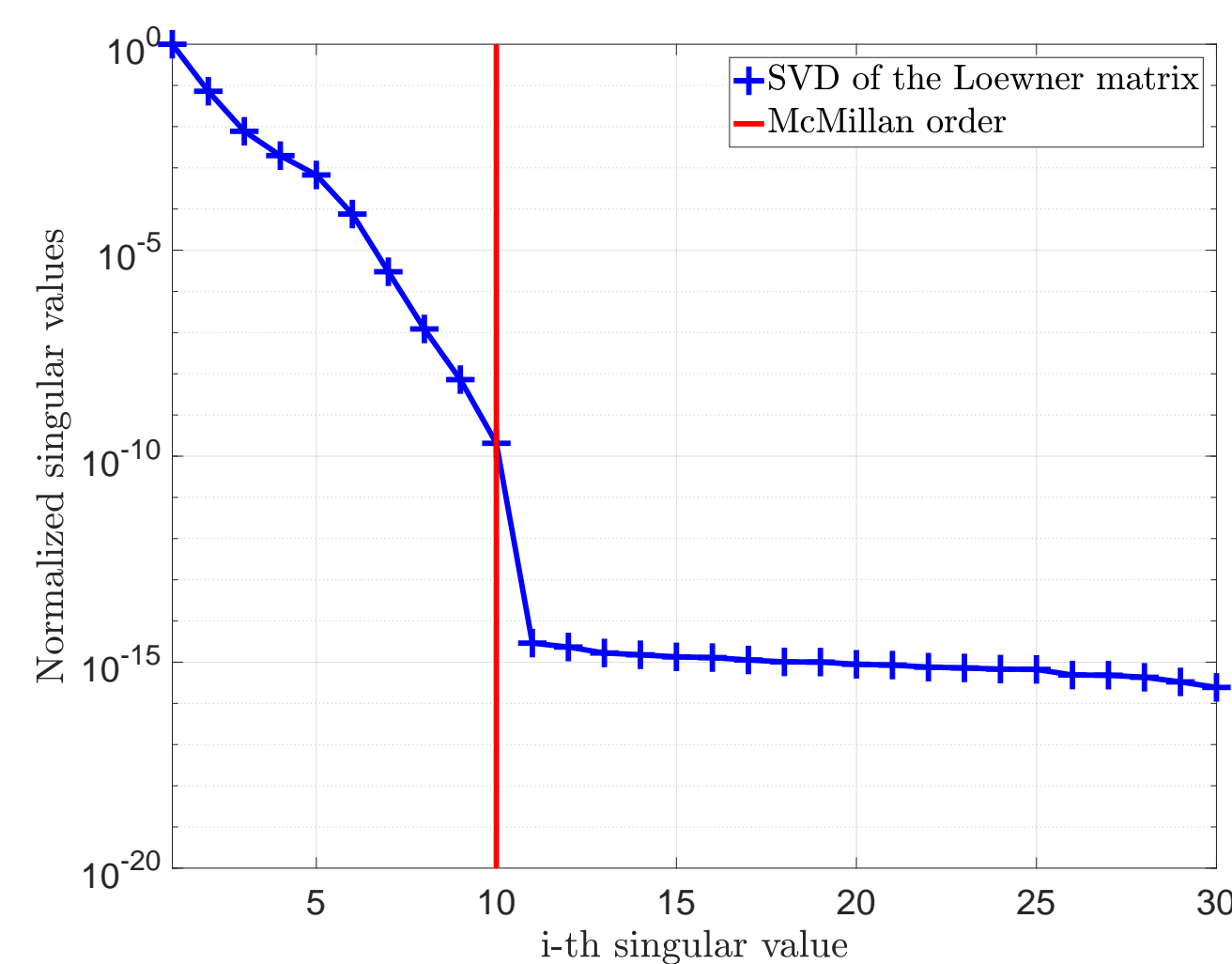
CONTROLLER IDENTIFICATION

The objective is to obtain a rational model $K = (E, A, B, C, D)$ such that:

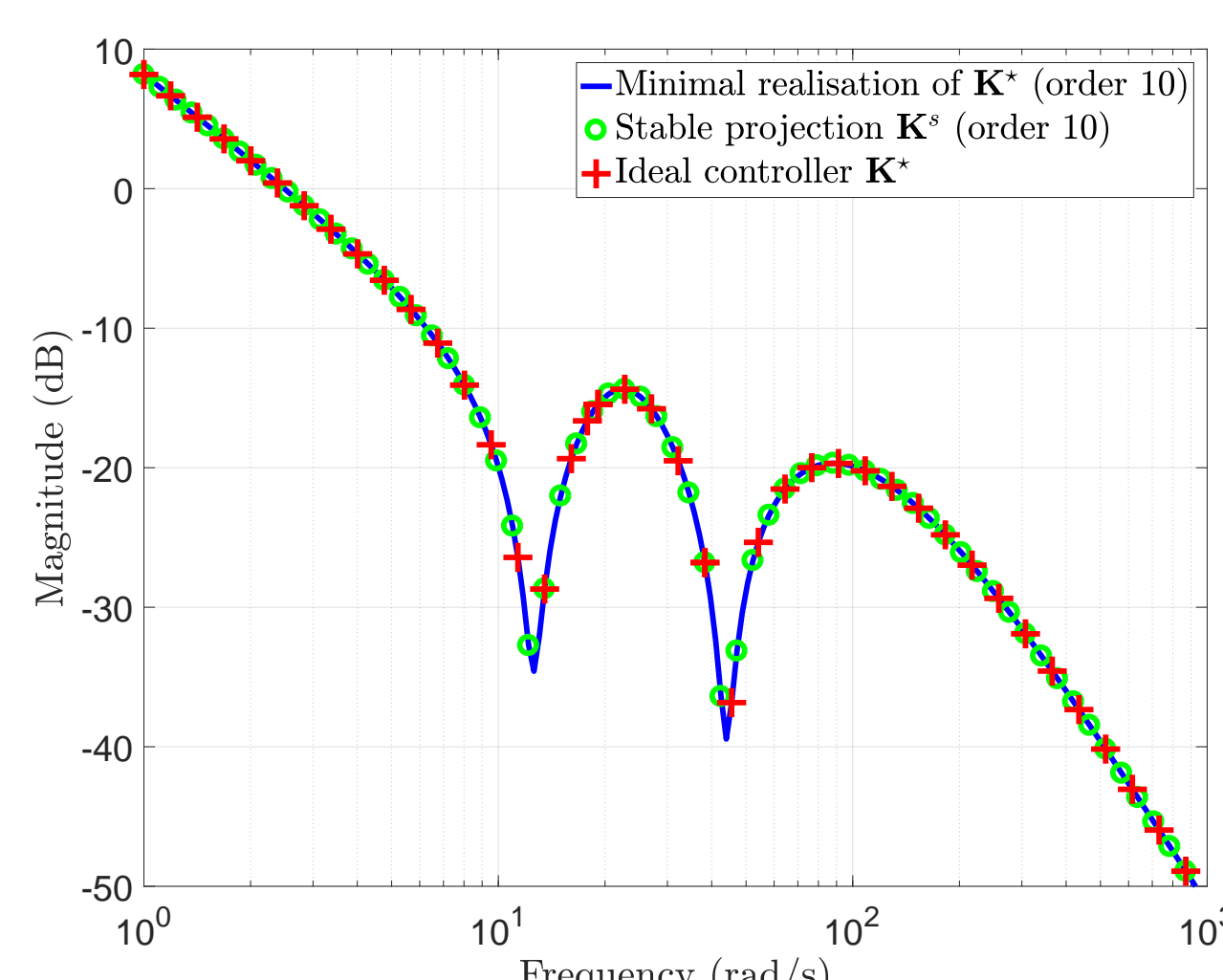
$$\forall i = 1 \dots N, K(\omega_i) = K^*(\omega_i).$$

→ Use of the Loewner pencil $[\mathbb{L}, \mathbb{L}_\sigma]$

1. Embedded order reduction

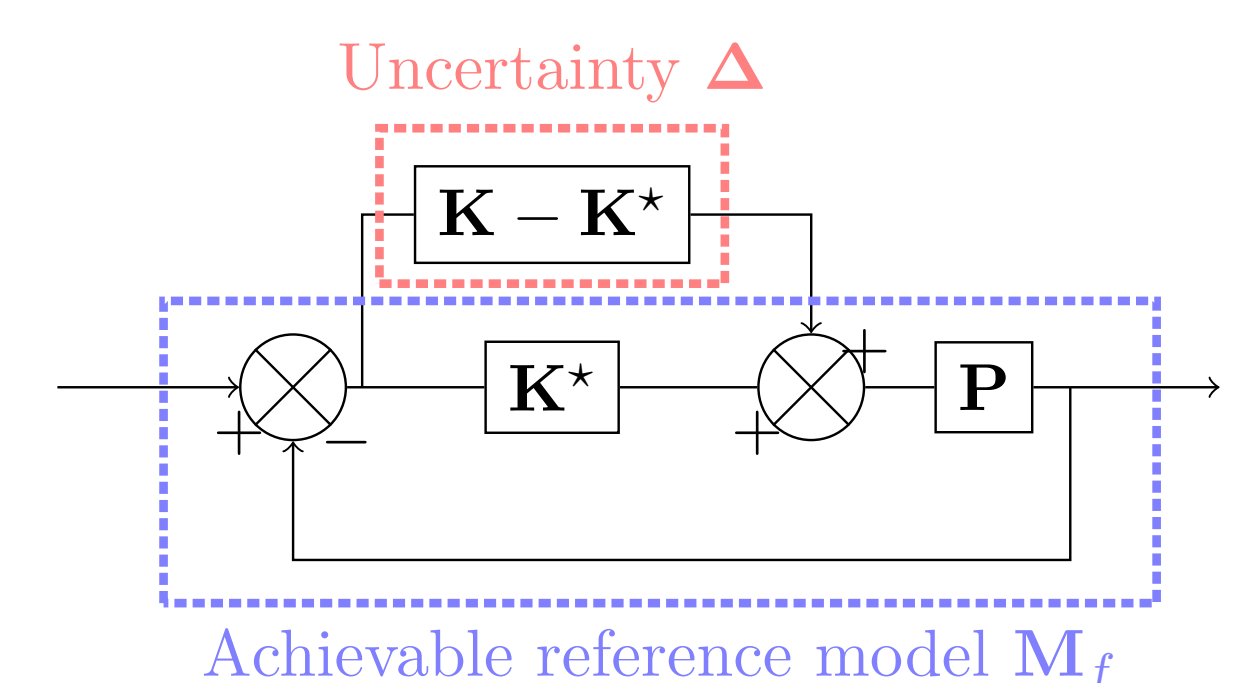


2. Stability of the identified model K



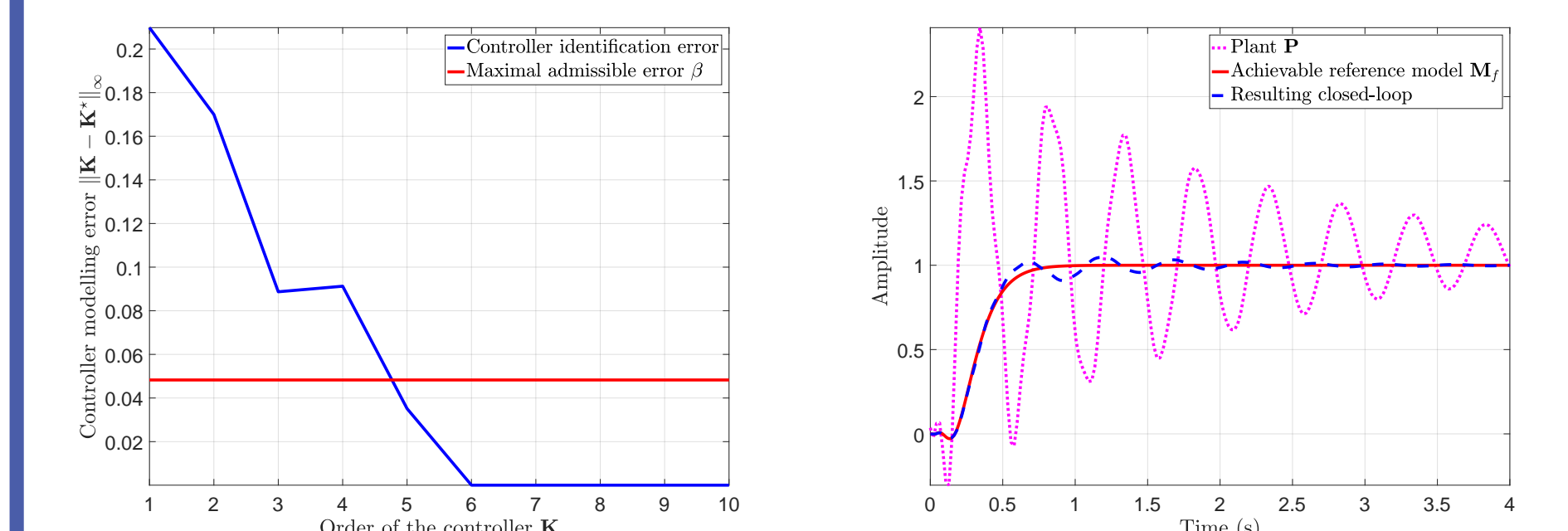
Proposed choice of M → no more compensation of instabilities in the open-loop!

CONTROLLER REDUCTION



The resulting closed-loop is well-posed and internally stable for all stable Δ such that $\|\Delta\|_\infty \leq \beta$ if and only if $\|(1 - M_f)P\|_\infty < \frac{1}{\beta}$.

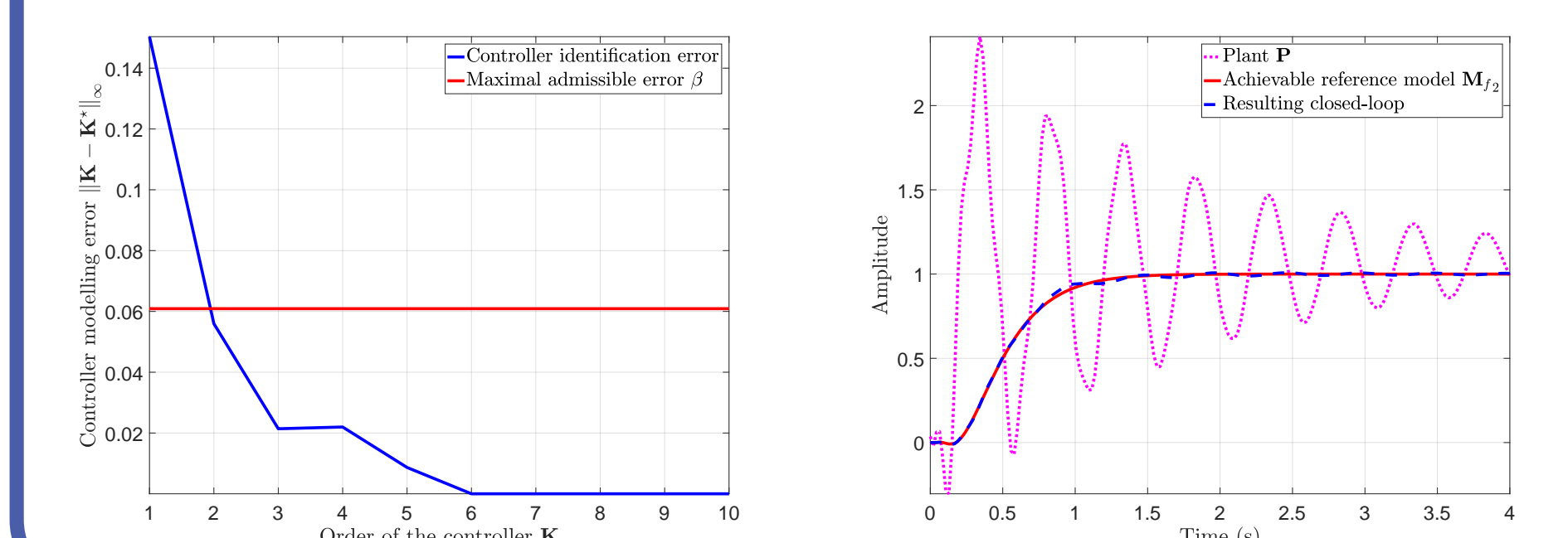
→ Limiting the controller modelling error allows to ensure closed-loop internal stability!



→ Conservatism of the small-gain theorem and importance of the choice of the initial specifications

$$M_2(s) = \frac{1}{0.04s^2 + 0.4s + 1}$$

$$M_{f2} = M_2 B_z$$



STRUCTURED LDDC DESIGN

General structuration of the controller:

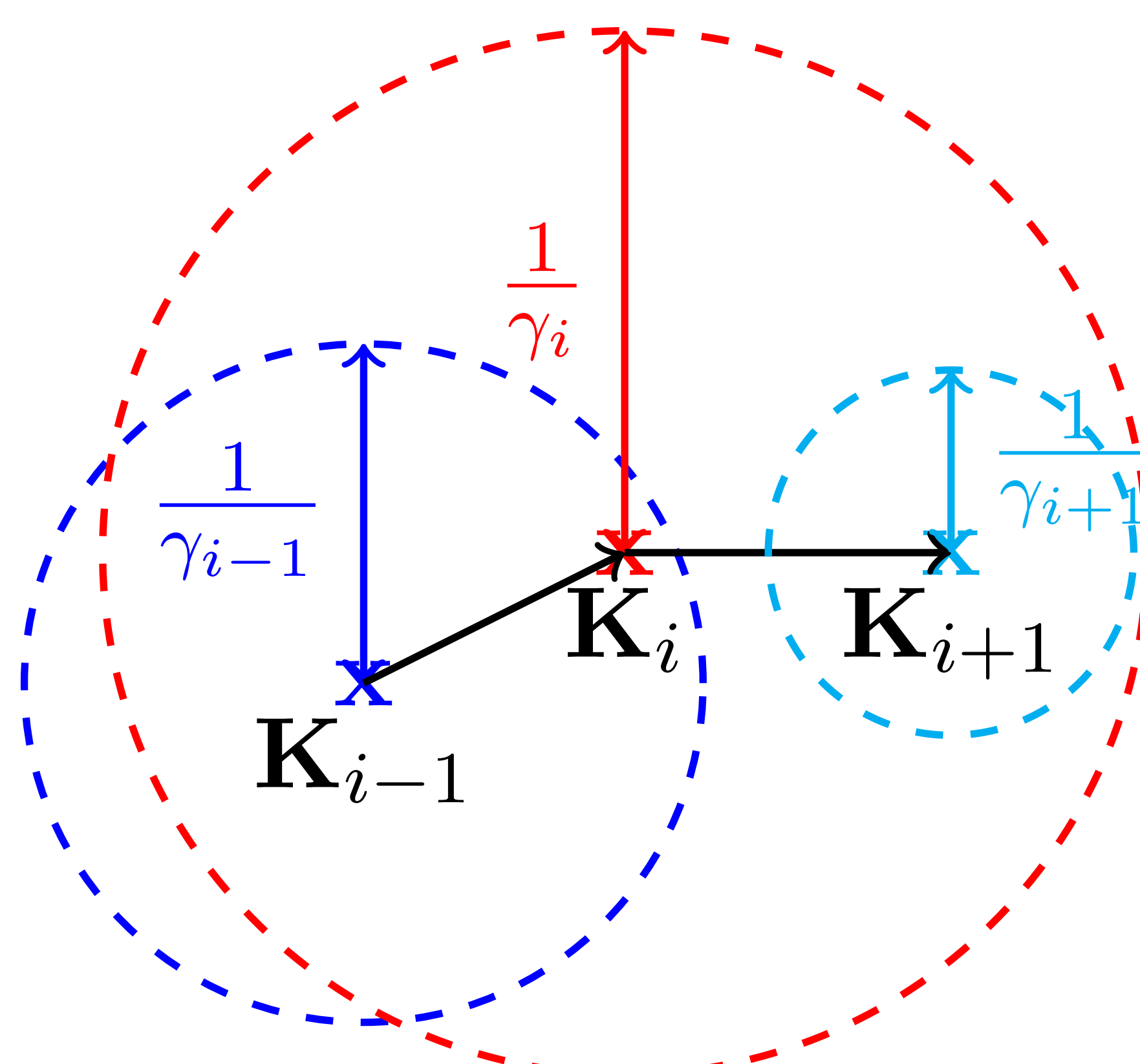
$$K(s, \theta) = \frac{1}{D(s, \theta)} N(s, \theta)$$

At iteration k , problem \mathcal{P}_k is solved:

$$\begin{aligned} \min_{\theta_k} & \|\mathbf{M}(\omega_i) - \mathbf{H}(\theta_k, \omega_i)\|_F^2 \\ \text{s.t.} & \|\mathbf{K}(\theta_k) - \mathbf{K}(\theta_{k-1})\|_\infty < \frac{1}{\beta_{k-1}} \\ & P\theta < 0 \end{aligned}$$

where

$$\mathbf{H}(\theta_k, \omega_i) = (I + \Phi_i \mathbf{K}(\theta_k, \omega_i))^{-1} \Phi_i \mathbf{K}(\theta_k, \omega_i)$$



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