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# Finite element modelling of Electroseismics and Seismoelectrics

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## 8 Abstract

9 In this chapter we describe a set of finite element formulations employed to approximate the  
10 solution to an extension to partially saturated porous media of Pride's equations, which is one  
11 of the most widespread theoretical frames that model the coupled electromagnetics and acous-  
12 tics of saturated porous media. We also show how these numerical algorithms can be imple-  
13 mented on multi-core computers, and analyse their performance, observing that the parallel  
14 efficiency of the algorithms decays slower when the resources of each computing core are used  
15 as close to their maximum as possible. Further, we perform a study of two dimensional PSVTM  
16 electroseismic conversions in a time-lapse carbon dioxide geological deposition monitoring sce-  
17 nario, setting the electromagnetic source in depth, and measuring the seismic responses both  
18 in wells and at the surface. We observe that, contrary to what is known from standard field  
19 settings, the interface responses play no important role in elucidating the CO<sub>2</sub> plume behaviour,  
20 and that the in-depth converted signals convey information about the carbon dioxide satura-  
21 tion in the plume.

## 22 1 Introduction

23 Geophysical methods of subsurface exploration are based on either seismic or electri-  
24 cal geophysical principles. The seismo-electromagnetic method combines both approaches, with  
25 the resolution of the seismics and the sensitivity of the electromagnetic methods to the fluids.  
26 It offers a non-invasive structure characterisation of the near surface earth from first few hun-  
27 dred meters up to the order of one thousand meters depth, in terms of fluids (water, oil, gas)  
28 [Dupuis *et al.*, 2007; Haines *et al.*, 2007a; Thompson *et al.*, 2007]. It is usual to use different  
29 terms, according to the used source: Seismoelectrics (SE) involves generating a seismic wave  
30 and measuring the electrical field contained within or generated by it [Haartsen and Pride, 1997;  
31 Haines *et al.*, 2007a,b], while electroseismics (ES) does the opposite by injecting a large amount  
32 of current into the ground and measuring the resulting seismic energy [Thompson *et al.*, 2005,  
33 2007]. First attempts on the seismic-electromagnetic effect were actually ES measurements,  
34 since an electric current was injected through the earth. The observations were thought to be  
35 due to changes in the earth resistivity under the influence of seismic waves. A first explana-  
36 tion was proposed to be linked to the fluctuations in the current through the electrolytic cell  
37 because of variations of the electro-chemical conditions at the surface of the electrodes, in-  
38 duced by the mechanical vibrations [Thyssen *et al.*, 1937]. Then different experimental set-ups  
39 could eliminate the effect of electrode surface [Thompson, 1939]; and later on Pride [1994];

40 *Butler et al.* [1996] showed that the resistivity modulation was not the relevant mechanism of  
41 the observed SE signals. Decades ago, *Thompson and Gist* [1993] observed conversions from  
42 electromagnetic to seismic energy at the siliciclastics Friendswood test site (Texas), with the  
43 presence of a sequence of high permeability water sands and low permeability shales over 300  
44 m depth. They showed through modelling that electroseismics are more sensitive to low per-  
45 meability formations whereas seismoelectrics are most sensitive to high permeability forma-  
46 tions. Over the last two decades some observations showed that the electroseismic conversions  
47 could yield conversions of higher energy efficiency. First successful demonstration that elec-  
48 troseismic conversions can distinguish between aquifers and gas sands and can be used at depths  
49 up to 1000 m using geophones placed on the surface of the earth were provided by [*Thomp-*  
50 *son et al.*, 2007; *Hornbostel and Thompson*, 2007].

51 The pioneering studies of *Reuss* [1809], *Wiedemann* [1852], *von Helmholtz* [1879], *Smolu-*  
52 *chowski* [1903] and *Frenkel* [1944] were completed in more recent times with the development  
53 of the corresponding theory. *Thompson and Gist* [1993] and *Pride* [1994] explained the elec-  
54 trokinetic coupling mechanism within the double electrical layer at the solid-fluid interface.  
55 Electro-osmotic flow occurs when an electric field acts on the electrolytes present in the flu-  
56 ids. This generates pressure gradients and, therefore, fluid flow and mechanical macroscopic  
57 perturbations, yielding the SE technique. Equally important is the reciprocal case called electro-  
58 filtration. In this one, an applied pressure gradient creates fluid flow and consequently, an ionic  
59 convection current, which in turn produces an electric field, phenomenon upon which ES is  
60 based. *Pride* [1994] derived the equations for both phenomena (ES-SE) whose linking factor  
61 is the so-called electrokinetic coupling coefficient. The uncoupled Biot's equations for fluid-  
62 saturated porous media [*Biot*, 1956a,b] and Maxwell's equations take place if this coefficient  
63 is zero; for a review on this or other theoretical frames for the electroosmotic and electrofil-  
64 tration phenomena, and hence for SE and ES see *Jouniaux and Ishido* [2012]; *Jouniaux and*  
65 *Zyserman* [2016].

66 The computation of approximate solutions to any of the several model equations gov-  
67 erning the rise of electromagnetic signals due seismic waves traversing a fluid saturated porous  
68 medium or conversely, the rise of seismic waves due to electromagnetic probes, is not a sim-  
69 ple task. We review now a small part of the literature on this topic, focusing ourselves mainly  
70 on the numerical treatment of *Pride's* equations, which are the ones we deal with in this chap-  
71 ter. *Haartsen and Pride* [1997] performed numerical modelling featuring seismic and electro-  
72 magnetic point sources on horizontally stratified media; they used a global matrix method to

73 obtain their results. They showed that Pride's equations can be decoupled in two modes, namely  
74 the SHTE (horizontal shear wave transverse electric field) mode, involving the seismic SH and  
75 transverse electric TE, and the PSVTM (transverse magnetic field of vertical P and S waves)  
76 mode, linking the seismic P-SV modes with the transverse magnetic TM mode; further they  
77 showed that the interface response was similar to the one of a vertical electric dipole situated  
78 right beneath the seismic source. *Mikhailov et al.* [1997] used this algorithm to analyse field  
79 measurements involving IR responses at a top soil-glacial till interface and later *Hu and Gao*  
80 [2011] extended this algorithm by including a moment tensor point source. *Han and Wang* [2001]  
81 proposed a finite element procedure in time-domain for modelling diffusive electric fields in-  
82 duced by pure shear waves. *Garambois et al.* [2002] presented a numerical simulation for SE  
83 events through the generalized reflection and transmission matrix method, extended afterwards  
84 by *Warden et al.* [2013] to unsaturated conditions to perform a study of the vadose zone. The  
85 works developed by *Pain et al.* [2005] and *Haines and Pride* [2006] describe a mixed finite  
86 element algorithm for ES in boreholes and a finite-difference procedure that applies ES in het-  
87 erogeneous media, respectively. Also, *Guan and Hu* [2008] introduced a finite-difference time-  
88 domain method with perfectly matched layer (PML) technique as boundary conditions in bore-  
89 hole geometries (cylindrical symmetry). They calculated ES and SE logging responses to es-  
90 timate the parameters of the porous medium around the borehole. A variation of this numer-  
91 ical technique was employed recently by *Zheng et al.* [2015] when simulating the electric field  
92 excited by the acoustic wave in a logging while drilling scenario. Two dimensional numer-  
93 ical tests about P-TM conversions using an implicit time stepping finite element algorithm in  
94 a commercial software were implemented by *Kröger et al.* [2014], while *Zhou et al.* [2014] stud-  
95 ied converted waves inside a borehole setting boundary conditions at the interface between fluid  
96 and porous media. Receivers along the vertical axis of the borehole detected the components  
97 of electric and magnetic field depending on the distance from the acoustic source. In a series  
98 of works *Grobbe and Slob* [2013, 2014]; *Grobbe et al.* [2014]; *Grobbe and Slob* [2016] pre-  
99 sented and applied the algorithm ESSEMOD, which is a layered-Earth analytically based sim-  
100 ulation code, implementing all existing possible seismo-electromagnetic and electro-magneto-  
101 seismic sourcereceiver combinations. This code can presently model fluid/porous medium/fluid  
102 transitions, thereby enabling studying typical seismo-electromagnetic laboratory wave prop-  
103 agation experiments, as the authors did in the third of these last references. In the last one,  
104 they studies the responses of different thin-bed packages, proving that the seismo- electromag-

105 netic responses are sensitive to changes in medium parameters in length scales much smaller  
106 than the seismic resolution.

107 Recently, using one dimensional versions of the numerical algorithms presented in this  
108 work, *Zyserman et al.* [2015] could demonstrate that the seismoelectric and seismomagnetic  
109 interface responses generated at boundaries of a layer containing carbon dioxide are sensitive  
110 to its CO<sub>2</sub> content; and *Munch and Zyserman* [2016] analysed how the seismoelectric inter-  
111 face responses vary when a superficial aquifer is contaminated with light or dense NAPLs.

112 In this chapter, the differential model consists in Maxwell and Biot equations; the nu-  
113 merical formulation uses a mixed finite element technique to approximate the electromagnetic  
114 wave fields and a non-conforming finite element procedure to approximate the solid displace-  
115 ments with the vector part of the Raviart-Thomas-Nedelec space of zero order to the fluid dis-  
116 placements for the seismic wave fields, see e.g. [*Raviart and Thomas*, 1975; *Nedelec*, 1980;  
117 *Douglas, Jr. et al.*, 1999; *Zyserman and Santos*, 2000; *Santos and Sheen*, 2007; *Santos*, 2009;  
118 *Zyserman et al.*, 2010; *Savioli et al.*, 2014] for other works where these spaces are used and  
119 analysed. The iterative non-overlapping domain decomposition algorithm proposed here [*San-*  
120 *tos and Sheen*, 2007; *Savioli et al.*, 2014] allows to solve problems with a large number of un-  
121 known and it is suitable to be implemented on computers with parallel architecture [*Zyserman*  
122 *and Santos*, 2000; *Gauzellino et al.*, 2009; *Zyserman et al.*, 2012]. We consider that events of  
123 feedback are neglected, and therefore Maxwell and Biot equations can be solved separately.  
124 Here, the fluid-saturated porous medium is characterized by effective models and viscoelas-  
125 tic behaviour [*Carcione*, 2001] also taken into account. With respect to the electromagnetic  
126 part, the electrical conductivity and electrokinetic coefficient are expressed as a function of the  
127 water saturation.

128 Our algorithm is applied to a case of time-lapse CO<sub>2</sub> geological sequestration scenario,  
129 where we analyse the seismic response to different saturations of CO<sub>2</sub>. **Finally, we would like**  
130 **to mention that although not included in this chapter, examples of applications of the presented**  
131 **algorithm to seismoelectric case studies can be found in [*Gauzellino et al.*, 2010a,b].**

## 132 **2 Simplifying and extending Pride's equations**

133 With the goal of making the SE/ES equations more tractable, it is usual to introduce sim-  
134 plifying assumptions by reducing the dimensionality of the equations and/or by decoupling the  
135 full set of Pride's equations in one of two different ways: if the electroosmotic feedback is ne-  
136 glected, the poroelastic equations can be decoupled from the electromagnetic ones; this is usu-

137 ally done in SE. If the electrofiltration feedback can be neglected, Maxwell's equations can  
 138 be decoupled from Biot's equations; this is the chosen way when dealing with ES problems.  
 139 In both cases the uncoupling can be safely carried out if the condition

$$\frac{\eta \mathcal{L}^2(\omega)}{k(\omega) \sigma^e} \ll 1 \quad (1)$$

140 is fulfilled [*Haines and Pride, 2006; Hu et al., 2007; Guan and Hu, 2008; Zyserman et al., 2010*];  
 141 here  $\eta$  stands for the fluid viscosity,  $\mathcal{L}(\omega)$  for the electrokinetic coupling coefficient,  $k(\omega)$  for  
 142 the dynamic permeability and  $\sigma^e$  for the electric conductivity. Most of the works present in  
 143 the literature take this condition as granted, and likewise we proceed in the present chapter.  
 144 In this way, to model a SE problem, Biot's equations are solved in a first instance to obtain  
 145 solid and fluid displacements generated by a seismic source, and in a second step, the elec-  
 146 tromagnetic responses are got; the fluid displacements are a constitutive part of the electro-  
 147 magnetic source. On the other hand, in an ES problem, Maxwell's equations are solved to get  
 148 the electric and magnetic fields created by electric currents, and the former is used in the source  
 149 term of the poroelastic equations to obtain, in a second step, the induced seismic wave fields.

## 150 **2.1 Problem dimensionality**

151 Considering the dimensionality of the problem, until now there has been, to the authors'  
 152 knowledge, just one attempt to solve Pride's equations involving finite sources and three di-  
 153 mensional geometries and fields [*Wang et al., 2013a*], and it has been done using the finite dif-  
 154 ferences technique. The difficulty in numerically modelling fully three dimensional problems  
 155 is the huge amount of unknowns involved, which implies the usage of very large computing  
 156 resources. Because of this reason, we restrict ourselves to show finite element algorithms for  
 157 two dimensional problems; that is, we consider in the next sections either seismic or electro-  
 158 magnetic sources generating two dimensional vector fields, which interact with two dimen-  
 159 sional Earth models and induce two dimensional responses.

## 160 **2.2 Extended models**

161 The extension of the original set of SE/ES equations to deal with partially saturated or  
 162 contaminated soils is usually carried out by having recourse to effective media models, as can  
 163 be seen in the works of [*Zyserman et al. [2010]; Warden et al. [2013]; Bordes et al. [2015]; Zy-*  
 164 *serman et al. [2015]; Munch and Zyserman [2016]*]. There exist also other approaches consid-  
 165 ering Biot-type formulations, such as the composite media approach proposed by [*Zyserman*

166 *et al.* [2012] to deal with the presence of methane hydrates, or the recent contribution by *Jar-*  
 167 *dani and Revil* [2015], where the theory is enlarged to deal with mixtures of immiscible flu-  
 168 ids. We use the effective media approach in the case study we present in this chapter -involving  
 169 a  $CO_2$  geological deposition situation-, so we show now how the model parameters are dealt  
 170 with. The rock physics models we show below are widely employed in the literature, but the  
 171 reader should recall that they are approximations and their validity must be tested, as we did  
 172 in this work. Moreover, other approximations do exist for most of the effective parameters [*Mavko*  
 173 *et al.*, 2009].

174 For the effective fluid mass density, calculated in terms of the water density  $\rho^w$ ,  $CO_2$   
 175 density  $\rho^{CO_2}$  and their respective saturations  $S_w$  and  $S_{CO_2}$  we use

$$\rho^E = \rho^w S_w + \rho^{CO_2} (1 - S_w), \quad (2)$$

176 where  $S_w + S_{CO_2} = 1$  is assumed. For the effective bulk modulus of such fluid mixture we  
 177 use *Brie et al.* [1995]

$$K^f = (K^w - K^{CO_2}) S_w^5 + K^{CO_2}; \quad (3)$$

178 the power five in this expression is chosen following *Carcione et al.* [2006a]. Here the  $CO_2$   
 179 is supercritical, so there is no gaseous phase in our model. The effective viscosity is computed  
 180 in terms of the mixture components viscosities  $\eta^l$ ,  $l = w, CO_2$  and water saturation  $S_w$  us-  
 181 ing *Teja and Rice* [1981]

$$\eta = \eta^{CO_2} \left( \frac{\eta^w}{\eta^{CO_2}} \right)^{S_w}. \quad (4)$$

182 In order to compute the saturated rock matrix properties, we proceed as follows: we consider  
 183 three different materials building the solid matrix, namely sand, silt and clay, and we call  $\gamma_{sand}$ ,  
 184  $\gamma_{silt}$  and  $\gamma_{clay}$  their respective volume fractions;  $\gamma_{sand} + \gamma_{silt} + \gamma_{clay} = 1$ . The mass density of the  
 185 aggregate  $\rho^s$  is given by the volume weighted mean of the respective components mass den-  
 186 sities,

$$\rho^s = \gamma_{sand} \rho^{s,sand} + \gamma_{silt} \rho^{s,silt} + \gamma_{clay} \rho^{s,clay}, \quad (5)$$

187 and the bulk mass density is calculated as usual,

$$\rho^b = \rho^s \phi + \rho^f (1 - \phi), \quad (6)$$

188 where  $\phi$  is the rock porosity. The bulk modulus  $K^s$  and shear modulus  $G^s$  of the mixture of  
 189 mineral grains are given by the Reuss average of the components bulk and shear moduli re-  
 190 spectively [*Mavko et al.*, 2009]

$$K^s = \left( \frac{\gamma_{sand}}{K^{s,sand}} + \frac{\gamma_{silt}}{K^{s,silt}} + \frac{\gamma_{clay}}{K^{s,clay}} \right)^{-1}, \quad (7)$$



191 and

$$G^s = \left( \frac{\gamma_{\text{sand}}}{G_{s,\text{sand}}} + \frac{\gamma_{\text{silt}}}{G_{s,\text{silt}}} + \frac{\gamma_{\text{clay}}}{G_{s,\text{clay}}} \right)^{-1}. \quad (8)$$

192 The solid matrix bulk modulus  $K^{fr}$  and shear modulus  $G^{fr}$  are then calculated by Krief's model  
193 as

$$K^{fr} = K^s(1 - \phi)^{3/(1-\phi)}, \quad G^{fr} = G^s(1 - \phi)^{3/(1-\phi)}. \quad (9)$$

194 The expressions in this equation have been used to deal with shaley sand data [Mavko *et al.*,  
195 2009].

196 The shear modulus of the effective fluid saturated rock matrix  $G_u$  is considered, as usual,  
197 equal to  $G^{fr}$ , but for the saturated matrix bulk modulus  $K_u$  we use Gassman's approach as  
198 follows [Gassmann, 1951; Santos *et al.*, 1992]

$$K_u = K^{fr} + \alpha^2 K^{av}, \quad K^{av} = \left[ \frac{\alpha - \phi}{K^s} + \frac{\phi}{K^f} \right]^{-1}, \quad \alpha_B = 1 - \frac{K^{fr}}{K^s}. \quad (10)$$

199 In this equation,  $K^{av}$  and  $\alpha_B$  are the so called fluid-storage and Biot-Willis coefficients re-  
200 spectively. The reader should recall here that Gassmann's model is valid for seismic frequen-  
201 cies, but for sonic or ultrasonic ones, other models taking into account squirt flow [Mavko and  
202 Jizba, 1991; Dvorkin and Nur, 1993; Dvorkin *et al.*, 1995] should be employed.

203 The model can be further extended if the diverse energy loss mechanisms present in the  
204 subsurface, which are not explicitly considered in Biot's equations are taken into account. One  
205 example of these mechanisms is the so called wave induce fluid flow, studied by several au-  
206 thors [Pride *et al.*, 2004; Rubino *et al.*, 2008; Picotti *et al.*, 2007; Song *et al.*, 2016]. With this  
207 goal, instead of considering particular loss processes, we consider that the studied region presents  
208 a viscoelastic behaviour, and we use Biot's correspondence principle [Biot, 1956c, 1962], re-  
209 placing the (real) relaxed elastic moduli  $G$  and  $K$  by complex frequency dependent viscoelas-  
210 tic moduli. To obtain their expressions we employ Liu's linear viscoelastic model [Liu *et al.*,  
211 1976], which reads for  $G$  (we deal with  $K$  in the same fashion):  $\widehat{G}(\omega) = G/(R(\omega) - iT(\omega)) =$   
212  $G_r(\omega) + iG_i(\omega)$ . The functions  $R(\omega)$  and  $T(\omega)$ , associated with a continuous spectrum of  
213 relaxation times, characterize the viscoelastic behaviour and are given by

$$R(\omega) = 1 - \frac{1}{\pi\widehat{Q}} \ln \frac{1 + \omega^2 T_1^2}{1 + \omega^2 T_2^2}, \quad T(\omega) = \frac{2}{\pi\widehat{Q}} \tan^{-1} \frac{\omega(T_1 - T_2)}{1 + \omega^2 T_1 T_2}.$$

214 The model parameters  $\widehat{Q}$ ,  $T_1$  and  $T_2$  are taken such that the quality factor  $Q(\omega) = \frac{G_r(\omega)}{G_i(\omega)}$   
215 is approximately equal to the constant  $\widehat{Q}$  within the frequency range we deal with in this work.  
216 Values of  $\widehat{Q}$  range from  $\widehat{Q} = 10$  for highly lossy materials to about  $\widehat{Q} = 1000$  for almost  
217 elastic ones. We used  $\widehat{Q} = 100$ ,  $T_1 = \frac{1}{2\pi} 10^6 s$  and  $T_2 = \frac{1}{2\pi} 10^{-7} s$  in all the examples shown

218 below. Although there are other models to account for the energy loss, e.g. [Chotiros and Isak-  
 219 son, 2004], we used Liu's model because it provides a constant quality factor over the frequency  
 220 range considered, the moduli are well behaved at  $\omega \rightarrow 0$  and ensures causality, which is a  
 221 reasonable behaviour for geophysical applications.

222 For the absolute permeability  $k_0$  of the porous medium we use [Mavko *et al.*, 2009]

$$k_0 = B \frac{\phi^3}{(1-\phi)^2} d_k^2 \quad \text{where} \quad \frac{1}{d_k} = \frac{\gamma_{sand}}{r_{sand}} + \frac{\gamma_{silt}}{r_{silt}} + \frac{\gamma_{clay}}{r_{clay}}. \quad (11)$$

223 Here  $B$  is a geometric constant (we assume  $B = 0.003$  [Carcione and Picotti, 2006]) and  
 224  $r_*$  is the radius of the different rock constituent particles.

225 In order to characterize the electric conductivity of the effective fluid saturated solid ma-  
 226 trix we use the expression recently proposed by Warden *et al.* [2013], extending Pride's orig-  
 227 inal formula [Pride, 1994, Eq.(242)] to the realm of partially saturated media:

$$\sigma_{eff}^*(S_w, \omega) = \frac{S_w^n}{F} \sigma^w + \frac{2}{F} \frac{C_{em} + C_{os}^*(\omega)}{\Lambda} \quad (12)$$

228 The first term in this equation -where  $F = \phi^{-m^c}$  stands for the formation factor,  $m^c$  being  
 229 the cementation coefficient- is Archie's law for a partially saturated medium, while the sec-  
 230 ond term accounts for the surface conductivity. The water electrical conductivity

231  $\sigma^w = \sum_{l=N_{a^+}, Cl^-} (e z_l)^2 b_l N_l$ , where  $e = 1.6 \times 10^{-19}$  C is the electron electric charge,  
 232 and  $z_l$  is the ions' valence, taken to be one for both species. The ions' mobility  $b_l$  and con-  
 233 centration  $N_l$  (depending on the salinity  $C_0$ ) are calculated following Carcione *et al.* [2003].

234 In the second term, accounting for the surface conductivity, the factor  $C_{em}$  [S] is the excess  
 235 conductance associated with the electromigration of double layer ions;  $C_{os}^*(\omega)$  [S] is the frequency-  
 236 dependent electro-osmotic conductance due to electrically induced streaming of the excess double-  
 237 layer ions and  $\Lambda$  [m] is the above presented pore-geometry dependent factor. We remark here  
 238 that, as in Brovelli *et al.* [2005] and Warden *et al.* [2013], the surface conductivity is assumed  
 239 to be independent of water saturation  $S_w$ , because under realistic saturation ranges (residual  
 240 water saturation  $S_{wr} \geq 10\%$ ) the thickness of the wetting phase layer on the pore surface  
 241 is always larger than the Debye length  $d^l$ . This also means that all fluid related properties in-  
 242 volved in the calculation of the surface conductivity and of the electrokinetic coupling -see  
 243 below- are just those of water.

244 Again, following Warden *et al.* [2013], we propose for the effective fluid saturated me-  
 245 dia the following electrokinetic coupling:

$$\mathcal{L}_0(S_w) = -\frac{\phi}{\alpha_\infty} \frac{\varepsilon^w \zeta^p}{\eta^w} \left(1 - 2 \frac{d^l}{\Lambda}\right) S_w^n \mathcal{C}(S_w), \quad (13)$$

246 In this equation  $\alpha_\infty = \phi F$  is the tortuosity,  $\varepsilon^w$  is the water electric permittivity,  $n$  is Archie's  
 247 saturation exponent (taken to be equal to the cementation exponent  $m^c$ ),  $d^l$  is the Debye length  
 248 and  $\mathcal{C}(S_w)$  is a function relating the streaming potential coefficient obtained under partial sat-  
 249 uration conditions to the one corresponding to full saturation conditions. Several authors have  
 250 investigated this relation from both theoretical and experimental viewpoints. In the study of  
 251 *Strahser et al.* [2011], different models for the streaming potential coefficient (SPC) depen-  
 252 dence on  $S_w$  were considered, namely the ones by *Perrier and Morat* [2000], *Revil et al.* [2007],  
 253 *Guichet et al.* [2003] and *Allègre et al.* [2010]. Characterizing this dependence by a power law  
 254 has been also proposed to model the relative electrokinetic coefficient in an imbibition exper-  
 255 iment [*Saunders et al.*, 2008]. *Jackson* [2010] also used a capillary tubes model making ex-  
 256 plicit the SPC dependence with water saturation, relative permeability and relative charge den-  
 257 sity. Further models derived from considerations on how the excess charge dragged by the wa-  
 258 ter varies with water saturation may be mentioned, as those derived by *Mboh et al.* [2012] and  
 259 *Jougnot et al.* [2015]. Recently, *Allègre et al.* [2012] modelled both Richards' equation for hy-  
 260 drodynamics and Poisson's equation for electrical potential for unsaturated conditions, using  
 261 a 1-D finite element method. They concluded, based on laboratory experiments [*Allègre et al.*,  
 262 2010, 2011] and using these equations, that the unsaturated electrokinetic coefficient should  
 263 have a non-monotonous behaviour. Moreover experimental results from periodic succession  
 264 of drainage and imbibition cycles on sand suggested that the airwater interface that develops  
 265 for unsaturated conditions polarizes and therefore generates an electrical response [*Allègre et al.*,  
 266 2014]. Finally *Allègre et al.* [2015] showed that the interface between water and air should also  
 267 be taken into account, since this interface is negatively charged, as the interface between the  
 268 rock matrix and the water. Moreover during a drainage the amount of this interface does not  
 269 decrease with decreasing water saturation, but first increases before decreasing, leading to a non-  
 270 monotonic behaviour of the resulting SPC [*Allègre et al.*, 2015].

For the present we select the following relation, displaying a non monotonic dependence  
 with water saturation [*Allègre et al.*, 2010]:

$$\mathcal{C}(S_w) = \left( \frac{S_w - S_{wr}}{1 - S_{wr}} \right) \left[ 1 + 32 \left[ 1 - \left( \frac{S_w - S_{wr}}{1 - S_{wr}} \right) \right]^{0.4} \right], \quad S_{wr} = 0.20, \quad (14)$$

271 in Fig. 1 the behaviour of  $\mathcal{C}(S_w)$ , where its sharp decrease for  $S_w \geq 0.8$  is remarkable.

272 Finally, notice that in order to write Eq. (13), it was assumed in *Warden et al.* [2013]  
 273 that the surface electrical conductivity is negligible against the electrical bulk conductivity; a  
 274 requirement fulfilled in this work due to chosen values for the salinity  $C_0$ . On the other hand,  
 275 we compute the zeta potential  $\zeta^P$  as  $\zeta^P = 0.008 + 0.026 \log_{10}(C_0)$  [*Pride and Morgan*, 1991].

### 3 Domain decomposition and Finite elements formulation

In this section the treatment of the ES PSVTM-mode is described; the ES SHTE-mode and both modes of the SE case treated in similar way. We assume that the chosen soil properties and seismic frequency range guarantee that any considered angular frequency  $\omega$  renders displacements currents that are negligible against conduction currents, i.e.  $\epsilon\omega \ll \sigma$ , and that it is well below Biot's critical frequency. Because of this reason, we use the static values  $k_0$  and  $\mathcal{L}_0$  instead of the dynamic ones  $k(\omega)$  and  $\mathcal{L}(\omega)$  for the permeability and electrokinetic coupling coefficient respectively.

We present below different kinds of mixed finite elements, which we use to approximate the solutions to both Maxwell's equations and Biot's equations; noting that previously to this step we apply to them domain decomposition (DD) procedures. The DD technique can be used at different levels [Toselli and Widlund, 2005]: continuous, discrete, or in the solution of the linear systems arising from the approximation of different partial differential equations. The domain decomposition procedures (DD) here presented belong to the continuous level, and are iterative methods that converge to the solution of the respective global procedures [Santos, 1998, 2009]; for different implementations, mainly involving the first and third mentioned levels, the reader can see, for example, references Japhet and Nataf [2001]; Gander *et al.* [2002, 2004].

The idea is to solve in parallel a collection of elliptic problems in the space-frequency domain with absorbing boundary conditions at artificial boundaries. Then, the solution in the time domain is obtained using the inverse Fourier transform.

Among the advantages of the presented procedure to solve wave propagation phenomena we can indicate:

- DD combined with any finite element method bypasses storage and solution of the large linear systems.
- In structured finite element meshes, DD is specifically designed to profit of the parallel architecture.
- Nonconforming finite element space significantly reduces the amount of information exchanged among processors.
- The rate of convergence of the iterative algorithm can be estimate using nonconforming finite element spaces.

### 3.1 Maxwell's equations

Let us then start by considering a domain  $\Omega = \Omega^a \cup \Omega^s$  comprising air and sub-  
 surface, with boundary  $\Gamma$ . The domain  $\Omega$  is partitioned into non overlapping subdomains  $\Omega_j$  with  
 boundary  $\Gamma_j$  as shown in Fig.2(a). Let also  $\Omega_j$  and  $\Omega_k$  be two adjacent subdomains of the do-  
 main decomposition, and  $\Gamma_{jk}$  their common boundary. Assuming that we are dealing with the  
 TM mode, where the present electromagnetic fields are  $E \equiv (E_x(x, z, \omega), E_z(x, z, \omega))$  and  
 $H \equiv H_y(x, z, \omega)$ , the domain decomposed formulation of Maxwell's equations reads [Zy-  
 serman *et al.*, 1999; Zyserman and Santos, 2000]:

$$\sigma^e E_j - (-\partial_z H_j, \partial_x H_j) = J^m \quad \text{in } \Omega_j, \quad (15)$$

$$\partial_z E_{x,j} - \partial_x E_{z,j} + i\omega\mu H_j = 0 \quad \text{in } \Omega_j, \quad (16)$$

$$H_j + \beta_{jk} E_j \cdot \chi_j = -\beta_{jk} E_k \cdot \chi_k + H_k \quad \text{on } \Gamma_{jk} \quad (17)$$

$$H_k + \beta_{jk} E_k \cdot \chi_k = -\beta_{jk} E_j \cdot \chi_j + H_j \quad \text{on } \Gamma_{kj} \quad (18)$$

$$(1-i)\sqrt{\frac{\sigma^e}{2\omega\mu}} E_j \cdot \chi_j + H_j = 0 \quad \text{on } \Gamma. \quad (19)$$

The first two of this set of equations are Ampere-Maxwell's and Faraday's respectively; in them  
 we denote the partial derivative with respect to coordinate  $x$  as  $\partial_x$ ;  $J^m$  is the external source  
 creating the TM polarized fields and  $\mu$  is the magnetic permeability. Eqs. (17)-(18) are Robin  
 boundary conditions [Douglas *et al.*, 1993; Kim, 1995] imposed on the boundaries between  
 any two domains to ensure consistency of the global solution;  $\chi$  is a vector tangent to the con-  
 sidered boundary and  $\beta$  a complex parameter; it is used to improve the efficiency of the it-  
 erative algorithm defined below. Finally, Eq. (19) is an absorbing boundary condition (ABC)  
 approximating the Silver-Müller radiation condition for infinite domains imposed on the bound-  
 ary  $\Gamma$  of the computational domain  $\Omega$  [Sheen, 1997]. In order to approximate the solution to  
 these equations an iterative hybridized mixed domain decomposed finite element procedure  
 is implemented [Zyserman *et al.*, 2010, 2012]. As already mentioned, the main concept un-  
 derlying this method is to split the problem in a collection of small ones whose individual so-  
 lutions can be easily computed.

Consider then, as approximating mixed finite element spaces the following ones

$$V^h = \{E^h \in L^2(\Omega) : E^h|_{\Omega_j} \in P_{0,1} \times P_{1,0}\}, \quad (20)$$

$$W^h = \{H^h \in L^2(\Omega) : H^h|_{\Omega_j} \in P_{0,0}\}, \quad (21)$$

$$V_j^h = V^h|_{\Omega_j}, \quad W_j^h = W^h|_{\Omega_j}. \quad (22)$$

328 Here  $P_{1,0}$  denotes a polynomial of degree less or equal 1 in  $x$  and less or equal 0 in  $z$ . Then,  
 329 in each element, we approximate the magnetic field with a two dimensional vector, whose com-  
 330 ponents are first order polynomials and the electric field with a constant. Therefore, these spaces  
 331 have five degrees of freedom associated with each element, four to the electric field and one  
 332 to the magnetic field, respectively. These degrees of freedom for TM mode are at the centres  
 333 of the faces and centre of each element as shown in Fig.2(b).

334 Let us assume that the finite element partition exactly coincides with the domain decom-  
 335 position, and denote with  $(\cdot, \cdot)$  and with  $\langle \cdot, \cdot \rangle$  the inner product in an element and on the bound-  
 336 ary of an element respectively. Using these conventions, the iterative algorithm implemented  
 337 for each frequency  $\omega$  reads:

338 Find  $(E_j^{h,n+1}, H_j^{h,n+1}, \ell_{jk}^{h,n+1}) \in V_j^h \times W_j^h \times \Psi_{jk}^h$  such that

$$\begin{aligned} & (\sigma^e E_j^{h,n+1}, \partial_z \psi - \partial_x \psi)_{\Omega_j} - (H_j^{h,n+1}, \psi)_{\Omega_j} + \sum_{\Gamma_{jk} \cap \Gamma = \phi} \langle \beta_{jk} E_j^{h,n+1} \cdot \chi_j, \psi \cdot \chi \rangle_{\Gamma_{jk}} \\ & + \langle (1-i) \sqrt{\frac{\sigma^e}{2\omega\mu}} E_j^{h,n+1} \cdot \chi_j, \psi \cdot \chi_j \rangle_{\Gamma \cap \Gamma_j} = \\ & - (J^m, \psi)_{\Omega_j} - \sum_{\Gamma_{jk} \cap \Gamma = \phi} \langle \beta_{jk} E_k^{h,*} \cdot \chi_k - H_k^{h,*}, \psi \cdot \chi \rangle_{\Gamma_{jk}}, \quad \psi \in V_j^h, \end{aligned} \quad (23)$$

$$((\partial_z E_j^{h,n+1} - \partial_x E_j^{h,n+1}), \varphi)_{\Omega_j} + (i\omega\mu H_j^{h,n+1}, \varphi)_{\Omega_j} = 0, \quad \varphi \in W_j^h, \quad (24)$$

$$\lambda_{jk}^{h,n+1} = \lambda_{kj}^{h,*} - \beta_{jk} (E_j^{h,n+1} \cdot \chi_j + E_k^{h,*} \cdot \chi_k) \quad \text{on } \Gamma_{jk}, \quad \Gamma_{jk} \cap \Gamma = \phi. \quad (25)$$

339 The mentioned complex parameter  $\beta$  is chosen as  $\beta_{jk} = \frac{1}{2}(\sqrt{\frac{\sigma_j^e}{2\omega\mu}} + \sqrt{\frac{\sigma_k^e}{2\omega\mu}})$ . The system  
 340 of equations (23)-(25) yields an *algebraic system* of nine equations in each element/subdomain.  
 341 The Lagrange multipliers appearing in (25) are approximations to the magnetic field *on the*  
 342 *boundaries* of the elements; they are introduced following a technique called *hybridization*, see  
 343 *Santos [2009]* and references therein. As the magnetic field is assumed to be piecewise con-  
 344 stant, they are necessary to avoid -because of the continuity condition of the tangential com-  
 345 ponent of the magnetic field- the constant solution throughout the computational domain. With  
 346 this in mind, the space

$$\Psi^h = \{\ell^h : \ell^h|_{\Gamma_{jk}} = \ell_{jk}^h \in [P_0(\Gamma_{jk})]^2 \equiv \Psi_{jk}^h, \ell_{jk}^h + \ell_{kj}^h = 0\}$$

347 is introduced, where  $P_0(\Gamma_{jk})$  denotes the constant functions defined on  $\Gamma_{jk}$ .

348 From one of the five equations originated in Eqs. (23)-(24) the magnetic field can be cal-  
 349 culated; from the other four equations the electric field unknowns can be obtained, each one  
 350 of the them computed just in terms of the calculated magnetic field unknown.

351 Further, notice that to the right hand side of each element contribute both the external  
 352 source -evaluated in the centroid of the element- and the (already known) unknowns coming  
 353 from adjacent elements (see last term of equation (23)).

### 354 **3.1.1 Iterative schemes**

355 The  $*$  appearing as superscript of the fields and Lagrange multipliers in the right hand  
 356 side of the iterative procedure given by (23)-(25) has different meanings, depending on how  
 357 the iterative procedure is effectively implemented. For a Jacobi-type iterative algorithm,  $*$  =  
 358  $n$ , a Seidel-type algorithm is obtained if for all subdomains  $\Omega_k$  such that  $j < k$ ,  $*$  =  $n + 1$   
 359 and for the ones such that  $j > k$ ,  $*$  =  $n$ . Other possible choice is the so called "Red-Black"  
 360 scheme; in it half of the subdomains of the partition are labelled "Red", the other half "Black",  
 361 and are disposed such that a Red subdomain has Black ones as first neighbours. In this scheme,  
 362  $*$  =  $n + 1$  if  $\Omega_j$  is a Red subdomain, and  $*$  =  $n$  if it is a Black one. In Figure 5 we dis-  
 363 play two different implementations of this scheme; in Fig. 5a the domain decomposition co-  
 364 incides with the finite element partition, so that the Red-Black scheme, depicted using black  
 365 and white squares, has the pattern of a chess-board, we call this option "Massive domain de-  
 366 composition (MDD)". On the other hand, Fig. 5b displays a situation in which each domain  
 367 involves several elements of the finite element partition; this is the "Stripes domain decom-  
 368 position (SDD)". The number of iterations necessary to reach convergence is dependent of the  
 369 chosen implementation [*Gauzellino et al.*, 2009].

370 In summary, if the MDD implementation of the Red-Black scheme is chosen, for each  
 371 frequency  $\omega$  the algorithm reduces to calculate in each iteration, nine algebraic equations (five  
 372 for the fields and four for the Lagrange multipliers) in each one of the  $n_x \cdot n_z$  elements. On  
 373 the other hand, the number of degrees of freedom of the SDD implementation of the Red-Black  
 374 scheme depends on how the discrete problem is solved within each stripe. Several options have  
 375 been investigated in *Gauzellino et al.* [2009] when solving a three dimensional Helmholtz equa-  
 376 tion; it was there concluded that the MDD implementation of the Red-Black scheme is the most  
 377 efficient.

### 378 **3.2 Biot's equations with electroosmotic source**

379 Once the electric field  $E_j$  is known in all domains  $\Omega_j \in \Omega$ , i.e., when the iterative pro-  
 380 cedure defined in the previous section has converged, we are in a position to calculate the in-  
 381 duced fluid filtration in the partially saturated poroelastic medium  $\Omega^s$ . We start by adding some

382 notation; let  $\nu_{jk}$  be the outer normal on  $\Gamma_{jk}$  from  $\Omega_j$  to  $\Omega_k$  and  $\nu_j$  the outer normal to  $\Gamma_j$ , see  
 383 Fig.3(a). We continue by writing the domain decomposed Biot's equations, in which we want  
 384 to find for every subdomain  $\Omega_j \in \Omega^s$  the displacements  $u_j = (u_j^s, u_j^f)$ , where  $u_j^s \equiv (u_x^s(x, z, \omega), u_z^s(x, z, \omega))_{\Omega_j}$   
 385 is the solid displacement and by  $u_j^f \equiv (u_x^f(x, z, \omega), u_z^f(x, z, \omega))_{\Omega_j}$  the relative fluid displace-  
 386 ment per unit volume of bulk material in the subdomain  $\Omega_j$  solutions to

$$-\omega^2 \rho^b u_j^s - \omega^2 \rho^E u_j^f - \nabla \cdot \tau^b(u_j) = 0, \quad (26)$$

$$-\omega^2 \rho^E u_j^s - \omega^2 g_0 u_j^f + i\omega \frac{\eta}{k_0} u_j^f + \nabla p_f = \frac{\eta}{k_0} \mathcal{L}_0 E_j, \quad (27)$$

$$\tau_{lm}(u_j) = 2G^{fr} \varepsilon_{lm}(u_j^s) + \delta_{lm} \left( \lambda_c \nabla \cdot u_j^s + \alpha K^{av} \nabla \cdot u_j^f \right), \quad (28)$$

$$p_f(u_j) = -\alpha_B K^{av} \nabla \cdot u_j^s - K^{av} \nabla \cdot u_j^f, \quad (29)$$

$$\mathcal{G}_{jk}(u_j) + i\omega \tilde{\beta}_{jk} \Pi_{\Gamma_{jk}}(u_j) = \mathcal{G}_{kj}(u_k) - i\omega \tilde{\beta}_{jk} \Pi_{\Gamma_{kj}}(u_k), \text{ on } \Gamma_{jk} \subset \Gamma_j, \quad (30)$$

$$\mathcal{G}_{kj}(u_k) + i\omega \tilde{\beta}_{jk} \Pi_{\Gamma_{kj}}(u_k) = \mathcal{G}_{jk}(u_j) - i\omega \tilde{\beta}_{jk} \Pi_{\Gamma_{jk}}(u_j), \text{ on } \Gamma_{jk} \subset \Gamma_k, \quad (31)$$

$$-\mathcal{G}(u_j) = i\omega \mathcal{B} \Pi_{\Gamma_j}(u_j) \text{ on } \Gamma_j \cap \partial\Omega^s. \quad (32)$$

387 In Eq. (27) the coefficient  $g_0 = 1.5\alpha_\infty \rho^E / \phi$  stands for the mass coupling coefficient; in it  
 388  $\alpha_\infty = \phi^{-m^c}$ , being the exponent  $m^c$  the so called Archie's consolidation factor, is the struc-  
 389 ture or tortuosity factor. The first of these coefficients represents the inertial effects associated  
 390 with dynamic interactions between solid and fluid phases, and is sometimes referred to as ef-  
 391 fective fluid inertia [Haines and Pride, 2006]. Notice that the coupling between electromag-  
 392 netic and mechanical processes in this equation is expressed in the right hand side, involving  
 393 the electrokinetic coupling coefficient  $\mathcal{L}_0$ . In the constitutive relations (28)-(29),  $\tau_{lm}^b$  and  $\varepsilon_{lm}^b$   
 394 denote the two dimensional stress and strain tensors,  $\lambda_u = K_u - \frac{2}{3}G^{fr}$  and the other coef-  
 395 ficients have been defined in Section 2.2. Eqs. (30)-(31) express, in terms of Robin transmis-  
 396 sion conditions, the consistency conditions at the interior boundaries  $\Gamma_{jk}$ , meaning the con-  
 397 tinuity of the solid displacement, the normal component of the fluid displacements and the gen-  
 398 eralized stresses [Santos *et al.*, 2005]. In these expressions,  $\tilde{\beta}_{jk}$  is a positive definite matrix  
 399 [Santos *et al.*, 2004b,a]. On the boundary of the domain  $\Omega^s$  the first order ABC given by the  
 400 Eq. (32) is employed; see the just mentioned references for details. In Eqs. (30)-(32) we have



401 denoted

$$\mathcal{G}_j(u_j) = (\tau(u_j)\nu_j \cdot \nu_j, \tau(u_j)\nu_j \cdot \chi_j, -p_f((u_j))) \text{ on } \Gamma_j, \quad (33)$$

$$\mathcal{G}_{jk}(u_j) = (\tau(u_j)\nu_{jk} \cdot \nu_{jk}, \tau(u_j)\nu_{jk} \cdot \chi_{jk}, -p_f((u_j))) \text{ on } \Gamma_{jk}, \quad (34)$$

$$\Pi_{\Gamma_j}(u_j) = (u_j^s \cdot \nu_j, u_j^s \cdot \chi_j, u_j^f \cdot \nu_j), \text{ on } \Gamma_j, \quad (35)$$

$$\Pi_{\Gamma_{jk}}(u_j) = (u_j^s \cdot \nu_{jk}, u_j^s \cdot \chi_{jk}, u_j^f \cdot \nu_{jk}) \text{ on } \Gamma_{jk}, \quad (36)$$

$$\mathcal{B} = \mathcal{R}^{1/2}\mathcal{D}^{1/2}\mathcal{R}^{1/2}, \text{ where } \mathcal{D} = \mathcal{R}^{-1/2}\mathcal{H}\mathcal{R}^{-1/2}, \quad (37)$$

402 with

$$\mathcal{R} = \begin{pmatrix} \rho^b & 0 & \rho^E \\ 0 & \rho^b - \frac{\rho^E 2}{g_0} & 0 \\ \rho^E & 0 & g_0 \end{pmatrix}, \quad \mathcal{H} = \begin{pmatrix} \lambda_c + 2G^{fr} & 0 & \alpha K^{av} \\ 0 & G & 0 \\ \alpha_B K^{av} & 0 & K^{av} \end{pmatrix}. \quad (38)$$

403 We can introduce now the iterative hybridized domain decomposed finite element procedure,  
 404 mentioning different versions of this method have been previously employed to simulate wave  
 405 propagation in saturated porous media with composite matrices [*Santos et al.*, 2004b] and prop-  
 406 agation of ultrasonic waves in media with patchy saturation [*Santos et al.*, 2005]. Here a brief  
 407 description of the method is given; for details the reader is encouraged to read these last ref-  
 408 erences.

409 To approximate each component of the solid displacements a nonconforming finite el-  
 410 ement space is used; while the fluid displacements are approximated by the vector part of the  
 411 Raviart-Thomas-Nedelec space of zero order [*Nedelec*, 1980]. Specifically, set  $\theta(x) = x^2 -$   
 412  $\frac{5}{3}x^4$ ,  $R = [-1, 1]^2$  and

$$\varrho^L(x, z) = \frac{1}{4} - \frac{1}{2}x - \frac{3}{8}(\theta(x) - \theta(z)), \varrho^R(x, z) = \frac{1}{4} + \frac{1}{2}x - \frac{3}{8}(\theta(x) - \theta(z)), \quad (39)$$

$$\varrho^B(x, z) = \frac{1}{4} - \frac{1}{2}z + \frac{3}{8}(\theta(x) - \theta(z)), \varrho^T(x, z) = \frac{1}{4} + \frac{1}{2}z + \frac{3}{8}(\theta(x) - \theta(z)); \quad (40)$$

413 we define  $\mathcal{Y}(R) = \text{Span}\{\varrho^L, \varrho^R, \varrho^B, \varrho^T\}$  and  $\mathcal{W}(R) = \mathcal{Y}(R) \times \mathcal{Y}(R)$ . Also, if  $\varphi^L(x) =$   
 414  $-1 + x$ ,  $\varphi^R(x) = x$ ,  $\varphi^B(z) = -1 + z$ ,  $\varphi^T(z) = z$ , set

$$\mathcal{Z}(R) = \text{Span}\{(\varphi^L(x), 0), (\varphi^R(x), 0), (0, \varphi^B(z)), (0, \varphi^T(z))\}, \quad (41)$$

415 and the finite element spaces  $\mathcal{W}_j^h$  and  $\mathcal{Z}_j^h$  are defined as usual by scaling and translating to  
 416 the element  $\Omega_j$ . Notice that in each domain of the finite element partition there exist twelve  
 417 unknowns, four for each solid displacement component, and two for each component of the  
 418 fluid displacement. A scheme of the degrees of freedom for this space of nonconforming fi-  
 419 nite element is illustrated in Fig.3(b). The hybridization implies also here the introduction of

420 Lagrange multipliers, this time associated to generalized forces at the midpoints  $\gamma_{jk}$  on the inter-  
 421 element boundaries  $\Gamma_{jk}$ , in the sense that  $\lambda_{jk} \sim \mathcal{G}(u_j)(\gamma_{jk})$  [Santos *et al.*, 2005]. They be-  
 422 long to the following space of functions defined on the interior interfaces:

$$\mathcal{L} = \left\{ \lambda : \lambda|_{\Gamma_{jk}} = \lambda_{jk} \in [P_0(\Gamma_{jk})]^3 = \mathcal{L}_{jk}^h \forall j, k \right\} \quad (42)$$

423 Setting initial values  $(u_j^{s,h,0}, u_j^{f,h,0}, \lambda_{jk}^{h,0})$  the iterative algorithm reads:

424 Find  $(u_j^{s,h,n+1}, u_j^{f,h,n+1}, \lambda_{jk}^{h,n+1}) \in \mathcal{W}_j^h \times \mathcal{Z}_j^h \times \mathcal{L}_{jk}^h$  such that

$$\begin{aligned} & -\omega^2 \left( \rho^b u_j^{s,h,n+1} + \rho^E u_j^{f,h,n+1}, v^s \right)_{\Omega_j} - \omega^2 \left( \rho^E u_j^{s,h,n+1} + g_0 u_j^{s,h,n+1}, v^f \right)_{\Omega_j} \\ & + i\omega \left( g_0 u_j^{f,h,n+1}, v^f \right)_{\Omega_j} + \sum_{lm} \left( \tau_{lm}^b(v_j^{h,n+1}), \varepsilon_{lm}^b(v^s) \right)_{\Omega_j} \\ & - \left( p_f(u_j^{h,n+1}), \nabla \cdot v^f \right)_{\Omega_j} + \left\langle i\omega \mathcal{B} \Pi_{\Gamma_j}(u_j^{s,h,n+1}), \Pi_{\Gamma_j}(v) \right\rangle_{\Gamma_j \cap \partial \Omega^s} \\ & + \sum_{\substack{k \\ \Gamma_{jk} \cap \partial \Omega^s = \phi}} \left\langle i\omega \tilde{\beta}_{jk} \Pi_{\Gamma_{jk}}(u_j^{h,n+1}), \Pi_{\Gamma_{jk}}(v) \right\rangle_{\Gamma_{jk}} = \left( \frac{\eta}{k_0} \mathcal{L}_0 E_j, v^f \right)_{\Omega_j} \\ & - \sum_{\substack{k \\ \Gamma_{jk} \cap \partial \Omega^s = \phi}} \left\langle i\omega \tilde{\beta}_{jk} \Pi_{\Gamma_{kj}}(u_k^{h,*}), \Pi_{\Gamma_{jk}}(v) \right\rangle_{\Gamma_{jk}} + \sum_{\substack{k \\ \Gamma_{jk} \cap \partial \Omega^s = \phi}} \left\langle \lambda_{kj}^{h,*}, \Pi_{\Gamma_{jk}}(v) \right\rangle_{\Gamma_{jk}} \quad (43) \end{aligned}$$

$$\lambda_{jk}^{h,n+1} = \lambda_{kj}^{h,*} - i\omega \tilde{\beta}_{jk} \left[ \Pi_{\Gamma_{jk}}(u_j^{h,n+1}) + \Pi_{\Gamma_{kj}}(u_k^{h,*}) \right] (\gamma_{jk}). \quad (44)$$

425 The same different choices for selecting the meaning \* present the iterative scheme Eqs. (23)-  
 426 (25) are available in this case. Whatever the decision, to solve Eqs. (43)-(44) the calculations  
 427 are more involved than in the former case. For example, if the chess-board implementation  
 428 of the Red-Black scheme is selected, equation (43) implies that in each iteration and for each  
 429 one of the elements into which the domain  $\Omega^s$  is divided a  $12 \times 12$  linear system of equa-  
 430 tions needs to be solved.

431 Notice also that the iteration parameter  $\tilde{\beta}_{jk}$  is chosen as the average  $\frac{1}{2}(\mathcal{B}_j + \mathcal{B}_k)$ , where  
 432  $\mathcal{B}$  is the matrix appearing in the absorbing boundary conditions. Furthermore, notice that the  
 433 Lagrange multipliers, associated to generalized forces on the inter-element boundaries  $\Gamma_{jk}$  -  
 434 but evaluated at the mid-points  $\gamma_{jk}$  - are calculated in each element by (44), which represents  
 435 twelve scalar equations. Thus, for each frequency  $\omega$  this algorithm reduces to solve in each  
 436 iteration and for each one of the elements into which  $\Omega^s$  is divided, a linear system of twelve  
 437 unknowns (eight from the solid and four from the fluid) plus twelve scalar equations to get the  
 438 Lagrange multipliers.

439 As a final remark, we mention that the spacetime solution is obtained by solving Eqs.  
 440 (43)-(44) for a finite number of frequencies and taking the inverse Fourier transform. Of course,  
 441 this comment is valid also for the iterative procedure used to get the electric field.

#### 442 **4 Parallel implementation**

443 For the implementation of the parallel code we used the MPI standard [Pacheco, 2011].  
 444 The most efficient way to perform the calculations is to assign to each processor, as close as  
 445 possible, the same number of unknowns [Alumbaugh *et al.*, 1996]. In our case, that means to  
 446 assign the same number of subdomains  $\Omega_j$  to each processor. If the load of the processors is  
 447 not balanced, some will remain idle while others are still computing, reducing the efficiency  
 448 of the algorithm. In order to fix ideas, let us work with nine processors (Fig5a), or three pro-  
 449 cessors (Fig5b). Each one runs exactly the same copy of the program, and gets the input data  
 450 from a single data file. Local variables are converted to global when necessary within the code;  
 451 we preferred this strategy to splitting the input file in multiple ones to be read by each pro-  
 452 cessor [Newman and Alumbaugh, 1997]. In Fig. 5 we display, by using different colors, the  
 453 portion of the computational domain  $\Omega$  assigned to each processor; the solid lines represent  
 454 the virtual boundaries among them. Naturally, it is possible to do this assignment in differ-  
 455 ent ways, we chose to display a very simple one for the sake of clarity. The processor  $\mathbf{P}_0$  solves  
 456 only in red (Fig5a) or green (Fig5b) regions, and simultaneously the other processors perform  
 457 their calculations in their respective regions.

458 The time needed to get the solution is usually longer than one-ninth (Fig5a) or one-third  
 459 (Fig5b) of the time with a serial code on one processor (assuming that processors of the same  
 460 kind are used). This happens because in each iteration ‘adjacent’ processors must interchange  
 461 information, so that the right hand sides of the elements situated on the borders of the region  
 462 corresponding to each processor is correctly set: In Fig.5 these regions are signaled with stronger  
 463 colours and delimited with dashed lines; they also show the processor number involved in the  
 464 data exchange; e.g., in Fig.5a 0-3 indicates the twelve elements, six assigned to processor  $\mathbf{P}_0$   
 465 (blue) and six assigned to processor  $\mathbf{P}_3$  (green) which must be selected to interchange data  
 466 between them. The information to be transmitted involves all the coefficients building the ap-  
 467 proximate solutions  $(E_j^n, H_j^n, \ell_{jk}^n)$  for Maxwell’s equations,  $(u_j^{s,n}, u_j^{f,n}, \lambda_{jk}^n)$  for Biot’s equa-  
 468 tions. Clearly, the same is valid for all the other regions; the interchange of information among  
 469 processors is performed simultaneously, once Eqs. (23)-(25) (Maxwell) (43)-(44) (Biot) are  
 470 solved in all nine (Fig. 5a) domains or three (Fig. 5b) domains.

471 We asserted that these domain decomposed algorithms are naturally rendered parallel  
 472 not only because of the description given above, but also due to the fact that the amount of  
 473 data to be transferred is not large. As sketched in Fig.5a and Fig. 5b the strong coloured re-

474 gions lay on only a single finite element width, because, as we explained above, the right hand  
 475 side of the iterative procedures involve only nearest neighbours.

## 476 **5 Case study: CO<sub>2</sub> sequestration**

477 Although previous field results in electroseismics employed surface sources and surface  
 478 receivers [Thompson *et al.*, 2007], in a more recent report on CO<sub>2</sub> geological storage moni-  
 479 toring, synthetic and field geoelectrical methods were applied to study possible gas migration  
 480 [Kiessling *et al.*, 2010]. Moreover Ishido *et al.* [2013] have numerically investigated the ap-  
 481 plication of self potential methods to monitor the migration of CO<sub>2</sub> sequestered into saline  
 482 aquifers, concluding that the used methods are effective for sensing the approach of CO<sub>2</sub> to  
 483 the well casings deep within the subsurface. On the other hand, Kim *et al.* [2013] have shown  
 484 that seismics was useful to detect CO<sub>2</sub> saturation below 15% and that electrical resistivity was  
 485 useful to detect CO<sub>2</sub> saturation above 15%.

486 Inspired by these results, we analyse the behaviour of seismic responses to an electro-  
 487 magnetic source deployed in a well, locating it beneath a CO<sub>2</sub> plume, trapped by an overly-  
 488 ing seal layer, as we depict in Fig. 4. We simulate a time-lapse monitoring scenario, by tak-  
 489 ing three different situations; the first one when no CO<sub>2</sub> has been pumped into the subsurface,  
 490 and therefore its saturation is 0%, a second one with a CO<sub>2</sub> saturation of 35%, and a third one  
 491 with a CO<sub>2</sub> saturation of 60%. We consider a PSVTM case, so, in order to get the appropri-  
 492 ate fields, the selected electromagnetic source is a magnetic dipole of infinite length in the strike  
 493 direction ( $y$ -axis), located right beneath a CO<sub>2</sub> plume at 220 m depth. Its expression, to be  
 494 set in Eq. (23) is  $J^m = -i\omega\mu\widehat{S}I(\omega)\delta(x - x_s)\delta(z - z_s)\check{y}$ ; here  $\widehat{S}$  is the area of the current  
 495 loop,  $I(\omega)$  is the current and  $(x_s, z_s)$  is the center of the loop, assumed to be the source lo-  
 496 cation. Seismic receivers are set in two different uncased wells, located at 10 m to the right  
 497 (Well #1) and 100 m to the left (Well #2) of the horizontal position of the electromagnetic source.  
 498 Notice that no conversions at the well walls are considered in our approach. This effect has  
 499 been studied by [Hu and Liu, 2002], who analyzed the converted electric field during acous-  
 500 toelectric logging. Surface receivers are also set, ranging from 300 m to the left to 300 m to  
 501 the right of  $x_s$ , separated 4 m from each other. The time signature of the source is a Ricker  
 502 wavelet with peak frequency of 60 Hz; its peak amplitude in time is located at  $t = 0.16$  s.

503 This source is Fourier transformed, and 200 equally spaced samples  $I(\omega)$  are used in  
 504 the calculations; each one of them is used in the right hand side of Ampere-Maxwell's equa-  
 505 tions (23) in the above described PSVTM algorithms. Once all the results in the space-frequency

506 domain are obtained, they are inverse Fourier transformed to get the space-time responses that  
507 we show and analyse below.

508 The physical properties of the top layer, seal layer, bottom layer and CO<sub>2</sub> plume are de-  
509 rived from basic parameter values given in Tables 1 and 2 using the different effective prop-  
510 erties described in Section 2. Notice that the effective fluid properties in the CO<sub>2</sub> plume are  
511 calculated assuming that the CO<sub>2</sub> is in its supercritical state, as is usually pumped into the sub-  
512 surface [*Kiessling et al.*, 2010; *Cairns et al.*, 2012]. Although the depth at which we situated  
513 the reservoir is not deep enough to create the conditions for the CO<sub>2</sub> to remain supercritical  
514 [*Kazemeini et al.*, 2010], we retain the mentioned depth value to keep a reasonable computa-  
515 tional cost, because of the size of the model. Note that the following analysis would remain  
516 exactly the same if we increased the depth of the bottom of the top layer as much as neces-  
517 sary to reach the pressure and temperature conditions for the CO<sub>2</sub> to be in supercritical state.

518 Concerning the calculation of the bulk electrical conductivity by means of Eq. (12) in  
519 the region occupied by the plume, we mention here that we consider the electrical conductiv-  
520 ity of carbon dioxide negligible compared to that of the brine partially saturating it. The pres-  
521 ence of CO<sub>2</sub> diminishes the bulk electrical conductivity as it would happen if air is present;  
522 however in supercritical state the reduction is not so strong as it would be if the CO<sub>2</sub> is gaseous  
523 [*Borner et al.*, 2013]. As a final remark concerning the computation of the electrokinetic cou-  
524 pling  $L_0$  in the plume, we want to mention that when CO<sub>2</sub> is pumped into a reservoir a small  
525 portion dissolves in water [*Carcione et al.*, 2006b; *Wang et al.*, 2013b], forming weak carbonic  
526 acid which reacts with the present dissolved salt ions [*Darwish and Hilal*, 2010]. This process  
527 alters the  $\zeta$  potential [*Moore et al.*, 2004], which in turn changes the electrokinetic coupling;  
528 in the present work the  $\zeta$  potential does not depend on the presence of carbon dioxide. How-  
529 ever, the latter is taken into account, as described above, by making  $L_0$  saturation dependent.  
530 We assumed that the electrokinetic coupling is changed when the amount of CO<sub>2</sub> is increased  
531 and water expelled, as it changes when water-saturation is decreased, replaced by air.

## 532 **5.1 Well gathers**

533 Let us now turn our attention to our results; in Fig. 6 we display the x-component of the solid  
534 acceleration records of the receivers located in a portion of well #1, between 100 m depth and  
535 280 m depth, before pumping CO<sub>2</sub> into the reservoir (Fig. 6(a)) and when the carbon diox-  
536 ide saturation of the plume reached a 35 % saturation (Fig. 6(b)). The ES source is a mag-  
537 netic dipole located at 220 m depth and originates the seismic signals from the interfaces be-

538 tween different porous media due to the electroosmotic coupling [*Pride, 1994; Pride and Garam-*  
539 *bois, 2005; Haines and Pride, 2006*]. In the Fig. 6(a), P and SV waves are clearly observed.  
540 They travel upwards and downwards from the seal layer and their velocities can be calculated  
541 by measuring the slopes; they are consistent with their values obtained from the dispersion re-  
542 lations. SV waves are well identified in the x-component for shallow and deep receivers. In  
543 Fig. 6(b) it is possible to notice changes in the character of the signals by the presence of the  
544 partially saturated CO<sub>2</sub> plume. The receivers located between 210 m and 220 m depth, i.e.,  
545 the thickness of the plume, get a stronger signal which also lasts longer, due possible to re-  
546 flections on the bases of the seal layer and plume respectively. The polarization change in the  
547 x-component of the solid acceleration observed at the depth at which the electromagnetic source  
548 is located is consistent with the fact that the x-component of the electric field, being tangent  
549 to the electric current circulating in the wire loop, changes its sign at  $z = z_s$ .

550 In Fig. 7 we display the z-component of the solid acceleration records, also for the re-  
551 ceivers located in well #1, and for the same depth interval as in the previous figure. We ob-  
552 serve also here that the presence of the CO<sub>2</sub> plume generates stronger signals for receivers lo-  
553 cated at its same depth interval, somehow "focusing" the converted seismic signals within the  
554 plume. In the case (a) of both figures, the signals are reinforced under the seal layer due to  
555 the influence of top and base of it.

556 Turning now our attention to the comparison of the relative amplitudes of the converted  
557 signals, we show, for both solid acceleration components, single traces measured at 190 m depth,  
558 i.e. above the seal layer in Fig. 8, and at 230 m depth, i.e. below the CO<sub>2</sub> plume (see Fig. 7),  
559 for the three CO<sub>2</sub> concentrations we are considering. It can be observed in all cases that the  
560 response for the 0% saturation is the weakest one and that the 35% saturation case is the strongest.  
561 The fact that the amplitude of the traces corresponding to the 35% saturation case is stronger  
562 than the ones corresponding to the 60% saturation case is compatible with the behaviour of  
563 the  $\mathcal{C}(S_w)$  function, because in the saturation interval considered, it grows with the water sat-  
564 uration (40% to 65% of water, see Fig. 1), i.e., it diminishes with CO<sub>2</sub> saturation (from 35%  
565 to 60% of CO<sub>2</sub>). Therefore, the electrokinetic coupling coefficient Eq. (13), and correspond-  
566 ingly the amplitude of the seismic source is smaller for the 60% CO<sub>2</sub> case than for the 35%  
567 CO<sub>2</sub>. Note that the difference of amplitude in the traces is bigger than the corresponding in  
568  $\mathcal{C}(S_w)$ , due to the factor  $S_w^n$  in this equation. It can also be observed that the amplitude of the  
569 traces measured below the plume, see Fig.7, are larger than the ones located above the seal  
570 layer, see Fig.6, for all CO<sub>2</sub> concentrations and both acceleration components, because pre-

571 precisely of the presence of the seal, which partially reflects the incident waves impinging into  
572 its base from below, due to the contrast in mechanical properties between them.

573 Finally in Fig. 10 we compare, for a single receiver located at 216 m depth, i.e. level  
574 with the plume, the amplitudes of the traces, for both acceleration components, recorded at  
575 well #1 and well #2. It can be clearly observed how the amplitude of the signal decays, be-  
576 cause both of spherical divergence and viscoelasticity as it travels out of the source since the  
577 well #1 is situated at 10 m from the source whereas the well #2 is situated at 100 m from the  
578 source.

## 579 5.2 Surface gathers

580 Let us now consider the responses measured by the surface accelerometers, depicted in  
581 Fig. 11 for the z-component. Fig. 11(a) corresponds to the model without CO<sub>2</sub>, while Fig. 11(b)  
582 show the results for the 35% saturation case. In the former case, two interface responses (IR)  
583 are observable. These IR arrive at the same time on surface receivers and are shown as hor-  
584 izontal arrivals in Figure 11(a). The topmost one, namely IR<sub>1</sub>, corresponds to the conversion  
585 of the electromagnetic signal hitting the surface (almost) at the same time as the magnetic dipole  
586 is turned on at 220 m depth; recall that the Ricker wavelet we are using as the source signa-  
587 ture is not centred at  $t=0$  s, but it has a time delay of  $t = 0.016$  s. Of course, this IR con-  
588 veys no information of the subsurface structure. The other interface response, labelled IR<sub>2</sub> in  
589 the figure, is originated at the interfaces defined by base and top of the seal layer; notice that  
590 they are not resolved in the measured traces. The fact that the two IRs are not observable in  
591 Fig. 11(b) is just because the amplitude of hyperbolic-shaped signal is much higher than the  
592 one corresponding to the no CO<sub>2</sub> case. We show that this is indeed the situation in Fig. 12,  
593 where we have plotted a single trace from the surface gather of both CO<sub>2</sub> concentrations here  
594 considered, located at 260 m offset; the arrival times of both interface responses and the up-  
595 wards travelling signal are at this offset easily individualized. The topmost interface response  
596 is, as expected, exactly the same for both situations, while the time arrivals of the second one  
597 for the two saturation cases do differ; this happens because at 35% CO<sub>2</sub> saturation there is a  
598 new interface response originated at the plume base, located at 220 m depth, which construc-  
599 tively interferes with the ones arising at the top and bottom interfaces of the seal layer. Finally,  
600 it can be noticed that the amplitude of the hyperbola is much bigger for the case with partial  
601 carbon dioxide saturation than for the case at zero saturation.

602 The other relevant feature, which does contain information about the carbon dioxide con-  
603 tent, as we explain below, is the hyperbolic-shaped signal with vertex at about 0.046 s.

604 As we described when explaining the well gathers, this signal is due to the electroos-  
605 motic coupling at the electromagnetic source position, spreads outwards from it and hits the  
606 seal layer and CO<sub>2</sub> plume, generating multiple reflections inside them. Some of this reflected  
607 waves travels towards the surface, and are detected by the receivers there.

608 That this signal contains information of the CO<sub>2</sub> content can be justified as follows: on  
609 one hand, the amplitude of the signals measured for the 35% saturation case is bigger than  
610 for the 0% saturation case, as we have just shown. On the other hand, in Fig. 13 we depict  
611 surface traces recorded at 20 m offset, before pumping CO<sub>2</sub> into the reservoir and with CO<sub>2</sub>  
612 saturations of 35% and 60%. It can be here clearly seen that the amplitudes depend on the sat-  
613 uration in the plume, and that the amplitude of the trace for the no carbon dioxide case is weaker  
614 than the other ones, which make the IRs, detectable in the former case, invisible for the lat-  
615 ter cases. Yet another point to be noticed is that the amplitude of the traces at the surface is  
616 about one order of magnitude weaker than the ones recorded at depth.

### 617 5.3 Parallel performance

618 In order to analyse the parallel performance of our algorithm, we compute the time needed  
619 to reach convergence by the iterative algorithms described by (23)-(25) and (43)-(44) for a sin-  
620 gular frequency and different number of cores, using the MDD implementation. We run the model  
621 previous to the CO<sub>2</sub> injection and select the source peak frequency. The number of iterations  
622 to reach convergence is not a function of the number of computing cores; i.e., the numerical  
623 procedure for Maxwell's equations ends at the same iteration number irrespective of the num-  
624 ber of cores employed. The same happens for Biot's equations, but of course, the number of  
625 iterations to reach the convergence threshold it is not the same as Maxwell's. In our exam-  
626 ples the chosen convergence threshold, being the value that the added relative error between  
627 successive solutions for all unknowns and for all the finite elements must reach, was set to 10<sup>-3</sup>.

628 In Table 3 we display, for two different meshes, namely a "small" one containing 2048  
629 × 1048 elements ( $\approx 3.56 \times 10^7$  unknowns) and a "big" one containing 4096 × 2048 ele-  
630 ments ( $\approx 1.43 \times 10^8$  unknowns) an analysis of the performance of our numerical algorithm  
631 in a parallel computing environment. We display, for each mesh and as a function of the num-  
632 ber of computing cores, the computing time, the ratio of the computing time with one core  
633 to the computing time with  $N$  cores, i.e., the so called "speedup"  $\mathcal{S}_p = T_1/T_N$ , and finally,



634 we display the so called "parallel efficiency"  $\mathcal{E}_p = \mathcal{S}_p/N$  [Foster, 1995]. If the parallel al-  
 635 gorithm was perfect, the speedup  $\mathcal{S}_p$  would be a linear function of the number of computing  
 636 cores; this is not what happens, mainly because the cores must interchange information to per-  
 637 form their computations, as explained in section 4 Parallel implementation. From the point of  
 638 view of the parallel efficiency,  $\mathcal{E}_p = 1$  would mean a perfectly efficient parallel algorithm;  
 639 the reason of its decay with increasing number of computing cores is the same as before: the  
 640 overhead due to the data transmission among processors. It is worth to mention that for the  
 641 big mesh, the computation with four cores show superlinear behaviour, or parallel efficiency  
 642 greater than one. Although it has been argued that this is not possible [Faber *et al.*, 1986], it  
 643 has been later demonstrated that this behaviour can be observed [Agrawal *et al.*, 1994; Shan,  
 644 2002] if resources are used more efficiently, for example by reducing the RAM access time.

645 Notice also that the efficiency decays faster when more than 64 cores are used. This hap-  
 646 pens because the cluster we employed consists of nodes containing this core number, being  
 647 the nodes interconnected through an InfiniBand network. Then, communication among cores  
 648 of a single node is faster than when all cores belonging to two or four nodes have to trans-  
 649 mit data among them.

650 It is also worth noticing that the efficiency decay is slower for the bigger mesh, i.e., our  
 651 numerical algorithm is scalable [Foster, 1995; Pacheco, 2011]. Of course, the absolute com-  
 652 puting time will still decay while increasing the number of cores, but if the size of the prob-  
 653 lem is not increased, the efficiency of the computation will decrease.

## 654 **6 Conclusions**

655 We have implemented a set of domain decomposed iterative finite element algorithms  
 656 to approximate the solution to the Pride's equations. These algorithms can be used to model  
 657 the two dimensional PSVTM and SHTE modes of both electroseismics and seismoelectrics;  
 658 swapping between both techniques implies in our numerical formulation just to choose the na-  
 659 ture of the employed source, and the order in which Maxwell's and Biot's equations are solved.  
 660 In this chapter we have modelled a time lapse monitoring of a CO<sub>2</sub> deposition site by means  
 661 of PSVTM electroseismics. Our results suggest that the wells used to pump carbon dioxide  
 662 into the subsurface could be used to set magnetic dipoles as sources and accelerometers as record-  
 663 ing devices, and the measured signals would be sensitive to the CO<sub>2</sub> concentration. Moreover,  
 664 this setting would avoid the necessity of filtering the coseismic signal, a well known difficulty  
 665 arising when both sources and receivers are set on the surface.

666           From the strictly numerical point of view, we have demonstrated that our algorithms can  
667 handle large number of unknowns, and proved to be scalable. We deem that these character-  
668 istics make our code a competitive one as a modelling tool both in electroseismics and seis-  
669 moelectrics.

Common model parameters	
$m^c$	1.85
$n$	1.85
$K^{s,\text{sand}}$ [GPa]	35
$K^{s,\text{silt}}$ [GPa]	34
$K^{s,\text{clay}}$ [GPa]	12.3
$G^{s,\text{sand}}$ [GPa]	44
$G^{s,\text{silt}}$ [GPa]	43
$G_{s,\text{clay}}$ [GPa]	15.6
$\rho^{s,\text{sand}}$ [Kg/m <sup>3</sup> ]	2600
$\rho^{s,\text{silt}}$ [Kg/m <sup>3</sup> ]	2600
$\rho^{s,\text{clay}}$ [Kg/m <sup>3</sup> ]	2580
$D_{s,\text{sand}}$ [m]	$8 \times 10^{-5}$
$D_{s,\text{silt}}$ [m]	$1 \times 10^{-5}$
$D_{s,\text{clay}}$ [m]	$5 \times 10^{-7}$
$K^w$ [GPa]	2.25
$\eta^w$ [Pa.s]	$1 \times 10^{-3}$
$\rho^w$ [Kg/m <sup>3</sup> ]	1000
$\varepsilon^w$ [F/m]	$80 \varepsilon_0$
$T$ [K]	298

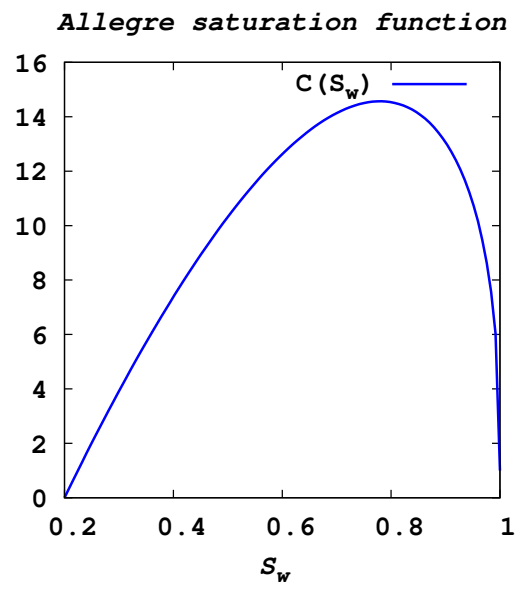
670 **Table 1.** *Values of model parameters used in the present work. Those not shown in this table can be ob-*  
671 *tained from the present ones using the formulae described throughout the text.*

Layer parameters			
Parameter	Top Layer	Seal	Bottom Layer
$\gamma_{\text{sand}}$ [-]	0.5	0.3	0.7
$\gamma_{\text{silt}}$ [-]	0.4	0.45	0.25
$\gamma_{\text{clay}}$ [-]	0.1	0.25	0.05
$\phi$ [-]	0.15	0.2	0.33
$C_0$ [mol/L]	$1. \times 10^{-3}$	$4. \times 10^{-2}$	$4. \times 10^{-1}$
$K^{CO_2}$ [Pa]	-	-	$2.5 \times 10^7$
$\eta^{CO_2}$ [Pa.s]	-	-	$1.5 \times 10^{-5}$
$\rho^{CO_2}$ [Kg/m <sup>3</sup> ]	-	-	505

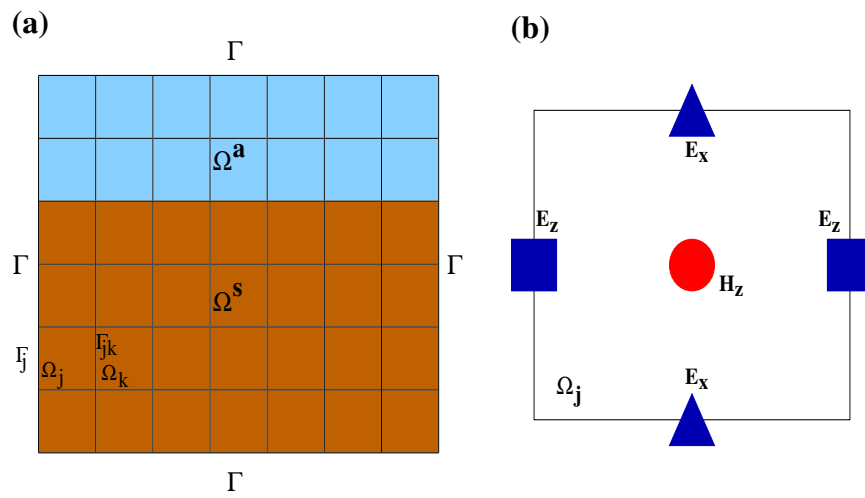
672 **Table 2.** *Dissimilar model parameters for the top, seal and bottom layers. The CO<sub>2</sub> values are used to ob-*  
673 *tain, together with the water corresponding ones, effective fluid parameters for the CO<sub>2</sub> plume, as explained*  
674 *in Section 2.*

Parallel performance						
# of Cores	Mesh 2048×1024			Mesh 4096×2048		
	Time [s]	$\mathcal{S}_p$	$\mathcal{E}_p$	Time [s]	$\mathcal{S}_p$	$\mathcal{E}_p$
1	12956	1	1	57572	1	1
4	3252	3.98	0.99	13581	4.23	1.05
16	1171	11.06	0.69	5034	11.43	0.71
64	392	33.05	0.52	1600	35.98	0.56
128	288	44.98	0.35	1016	56.67	0.44
256	320	40.49	0.16	948	60.73	0.24

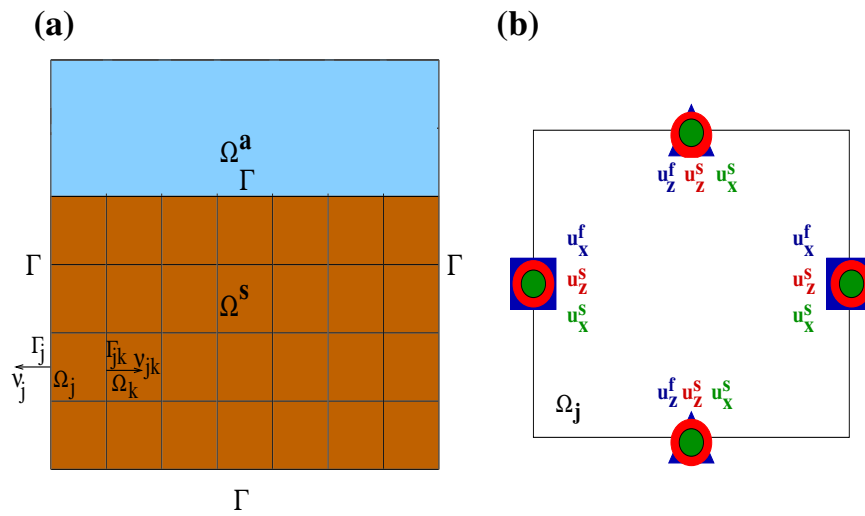
675 **Table 3.** *Computation times, speedup and parallel efficiency when solving a single frequency of the PSVTM*  
676 *algorithm, MDD implementation, for the model without CO<sub>2</sub>. Two different discretizations are considered,*  
677 *one involving 2048 × 1024 finite elements, i.e., approximately 3.56 × 10<sup>7</sup> unknowns, the other one involving*  
678 *4096 × 2048 finite elements, i.e., approximately 1.43 × 10<sup>8</sup> unknowns.*



679 **Figure 1.** The saturation function  $C(S_w)$  employed to model the saturation dependent behaviour of the  
680 electrokinetic coupling coefficient  $L_0$ .

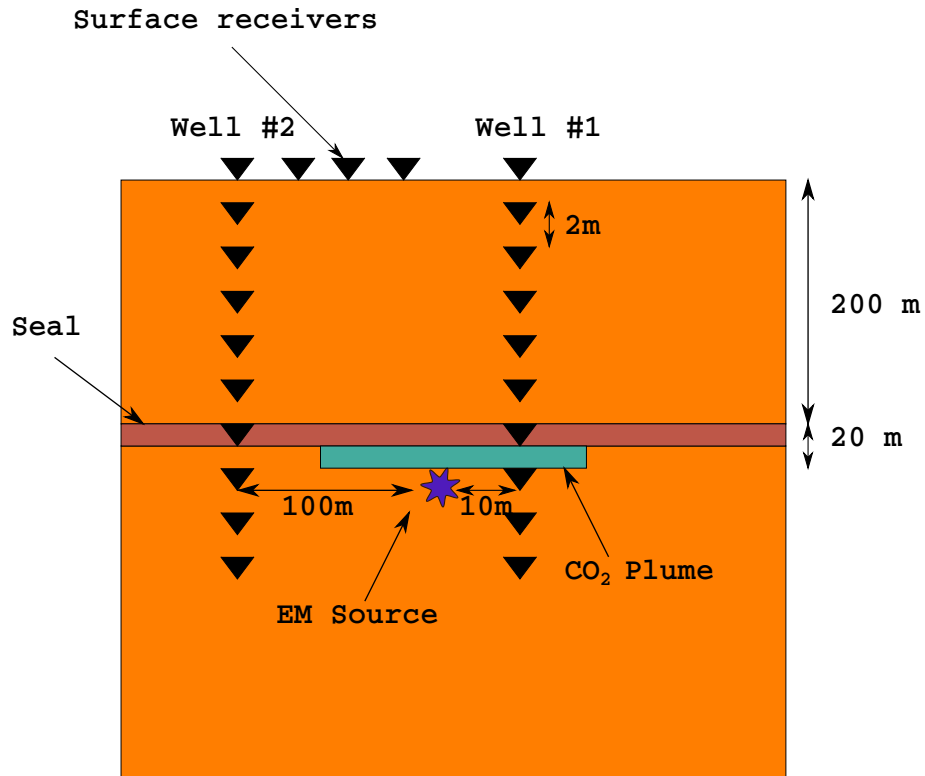


681 **Figure 2.** Electromagnetic equations case. (a) The computational domain  $\Omega = \Omega^a \cup \Omega^s$  integrating air  
 682 and subsurface, with external boundaries  $\Gamma_j$  and internal boundaries  $\Gamma_{jk}$ . (b) Scheme for the five degrees of  
 683 freedom associated with each element, four to the electric field and one to the magnetic field.

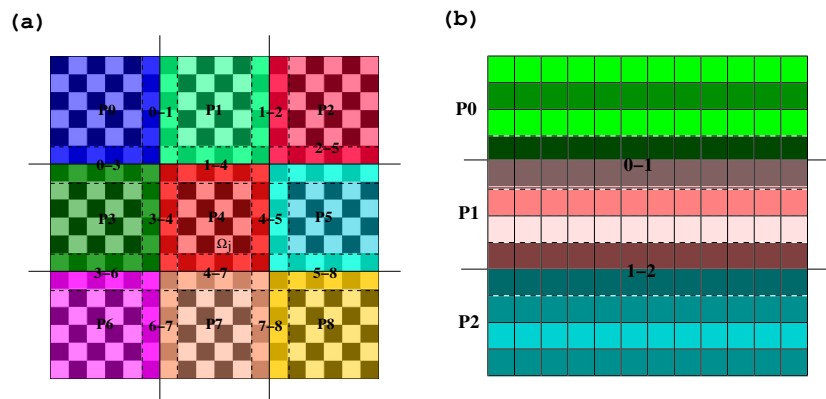


684 **Figure 3.** Biot's equations case. (a) The computational domain is only the subsurface  $\Omega = \Omega^s$ , with exter-  
 685 nal boundaries  $\Gamma_j$  and internal boundaries  $\Gamma_{jk}$ . The outer normals  $\nu_j$  and  $\nu_{jk}$  are also indicated. (b) Scheme  
 686 for the twelve degrees of freedom associated with each element, eight to the solid displacements and four to  
 687 the fluid displacements.

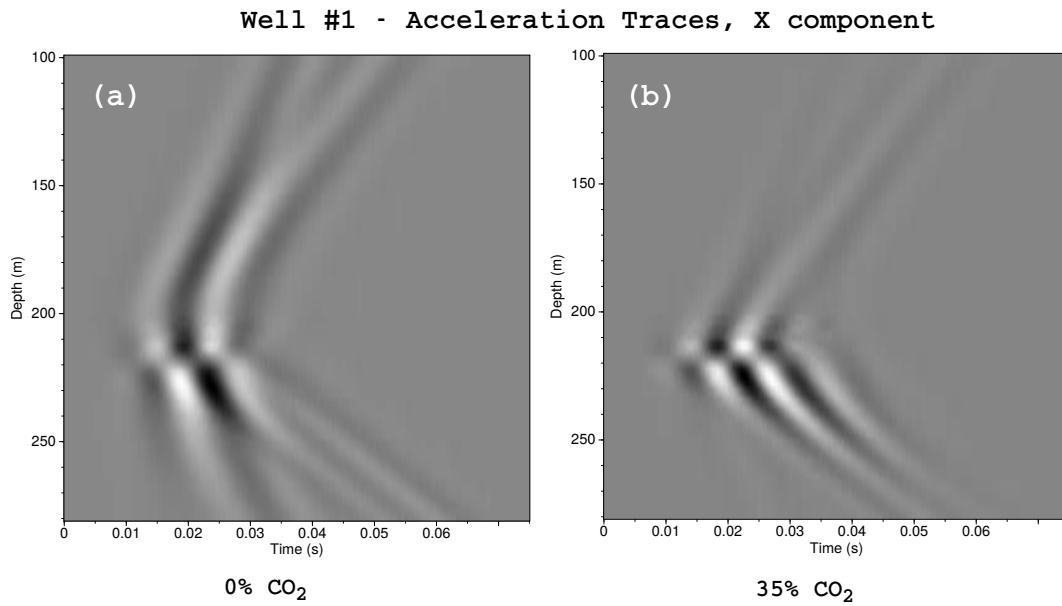




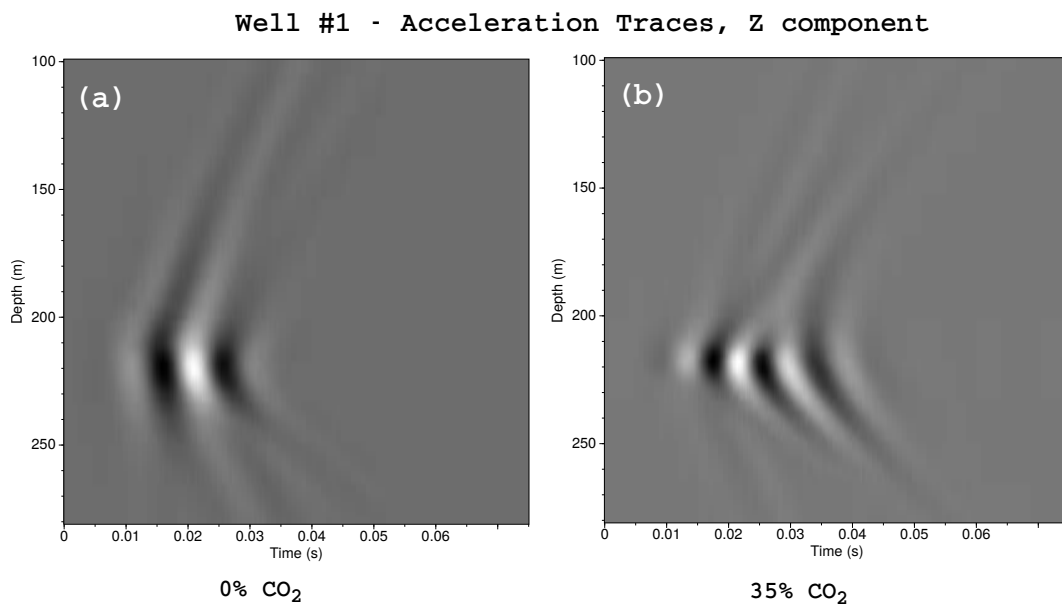
688 **Figure 4.** The CO<sub>2</sub> geological deposition model. The electromagnetic source is at 220 m depth, below  
689 the CO<sub>2</sub> plume, whose top is located at 210 m and base at 220 m (being the CO<sub>2</sub> in supercritical state). The  
690 seismic receivers are on the surface and in two wells.



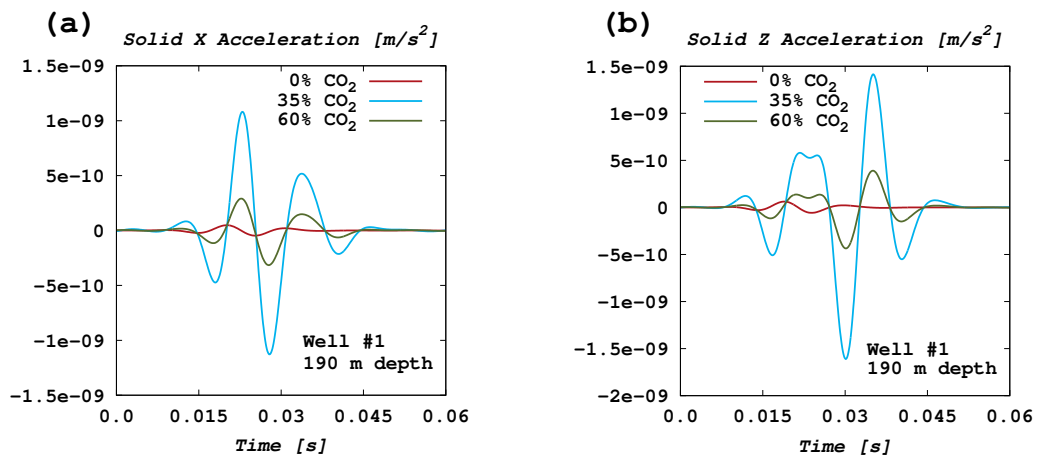
691 **Figure 5.** Each one of the displayed figures involves two layers, namely bottom layer and top layer. The  
 692 former depicts the finite element partition, and domain decomposition of the computational domain  $\Omega$ : (a)  
 693 The MMD scheme, (b) The SDD scheme. The latter (coloured) shows the distribution of the computational  
 694 domain according to the selected DD scheme: (a) nine computing cores, (b) three computing cores.



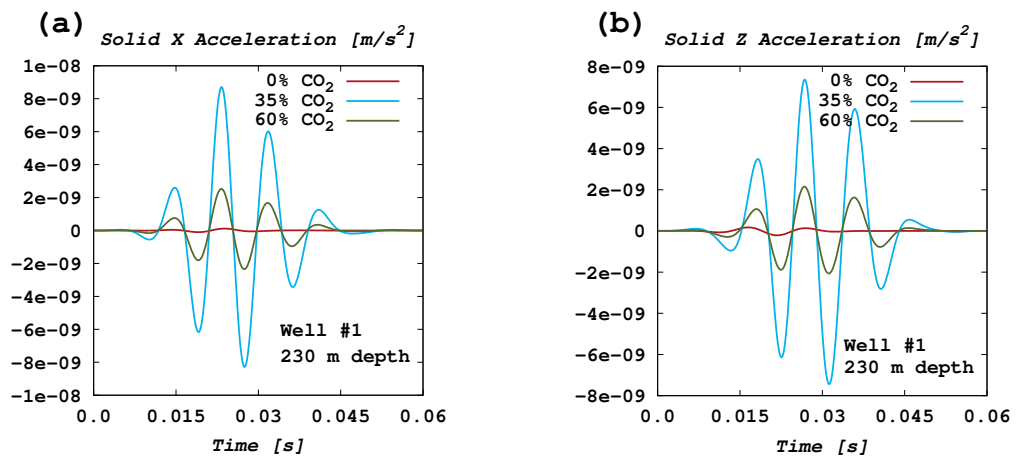
695 **Figure 6.** Well #1 x-component acceleration gather, before CO<sub>2</sub> pumping (a), and when CO<sub>2</sub> saturation  
696 reaches 35%. The electromagnetic source is at depth 220m just below the CO<sub>2</sub> plume which is between 210  
697 and 220 m depth.



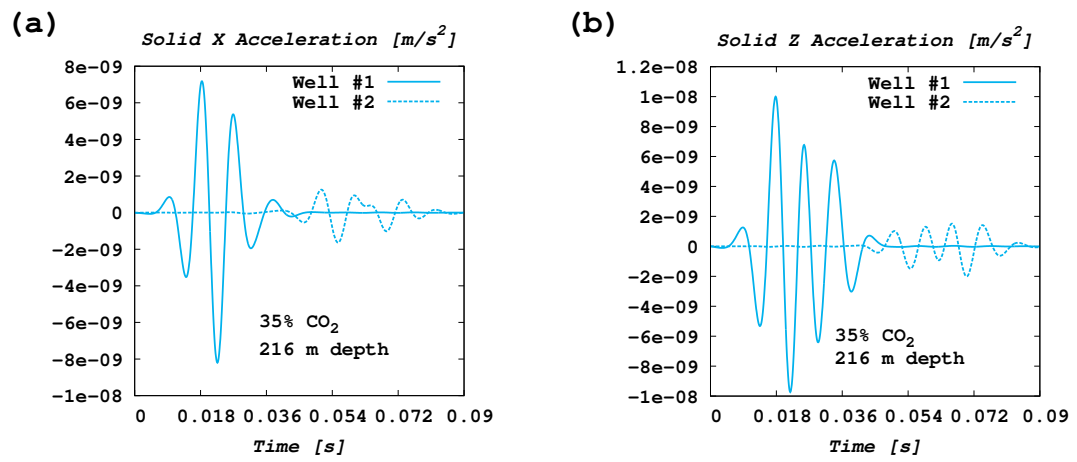
698 **Figure 7.** Well #1 z-component acceleration gather, before CO<sub>2</sub> pumping (a), and when CO<sub>2</sub> saturation  
699 reaches 35%.



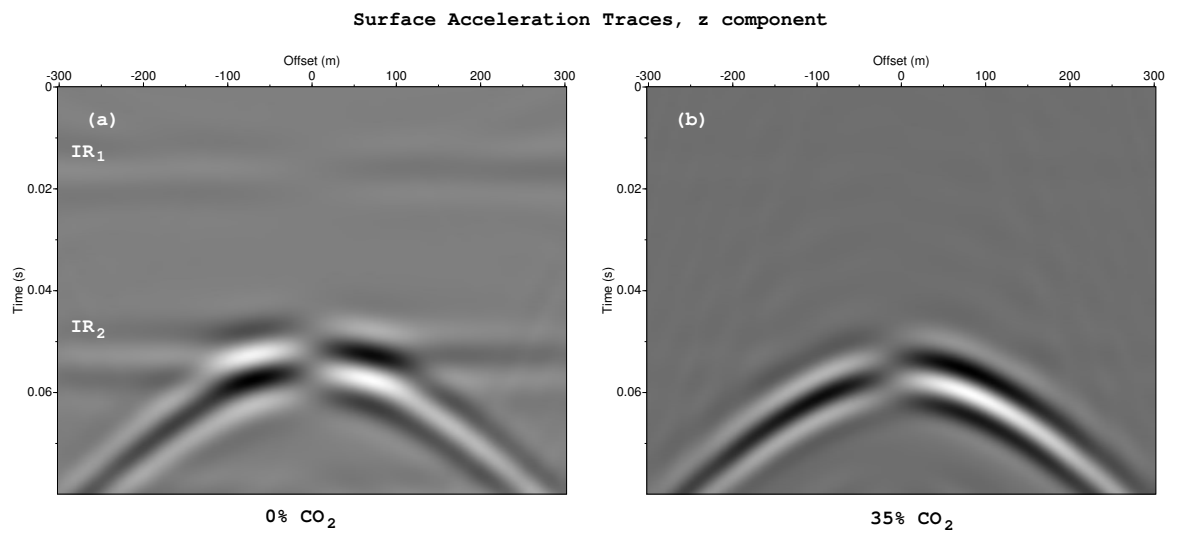
700 **Figure 8.** Well #1 acceleration traces, (a) x-component (b) z-component, before pumping CO<sub>2</sub> into the  
 701 reservoir and with CO<sub>2</sub> saturations of 35% and 60%, for a receiver located at 190 m depth, i.e., above the seal  
 702 layer.



703 **Figure 9.** Well #1 acceleration traces, (a) x-component (b) z-component, before pumping CO<sub>2</sub> into the  
 704 reservoir and with CO<sub>2</sub> saturations of 35% and 60%, for a receiver located at 230 m depth, i.e., below the  
 705 CO<sub>2</sub> plume.



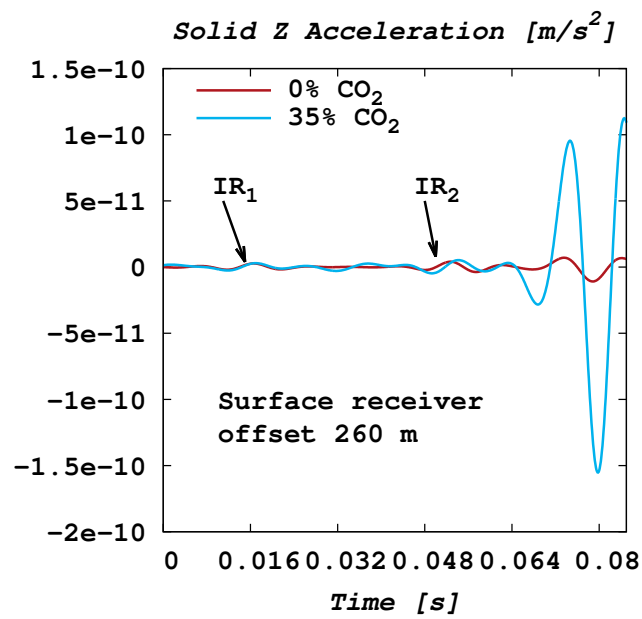
706 **Figure 10.** Comparison of Well #1 and Well #2 solid acceleration traces traces, for a receiver located at  
 707 216 m depth, i.e. at the same level as the CO<sub>2</sub> plume, for (a) x-component (b) z-component. CO<sub>2</sub> saturation is  
 708 35%.



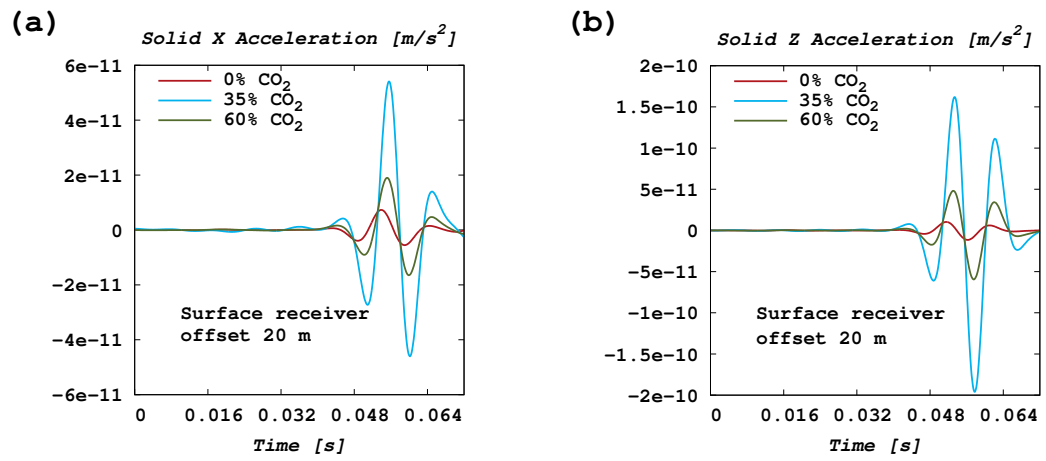
709 **Figure 11.** Surface acceleration traces, z component, (a) before pumping  $CO_2$  into the reservoir, (b) 35%

710  $CO_2$  saturation.





711 **Figure 12.** Surface acceleration traces, z component, taken at 260 m offset, before injection of CO<sub>2</sub> and at  
712 35% CO<sub>2</sub> saturation.



713 **Figure 13.** Surface acceleration traces, (a) x-component (b) z-component, before pumping CO<sub>2</sub> into the  
 714 reservoir and with CO<sub>2</sub> saturations of 35% and 60%, for a receiver located at 20 m offset.

715 **References**

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