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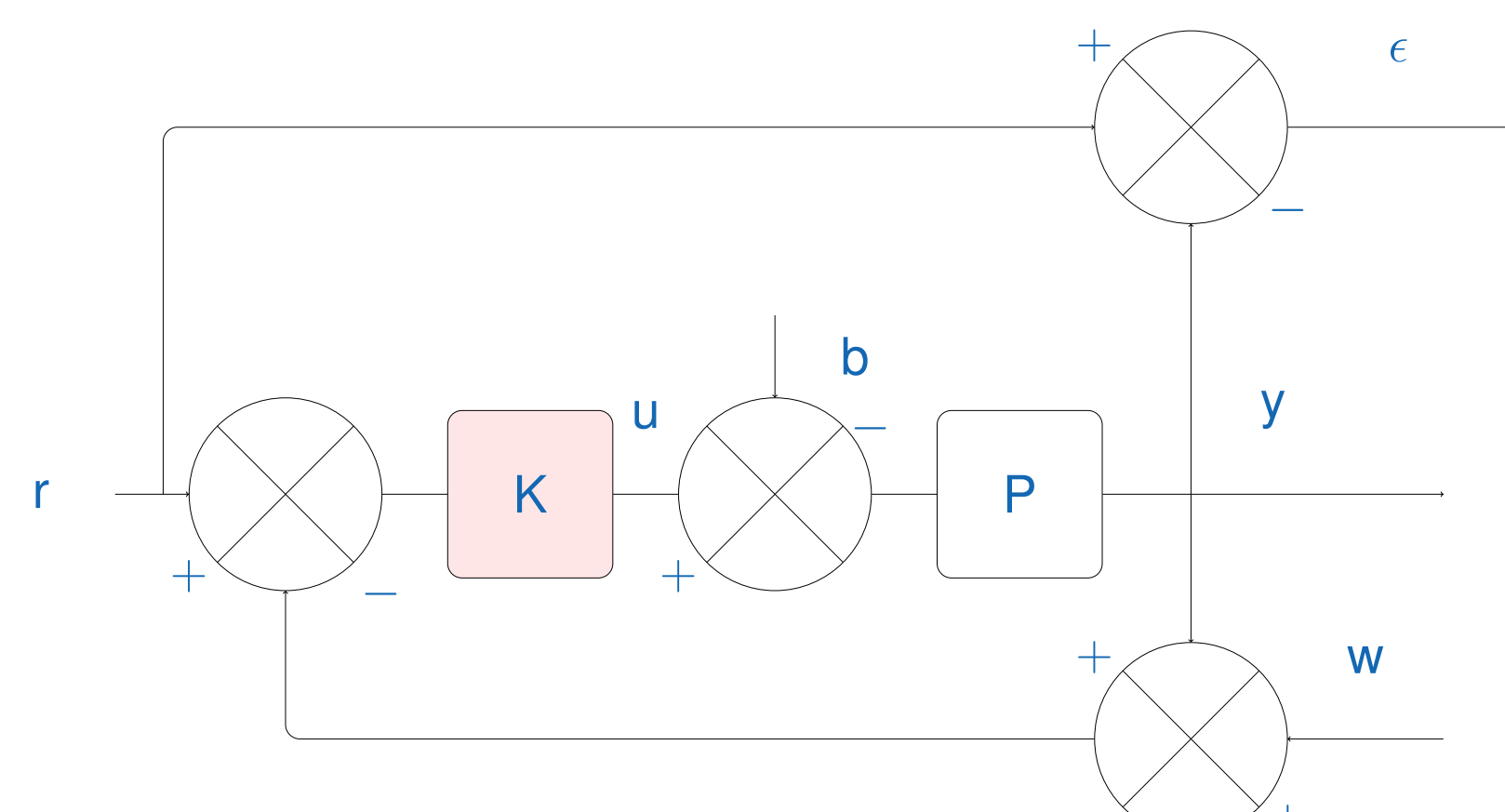
A control application to matching theory: Sensitivity minimization

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Background: sensitivity minimization



Sensitivity function: $S = T_{r \rightarrow \epsilon} = \frac{1}{1+PK}$

Transmission function: $T = T_{r \rightarrow y} = \frac{PK}{1+PK}$

Specifications:

1. **Closed-loop stability:** S is stable LTI and satisfies the following interpolatory conditions:

$$\begin{cases} S(z_k)=1 \quad \forall k = 1 \dots n_z \\ S(p_l)=0 \quad \forall l = 1 \dots n_p \end{cases}$$

where $\{z_k\}_{k=1 \dots n_z}$ and $\{p_l\}_{l=1 \dots n_p}$ are the unstable zeros and poles of the plant P .

2. **Limited overshoot:** $\|S\|_{\infty} \leq \gamma$

3. **Low static error:** $\min_S \max_{\theta \in (0, \theta_c)} |S(e^{j\theta})|$, θ_c is the cutoff frequency ($\lim_{t \rightarrow \infty} \epsilon(t) = \lim_{z \rightarrow 1} S(z)$)

Proposed method

Proposed method (algorithm in [1])

$$\min_f \max_{z \in I} |f(z)| \quad (1)$$

1. f is a Schur function

$f = BU$ with B inner and U outer of degree N

$|U| = \frac{P}{P+R}$ (Belevitch form with R fixed)

→ Parametrization by the polynomial $P \geq 0$ of degree $2N$ ($P \in \mathbb{P}_{2N}^+$)

2. f satisfies given interpolatory conditions $f(\alpha_i) = \beta_i$, $i = 1 \dots M$

i.e. $B(\alpha_i) = \frac{\beta_i}{U(\alpha_i)} = \gamma_i \iff \Delta(P) \geq 0$, $\Delta(p)_{i,k} = \frac{1-\gamma_i \bar{\gamma}_k}{i(\alpha_i - \bar{\alpha}_k)}$

→ The problem becomes a convex optimization with respect to P :

$$\min_{P \in \mathbb{P}_{2N}^+} \max_{z \in I} \frac{P(z)}{R(z)} \text{ s.t. } \Delta(P) \geq 0.$$

Finally, we have $\deg f \leq N + M - 1$

$$f(z) = \frac{1}{\gamma} S\left(\frac{1}{z}\right)$$

f is solution of (1) with $\begin{cases} f(\frac{1}{z_k}) = \frac{1}{\gamma} \quad \forall k = 1 \dots n_z \\ f(\frac{1}{p_l}) = 0 \quad \forall l = 1 \dots n_p \end{cases}$

→ S satisfies the specifications

A similar paradigm has been proposed in [2], the problem is formulated as a shaping one, S should be stable and satisfies:

$$\begin{cases} S(z_k)=1 \quad \forall k = 1 \dots n_z \\ S(p_l)=0 \quad \forall l = 1 \dots n_p \end{cases} \quad \|S\|_{\infty} \leq \gamma \quad \max_{\theta \in (0, \theta_c)} |S(e^{j\theta})| \leq \varepsilon$$

→ The static gain of S is only bounded, not minimized.

The resolution is done as follows:

1. Compute a first infinite dimensional guess that perfectly shape the criterion.
2. Approximate it with a reduced order using inverse problem of analytic interpolation.

Numerical example used in [2]

Inputs:

* The model of the plant P (to define the interpolatory conditions): $P(z) = \frac{1}{z-2}$;

* The interval I (cutoff frequency): $\theta_c = 0.25 \text{ rad.s}^{-1}$;

* The desired upper bound γ for $\|S\|_{\infty}$: $\gamma = 2.5$.

* The desired degree N for the sensitivity function S ($n = 3$ or $n = 5$ here).

$$\begin{cases} S(\infty)=0 \\ S(2)=1 \end{cases} \Rightarrow \begin{cases} f(0)=0 \\ f(0.5)=0.4 \end{cases}$$

→ This method avoid any weights selection, which is not immediate in traditional H_{∞} design.

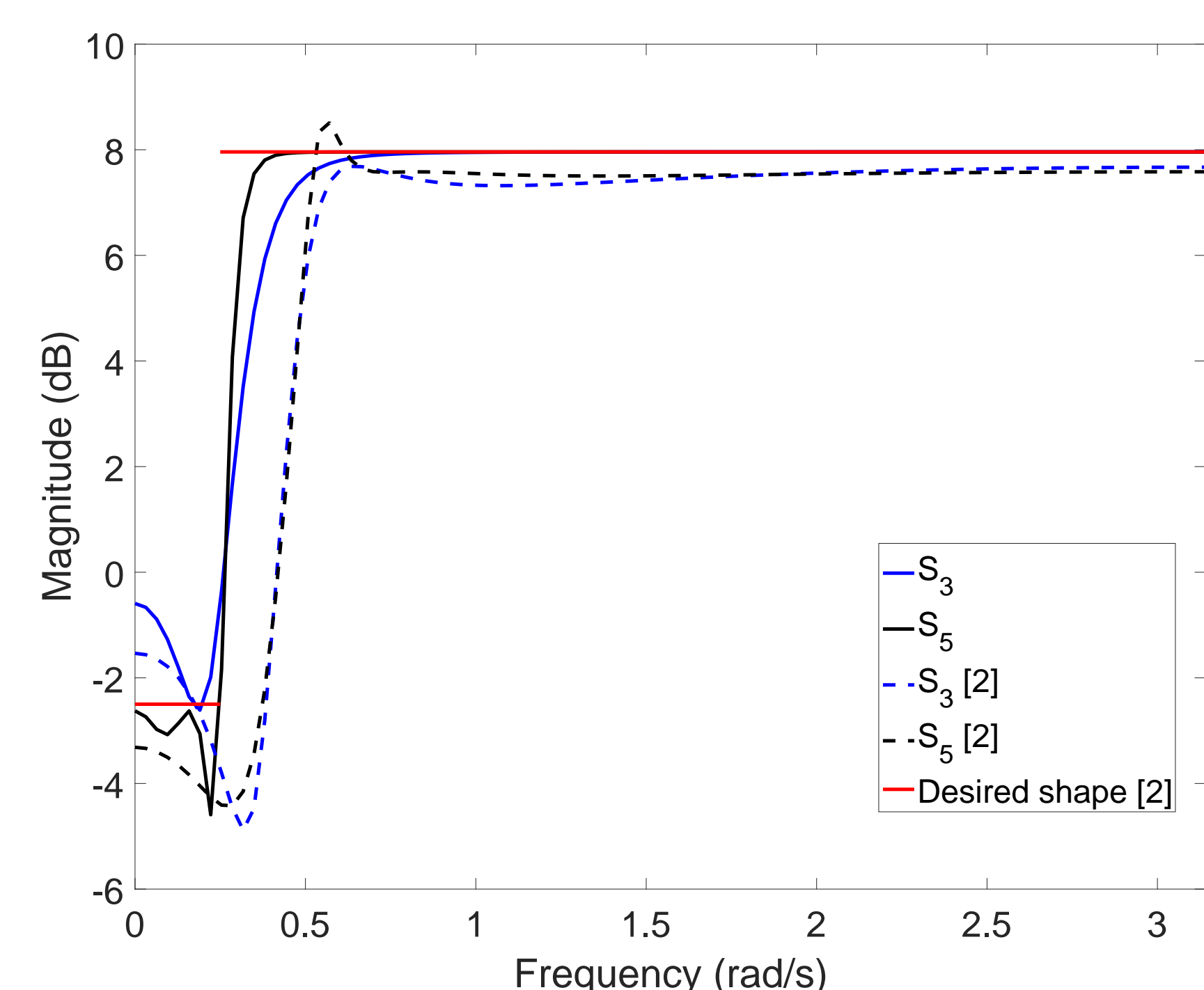


Figure 1: Sensitivity functions of degree 3 and 5 obtained in [2] and through the proposed method.

Outlooks

- ✓ Efficient way to design a closed-loop considering a LTI model of the plant
- ✓ Allows to find the lowest static error for a given order
 - Stability of the controller is not ensured
 - Generalization to MIMO cases
 - How to consider frequency dependant robustness?

Bibliography

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