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Field-induced circulation flow in magnetic fluids.

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Summary

We present results of theoretical study of circulation flow in ferrofluids under the action of an alternating inhomogeneous magnetic field. The results show that the field with the amplitude about 17 kA/m and frequency 10s^{-1} can induce the mesoscopic flow with the velocity amplitude about 0.5mm/s. This mechanism can be used for intensification of drug delivery in blood vessels.

1. Introduction

The main problem of treatment of brain strokes is related to very slow diffusion of the thrombolytic drugs toward blood clots through blocked vessels. An American company Pulse Therapeutics has proposed a smart solution to this problem using magnetic micro- or nanoparticles entrained in motion by alternating magnetic fields and able to create recirculating flows in the blocked vessels [1]. These recirculating flows considerably enhance convective transport of the drug towards the clots. Only a few works have been published on this topic [2-4], so, the physical understanding of the origin of the recirculating or oscillatory flows created by moving and rotating magnetic particles is still lacking. To shed more light onto this problem, in this paper, we propose a theoretical model considering ferroparticle motion

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and induced flows inside a channel under applied alternative non-homogeneous magnetic fields. The obtained amplitudes of the velocity and fluid displacement are compared to those required for efficient drug delivery to the blood clots in a real situation.

The aim of this work is theoretical study of circulation flows induced by an alternating magnetic field in a magnetic fluid filling a flat gap, which size in its plane is much more than the distance between the gap walls.

2. Mathematical model and the main approximations.

We consider an infinite flat gap of the thickness l filled with a ferrofluid, containing identical spherical ferroparticles. The alternating magnetic field is created by four solenoids, illustrated in Fig. 1.

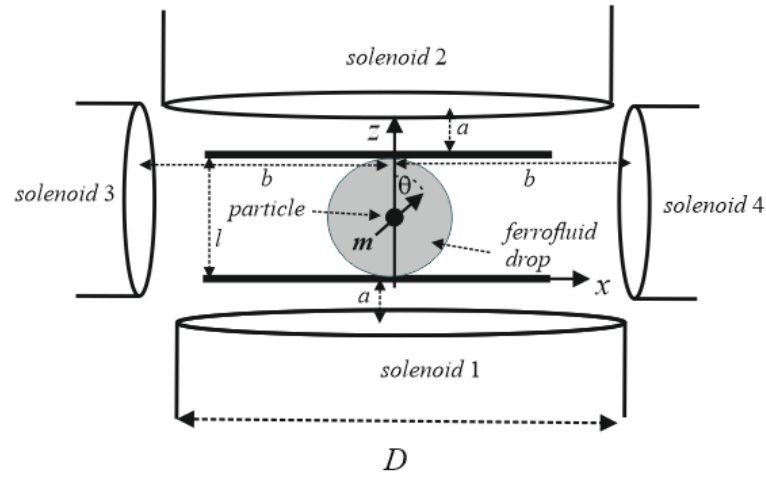


Fig.1. Illustration of the model system

Let m and M be the particle magnetic moment and saturated magnetization of its material; V_p is the particle volume. It is supposed that the gap thickness l is much less than the solenoid diameter D ($l \ll D$). We suppose that, initially, some drop of a ferrofluid is injected in the center of the considered model.

We will denote the local volume concentration of the particles as $\Phi(x,z,t)$ and, for maximal simplification of mathematics, consider the 2D approximation, when all physical events take place in the plane (x,z) shown in Fig.1. Additionally, we will neglect effects of the particles Brownian rotation. This means, we suppose that the Zeeman energy of the particle interaction with the field \mathbf{H} is significantly more than the thermal energy kT . Note that, from the practical point of view, this case is the most interesting. Note, for the magnetite particles with diameter about 10-20nm this condition is fulfilled if the local H exceeds 10kA/m. That is easy achievable range of the field. Equations of flow of the magnetizable fluid at low Reynolds number can be presented as (see [5,6]):

$$\begin{aligned} \rho \frac{\partial v_x}{\partial t} &= -\frac{\partial p}{\partial x} + \eta \Delta v_x + \frac{1}{2} \frac{\partial}{\partial z} \Gamma + \mu_0 M \Phi \left[\cos \theta \frac{\partial}{\partial z} + \sin \theta \frac{\partial}{\partial x} \right] H_x \\ \rho \frac{\partial v_z}{\partial t} &= -\frac{\partial p}{\partial z} + \eta \Delta v_z - \frac{1}{2} \frac{\partial}{\partial x} \Gamma + \mu_0 M \Phi \left[\cos \theta \frac{\partial}{\partial z} + \sin \theta \frac{\partial}{\partial x} \right] H_z \\ \frac{\partial}{\partial x} v_x + \frac{\partial}{\partial z} v_z &= 0 \end{aligned} \quad (1)$$

Here $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator, $\Gamma = \mu_0 M \Phi (H_z \sin \theta - H_x \cos \theta)$ is the magnetic torque, acting by unit volume of the ferrofluid; \mathbf{H} is the local magnetic field in the fluid; θ is the angle between the particle magnetic moment m and the axis Oz , normal to the gap plane (see Fig.1). The third terms in the first two equations of (1) present the stress, which appear because of the magnetic torque Γ ; the fourth one is the ponderomotive force, acting on the fluid from the side of the non-uniform field \mathbf{H} .

The boundary conditions for (1) are:

$$\begin{aligned} v_x = v_z = 0 \text{ at } z = 0, l \\ v_x, v_z \rightarrow 0 \text{ at } x \rightarrow \infty \end{aligned} \quad (2)$$

In the “no Brownian” approximation, equation for the angle θ reads [1]

$$\frac{\partial \theta}{\partial t} = \frac{1}{2} \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) - \frac{1}{6\eta\Phi} \Gamma \quad (3)$$

Note, the ratio Γ/Φ is magnetic torque with acts on the unite volume of the particle.

Equation of continuity for the particles concentration can be written as:

$$\frac{\partial}{\partial t} \Phi + \text{div} \left[\Phi \left(\mathbf{v} - \frac{1}{3\pi\eta d} \nabla U \right) \right] = D \Delta \Phi \quad (4)$$

$$U = -\mu_0 M V_p (H_z \cos \theta + H_x \sin \theta)$$

Here d is the particle diameter; U is the particle potential energy in the field \mathbf{H} .

Below we will use the following estimates for the system parameters. We suppose that the particles are magnetite, therefore $M \approx 500 \text{ kA/m}$; diameter of the particle $d \approx 15 - 20 \text{ nm}$; volume concentration of the particles $\Phi \sim 0.01$; the angular frequency of the alternating field, *created by the colenoid* $\omega \sim 10 \text{ s}^{-1}$; the solenoid provides the field \mathbf{H} inside the gap with the strength absolute value $H > 10 \text{ kA/m}$. The viscosity and density of the host medium are close to those of water ones, i.e. we suppose $\eta \sim 10^{-3} \text{ Pa} \cdot \text{s}$; $\rho \sim 10^3 \text{ kg/m}^3$. The gap thickness is estimated as $l \sim 1 \text{ mm}$.

Analytical solution of the whole system (1-4) is impossible. However, serious simplifications can be achieved by using the mentioned estimates for the system characteristics. First of all, from the mathematical point of view, the fluid flow is provoked by the last two terms in the Navier-Stocks equations of (1). These terms are proportional to the small concentrations Φ of the particles. Therefore, roughly, the scaling relation $v \sim \Phi$ must be held and the terms $\frac{1}{2} \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right)$, Φv can be considered as the smallest ones in eqs. (3), (4). We will neglect these terms.

Let us present the local concentration of the particles as $\Phi(x, z, t) = \Phi_0(x, z) + \varphi(x, z, t)$ where $\Phi_0 \sim 0.01$ is the initial volume concentration of the particles in the drop. Estimates show that in the field with the strength

$H \sim 10 \text{ kA/m}$ and the spatial scale $D \sim 10^{-2} \text{ m}$ the characteristic time $\tau = \frac{3\pi\eta d L^2}{\mu_0 M V_p H}$ of magnetophoretic migration of

the magnetite particle with diameter 20 nm at the distance D is more than $5 \cdot 10^5 \text{ s} \approx 140 \text{ hours}$. That significantly exceeds the time, presenting interest from the viewpoint of the drug delivery technology. Therefore, at least in the first approximation, one can neglect the term φ as compared with Φ_0 and suppose that for the time, presenting interest, the equality $\Phi = \Phi_0(x, z)$ is held. In principle, the drop shape can be varied because of the convective motion of the ferrofluid particles in the generated circulation flow. Discussion of this factor is given in the Conclusion.

Neglecting the term $\frac{1}{2} \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right)$ and by using the explicit form for the torque Γ , one can rewrite eq. (3) as:

$$\frac{\partial \theta}{\partial t} = -\frac{\mu_0 M}{6\eta} (H_z \sin \theta - H_x \cos \theta) \quad (6)$$

For convenience one can present the first two equations of (1) as:

$$\begin{aligned} \rho \frac{\partial v_x}{\partial t} &= -\frac{\partial p}{\partial x} + \eta \Delta v_x + \mu_0 M \Phi G_x \\ \rho \frac{\partial v_z}{\partial t} &= -\frac{\partial p}{\partial z} + \eta \Delta v_z + \mu_0 M \Phi G_z \\ G_x &= \frac{1}{2} \frac{\partial}{\partial z} (H_z \sin \theta - H_x \cos \theta) + \left[\cos \theta \frac{\partial}{\partial z} + \sin \theta \frac{\partial}{\partial x} \right] H_x \\ G_z &= -\frac{1}{2} \frac{\partial}{\partial x} (H_z \sin \theta - H_x \cos \theta) + \left[\cos \theta \frac{\partial}{\partial z} + \sin \theta \frac{\partial}{\partial x} \right] H_z \end{aligned} \quad (7)$$

We will suppose that the solenoids create the field with the same angular frequency ω and the components

$$H_x = (H_{01x}(x, z) - H_{04x}(x, z)) \cos \omega t + (H_{02x}(x, z) - H_{03x}(x, z)) \sin \omega t \quad (8)$$

$$H_z = (H_{01z}(x, z) - H_{04z}(x, z)) \cos \omega t + (H_{02z}(x, z) - H_{03z}(x, z)) \sin \omega t$$

Here $\mathbf{H}_{01}, \mathbf{H}_{02}, \mathbf{H}_{03}$ and \mathbf{H}_{04} are amplitudes of the fields created by the lower, upper, left and right solenoids in Fig.1. The relations (8) mean that the north pole of the left solenoid in Fig.1 is situated in front of the south pole of the right one. The same is for the magnets which are above and below the gap with the fluid.

It follows from eq.(6) that, in the order of magnitude, the characteristic time τ_H of the angle θ relaxation to the given field \mathbf{H} is $\tau_H \sim \eta / \mu_0 M H$. Suppose that absolute value of the field in the gap has the typical magnitude $H \sim 10 \text{ kA/m}$, the particles are magnitude ($M \sim 500 \text{ kA/m}$) and the carrier liquid is water ($\eta \sim 10^{-3} \text{ Pa} \cdot \text{s}$). In this case $\tau_H \sim 10^{-6} \text{ s}$. The typical value of the angular frequency for the experiments like that is $\omega \sim 10 \text{ s}^{-1}$, therefore, the field alternation period is about 0.1 s^{-1} . Therefore, the rate of the field alternation is about five decimal orders less than the rate of θ relaxation. This means that in any moment of time the angle θ has practically the value θ^0 equilibrium with respect to the field \mathbf{H} at the given time t ; this angle is determined by the relations

$$\cos \theta^0 = \frac{H_z}{H}, \quad \sin \theta^0 = \frac{H_x}{H} \quad (9)$$

$$H = \sqrt{H_x^2 + H_z^2}$$

Taking into account, we use $\theta = \theta^0$ in eq. (9). In this approximation, with account of the standard magnetostatic relation $\text{rot} \mathbf{H} = 0$, one gets from (9):

$$G_x = \frac{\partial}{\partial x} H; \quad G_z = \frac{\partial}{\partial z} H \quad (10)$$

This is convenient to introduce the current function Ψ so that

$$v_x = \frac{\partial}{\partial z} \Psi, \quad v_z = -\frac{\partial}{\partial x} \Psi \quad (11)$$

Combining (7) and (11), taking into eq. (10), after transformations we come to the following equation with respect to Ψ :

$$\rho \frac{\partial Q}{\partial t} = \eta \Delta Q + \mu_0 M \left[\frac{\partial}{\partial x} H \cdot \frac{\partial}{\partial z} \Phi - \frac{\partial}{\partial z} H \cdot \frac{\partial}{\partial x} \Phi \right] \quad (12)$$

$$Q = \Delta \Psi$$

The boundary conditions for equation (12) are:

$$z=0, l \quad \frac{\partial \Psi}{\partial x} = \frac{\partial \Psi}{\partial z} = 0 \quad (13)$$

$$t=0, \Psi = 0$$

Taking into account the relation $D \gg l$ between the velocity v scales in the x (D) and z (l) directions, we get the strong inequalities

$$\frac{\partial^2 \Psi}{\partial z^2} \gg \frac{\partial^2 \Psi}{\partial x^2}$$

Therefore, in eq. (9) one can put

$$Q = \frac{\partial^2 \Psi}{\partial z^2}; \quad \Delta Q = \frac{\partial^4 \Psi}{\partial z^4} \quad (14)$$

And to present this equation as:

$$\frac{\partial Q}{\partial t} = \nu \frac{\partial^2 Q}{\partial z^2} + f; \quad (15)$$

$$f = \frac{\mu_0 M}{\rho} \left[\frac{\partial}{\partial x} H \cdot \frac{\partial}{\partial z} \Phi - \frac{\partial}{\partial z} H \cdot \frac{\partial}{\partial x} \Phi \right]$$

3. Solution of equation (12); results and discussions.

We suppose that the spatial scale of the function f in the x direction is much more than that in z direction.

This is justified by configuration of the solenoids, which create the field \mathbf{H} and the ferrofluid drop distribution in the gap. Indeed, the spatial scale of the function f is determined by the solenoid diameter D and their distances a, b from the ferrofluid drop. All these scales usually are much more than diameter of the blood vessel, which is modeled here by the gap with the thickness l . Thus, at the distance l , the function f changes very weakly and, in the first approximation, one can neglect dependence of f on the coordinate z .

Taking into account that the function f is periodical with the period $2\pi/\omega$, we can expand this function in the Fourier series as (see, for example, ref. [7]):

$$f = \frac{1}{2}fC_0 + \sum_{n=1}^{\infty}(fC_n \cos(\omega nt) + fS_n \sin(\omega nt)) \quad (16)$$

$$fC_n = \frac{1}{T} \int_{-T/2}^{T/2} f(x,t) \cos(\omega nt) dt \quad fS_n = \frac{1}{T} \int_{-T/2}^{T/2} f(x,t) \sin(\omega nt) dt$$

$$\Psi = \frac{1}{2}\Psi C_0 + \sum_{n=1}^{\infty}(\Psi C_n \cos(\omega nt) + \Psi S_n \sin(\omega nt))$$

By using (16) in (14) and (15), we come to the relations:

$$\Psi C_n = -A_n \frac{e^{kz} \cos kz}{2k^2} - B_n \frac{e^{kz} \sin kz}{2k^2} - C_n \frac{e^{-kz} \cos kz}{2k^2} - D_n \frac{e^{-kz} \sin kz}{2k^2} + L1_n z + L2_n - \frac{fS_n}{2(\omega n)} z^2 \quad (17)$$

$$\Psi S_n = A_n \frac{e^{kz} \sin kz}{2k^2} - B_n \frac{e^{kz} \cos kz}{2k^2} - C_n \frac{e^{-kz} \sin kz}{2k^2} + D_n \frac{e^{-kz} \cos kz}{2k^2} + N1_n z + N2_n + \frac{fC_n}{2(\omega n)} z^2$$

$$k = \sqrt{\frac{\omega}{2\nu}}$$

Here $A_n, \dots, N2_n$ are constants of integration, to be determined.

The boundary conditions (13) can be rewritten in the form

$$z=0,l \quad \frac{\partial \Psi C_n}{\partial z} = 0, \quad \frac{\partial \Psi S_n}{\partial z} = 0, \quad \Psi C_n = 0, \quad \Psi S_n = 0 \quad (18)$$

Combining eqs. (17) and (18), we come to a system of eight linear equations with respect to eight variations $A_n, \dots, N2_n$. Analytical solution of this systems is too cumbersome; however, the numerical calculations can not present a problem.

Having the coefficients $A_n, \dots, N2_n$ found, by using eq. (11), we determine the velocity components $v_{x,z}$.

Results and discussion.

For definiteness we supposed the the spatial distribution of the particles obey to the Gaussian distribution

$$\Phi = \Phi_0 \exp \left[-\frac{\left(x^2 + \left(z - \frac{l}{2} \right)^2 \right)}{\sigma^2} \right] \quad (19)$$

This formula means that in the vicinity of the middle of the gap there is a dense cloud of ferrofluid with the maximal *volume concentration* Φ_0 at point $(0, l/2)$; σ is dispersion of this distribution. It should be noted that the spatial distribution (19) is used here just as a model one. Any other, more relevant to a concrete experimental situation, can be used for the calculations in the frame of the suggested approach.

To calculate the components of amplitude \mathbf{H}_{01} of the field, created by the solenoid 1 in Fig.1, using the Biot-Savart law. In cylindrical coordinates it has the form

$$H_{01x}(x, z) = \frac{I \cdot D \cdot N}{8\pi h} \int_{-(h+a)}^{-a} dz' \left(\int_0^{2\pi} \frac{(z - z') \cos \varphi}{\left[(z - z')^2 + \left(\frac{D}{2} \sin \varphi \right)^2 + \left(x - \frac{D}{2} \cos \varphi \right)^2 \right]^{3/2}} d\varphi \right)$$

$$H_{01z}(x, z) = \frac{I \cdot D \cdot N}{8\pi h} \int_{-(h+a)}^{-a} dz' \left(\int_0^{2\pi} \frac{\frac{D}{2} - x \cos \varphi}{\left[(z - z')^2 + \left(\frac{D}{2} \sin \varphi \right)^2 + \left(x - \frac{D}{2} \cos \varphi \right)^2 \right]^{3/2}} d\varphi \right)$$

where I is the current in the solenoid, h is its height, N is the number the solenoid coils and a is the distance from solenoid to the gap. This formula represents the field components for the lower solenoid, for the other solenoids these components have been determined similarly

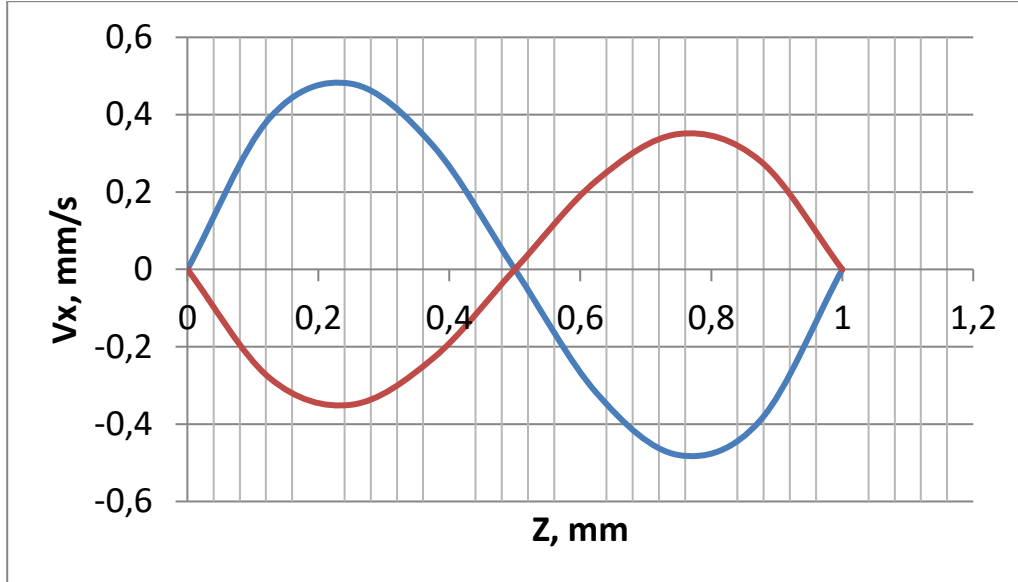


Fig. 2. Longitudinal velocity component v_x vs. z coordinate at a fixed $x = 10$ mm in Fig. 1.

Red line: time $t = 0.5$ s; blue one: $t = 1$ s. The field angular frequency $\omega = 10s^{-1}$. Volume concentration of the particles in the drop center $\Phi_0 = 0.01$; the dispersion $\sigma = 1$ cm; the solenoid characteristics are identical for all solenoids: diameter $D=1$ cm; current $I = 10$ A; height $h=1$ cm; number of coils $N = 10^4$.

The distances, shown in Fig. 1:

$$a = b = 5 \text{ cm}; \text{ the gap thickness } l=1\text{mm}.$$

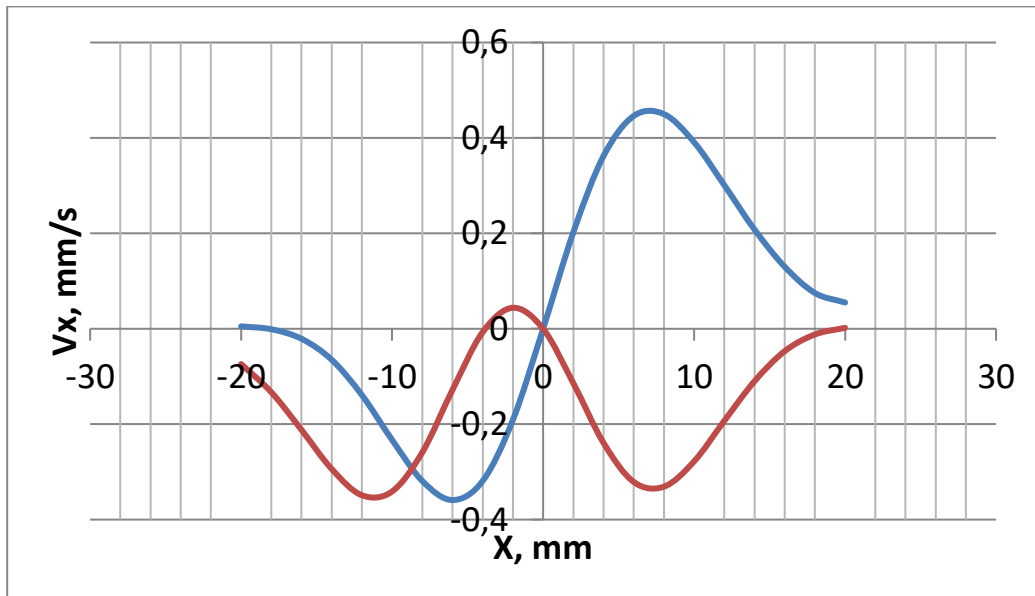


Fig. 3. Longitudinal velocity component v_x vs. x coordinate at a fixed $z = l/8$ in Fig. 1.

Red line: time $t = 0.5$ s; blue one: $t = 1$ s. The other parameters are the same as in Fig. 2

Note that for the system parameters, corresponding to Figs.2,3, the amplitude of absolute value of the field in the gap center is about 17kA/m

4. Conclusions

We present theoretical model of magnetically induced circulation flow in a flat channel filled by a Newtonian liquid and a drop of ferrofluid. The results show that oscillating magnetic field with amplitude about 17 kA/m and frequency about 10s^{-1} in the channel with the width 1mm can induce circulating flow with the velocity amplitude about 0.5mm/s.

Let us discuss now the approximation of the permanent shape of the ferrofluid drop. Since the magnetophoretic displacement of the ferrofluid particles is neglected, the drop shape can be changed only because of the convective effects. In the order of magnitude, the displacement δr of the drop particles for the field period satisfies to the relation $\delta r \leq v_{max}2\pi/\omega$, where v_{max} is maximal value of the velocity. Our results give $v_{max} \sim 0.5\text{mm/s}$. By using $\omega \sim 10\text{s}^{-1}$, one gets $\delta r \leq 0.3\text{mm}$. This is much less than the used characteristic size $\sigma \sim 1\text{cm}$ of the drop. Therefore, the simplification of the permanent shape of the drop is justified, at least, as the first approximation.

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Data Accessibility

Competing Interests

We have no competing interests

We have no competing interests.

References

1. Creighton, Francis M. 2012. "Magnetic-based systems for treating occluded vessels." U.S. Patent No. 8,308,628. 13 Nov.
 2. Clements, M. J. 2016. A mathematical model for magnetically-assisted delivery of thrombolytics in occluded blood vessels for ischemic stroke treatment (Doctoral dissertation, Texas University).
 3. Gabayno, J. L. F., Liu, D. W., Chang, M., & Lin, Y. H. 2015. Controlled manipulation of Fe₃O₄ nanoparticles in an oscillating magnetic field for fast ablation of microchannel occlusion. *Nanoscale*, 7(9), 3947-3953. (doi: 10.1039/x0xx00000x)
 4. Li, Q., Liu, X., Chang, M., & Lu, Z. (2018). Thrombolysis Enhancing by Magnetic Manipulation of Fe₃O₄ Nanoparticles. *Materials*, 11(11), 2313, 12p.
 5. Rosensweig, R, 1985, *Ferrohydrodynamics*. (Cambridge, New York)
 6. Landau, L., Lifshitz, E., 1960, *Electrodynamics of Continuum Media*. (Pergamon Press, London)
 7. Farlow, S.J., Partial Differential Equations for Scientists and Engineers (J.Wiley and Sons, 1982)
- Creighton, Francis M. "Magnetic-based systems for treating occluded vessels." U.S. Patent No. 8,308,628. 13 Nov. 2012.

Figure captions

Figure 1 Illustration of the model system

Figure 2. Longitudinal velocity component v_x vs. z coordinate at a fixed $x = 10\text{mm}$ in Fig. 1.

Red line: at time $t = 0.5$ s; blue one: $t = 1$ s. The field angular frequency $\omega = 10s^{-1}$. Volume concentration of the particles in the drop center $\Phi_0 = 0.01$; the dispersion $\sigma = 1$ cm; the solenoid characteristics: diameter $D=1$ cm; current $I = 10$ A; height $h=1$ cm; number of coils $N = 10^4$ for all solenoids. The distances, shown in Fig. 1:

$a = b = 5$ cm; the gap thickness $l=1$ mm.

Figure 3. Longitudinal velocity component v_x vs. x coordinate at a fixed $z = l/8$ in Fig. 1.

Red line: at time $t = 0.5$ s; blue one: $t = 1$ s. The other parameters are the same as in fig. 2

Note that for the system parameters, corresponding to Figs.2,3, the amplitude of absolute value of the field in the gap center is about 17kA/m