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Physical signs and minutes of days: mean motion computations in the *Parisian Alfonsine Tables*

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1. Introduction

A Latin tradition for Alfonsine astronomy emerged from 1320 onward in Paris. It was expressed in a range of tabular, instruments and textual works produced by a very active generation of astronomers including John Vimond, John of Lignères, John of Murs and John of Saxony among others. One of the most successful table set beginning its circulation from this Parisian milieu is known today as the *Parisian Alfonsine Tables*. Their precise origin, definition and content is yet a matter of historical research. In this paper, I survey the last decades of this historical debate on a specific aspect at the interface between history of astronomy and history of mathematics.

In the 1970's and early 1980's, following up on works of Duhem and Dreyer, John D. North and Emmanuel Poulle investigated the question of the emergence and transmission of a Latin tradition for Alfonsine Astronomy.¹ Although not defending the same historiographical thesis, they identified a number of “characteristic” elements distinguishing the *Parisian Alfonsine Tables*, as presented in the 1483 princeps edition by Ratdolt, and the *Castillian Alfonsine Canons* edited by Rico y Sinobas in the nineteenth century.² The appeal for such analysis derives ultimately from the state of the known manuscript sources on the early development of Alfonsine astronomy. There is only one manuscript witness for the *Castillian Alfonsine Canons*, while all known canons to the *Parisian Alfonsine Tables* were compiled in Latin after 1320 first in Paris and later elsewhere around Europe. There are no known copies of a table set produced in the court of Alfonso X and corresponding to the *Castillian Alfonsine Canons*, while there are hundreds of witnesses of the *Parisian Alfonsine Tables*, which do not correspond strictly to the *Castillian Alfonsine Canons*. The noted divergences between the *Castillian Alfonsine Canons* and the *Parisian Alfonsine Tables* include:

1. specific values for the equation of the Sun, Venus and Jupiter;

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¹ J. D. North, “The Alfonsine Tables in England”, in *Prismata: Festschrift für Willy Hartner*, Wiesbaden 1977, 327-59 ; E. Poulle, “John of Murs”, in *Dictionary of Scientific Biography*, New York 1973, 128-33; E. Poulle, “John of Lignères”, in *Dictionary of Scientific Biography*, New York 1973, 122-8; E. Poulle, “John of Saxony”, in *Dictionary of Scientific Biography*, New York 1973, 139-41; E. Poulle, “Jean de Murs et les tables alfonsines”, *Archives d'histoire doctrinale et littéraire du moyen âge*, 47 (1980), 241-71; E. Poulle, *Les tables alfonsines avec les canons de Jean de Saxe*, Paris 1984; J. D. North, “Just whose were the Alfonsine Tables”, in *From Bagdad to Barcelona, studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*, Barcelona 1996, 453-77

² R. Sinobas, *M Libros del Saber de Astronomia del Rey D. Alfonso X de Castilla*, Madrid 1863-7; E. Ratdolt, *Tabule astronomice illustrissimi Alfonsij regis castelle*, Venice 1483.

2. reliance on tropical rather sidereal coordinates;
3. specific double motion of the eighth sphere;
4. concomitant use of physical signs of 60° and sexagesimal numbers of days in mean motion tables.

Emmanuel Poulle, considering these elements a whole, went further and argued that the *Parisian Alfonsine Tables* were historically independent from the work of the Castilian astronomers under the patronage of Alfonso X. He maintained that the Parisian astronomers, and among them chiefly John of Murs in his *Expositio*, constructed the link to Alfonso X as a “literary fiction” in order to legitimise their creation of an entirely new table set.³

In the late 1990’s and early 2000’s, this radical interpretation was strongly challenged in a series of publications mainly by José Chabás and Bernard R. Goldstein where the whole constituted by Emmanuel Poulle is analysed and questioned piece by piece. More significantly than this argumentation strategy, the strength of José Chabás and Bernard R. Goldstein’s intervention rely on a study of many new sources far beyond the 1483 princeps edition. These new sources comprise the most important table sets from the early Parisian Alfonsine corpus and a new commented edition of the Castilian canons to the Alfonsine tables.⁴ Emmanuel Poulle, while reacting to these publications in different articles and book reviews, did not provide new studies based on primary sources or fundamentally different arguments to the discussion than what he had produced in the 1970’s and 1980’s.⁵ In a review of José Chabás and Bernard R. Goldstein’s edition of the *Castillian Alfonsine Canons* published in 2004 Noël Swerdlow push a different hypothesis: Alfonsine tables did rich the Parisian astronomers of the early fourteenth century but they are not the version described in the *Castillian Alfonsine Canons*.⁶ The historiographical debate toned down since about ten years. However, in recent publications, Julio Samsó produces new evidences of dependencies between the *Parisian Alfonsine Tables* and astronomical traditions from *al-Andalus*.⁷ He argues strongly for the existence of a second version of the Alfonsine Tables, produced in Alfonso court sometime after the canons were compiled, and sharing at least some of the characteristic of the *Parisian Alfonsine Tables*.⁸

³ Poulle, “Jean de Murs...”, 1980

⁴ The bibliography of J. Chabás and B. R. Goldstein on the topic is too large to be quoted here extensively, please refer to J. Chabás, *Computational Astronomy in the Middle Ages, Sets of Astronomical Tables in Latin*, Madrid 2019, 400-1.

⁵ E. Poulle, “Les tables Alphonsines sont-elles d’Alphonse X?”, in *De astronomia Alphonsi Regis*, Barcelona 1987; E. Poulle, *Astronomie Planétaire au Moyen Âge Latin*. Aldershot 1996.

⁶ “The common parameters must therefore come from a common earlier source, and, unless one wishes to hypothesize some completely unknown tables, this can only be some earlier Alfonsine tables, although not strictly the version described in the Castilian canons.” N. Swerdlow, “Alfonsine Tables of Toledo and later Alfonsine Tables”, *Journal for the History of Astronomy*, (35) 2004, 479-84, 482. This hypothesis was presented first by Julio Samsó in 1987: J. Samsó, “Alfonso X and Arabic Astronomy”, in *De Astronomia Alphonsi Regis*, Barcelona 1987, 23-58, 31 quoted from J. D. North, “Just whose were the Alfonsine Tables”, in *From Bagdad to Barcelona, studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*, Barcelona 1996, 453-77, 456 (n. 9). In 2011, analysing the references to Alfonso in John of Murs *Expositio*, I came to a similar conclusion: M. Husson, “L’astronomie alphonsine dans l’*Expositio intentionis regis Alfonsii circa tabulas ejus* de Jean Murs”, *Archives d’histoire doctrinale et littéraire du Moyen âge*, (78) 2011, 229-45.

⁷ J. Samsó, “Al-Zarqal, Alfonso X and Peter of Aragon on the solar equation”, in *From deferent to equant: a volume of studies in the history of science in the ancient and medieval Near East in honor of E. S. Kennedy*, New York 1987; J. Samsó, “Ibn Ishaq and the Alfonsine Tables”, *Journal for the History of Astronomy*, (50) 2019, 360-5; J. Samsó, *On Both Side of the Strait of Gibraltar*, Leiden 2020, in particular 33-44, 642-654, 827-858.

⁸ “My hypothesis is, therefore, that there were two different versions of the original AT. The first one, represented by the Castilian canons, followed the model of the *Toledan Tables*, and we do not know whether the corresponding numerical

Beginning in the nineteenth century, the complex historiographical discussion on the first step of the transmission of Alfonsine astronomy in Europe has brought to light many new sources in the four last decades and thus created new research opportunities. In this respect, one may note that, among the different “characteristics” listed above, the last is distinct, as it does not point to astronomical theory but to the astronomers’ computation practices. Consequently, issues related to the history of mathematics emerge related to the advent, sources and posterity of this system for mean motion computations. It also opens a range of questions about how quantities and numbers were addressed in mathematical astronomy during the fourteenth and fifteenth centuries. The importance of these issues appears even more acutely if one considers two supplementary points. First, among the scholarly disciplines of the late medieval period, astronomy was probably the most important producer of numbers. These numbers appeared in tables, ephemerides and calendars, but also instruments, clocks and sundials. Second, through these various devices, astronomy contributed greatly to a growing cultural significance of numbers in the late medieval period.⁹

My intervention in this historiographical discussion is limited. I will focus on the significance of the format of mean motion computations in the *Parisian Alfonsine Tables*. Reviewing the historiographical debates on this specific aspect and taking into account the latest and ongoing research on the Alfonsine corpus, I will address successively the emergence of this computation practice in early Parisian Alfonsine documents, its possible sources and antecedent in *al-Andalus*, and its posterity in the later Alfonsine traditions.

2. Fourteenth century Parisian sources

2.1 Mean motions computations in the *Parisian Alfonsine Tables*

In mathematical astronomy traditions ultimately depending on Ptolemy’s *Almagest*, computations of planetary positions are usually decomposed in two main steps. In the first step, one is required to compute the position the celestial object would have if its motion were occurring at a constant mean velocity over time. In the second step one computes and applies different types of corrections in order to take into account the changing velocity of the celestial object, changes in the sphere of reference or effects caused by the position of the observer. This article concerns the first of these two steps: computing mean positions.

In order to achieve this first step of the computation one needs to know how much a celestial object travelled at constant mean velocity in a given amount of time. This is done by multiplying this amount of time with a fixed quantity representing the mean velocity of that celestial object. Thus, from a mathematical standpoint this first step implies the application of a proportionality rule. However this is done in a context where, first the type of quantities implied in the computation are of different nature (temporal and angular), and second, celestial motions are measured on circles and are thus cyclic (i.e. one must discard full revolutions from the final product of the multiplication).

tables were ever compiled. The second recension corresponds to the Latin tables, which, apparently, reached Paris without canons.” Samsó, *On Both Side*, 2020, 857.

⁹ A. J. Crosby, *The Measure of Reality: Quantification and Western Society, 1250-1600*. Cambridge 1997 ; J. Kaye, *Economy and Nature in the Fourteenth Century: Money, Market Exchange, and the Emergence of Scientific Thought*. Cambridge 1998 ; D. Glimp and M. R. Warren (eds), *Arts of Calculation: Quantifying Thought in Early Modern Europe*. New York 2004 ; A. Tihon, “Numeracy and Science.” In *Oxford Handbook of Byzantine Studies*, Oxford 2008, 803-19.

The most common way to present and arrange this computation in a tabular procedure relies heavily on the temporal nature of one of the quantities considered in the proportionality rule. The temporal quantity is expressed according to a given calendar (e.g. Arabic, Hebrew or Julian). Then for each celestial object, tables for years, months, days, hours and minutes of this calendar tabulate the portion of the circle travelled by the celestial object. Along with these tables providing motion of the celestial object others, usually only for years, provide fixed positions at given dates for the celestial object. This arrangement is adopted in *Almagest* IX.4 and for instance by the *Toledan Tables* which rely on the Arabic calendar.¹⁰ All Latin adaptations of the *Toledan Tables* have maintained this system. However, they often changed the underlying calendar to the Julian one.¹¹ According to the *Castillian Alfonsine Canons* this was the choice made by Alfonso's astronomers as well.¹²

Mean motion computations are arranged in a very different way in the *Parisian Alfonsine Tables*. The temporal quantity is not expressed according to a given calendar but directly in days. Moreover, this number of days is expressed in an integral sexagesimal way. That is days are grouped by sets of successive power of sixty and divided in minutes of day (equal to a sixtieth of day) rather than in hours. The meaning of this is best grasped with an example. In this system, a quantity like 365 days, 12 hours is expressed as 6, 5; 30 because $6 \cdot 60 + 5$ equal 365 days, 12 hours is equal to half a day just as 30 is half of 60. Matching this sexagesimal notation of temporal quantities, angular quantities are also expressed sexagesimally. Zodiacal signs of 30° are the only non-sexagesimal division in the usual system. "Physical" signs of 60° replace them to obtain a consistent sexagesimal notation system for angular quantities.¹³ These choices with respect to the kind of number used to express both temporal and angular quantities allow a radical transformation of mean motion tables.

Figure 1 displays the table related to the mean motion of the sun, venus and mercury. All the other tables are arranged according to a similar model. The first column runs from 1 to 60 and can be used for any of the sexagesimal components of a temporal quantity. Thus, the table is no longer dependent on any calendar. In front of each of these sixty values for the argument, one finds sixty corresponding angular values expressed sexagesimally. The relation between these angular values are simple as the table is a straightforward multiplication table. Moreover, because the sexagesimal division of temporal and angular quantities correspond to each other, simple positional shifts in the angular quantity allow to handle computation with complex temporal quantities in a simple way. For instance, One reads "0, 0, 59, 8, 19, 37, 19, 13, 56" in front of argument "1". If this argument "1" is taken to be a number of days then the corresponding entry will be 0; 59, 8, 19, 37, 19, 13, 56°. ¹⁴ On the other hand, if this argument "1" is taken to be a number of minutes of a day then the corresponding entry will be 0; 0, 59, 8, 19, 37, 19, 13, 56°, that is the same series of sexagesimal digits shifted by one position to the right. Small triangular tables of positional units below the numerical table itself indicate how these shifts should be applied.

¹⁰ F. S. Pedersen, *The Toledan Tables: A Review of the manuscripts and the textual versions with an edition*, Copenhagen 2002.

¹¹ E. Poulle, "Un témoin de l'astronomie latine du XIIIe siècle: les tables de Toulouse", in *Comprendre et maîtriser la nature au moyen âge: Mélanges d'histoire des sciences offerts à Guy Beaujouan*, Geneva 1994 ; J. Chabás and B. R. Goldstein, *The Alfonsine Tables of Toledo*, Dordrecht 2003; J. Chabás and B. R. Goldstein, *A survey of European Astronomical Tables in the Late Middle Ages*, Leiden 2012; J. Chabás, *Computational Astronomy in the Middle Ages*, Madrid 2019.

¹² Chabás and Goldstein, *The Alfonsine...*, 2003

¹³ The expression "physical" for this kind of integral sexagesimal division is first found in John of Lignères *Algorismus minuciarum* and John of Murs *Tabula tabularum* both arithmetical rather than astronomical texts written around 1321. The expression is also present in the arithmetical parts of the canons to the *Tabule magne* by John of Lignères (around 1325). It is not found in John of Lignères *Quia ad inveniendum*, nor in John of Saxony *Tempus est mensura* neither in John of Murs canons of 1339.

¹⁴ Note that this value is typically tropical rather than sidereal according to the second "characteristic" element listed in introduction.

Tabula medij motus Solis Veneris et Mercurij.

4					4														
3					3														
2					2														
p ^a	p ^a	z	z	z	p ^a	p ^a	z	z	z										
1	o	0	59	8	19	37	19	13	56	31	0	30	33	18	8	16	56	11	56
2	c	1	58	16	39	14	38	27	52	32	0	31	32	26	27	54	15	25	52
3	o	2	57	24	58	51	57	41	48	33	0	32	31	34	47	31	34	39	48
4	o	3	56	33	18	29	16	55	44	34	0	33	30	43	7	8	53	53	44
5	o	4	55	41	35	6	36	9	40	35	0	34	29	51	26	46	13	7	40
6	o	5	54	49	57	43	55	23	36	36	0	35	28	59	46	23	32	21	36
7	o	6	53	58	17	21	14	37	32	37	0	36	28	8	6	0	51	35	32
8	o	7	53	6	36	58	33	51	28	38	0	37	27	16	25	38	10	49	28
9	c	8	52	14	56	35	53	5	24	39	0	38	26	24	45	15	30	3	14
10	c	9	51	23	16	13	12	19	20	40	0	39	25	33	4	52	49	17	20
11	c	10	50	31	35	50	31	33	16	41	0	40	24	41	24	30	8	31	16
12	o	11	49	39	55	27	50	47	12	42	0	41	23	49	44	7	27	45	12
13	o	12	48	48	15	5	10	1	8	43	0	42	22	58	3	44	16	59	8
14	o	13	47	56	34	42	29	15	4	44	0	43	22	6	23	22	6	12	4
15	o	14	47	4	54	19	48	29	0	45	0	44	21	14	42	59	25	27	0
16	o	15	46	13	13	57	7	42	56	46	0	45	20	23	2	36	44	40	56
17	o	16	45	21	33	34	26	56	52	47	0	46	19	31	22	14	3	54	52
18	o	17	44	29	53	11	46	10	48	48	0	47	18	39	41	51	23	8	48
19	o	18	43	38	12	49	5	24	44	49	0	48	17	48	1	28	42	22	44
20	o	19	42	46	32	26	24	38	40	50	0	49	16	56	21	6	1	36	40
21	o	20	41	54	52	3	43	52	36	51	0	50	16	4	40	43	20	50	36
22	o	21	41	3	11	41	3	6	32	52	0	51	15	13	0	20	40	4	32
23	o	22	40	11	51	18	22	20	28	53	0	52	14	21	19	57	59	18	28
24	o	23	39	19	50	55	41	34	2	54	0	53	13	29	39	35	18	32	24
25	o	24	38	28	10	33	0	48	20	55	0	54	12	37	59	12	37	46	20
26	o	25	37	36	30	10	20	2	16	56	0	55	11	46	18	49	57	0	16
27	o	26	36	44	49	47	39	16	12	57	0	56	10	54	28	27	16	14	12
28	o	27	35	53	9	24	58	30	8	58	0	57	10	2	58	4	35	28	8
29	o	28	35	1	29	2	17	44	4	59	0	58	9	11	17	41	54	42	4
30	o	29	34	9	48	39	36	58	0	60	0	59	8	19	37	19	13	56	0
m	g	m	z	z						m	g	m	z	z					
z	m	z	z							z	m	z	z						
z	z	z								z	z	z							
4	z									4	z								

Figure 1: mean motion tables for the Sun, Venus and Mercury from

Tabule astronomice illustrissimi Alfonsij regis Castellæ, ed. Erhard Ratdolt, Venice, 1483

2.2 Mean motions computations in the early Alfonsine Parisian sources

The contrast between this way to present mean motion computations in the *Parisian Alfonsine Tables* attested in a large number of manuscripts from the fourteenth and fifteenth century and the more usual calendrical way to organise the work implied by the *Castilian Alfonsine Canons* constitute the last of the four “characteristic” elements listed in introduction. John D. North in 1977 comments this contrast first in a mathematical perspective noting that the *Parisian Alfonsine Tables* system is mathematically more “consistent” than the calendrical one. He remarks also that the English sources with which he is concerned did not adopt the system and conclude that mathematical consistency and compactness of table were not values pursued by these historical actors who preferred “to copy more and think less”.¹⁵ Emmanuel Poulle, in 1984 affirm that

¹⁵ “They lived in a mathematically imperfect society which reckoned time in the same illogical way as we do today, and they were not looking for mathematical virtue, but for an easy life. The Parisians preferred to copy fewer tables of motions, and to add tables for the conversion of time intervals to sexagesimal parts. Alfonso, the Toulouse astronomers,

the presentation of mean motion tables is the “great innovation” of the *Parisian Alfonsine Tables*. In the line of North’s remark on the greater mathematical consistency of the *Parisian Alfonsine Tables* presentation, Emmanuel Poulle points to its “truly universal” nature.¹⁶ In 2003, José Chabás and Bernard R. Goldstein concede, “this presentation became a characteristic feature of Parisian Alfonsine astronomy as it spread all over Europe”. They ponder also this affirmation noting that the uses of zodiacal signs of 30° never disappeared and are present in the works of the most important early Parisian astronomers.¹⁷ In doing so they orient their reflection toward an analysis of transmission through primary sources. They also abstain of any remark with respect to intrinsic mathematical qualities of the presentation adopted in the *Parisian Alfonsine Tables*. José Chabás in 2019 is affirmative and proposes simply to discard this feature as distinctive of *Parisian Alfonsine Tables* “because this format was not followed by many Alfonsine table-makers or copyists”. He also introduce, as an indication, some sort of quantitative argument based on extent sources stating that, in the Vatican and Casanatense Libraries in Rome about two out of five Alfonsine mean motion tables follow the calendrical presentation while the rest follow the sexagesimal presentation.¹⁸

While they set the terms of the debate, the *Parisian Alfonsine Tables* are not the only document appearing in the early Parisian Alfonsine milieu playing a role in the historiographical discussion concerning the organisation of mean motion computations. In 1980, Emmanuel Poulle published a critical edition of John of Murs *Expositio intentionis regis Alfontii circa tabulas ejus* which he dates 1321. John of Murs *Expositio* is a text where different aspects of the *Parisian Alfonsine Tables* are discussed especially with respect to the motion of the sun, the length of the year and the motion of the eighth sphere. John of Murs evaluates information found in a set of tables he ascribes to Alfonso by assessing their internal consistency, comparing them with information found in other table sets and sources, and checking them against observations. The format of mean motion tables is not a topic John of Murs addresses directly. However Emmanuel Poulle remarks that John of Murs uses signs of 60° especially at the beginning of the text and that later he uses signs of 30°. For him, this mixed situation results from the sources used by John of Murs: when quoting the tables ascribed to Alfonso he uses sign of 60°, while sign of 30° are used in connection to the table of Toulouse. He prefers however an interpretation of this feature of the text in terms of successive redaction phases.¹⁹ John of Murs’s contribution

John of Lignères for a time, and most of the English, preferred to copy more and think less”, J. D. North, “The Alfonsine...”, 1977, 272

¹⁶ “Une telle presentation constitue, quant à la forme, la grande nouveauté des tables alphonsines [...]. Introduite jusqu’aux tables des coordonnées à variations régulières, elle en établit le caractère véritablement universel.”, E. Poulle, *Les Tables Alphonsines...*, 1984, 194

¹⁷ “This presentation became a characteristic feature of Parisian Alfonsine astronomy as it spread all over Europe, and it parallels the sexagesimalization of days. However, zodiacal signs of 30° were never entirely displaced and even appear in works by the principal collaborators who produced the Parisian Alfonsine Tables.” Chabás and Goldstein, *The Alfonsine Tables...*, 2003, 251.

¹⁸ “It has been argued that the latter is a distinctive feature of the tables compiled by Alfonsine astronomers, but this claim does not hold, because this format was not followed by many alfonsine table-makers or copyist. To be sure, from a recent survey of sets of Parisian Alfonsine tables in manuscripts at the Vatican Library and Casanatense Library in Rome, in which about 25 mean motion tables have been identified, it follows that about two out of five mean motion tables are arranged according to years, months, etc. whereas the presentation of the rest is based on sexagesimal multiples”, Chabás, *Computational Astronomy...*, 2019, 251.

¹⁹ “Mais d’autres éléments plus techniques incitent à voir dans l’*Expositio* des redactions successives: alors que le début du texte pratique la numération sexagesimale systématique, qui va devenir une des caractéristiques de l’astronomie alphonsine, tant des jours que des degrés, on trouve, à compter du bas du fol. 158 (c’est-à-dire bien avant l’endroit où Duhem situait la fin de l’*Expositio* et le début de l’observation d’Eveux), l’emploi des signes ‘naturels’ de trente degrés pour exprimer toutes les positions dans le zodiaque; et pas seulement quand il est question de positions découlant des tables de Toulouse, où les signes naturels sont de mise, mais même pour des positions alphonsines.”, Poulle, “Jean de Murs...”, 1980, 246.

to the early Parisian Alfonsine corpus goes beyond the *Expositio*. He compiled at least four different table sets known as the *Kalendarium solis et lune* (around 1321?), the *Patefit* (before 1335), the *Tables of 1321* (around 1321) and the *Tabulae permanentes* (after 1339 and before 1345).²⁰ None of these table sets proposes mean motion tables following the format of the *Parisian Alfonsine Tables*. Moreover, as was the case in the *Expositio*, John of Murs fluctuates in his uses of sexagesimal notation. The *Kalendarium* uses signs of 30° while the *Tables of 1321* uses signs of 60°. The *Patefit* relies mostly on signs of 30° except for one very original table subset for true syzygies using signs of 60°. Finally, the *Tabulae permanentes* relies on signs of 30°. In 2019, José Chabás noted these fluctuations but did not specifically comment upon them. Earlier scholarship on these texts mostly studied them separately and did not reflect on this particular aspect. Finally, John of Murs composed in 1339 a set of canons for planetary and eclipse computation. For its planetary part, the canons of John of Murs follow the organisation of the *Parisian Alfonsine Tables*. This text, remaining unedited, was recently studied by Philipp E. Nothaft but with a focus on the eclipse part of the canons. The description of mean motion tables and uses of signs of 60° was mostly out of the scope of Philipp's article.²¹ The text is composed of different strata (some of which come directly from the *Expositio*). John of Murs does not attempt to uniformise them in any way. The resulting effect is that according to the topic signs of 60° and signs of 30° are found in the canons. When treating mean motion computation, John of Murs is attentive to explain how conversions between usual and sexagesimal expression of temporal and angular quantities can be performed in both directions. For instance, instructions are given to convert hours into minutes of days but also minutes of days into hours.²²

In his 1980 publication Emmanuel Poulle affirmed that John of Murs *Expositio* marks the birth of Alfonsine astronomy in Latin.²³ José Chabás and Bernard R. Goldstein challenged this affirmation in a series of publications from 2003 and 2004 pushing forward a new source in the discussion: John Vimond table set.²⁴ Two points are especially important for their argumentation. First, Vimond tables share several “characteristic” elements of the *Parisian Alfonsine Tables* (e.g. the first and third of the above list). Second they can be dated to 1320 on the basis of the tables' epoch. Vimond tables have a very peculiar organisation (they are based on mean conjunctions in a calendrical format) and do not propose mean motion tables in any usual meaning of the term. However they use sexagesimal division of days in equation tables for the luminaries in order to provide interpolation coefficient in a way reminiscent of some devices of the *Almagest* for the same purpose.

John D. North, already in 1977, was particularly attentive to John of Lignères works because of their importance in the transmission of Alfonsine astronomy to English milieus. With respect to mean motion computations and the uses of sexagesimal numbers, John D. North notes that in the *Tabule Magne* and in the

²⁰ J. Chabás and B. R. Goldstein, “John of Murs Tables of 1321”, *Journal for the History of Astronomy*, (40) 2009, 297-320; J. Chabás and B. R. Goldstein, “John of Murs Revisited: the *Kalendarium solis et lune* for 1321”, *Journal for the History of Astronomy*, (43) 2012, 411-437; R. L. Kremer, “John of Murs, Wenzel Faber and the Computation of True Syzygy in the Fourteenth and Fifteenth Centuries”, *Acta Historica Leopoldina*, (54) 2008, 147-60; B. Porres and J. Chabás, “John of Murs *Tabulae permanentes* for finding true syzygies”, *Journal for the History of Astronomy*, (32) 2001, 63-72; Chabás, *Computational...*, 2019, 147-73

²¹ P. E. Nothaft, “Jean des Murs's *Canones Tabularum Alfonsii* of 1339”, *Erudition and the Republic of Letters*, (4) 2019, 98-122 and Nicholas Jacobson, Forthcoming.

²² N. Jacobson, forthcoming

²³ “En tant qu'acte de naissance de celle-ci dans le monde latin l'*Expositio* est un document essentiel”, Poulle, “Jean de Murs...”, 1980, 247

²⁴ J. Chabás and B. R. Goldstein, “John Vimond and the Alfonsine Trepidation Model”, *Journal for the History of Astronomy*, (34) 2003, 163-70; J. Chabás and B. R. Goldstein, “Early Alfonsine Astronomy in Paris: The Tables of John Vimond (1320)”, *Subhajl*, (4) 2004; Chabás, *Computational...*, 2019, 133-46

tables of 1322 John of Lignères rely on the calendrical type of presentation and usual notation for temporal and angular quantities. In 2019, José Chabás and Bernard R. Goldstein analysed an Almanac for the epoch 1341 computed by John of Lignères. In this work also the Parisian astronomer uses the usual notation for temporal and angular quantities. There is only one work of John of Lignères where he adopts the sexagesimal system of the Parisian Alfonsine table. This work is known from its incipit as *Quia ad inveniendum*. It is a brief canon exposing the use of the *Parisian Alfonsine Tables* to compute planetary positions. Emmanuel Poulle and John D. North signalled the *Quia ad inveniendum* in publications from the 1970's.²⁵ Their appreciation of the text is convergent. Because of its characteristics they date the text from after 1322 (John of Lignères tables of 1322) and probably before 1327 (John of Saxony *Tempus est mensura motus*). In a similar fashion as John of Murs in 1339, John of Lignères already in the 1320s prepared instructions to convert temporal and angular quantities in between sexagesimal and usual expressions in both directions. However, contrary to the canons of John of Murs, the *Quia ad inveniendum* is consistent in its use of 60° signs, probably because it is restricted to planetary computations.²⁶

Discussions of John of Saxony's works are also central to the historiographical debates on early Parisian astronomy. Among them, the canons *Tempus est mensura motus*, printed in 1483 with the *Parisian Alfonsine Tables* are essential. In 1984, Emmanuel Poulle edited the text from the 1483 princeps edition. Along with the text of the canons he selected and edited what he considered to be the *Parisian Alfonsine Tables*. His selection of tables was restrictive and strongly dependent on his understanding of the *Tempus est mensura motus*. José Chabás and Bernard R. Goldstein criticised Emmanuel Poulle's restrictive selection of tables in several publications and pointed to several inconsistencies in the way Emmanuel Poulle applied his own selection rule.²⁷ Emmanuel Poulle discarded several tables that are nonetheless addressed by the canons especially in connection to syzygy computations or astrological issues. However, Emmanuel Poulle does not so silently and explains the reason for his choices.²⁸ In this debate on the relation of John of Saxony *Tempus est mensura motus* to the Parisian Alfonsine table, all parts agree on the fact that mean motion tables with their specific format belongs to the *Parisian Alfonsine Tables*. Thus, in spite of the fact that it touches upon the very definition of the limits and content of the *Parisian Alfonsine Tables* I will not dwell into it. John of Saxony, like John of Lignères shortly before him and John of Murs about twelve years later, is also attentive to provide procedures in order to switch in between the usual and the full sexagesimal notation of temporal and angular quantities in both directions. He even adds the supplementary notes that such bi-directional conversion procedures will be useful if one works also with an instrument.²⁹ John of Saxony is mostly faithful to full sexagesimal notation in the canons. The only clear exception to it is the mention of zodiacal signs of 30° in canon 23 addressing the issue of the sun entrance into zodiacal signs. When writing on mathematical astronomy topics in other contexts John of Saxony does not feel compelled to use full sexagesimal notation. Like John of Lignères, he authored an almanac with a short canon beginning *Cum adniverterem quam*

²⁵ Poulle, "John of Lignères", 1973; North, "The Alfonsine...", 1977, 272

²⁶ P. Hadrava and A. Hadravova, forthcoming

²⁷ Chabás and Goldstein, *The Alfonsine...*, 2003 243-4; Chabás, *Computational...*, 2019, 241

²⁸ Poulle, *Les Tables Alfonsines...*, 1984, 6-7, 209-213, 222

²⁹ "Necessitas hujus tabule fuit ut, cognitis horis equalibus transactis post aliquam diem completam que per operationes instrumentorum sciuntur vel per horologium, possemus motus eis correspondentes per istas tabulas invenire cum dies naturalis hic non distinguatur vel dividatur in horas sed in minutas dierum", Poulle, *Les Tables Alfonsines...*, 1984, 44

plurimos magistros... He used in this work the usual calendrical notation for temporal quantities and zodiacal signs of 30° for angular quantities.³⁰

For John D. North, the question of the format of the mean motion tables, and consequently the concomitant use of sexagesimal notation for temporal and angular quantities was an important issue insofar as it was one of the elements allowing him to argue for the strong influence of John of Lignères, and specifically of the *Tabule magne*, in the transmission of Alfonsine astronomy to England in general and Oxford in particular. In this general context, John D. North also commented upon the “mathematical virtue” of the sexagesimal system. Emmanuel Poulle wished to show that the Parisian Alfonsine Table were indeed a Parisian creation and had no connection to the Castilian court of Alfonso X. The format of mean motion tables and the use of sexagesimal notation, although a matter of computations rather than fundamental astronomy, offered him a supplementary opportunity to point a distance of the Parisian table set with the Castilian canons. Emmanuel Poulle also insisted on the mathematical consistency and “universality” of the sexagesimal system for temporal and angular quantities. He had also a tendency to order chronologically the different pieces produced by Parisian astronomers so as to construct a scenario of a progressive adoption of the sexagesimal paradigm which was supposed to be achieved with the redaction by John of Saxony of the *Tempus est mensura motus*. A text he used, in a restrictive (and retro-active?) way, to define the *Parisian Alfonsine Tables* themselves. José Chabás and Bernard R. Goldstein, on the other hand, while commenting on “user-friendliness”, mostly focused on issue of transmission and more specifically on clues of dependencies between the *Parisian Alfonsine Tables* (of which they have a more extensive understanding than Emmanuel Poulle) and the Castilian canons. In constantly bringing new sources into the debate, they have considerably enriched the discussion and its complexity.

Bringing more primary sources to light the historiographical debates of the last decades produced several clarification and important results. In particular, the issue of mean motion tables and sexagesimal notations appears now as a bundle of properties each having its own history: the format of the tables, the use of sexagesimal notation for temporal quantities and the use of sexagesimal notations for angular quantities are three different things. Beyond these mathematical aspects, but not entirely independent from them, mean motion tables in the *Parisian Alfonsine Tables* have also two other astronomical characteristics of importance: they are tropical and they have specific values for mean motions.³¹ On the issue of the emergence and transmission of these mean motion tables, as known to us through sources produced by early Alfonsine Parisian astronomers, the diversity of attitudes of the historical actors toward this aspect of their practices appears clearly. Therefore, the scenario proposed by Emmanuel Poulle of an invention and progressive adoption of a mathematically more consistent sexagesimal notation systems become very difficult to sustain.³²

If, at least for methodological reasons, we maintain a question mark on the origin of the *Parisian Alfonsine Tables* it appears from the evidence gathered so far that, apart from John of Murs tables for 1321, no other

³⁰ J. Chabás and B. R. Goldstein, “The Master and Disciple: The Almanac of John of Lignères and the Ephemerides of John of Saxony”, *Journal for the History of Astronomy*, (50) 2019, 82-96, 87.

³¹ Julio Samsó have argued recently that many of these values may well derive from Ibn Ishaq zij: J. Samsó, “Ibn Ishaq and the Alfonsine Tables”, *Journal for the History of Astronomy*, (50) 2019, 360-5.

³² Julio Samsó 2020 reach a similar conclusion, “The actual situation is not so clear: signs of 30° appear in John Vimond’s tables, in John of Murs *Patefit*, in John of Lignères’ tables for 1322, as well as in his canons *Cujuslibet* and *Multiplis philosophie variis*. They will continue to be used in William Batecombe’s Oxford tables (1348) and in the fifteenth-century *Tabulae rsolutae*. [...] Signs of 60° appear in several works of John of Murs generally, but not systematically: in his *Expositio* he uses signs of both kinds. John of Lignères’s *Quia ad inveniendum* uses signs of 60°”, Samsó, *On Both sides...*, 2020, 840

tables set produced by the four most important early Alfonsine Parisian astronomers makes a systematic use of sexagesimal notation. The two earlier references to Alfonsine astronomy in the Parisian milieu, that of Vimond tables and John of Murs *Expositio*, feature, in quite different ways, use of sexagesimal notation. It appears also that the different canons written to the *Parisian Alfonsine Tables* by John of Lignères, John of Saxony and John of Murs are all attentive to provide procedures allowing switching in both directions from sexagesimal to usual notation for temporal and angular quantities. These practitioners prepared an environment where both kinds of expressions for temporal and angular quantities could be used. Indeed, when dealing with computation other than mean motion and longitude equation most of these authors used a usual notation. All these features are hardly reconciled with the scenario of a group of astronomers consistently pursuing a program of reform of mathematical computation in astronomy where sexagesimal notation for temporal and angular quantities was to replace usual practices. A scenario where the early Parisian astronomers were confronted to a transmitted table set already organising mean motion computations as in the *Parisian Alfonsine Tables* seems more likely to me. Confronted to this table set they evaluated it, and being satisfied with the result included it in their practices by producing canons texts (and new table sets) in which they made sure to be flexible and accommodate different notation systems and ways to organise mean motion computations.

The topic of the specific format for mean motion tables and the related topic of sexagesimal notations is addressed mostly in canons to the *Parisian Alfonsine Tables*. However, we see also John of Murs, John of Lignères, and, as a student, John of Saxony, producing specific mathematical works in relation to sexagesimal computation.³³ I have studied these sources and their relation to different astronomical canons in other contexts and I will not iterate these analyses here.³⁴ It is relevant however to point some of the results of these publications. The variety of numerical notations in astronomy and their relation to common computation practices, as presented in university texts (e.g. Sacrobosco *Algorism*), prompted, for Jean de Murs and Jean de Lignères at least, a reflection on numbers, fractions and fundamental arithmetical operations. In some cases, early Alfonsine Parisian astronomers pushed as far as they could the resources offered for computation by some specific device and numerical notation. This is for instance the attitude of John of Murs for sexagesimal notation in the *Tabula tabularum*. However, the overall picture offered by these works is that of computational culture valuing a diversity of computations means adapted to different types of computations and different standards of accuracy and precision. In this context, computation procedures remaining at the level of sexagesimal notation for temporal and angular quantities, like that of mean motion table in the *Parisian Alfonsine Tables*, were not considered the most accurate and precise. When they needed precision and accuracy, early Alfonsine Parisian astronomers, relied on computational procedures where numbers were converted into large integers of a small fractional unit, operated upon as integers following common algorism practices and specific rules to handle fractional units, and converted back to sexagesimal notations, with eventual rounding, at the final step. In other words, as important as they may be, changes in the sexagesimal format of temporal and angular quantities numerical notations did not transform deeply accurate and expert astronomical computations.

³³ John of Lignères wrote an *algorismus minutiarum* (H. L. L. Busard, *Het rekenen met breuken in de middleleeuwen, in het bijzonder bij Johannes de Lineriis*, Brussels 1968) ; John of Murs his *Tabula tabularum* (M. Husson, “La *Tabula tabularum* de Jean de Murs: nombres et opérations arithmétiques en astronomie au début du quatorzième siècle”, *Cahiers de Recherches Médiévales et Humanistes*, (27) 2014, 97-122), and in the manuscript Erfurt F. 377 we see John of Saxony interacting as a student with John of Murs’s work and learning the manipulation of these numbers (M. Husson and M.-M. Saby, “Le manuscrit Erfurt F. 377 et l’astronomie Alphonsine Parisienne”, *Micrologus*, (27) 2019, 205-35)

³⁴ M. Husson, “Astronomers’ elementary computations in Paris between 1321 and 1327: the circle around John of Lignères and John of Saxony”, in *Mathematical Practices in the Astral Sciences*, Forthcoming.

3. Sexagesimal notations, before and after the early Alfonsine Parisian astronomers

3. 1 Antecedents

Historiographical discussions around the *Parisian Alfonsine Tables* also explored their potential connections to astronomical material produced in *al-Andalus*. Major contributions in this area of research come from Julio Samsó, José Chabás and Bernard R. Goldstein. I will briefly survey the elements brought by this scholarship to the issues of the potential antecedent in the Iberian peninsula of the format of mean motion tables in the *Parisian Alfonsine Tables*. Following the result of the first part I will consider first sexagesimal notation for temporal quantities, then sexagesimal notation for angular quantities and finally the format of the tables.

In 1984, Emmanuel Poulle makes a small note on the uses of sexagesimal notation for temporal quantities.³⁵ He points there to the fact that such a system was previously used for conversion between eras in the Table of Cremona. It is however in 2003, that José Chabás and Bernard R. Goldstein addressed specifically the issue of sexagesimal notation for temporal quantities in the *Parisian Alfonsine Tables*.³⁶ They remark that such notation system was of common use in the *Toledan Tables* and in al-Kwarizmi *zij*. They point also to occurrences of such a sexagesimal expression of a temporal quantity in the *Castillian Alfonsine Canons*. They indicate that al-Biruni's *Chronology of Ancient Nations* used sexagesimal notation for temporal quantities. Finally, they also remark that the use of sexagesimal fractions of days to express temporal quantities was also of common use in other areas of mathematical astronomy and points in this respect to the length of the synodic month in the *Almagest* or in Albatani *zij*. In 2020, Julio Samsó also addressed this specific issue.³⁷ He is able to give more depth to the use of this notation in the Iberian Peninsula by pointing several new sources including: Ibn al-Ha'im al-Ishbili's *Kamil zij*, Ibn Al-Kammad's *Muqtabas zij*.

The use of signs of 60° and thus of a sexagesimal notation for angular quantities is apparently more difficult to trace back to sources related to the Iberian Peninsula. For Emmanuel Poulle, in 1984, their Parisian origin is out of doubt.³⁸ In 2003, José Chabás and Bernard R. Goldstein point to a parallel in the Hebrew *zij* of Judah ben Asher II (d. 1391) produced in the Iberian peninsula. The same historiographical difficulty appear with respect to the structure of the mean motion tables themselves.³⁹ Julio Samsó noted already in 1987 that sexagesimal tables for mean motion appear very early in the history of Arabic astronomy mentioning the eight-century astronomer al-Fazari.⁴⁰ In 1999, Angel Mestres, remarked that in the Tunisian recension of Ibn Ishaq's *zij*, Ibn al-Bana's *Minhaj*, and Ibn al-Raqqam' *Shamil* a subtable compacted mean motion for hours and

³⁵ “la numeration sexagésimale du calendrier avait déjà été utilisée (tables de Crémone), mais seulement pour les changements d'ères.”, Poulle, *Les tables alphonsines...*, 1984, 193-4

³⁶ Chabás and Goldstein, *The Alfonsine...*, 2003, 249-51

³⁷ Samsó, *On Both Sides...*, 2020, 840-4

³⁸ “Pour les degrés, leur regroupement en six signes de soixante unités représente une innovation essentielle des tables alphonsines”, Poulle, *Les tables alphonsines...*, 1984, 187

³⁹ Chabás and Goldstein, *The Alfonsine...*, 2003, 251.

⁴⁰ Samsó, “Alfonso X...” 1987, 32

minutes in a single table with argument 1 to 60.⁴¹ In fact, common mean motion subtables have a multiplication table structure.

These historical inquiries indicate that there were ample sources for the use of sexagesimal notation with temporal quantities both in al-Andalus and in Latin sources. Sexagesimal notation for angular quantities seems more difficult to trace outside of the *Parisian Alfonsine Tables* themselves. The format of mean motion as a multiplication table is also quite standard although it is only with a concomitant use of sexagesimal notations for temporal and angular quantities that it can take the specific form found in the *Parisian Alfonsine Tables*.

Now, considering the question from a mathematical perspective, it appears that the key choice is to extend the use of a sexagesimal notation for temporal quantities from chronology to mean motion computations. Once this choice is made the two supplementary steps of adopting a multiplication format for mean motion tables and signs of 60° for angular quantity are not huge conceptual shifts.⁴² In term of organisation of astronomical computations the extension from chronology to mean motion of the use of a sexagesimal notation for temporal quantities appear as a simplification in a context where several eras and calendars are to be used concomitantly. One is thus able to skip a conversion from a sexagesimal expression of a temporal quantity to an expression of this same quantity in the calendar with which the mean motion tables are tabulated: he can directly use the sexagesimal expression with the mean motion tables.

I can see at least three kinds of motivations for medieval practitioners to handle concomitantly different calendars in astronomical computations. Many scholars noted the first: a multicultural environment like that of Alfonso court imply the use of many calendars.⁴³ This is certainly a strong argument but it does not explain why the *Parisian Alfonsine Tables* or the *Toledan Tables* among others keep record of ancient eras and calendars long out of uses by any active community. The presence of this kind of records points, in my opinion, to two different and interrelated practices: first, the ongoing pertinence and consideration of ancient observations, mostly found in the *Almagest*, in late medieval astronomical practices if only for discussions about precession and mean motion parameters; second, the practice of chronology. Among the early Parisian astronomers, John of Murs has demonstrated interest in both of these activities. He is also the author of the *Tabula tabularum* a treatise pushing as far as possible the potentiality of computation with a sexagesimal multiplication tables. Thus, he seems a good candidate for the design of the specific mean motion computations found in the *Parisian Alfonsine Tables*. Nonetheless, I think it unlikely that John of Murs was responsible for this. First the *Expositio* shows him confronted to a source where sexagesimal notation for temporal and angular quantities, in the context of mean motion, is present. Second, his considerations of ancient observations in the *Expositio* and in later chronological works are dependent on the *Parisian Alfonsine Tables*. Moreover, Astronomers in the Alfonso court were interested in chronology and observations thus, in my view, all clues point to an adoption in Castille of this specific format for mean motion tables, and the notation of temporal and angular quantities.

⁴¹ A. Mestres, *Materials andalusins en el zīj de Ibn Ishaq al-Tunisi*, Unpublished Ph.D. dissertation, University of Barcelona 1999, 27, 188 quoted from Samsó, *On Both sides...*, 2020, 841

⁴² Chabás and Goldstein, *The Alfonsine...*, 2003, 251.

⁴³ This point is expressed very clearly for instance there: “The discussion of a variety of calendars makes sense in Spain where many calendars were used. In Paris, however, dates were only given in the Christian calendar with the era of Incarnation; other calendars had no cultural context at the time.”, Chabás and Goldstein, *The Alfonsine...*, 2003, 249

3. 2 Posterity

In the early Alfonsine Parisian Milieu, apart from John of Murs tables for 1321, the newly compiled table sets did not adopt a systematic use of 60° signs or the peculiar format of mean motion tables of the *Parisian Alfonsine Tables*. This quite impressive phenomenon is only confirmed if the scope of the survey is extended to other scholarly milieus where Alfonsine astronomy was cultivated in the fourteenth and fifteenth century. For instance, the very innovative, but seldom diffused, tables for the seven planets (1340), for which we lack authorship and geographical context, also use 30° signs and calendrical mean motions.⁴⁴

The inclination of early Alfonsine English milieus toward calendrical presentations for mean motion tables was long noted by Dreyer who interpreted it as a fidelity to the original format of the Castilian tables. John D. North showed that one should see there the influence of John of Lignères tables of 1322 and *Tabule magne*.⁴⁵ The tables of William Rede and those of William Batecombe for Oxford both rely on calendrical presentation for mean motion tables and the use of 30° signs. The latter had a large diffusion attested by numerous manuscript copies produced in England, Germanic spaces and Italy.⁴⁶

First compiled for the meridian of Wroclaw in 1424 by Petrus Cruciferus, the *Tabulae Resolutae* entered the university curriculum of Cracow university by the middle of the fifteenth century. They use signs of 30° , calendrical notation for temporal quantities and for their mean motion tables. They had a wide diffusion in Germanic spaces, central Europe and Poland. They also reached Spain and actually initiated there the influence of the Latin tradition of Alfonsine astronomy.⁴⁷ Nicholas Polonius, holding the first chair of astrology at the university of Salamanca, adapted and wrote new canons for the *Tabulae Resolutae*. Along the course of their diffusion mostly in university contexts, they had no less than four different canons. None says a word about the fact that they do not follow the *Parisian Alfonsine Tables* setup for mean motion and notations of temporal and angular quantities.⁴⁸ About three quarter of a century before the compilation of the *Tabulae Resolutae*, Nicholas of Aquaeductu, probably trained in Paris but then moving back to Germanic spaces, composed an extensive canon set on the *Parisian Alfonsine Tables*. The first parts of his canons, dealing with chronology and mean motion, present the use of sexagesimal notation for temporal and angular quantities as well as the specific design of mean motion tables. On the other hand, the last portion of his canons, addressing the “accidents” of planetary motions (e.g. stations and retrogradations, but also latitude or eclipses) depend heavily on John of Lignères canons of 1322 and thus rely on usual notations for temporal and angular quantities.⁴⁹ A similar phenomenon happens for the slightly later composed canon sets to the Alfonsine tables by Heinrich Selder.⁵⁰ Johannes Gmunden used sign of 60° in the first version of his table set, but for the latter versions (e.g. the *Tabule maiores* and the *Tabule breviores*) he opted firmly for sign of 30° and

⁴⁴ J. Chabás and B. R. Goldstein “Displaced tables in Latin. The Table for the Seven planets for 1340”, *Archive for history of Exact Sciences*, (67) 2013, 1-42; Chabás, *Computational...*, 2019, 215-25

⁴⁵ J. L. E. Dreyer, “On the Original Form of the Alfonsine Tables”, *Monthly Notices of the Royal Astronomical Society*, (80) 1920, 243-62; North, “The Alfonsine Tables...”, 1977

⁴⁶ Chabás, *Computational...*, 2019, 207-15, 227-37

⁴⁷ “In spite of the fact that the Alfonsine Tables were known in Iberia towards the end of the fourteenth century, their influence was not felt until the second half of the fifteenth century and the reintroduction of the AT in Salamanca by Nicolas Polonius (1460)”. Samsó, *On Both Sides...*, 2020, 869

⁴⁸ Chabás, *Computational...*, 2019, 207-15, 311-21, Petr Hadrava and Alena Hadravova are currently working on a critical edition of the four main canons written to the *Tabulae Resolutae* in the context of ALFA.

⁴⁹ N. Jacobson, forthcoming

⁵⁰ P. E. Notafth, “*Vanitas vanitatum et super omnia vanitas*: The Astronomer Heinrich Selder and a Newly discovered fourteenth-Century Critique of Astrology”, *Erudition and the Republic of Letters*, (1) 2016, 261-304.

common expression of temporal quantities.⁵¹ For Gmunden, this choice is driven by astronomical instruments, which commonly used sign of 30°. ⁵² This is reminiscent of a remark made by John of Saxony in *Tempus est mensura motus*. Peurbach *Tabulae eclipsum* consistently uses signs of 30°. ⁵³ Regiomontanus *Tabule primi mobilis*, of trigonometrical rather than directly astronomical nature, abandon altogether the use of signs and express angular quantities directly in degrees. The more extensive *Tabulae directionum* by Regiomontanus and Martinus Bylica of Ilkush (first published in 1490) address spherical astronomy issues in connection with astrological problems. In this context, zodiacal signs of 30° are used and for mathematically oriented tables direct expression in degrees also, but no signs of 60° are found. ⁵⁴

In Italy, the situation is similar. Sancto Archangelo canons, and the related table set rely on signs of 30° and calendrical mean motions. ⁵⁵ Prodoscimo de Beldomandi, compiled a table set based on an earlier work of Jacopo Dondi. This Table set relies on the Parisian Alfonsine Tables. In this respect they use signs of 60°. However, the organisation of mean motion tables and the expression of temporal quantities are calendrical. ⁵⁶ Moreover, the canons compiled for this table set describe current 30° zodiacal signs and provide instructions to compute astrological squares and ephemerides where 30° signs are required. ⁵⁷ Giovanni Bianchini also adopt an eclectic attitude with respect to numerical notation in his various tables sets. In the *Tabulae astronomiae* for instance one finds sign of 60°, signs 30° or direct expression in degrees for angular quantities. ⁵⁸ The situation is similar in the different version of the *tabulae primi mobilis*. ⁵⁹ The more compact set of auxiliary tables known as *Tabulae magistrates* consistently use sign of 60°. ⁶⁰ This eclectic attitude of Bianchini probably informed Peurbach and Regiomontanus own decision on these matters.

Considering these evidences, it appears that the uses of sexagesimal notations remained a marginal phenomenon limited mostly to the *Parisian Alfonsine Tables* and a few canon sets related to them. This conclusion is certainly exact at the level of the compilation and composition of new table and canon sets in the Alfonsine tradition. I believe this view is also partially misleading if used as a means to study the actual practices of astrologers, astronomers and various students in the astral sciences in the fourteenth and fifteenth century Europe. A small survey of manuscripts in Casatenese and Vatican Library recently done by José Chabás and quoted above shows that 3 out 5 Alfonsine mean motions tables follow the format of the *Parisian Alfonsine Tables*, in other words, a majority. This result relies on a limited sample but is part of a larger enterprise of referencing the corpus of Alfonsine works at the manuscript level collectively undertaken in the context of the ERC project ALFA. ⁶¹ When this database is complete, it will be possible to ask such quantitative questions on a representative sample of more than 600 manuscripts and trace geographical and chronological variations in the diffusion of different Alfonsine works. It is however rather safe to believe that the lion share of manuscript witnesses will be for the *Parisian Alfonsine Tables* in association with John of Saxony *Tempus est mensura motus*. Thus, in spite of the indisputable fact that almost no other Alfonsine works

⁵¹ Chabás, *Computational...*, 2019, 221-37

⁵² Chabás, *Computational...*, 2019, 322, 335

⁵³ Chabás, *Computational...*, 2019, 365-77

⁵⁴ Chabás, *Computational...*, 2019, 377-91

⁵⁵ Jacobson, forthcoming

⁵⁶ Chabás, *Computational...*, 2019, 299-311

⁵⁷ Jacobson, forthcoming

⁵⁸ Chabás, *Computational...*, 2019, 337-65

⁵⁹ G. Van Brummelen, "The End of an Error: Bianchini, Regiomontanus, and the *Tabulation of Stellar Coordinates*", *Archive for the History of Exact Sciences*, (72) 2018, 547-63

⁶⁰ Chabás, *Computational...*, 2019, 349-53

⁶¹ CoG, 723085, 2017-2022, P.I. Matthieu Husson

truly adopted the sexagesimal notation for angular and temporal quantities and the related format of mean motion tables, this specific organisation of mean motion computation probably had a massive presence throughout Europe in manuscripts. This presence might in fact be a factor explaining why so few works in the later Alfonsine traditions actually re-iterated the gesture of the *Parisian Alfonsine Tables*.

Although the analysis of the Alfonsine astronomers practices and of the phenomenon occurring at the manuscript level are fairly recent trends of research it is already possible to point a few interesting facts with respect to the issue discussed in this paper. First, in the manuscript transmission of John of Lignères *Tabule magne*, of William Batecombe *Oxford Tables* or of John of Murs *Tabule permanentes*, all works composed with signs of 30° degrees, we find some witnesses converting these tables to signs of 60°. ⁶² This shows that one scribe could also silently make the choice of changing types of notations for temporal and angular quantities. The extension of this phenomenon and its consequences are yet to be analysed. Second, the manuscript tradition of a late fifteenth century family of equatorial instruments known as *Theorice nouvelle*, also attest an interesting phenomenon. Figure 2 shows an instance of this instrument. The outer scale, representing the ecliptic, is graduated with zodiacal signs of 30° each with their proper astrological name. The inner scale, representing the epicycle and actually not used in computation with the instrument, is graduated in sign of 60° each with their ordinal name. However, these 60° signs are divided in degrees but twice from 0° to 30° in alternating red and black scales. One could hardly think of a more explicit expression of the flexible and eclectic attitude of Alfonsine astronomers with respect to this issue! ⁶³ Once more, the extension and consequence of this type of phenomenon is yet to be appreciated.

⁶² Lisbon, Bibliotheca de Ajuda, MS 52-XII-35, 67r-92v for the *Tabule Magne* and Cambridge, University Library, MS Peterhouse 75.I, 45r-61r for the Oxford tables and Rome Bibl. Casanatense 1673, 89v-92r for the *Tabule permanentes* (I'm thankful to Camille Bui for attracting my attention to this manuscript of the Oxford Tables and to Richard Kremer for attracting my attention to the *Tabule permanentes* manuscript).

⁶³ Samuel Gessner attracted my attention to these scales. Other manuscripts for this family of instrument present variations of this phenomenon. See S. Gessner, forthcoming.

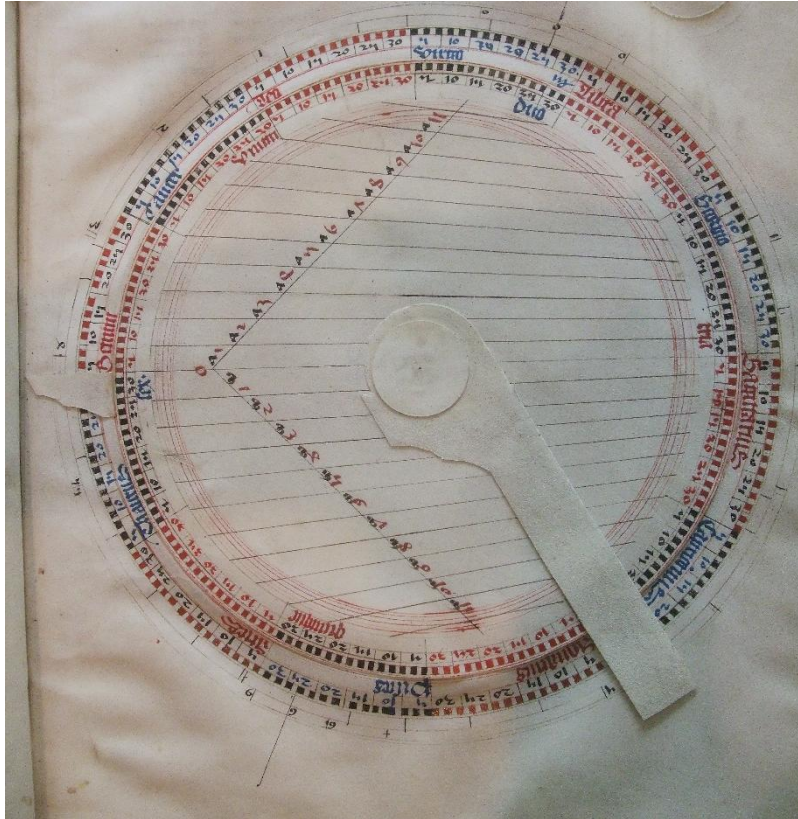


Figure 2 : Schweinfurt, Stadtarchiv / Museum Otto Schäfer AvS Ha 20, f. 5

Conclusion

This survey of the discussions around the organisation of mean motion computations in the Parisian Alfonsine tables produced four main historiographical results:

- The research around the early Alfonsine Parisian milieu shows more and more clearly that the Emmanuel Poulle scenario of the creation and progressive adoption of this system during the 1320's in Paris is not accounting for what we know of the sources.
- The more recently published research around possible antecedents of this system in the Iberian Peninsula in Arabic, Hebrew, Latin or Castilian sources also indicate that the Castilian court of Alfonso X was a context where such a system was more likely to emerge.
- The sexagesimal notation system was little diffused outside the *Parisian Alfonsine Tables* themselves and remained mostly confined to mean motion computations.
- Diffusion at the manuscript level however shows the endemic presence of the sexagesimal mean motion in the manuscript traditions, especially through the witnesses of the *Parisian Alfonsine Tables*. It also shows the agency of scribes on this issue both in tables and instruments design.

The analysis of this material also granted us interesting insight with respect to the mathematical practices of Alfonsine astronomers:

- The key decision from historical actors, which probably prompted the emergence of the whole system, was to extend the use of sexagesimal notation of temporal quantities from chronological to mean motion computations.
- The system prompted some mathematical reflection about numerical quantities and arithmetical operations, especially but not exclusively in the early Alfonsine Parisian milieu. However, it did not transform the heart of accurate expert astronomical computations that were made on large integer expressions of small subunit of the degree, the day or the hour.
- In the long run, the emergence of this system installed a situation where astronomers would commonly manipulate different types of numerical notations in different areas of mathematical astronomy in tables, canons and instruments. This creates a context for the emergence of more radical interventions on the conception of numerical quantities, their notation and manipulation.

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⁶⁴ See G. Van Brummelen, “Distinction of Magnitude: Numbers and Quantities in the 15th century and the Invention of Decimal Positional Arithmetic”, *Forthcoming*