

ON THE IMPACT OF HISTORY ON MODERN RESEARCH AND TEACHING

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PREVIOUS RESULTS

- **Paradox:** Brahmagupta, Baudhāyana, and Tartaglia recorded original results but seem not to have recorded derivations or even the proper assumptions of their own theorems.
- **Indeed,** did not use dogmatic form often considered standard.
- **Solution:** Derivations etc. are encoded in the discursive structure : *apodictic discourse* (= *motivated + conclusive*). Hence, there are several types of rigorous discourse.

Pb.: Overestimation of dogmatic discourse?

PROBLEM

Why was dogmatic discourse (prototype : Euclid's *Elements*) **taken as a standard** in the last two centuries, given that people like Tartaglia (first translator of the *Elements* into a vernacular) do not use it when they defend themselves?

Answer: (i) **Rediscovery and overestimation** of Euclid in the 19th c. **for teaching**, in Great Britain...

(ii) ...while **research** scrutinized the *Elements* and built modern Mathematics as **a critique** of them.

Upshot: Close reading of modern apodictic discourses should help in research. Example: reading L. de Broglie's thesis leads to new results.

OUTLINE

- A. History for (mostly British) **teaching** has led to an **overestimation of Euclid**.
- B. History for **research** has shown that
- 1) Euclidean Geometry is **not quite mathematically consistent and cannot be amended** ;
 - 2) modern Mathematics is “**multicultural**”, partly Indian, and Mesopotamian, and... .
 - 3) Mathematics is **not cumulative** (loss of content).
- C. Viewing rigorous M. as **apodictic discourse** helps solve ancient & modern open problems. (Not just a program...)

PREVIOUS WORK: RIGOROUS MATH. AS APODICTIC DISCOURSE

Refs.: <https://www.normalesup.org/~kichenassamy>

- (How *Brahmagupta* obtained his results on the *cyclic quadrilateral*) *Historia Mathematica*, 37(1) (2010) 28-61 and 39(4), (2012) 387-404.
- (How *Brahmagupta* obtained his results on *congruences*) *Gaṇita Bhāratī*, to appear
- (How *Tartaglia* obtained his results on *cubic equations*) *Historia Mathematica*, 2015, 42 (4), 407-435.
- (How *Baudhāyana* obtained his approx. *quadrature of the circle*) *Historia Mathematica*, 33, 2006, pp.149-183.

REFERENCES : DISCURSIVITY IN MATHEMATICS AND PHILOSOPHY

- « **L'analyse littéraire** au service de l'Histoire des mathématiques », *Comptes-rendus des séances de l'Académie des inscriptions et belles-lettres*, 2012-II (4), pp.781-796.
- (**sat & asat**) *Journal Asiatique*, 2018, 306 (1), 85-99.
- « L'emploi métonymique de l'arbre *kallāl* dans la philosophie médiévale en pays tamoul. » P.-S. Filliozat et M. Zink (eds.). *L'Arbre en Asie*, De Boccard, Paris, 2018, pp. 279-299.
- « L'irruption de l'infini : la légende de la colonne de lumière » (= **liṅgodbhava**), *Comptes-Rendus des Séances de l'Académie des Inscriptions et Belles-Lettres*, 2018 (4), to appear.

REFERENCES : “MODERN MATHEMATICS”

- “**Conformal Curvature Flows**: From Phase Transitions to Active Vision,” *Archive for Rational Mechanics and Analysis*, 134 (1996) 275-301 (with A. Kumar, P. J. Olver, A. Tannenbaum and A. Yezzi).
- **Fuchsian Reduction** : *Applications to Geometry, Cosmology and Mathematical Physics*, Birkhäuser, Boston, October 2007,
- “On a Conjecture of Fefferman and Graham,” *Advances in Mathematics*, 184 (2) (2004) 268-288.
- « Improving **Hölder’s inequality** », *Houston Journal of Mathematics*, 36 (1) (2010) 303-312,
- « Mécanique ondulatoire et **C-équivalence** », *Annales de la Fondation Louis de Broglie*, 45 (2020), 99-111.

A. *ELEMENTS* FOR TEACHING ?

- 1. “During the second half of the 18th c. England had come to be the only country where Euclid was practically the only geometrical text used.” (Cajori (1910) p. 193)**
- 2. “*Elements of Plane Geometry*” (AIGT) not used at home, but in the British colonies” (Cajori, p. 197). “...merely a smoothed down and polished presentation of the first six books of E.’s *Elements*” (Klein, quoted *ibid.*)**
- 3. Syllabus based on Euclid in Chennai (Senthil Babu, TIFR 2012, Table 1 p. 57-59),**

A. ELEMENTS FOR TEACHING ?

- 4. Reaction to “modern math.” : “A Euclidean course exists for some as an alternative to a modern course.” [Fielker \(1986\)](#)**
- 5. “The 11-16 geometry curriculum in England continues to concentrate on techniques for working in 2 dimensions, such as the plane geometry derived from Euclid, together with elements of transformation, vector and coordinate geometry.” [Royal Society \(2001\)](#).**

A. OVERESTIMATION OF EUCLID

Hence, **illusion** that (i) Euclid can be amended; (ii) Euclid is the standard of mathematical exposition (iii) **Modern M.** supposedly an outgrowth of Euclid's Elements, while it is a **critique** of E. based in part on input from **other cultures** (see below).

But : modern (university) teaching = essentially (toned down) Bourbaki + other math. (PDE, Differ. Geom...). Kept from Euclid the need to give a **connected, gapless argument** : may skip simple steps **but not significant ones.**

B1 : PROBLEMS WITH THE *ELEMENTS*

1. The notion of **angle** as magnitude attached to the meeting of lines or curves leads to a **paradox** (Proclus).

Conflation of **angle/solid angle** and **vertex** (Lakatos) hence, no good theory of polygons/polyhedrals.

Need to add and subtract angles and to measure them : measuring angles = **rectification** of an **arc of a circle** (d'Alembert, Dieudonné): **need real numbers and series.**

B1 : MODERN MATHEMATICS AS CRITIQUE OF THE *ELEMENTS*

- 2. The infinite is shunned** “The whole is greater than the part” excludes infinite sets.
- 3. Division with remainder**, improperly called “Euclidean division”, is not in Euclid. (Euclid’s algorithm is a mutual subtraction algorithm, not a mutual division algorithm.).
- 4. E. miss some axioms** (Pasch, Hilbert), etc.
- 5. Lack of algebra, appeal to figures, etc.**

B2. MODERN MATHEMATICS AS CRITIQUE OF THE *ÉLÉMENTS*: CHRONOLOGY

- 1. Legendre (late 18th c.)** : Hellenistic works not satisfactory for basic reasons (see his Number Th.) hence, new “Elements”
- 2. From 1819**, tr. + adopt. **French texts in the US**
- 3. Meanwhile, discovery of non-Euclidean Geometry** : Hellenistic Geometry is not a representation of reality
- 4. Gauss** : surveying needs **Differential Geometry [1827]**;

B2. MODERN MATHEMATICS AS CRITICISM OF EUCLID: CHRONOLOGY (CONT'D)

6. Riemann : same in 3D [1854].

7. Felix Klein, “Erlangen Program” [1872] +
lectures on “Elementary math. from a higher
standpoint” [1908]

8. General Relativity [1915] : need a Lorentzian
Geometry.

B3. MOD.MATH. ALSO FUELED BY NEW PRIMARY SOURCES

- **628 : Brahmagupta [1817]**
- **12th c. : Bhāskara II [1817]**
- **Kerala School [KV Sarma, MS Sriram, MD Srinivas, K Ramasubramanian, V. Pai... 21st c.]**
- **Al-Khwārizmī (Indian influence) [Rosen, 1831; Ruska, 1917; Gandz, 1936]**
- **499 : Āryabhaṭa [Kern, 1874] ; [Keller, 2006]**
- **Śulva-s. (c. 800-400 BC) “Treatises of the Cord” [Thibaut, 1875]. Indus [1924-1986...]**
- **Elt.s. [Peyrard, 1804, 1814-8; Heiberg, 1883-5]**

B3. MOD.MATH. ALSO FUELED BY NEW PRIMARY SOURCES

- Old Persian [Grotefend, 1802]
- **Egypt** : Champollion [**1822**]...
- ...Rhind Math. Papyrus [purchased **1858**]
- **China** : 19th c. [ex.: Biot, Zhou Bi 周髀, **1841**]
- M. in **Sumer & Akkad** [Thureau-Dangin, **1938**]
- Italian school (13th to 16th c.) [Cossali, 1797-9; Libri 1838-41, ..., Masotti, 1959; ...]
- **Archimedes** with figures [Netz, 2017 !]

B3. MATHEMATICS NOT CUMULATIVE: BREAKS IN THE CONTINUITY OF KNOWLEDGE

- Pacioli's & Tartaglia's *evidentiae* (*proto-identities*) forgotten after him.
- Pacioli's & Tartaglia's tradition of **multiple unknowns** of the « Ancients » cut off (used it but did not relate it to a tradition).

B3. MATHEMATICS NOT CUMULATIVE :

- 1. Heterometry** (Baudhāyana) : (i) number embedded in Geometry through the choice of units ; (ii) units are scalable (allometry) ; (iii) there may be several incommensurable units in one problem. (Hence, no prime numbers.)
- 2. The non-manifest** (avyakta = unknown) underlines the manifest (rejection of the Lokāyata) : perception is not the only pramāṇa : inference is needed.
- 3. Brahmagupta's 'triquadrilateral'** (*tricatorbhuja*) forgotten after him.

C. NEED HISTORY: ANALYSIS OF L. DE BROGLIE'S THESIS

1. His “**chunk of energy**” is represented by an extended wave with its own time/length units.
2. That means each “**chunk**” defines its own “**rest**” **system**.
3. This system is a pseudo-inertial system in the sense of **C-equivalence** (S.K., 1963) = inertial system with non-standard clocks/rods.
4. **Can measure** these new units (via Mössbauer or via measurements of acceleration).

CONCLUSIONS

- (i) Mathematics is **partially Indian, and Mesopotamian, and...)** and therefore **essentialism is futile.**
- (ii) **Modern mathematics has not incorporated all the results of the past. Math. is not cumulative.**
- (iii) **The internal development** of a cultural tradition may **fail to get rid of its blinkers** (e.g. needed input through the Arab world).
- (iv) Rigorous mathematics: **apodictic or dogmatic discourse.**

History is a dimension of Mathematical activity.

History alone gives meaning to the present and paves the way for the future.