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1 DART radiative transfer modelling for sloping landscapes

2
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9 10 **Abstract**

11
12 Topography is one of the key factors that impact remotely sensed data and their interpretation.
13 Indeed, combined with the viewing geometry and neighbour effects, it strongly affects the direct,
14 diffuse and multi-scattered scene irradiance, which in turn impacts the radiative budget and
15 remote sensing signals of the landscapes. The increased availability of digital elevation models
16 (DEM) and the advancement of 3D radiative transfer (RT) models allow us to better address
17 these topographic effects. DART (Discrete Anisotropic Radiative Transfer) is one of the most
18 accurate and comprehensive 3D RT models that simulate remote sensing observations of
19 natural and urban landscapes with topography and atmosphere. It simulates environmental
20 effects (*i.e.*, impact of adjacent landscape on the observed landscape) using a so-called infinite
21 slope mode that infinitely duplicates the observed landscape while ensuring the continuity of
22 slope and altitude at the DEM edges. Up to DART version 5.7.4, this mode was slightly
23 inaccurate and computer intensive, depending on the topography. This paper presents an
24 innovative modelling strategy that greatly improves it in terms of accuracy, image quality and

25 computer efficiency. For that, a fictive auxiliary oblique plane, adapted to the landscape
26 topography, is introduced for managing the scene illumination, the Earth-Atmosphere coupling
27 and the storage of the radiation that exits the scene before being projected onto the sensor plane.
28 Improvements and validations are illustrated both visually and quantitatively by DART images,
29 radiometric products and radiative budget. For example, the observed reflectance of a
30 Lambertian slope is equal to the expected analytical value. In addition, the solar plane
31 reflectance of a forest on a mountain slope (experimental scene) has an average error of about
32 0.01% relative to the reflectance of the same forest stand in the reference scene (*i.e.*, nine
33 duplications of the experimental scene). This new modelling is already integrated in the official
34 DART version (<https://dart.omp.eu>).

35

36 Key words

37

38 Topography, DART, Infinite slope, Radiative transfer, Environmental effects

39

40 **1 Introduction**

41

42 Mountain regions cover around a quarter of the Earth's land surface (Liang, 2005) and are home
43 to around one-tenth of the world's population (Denniston, 1996). They also provide vital
44 economic and ecological resources to the world's population, thanks to their rich biodiversity,
45 water-storing capacity, mineral reserve, *etc* (Price, 1998). Therefore, mountain areas are more
46 and more surveyed with remote sensing (RS) data (Boori et al., 2015; Barrachina et al. 2015;
47 Reinhold et al., 2016; Chimner et al., 2019). However, RS data are usually contaminated by
48 topographic effects, since slopes combined with the observation configuration (*e.g.*, viewing

49 and sun geometry, atmosphere composition) impact the direct, diffuse and multi-scattered
 50 irradiance, which in turn affects the RS signals and radiative budget (RB) of mountain regions
 51 (Schaaf et al., 1994; Mousivand et al., 2015; Pellarin et al., 2016; Wen et al., 2018). In the last
 52 four decades, a lot of work has been done to eliminate or to reduce these topographic effects
 53 (Holben and Justice, 1980; Proy et al., 1989; Sandmeier et al., 1997; Richter and Schläpfer;
 54 2011; Santini et al., 2019), notably through the development of numeric models (Table 1).

55

56 Table 1. Models that quantify topographic effects. Model category and references are indicated.

Model Category	Model Name	Model Reference
	Cosine Correction	Holben and Justice, 1980
Empirical	C correction	Teillet et al., 1982
	SCS	Gu and Gillespie, 1998

Geometric optical	GOMST	Schaaf et al., 1994
	GOST	Fan et al., 2013

3D radiative transfer	DART	Gastellu-Etchegorry et al., 1996; 2017
	LESS	Qi et al., 2018
	RAPID3	Huang, 2018
	Rayspread	Widlowski et al., 2007

57

58 Empirical models (*e.g.*, Cosine Correction, C correction, SCS) are the simplest and most
 59 practical for topographic correction, but the lack of generality and physical considerations limits
 60 their accuracy. Geometric optical models (*e.g.*, GOMST, GOST) use a geometric approach to
 61 quantify how canopy directional reflectance depends on slopes, solar direct irradiance and 3D
 62 tree crowns. However, they usually neglect sky diffuse illumination and multiple scattering in

63 landscapes. Also, the solo slope is a theoretical assumption that rarely exists in the nature. Being
64 more sophisticated and comprehensive, 3D radiative transfer (RT) models (*e.g.*, DART, LESS,
65 RAPID, Rayspread) are assumed to be the most accurate. They can simulate complex
66 topography, realistic 3D landscape elements, multiple scattering in the landscape and possibly
67 with diffused illumination from the atmosphere. Thanks to the development of computer
68 hardware and RT algorithms (*e.g.*, PBRT, LuxCoreRender, Mitsuba) (Grimaldi and Vergauwen,
69 2008; Jakob, 2014; Pharr et al., 2016), 3D RT models become more and more efficient, which
70 opens new opportunity for removing or reducing topographic effects in RS observations. Most
71 3D RT models were developed for the Earth surface RT modelling with atmosphere being
72 treated as an interface that is usually computed by an atmospheric RT model such as
73 MODTRAN (Berk and Hawes, 2017). It is typically the case of the recently developed LESS
74 model that can simulate very large forest landscapes (Qi et al., 2018). Atmospheric RT
75 modelling in presence of complex mountainous topography is still a challenge due to the
76 complex Earth-Atmosphere radiative coupling. To our knowledge, DART is the only 3D RT
77 model that simulates the Earth-atmosphere radiative coupling and environmental effects in
78 mountain areas. The so-called environmental effects are the impact of the adjacent landscapes
79 on the observed landscape.

80

81 The accuracy of DART Earth surface RT modelling for vegetation canopy over flat terrain was
82 already verified in the four phases of the RAMI project (RAdiative transfer Model
83 Intercomparison) (Widlowski et al., 2007; Widlowski et al., 2013; Widlowski 2015) and
84 experiments (Guillevic et al., 2003; Guillevic et al., 2013). DART atmospheric RT modelling
85 was successfully tested with MODTRAN5 simulations (Grau and Gastellu-Etchegorry, 2013;
86 Gastellu-Etchegorry et al, 2017; Wang and Gastellu-Etchegorry 2020), with reflectance
87 accuracy better than 0.004 and TIR brightness temperature accuracy better than 1.0 K, at bottom

88 (BOA) and top (TOA) of atmosphere. DART is continuously improved in terms of accuracy,
89 efficiency and modelling. For example, sun induced fluorescence modelling was introduced in
90 2016 (Gastellu-Etchegorry et al., 2017), and the Embree ray tracing library was recently
91 introduced for reducing simulation time and memory demand (Wald et al., 2014; Qi et al., 2019).
92 The work presented here is an improvement concerning RT modelling over sloping landscapes.
93
94 DART scene is a 3D array of cells that can contain turbid material, fluids and facets (*i.e.*,
95 triangles). The scene is either spatially finite (*i.e.*, “isolated scene” mode) or infinite (*i.e.*,
96 “repetitive scene” mode for simulating landscapes on flat terrain and “infinite slope” mode for
97 simulating landscapes on sloping terrain). A spatially infinite scene is an infinite repetition of
98 the actually simulated scene. Being designed for sloping landscapes, the infinite slope mode
99 ensures the altitude and slope continuity at the DEM edges of any duplicated scene. In DART,
100 RT is simulated by tracking rays along a finite number of discrete directions. The infinite slope
101 mode simulates the environmental effects by re-entering environmental rays into the observed
102 scene. These environmental rays are scattered from the neighbouring duplicated scenes. Rays
103 that exit the scene through its upper surface (*i.e.*, rays that constitute the observation) are spread
104 at their point of intersection with the horizontal plane at the scene bottom where they constitute
105 intermediate images, also called spread images (Yin et al., 2015). They are used to simulate the
106 Earth-atmosphere radiative coupling and RS images. The sloping topography explains that each
107 spread image has a specific polygon-shape and dimension per direction. Up to DART versions
108 5.7.4, scientists noted that the infinite slope mode could be inaccurate, as illustrated below in
109 this paper. An analysis of DART RT modelling allowed us to stress two major sources of
110 inaccuracy: (1) Scene illumination: depending on topography and illumination directions, it is
111 not uniform as expected. (2) Polygon-shape spread images: depending on topography and
112 viewing direction, these polygon-shapes can be very complex, which explains the presence of

113 border effects in simulated images and inaccurate Earth-Atmosphere radiative coupling. The
114 initial infinite slope mode was also constrained by technical issues: the required random access
115 memory (RAM) and simulation time could be unexpectedly large depending on the scene
116 topography. For example, compared to the simulation of an urban or vegetation landscape on a
117 horizontal surface, the simulation of this landscape on a 30° slope could increase the RAM
118 demand by a factor of 8, and computation time by a factor of 1.5.

119

120 This paper presents a new modelling strategy of DART infinite slope mode designed to improve
121 the accuracy and efficiency of DART RT modelling for sloping landscapes. Section 2 presents
122 major DART characteristics related to the present work. Section 3 describes the initial DART
123 infinite slope modelling method and its issues. Section 4 details the new infinite slope modelling
124 strategy, with additional information in the appendix. Section 5 firstly illustrates DART
125 improvements with simulations of a solo slope and a tree on a solo slope, and secondly presents
126 the validation of the new strategy with two simulated scenes: 1) a rugged slope, 2) a realistic
127 forest on a rugged slope. Finally, section 6 presents concluding remarks and perspectives.

128

129 **2 DART model and infinite slope mode**

130 **2.1 General description**

131

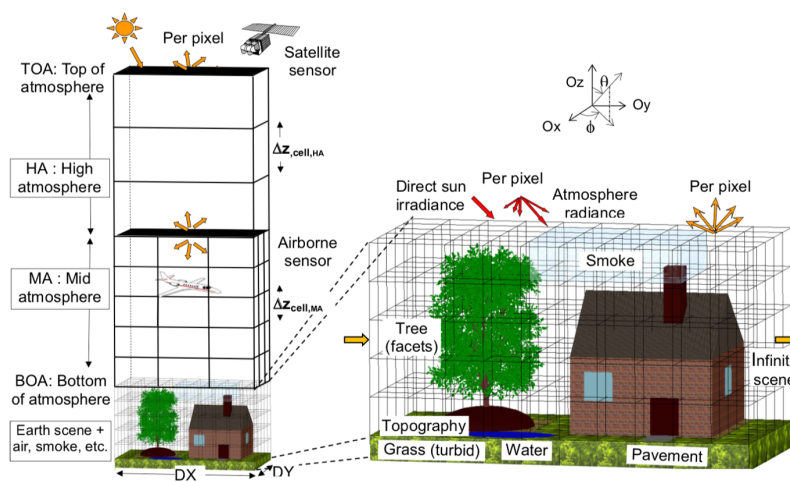
132 DART (<https://dart.omp.eu>) has been developed at CESBIO since 1992 (Gastellu-Etchegorry
133 et al., 1996; Gastellu-Etchegorry et al., 2017). It is one the most accurate and comprehensive
134 3D RT model in the optical remote sensing community. It simulates the RB, bi-directional
135 reflectance factor (BRF) and directional brightness temperature (BTF), images of
136 spectroradiometers at any altitude of 3D natural and urban landscapes, with topography and

137 atmosphere (Figure 1), for any viewing direction, from visible to thermal infrared domain. It
 138 simulates also the sun induced chlorophyll fluorescence of 3D vegetation canopies and the
 139 terrestrial and aero-spatial LiDAR signal (point cloud, waveform, photon counting).

140

141 In DART, the atmosphere is a layered horizontally homogeneous volume (Figure 1). A SQL
 142 database stores the spectral optical properties and vertical profiles of its constituents (*i.e.*, gases,
 143 aerosols, clouds). The Earth scene (*i.e.*, urban / vegetation landscape) is an array of cells (*i.e.*,
 144 voxels) (Figure 1) where elements are made of facets, also called triangles, possibly combined
 145 with cells filled with turbid vegetation medium or fluids (air, soot, water, *etc*). SQL databases
 146 store optical properties that can be assigned to scene elements. The integrated PROSPECT /
 147 FLUSPECT leaf model (Jacquemoud et al., 1990; Vilfan et al., 2016; Feret et al., 2017)
 148 computes leaf optical properties and fluorescence using leaf biochemistry and structure.
 149 Orthographic and perspective (pinhole, pushbroom, fisheye) cameras are also simulated in
 150 order to generate satellite, airborne and in-situ RS images (Yin et al., 2015). Up to DART 5.7.4,
 151 these images are generated by a bi-linear interpolation method.

152



153

154 Figure 1. DART 3D discrete mock-up. Landscape elements are made of triangles, and/or fluid and
 155 turbid vegetation cells. The landscape itself is a cell matrix in order to ease ray tracing. The

156 atmosphere has three levels: upper level made of layers, middle level made of cells of any size,
 157 and lower level in the landscape.

158 DART simulates the RT in the Earth-atmosphere system with an iterative discrete ordinate
 159 method (Gastellu-Etchegorry et al. 1996, 2004, 2008, 2017), *i.e.*, radiation intercepted at
 160 iteration n is scattered at iteration $n + 1$, by tracking steady-state energy flux $W(r, \lambda, \Omega)$ (unit:
 161 W/sr/ μm) along a finite number of directions Ω at position $r(x, y, z)$, at wavelength λ using the
 162 RT equation for a monochromatic radiation

163

$$\left(\xi \frac{d}{dx} + \eta \frac{d}{dy} + \mu \frac{d}{dz} \right) W(r, \lambda, \Omega) = -\alpha_e(r, \lambda, \Omega)W(r, \lambda, \Omega) + \alpha_a(r, \lambda, \Omega)W_B(T, \lambda, \Omega) + \int_0^{4\pi} \alpha_s(r, \lambda, \Omega') \frac{P(r, \lambda, \Omega' \rightarrow \Omega)}{4\pi} W(r, \lambda, \Omega') d\Omega' \quad (1)$$

164

165 where the term on the left side of equation (1) is the divergence of spectral energy, with (ξ, η, μ)
 166 being the unit vector of direction Ω . The three terms on the right side of equation (1) are the
 167 extinction and thermal emission along direction Ω , and the integrated scattering radiant flux
 168 into direction Ω over 4π space (Subrahmanyam Chandrasekhar, 2013). The total extinction
 169 coefficient α_e is the sum of absorption and scattering extinction coefficient ($\alpha_e = \alpha_a + \alpha_s$).

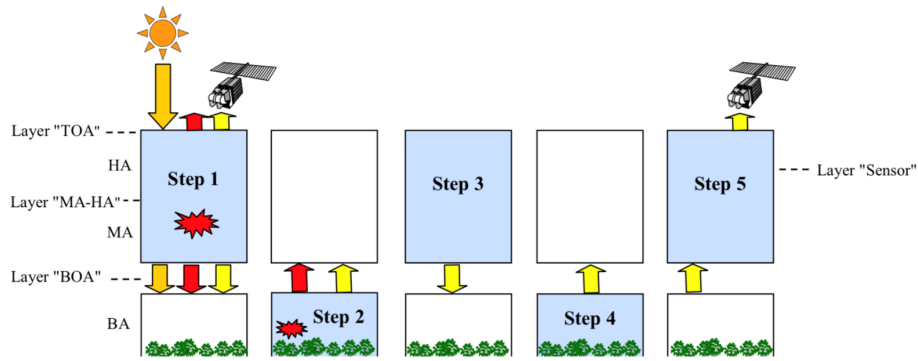
170 W_B is the thermal energy flux emitted by a black body. $\frac{P(r, \lambda, \Omega' \rightarrow \Omega)}{4\pi}$ is the scattering phase
 171 function. It represents the proportion of incident flux along Ω' that is scattered per solid angle
 172 along Ω .

173

174 Figure 2 illustrates the 5 major steps of DART RT modelling in the Earth-atmosphere system.
 175 (1) Atmosphere thermal emission and sun illumination. (2) Earth surface RT. (3) Earth-

176 atmosphere radiative coupling. (4) Earth surface RT of atmosphere backscattered radiation. (5)
 177 Transfer of BOA upward radiation to any altitude, including TOA.

178



179

180 Figure 2. Major steps for modelling RT in the Earth-atmosphere system. Thermal emission in mode
 181 (T) is shown by the red symbol (steps 1 and 2). Arrows indicate directly transmitted solar
 182 (orange) and thermal (red) radiation, and scattered radiation (yellow), either thermal or solar.

183

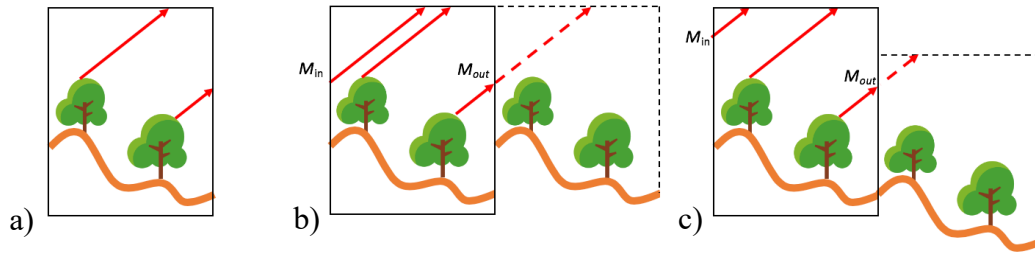
184 2.2 Infinite slope mode

185

186 Earth surface RT is simulated with one of three modes. (1) Isolated scene mode (Figure 3.a).
 187 The Earth surface RT does not depend on neighbouring landscape: a ray that exits the scene
 188 does not re-enter the scene. It corresponds to laboratory experiments where only the studied
 189 scene is illuminated. (2) Repetitive scene mode (Figure 3.b). The landscape is an infinite
 190 horizontal duplication of the actually simulated scene: a ray that exits the 3D scene through a
 191 point M_{out} on a vertical face re-enters the scene along the same direction through a point M_{in}
 192 symmetric to M_{out} , at the same altitude. (3) Infinite slope mode (Figure 3.c). The landscape is
 193 an infinite duplication of the actually simulated scene with a continuity of slope and altitude at
 194 the DEM edges: a ray that exits the scene at M_{out} on a scene vertical face re-enters the scene at
 195 M_{in} , with an altitude shift equal to the altitude difference of the actually simulated scene and
 9

196 the duplicated scene at ray exit face. The infinite slope mode was designed for the simulation
 197 of sloping landscapes. Conversely to the repetitive scene mode that leads to unrealistic border
 198 effects in presence of slope. Only the repetitive scene mode and the infinite slope mode can
 199 simulate environmental effect. Both verify the energy conservation.

200



201

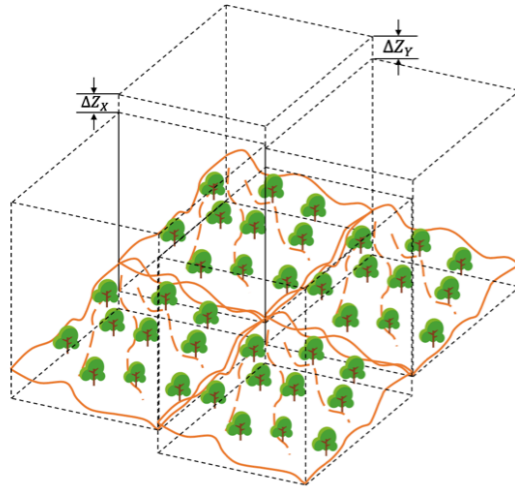
202 Figure 3. The three DART modes for simulating RT in 3D scenes. a) Isolated scene mode. b) Repetitive
 203 scene mode. c) Infinite slope mode. The actually simulated scene is framed by a black box.
 204 The dotted box frames one of the fictive neighbourhoods of the actually simulated scene.

205

206 2.3 Construction of infinite slope mock-up

207

208 In the infinite slope mode, the mean altitude of each edge of the DEM is computed to get the
 209 mean vertical shifts ΔZ_X and ΔZ_Y of the neighbour duplicated scenes, along the axes X and Y,
 210 respectively (Figure 4). Figure 4 displays the infinite spatial repetition of this Earth scene. If
 211 the altitudes of the opposite edges of a DEM are not parallel, the DEM edges and corners are
 212 smoothly connected by Bezier triangles (Gerald, 2014) to ensure the continuity of slopes and
 213 altitudes at the DEM edges of the actually simulated scene and its duplications. Whatever the
 214 Earth scene topography, the atmosphere above the BOA interface is horizontal, since density,
 215 pressure and temperature of DART atmosphere are assumed identical at the same altitude.



216

217 Figure 4. DART scene in infinite slope mode with its infinite duplications. Three duplications are

218 shown here. The mean altitude differences between neighbour Earth scenes are indicated (ΔZ_x

219 for axis X and ΔZ_y for axis Y). DEM edges are smoothly connected with Bezier triangles.

220

221 3 Initial results and problems

222 3.1 Initial strategy

223

224 In the initial infinite slope mode, illumination (Figure 5.a) ray sources depart from the top scene

225 and/or from a so-called “scene window”. A scene window is the part of the scene vertical face

226 through which an exit-ray will not re-enter the scene. Its spatial extent depends on the slope and

227 ray direction. It is computed per illumination direction. Figure 5.a shows a simple 2D example

228 with rays parallel to the paper plane. In 3D, the scene window geometry can be very complex

229 to manage, which in turn can lead to serious illumination flaws. During the Earth surface RT

230 modelling, a ray that exits the scene top/scene window along direction Ω_n is projected to the

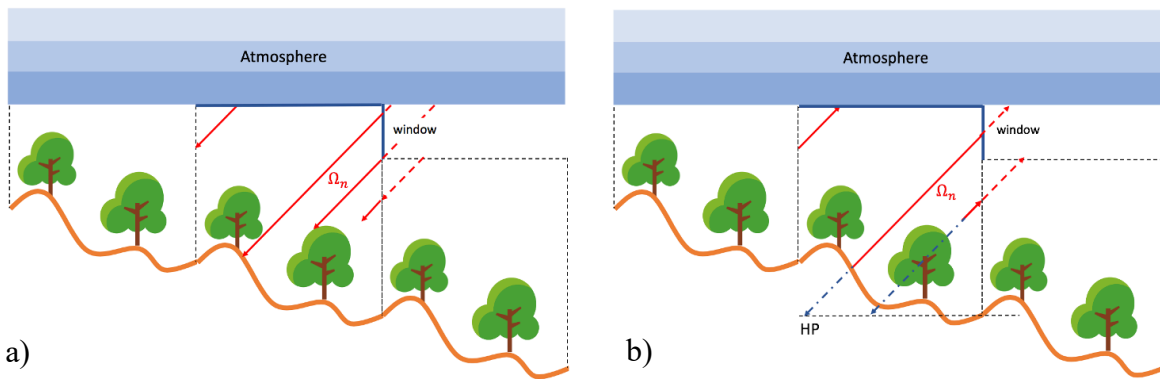
231 spread image on the Horizontal Plane (HP) (Figure 5.b), whereas a ray that crosses a scene

232 vertical face outside the scene window re-enters the scene from the opposite face with an

233 altitude shift (*i.e.*, ΔZ_x along axis X and ΔZ_y along axis Y as indicated in Figure 4). The shape

234 and dimension of spread images on HP depend on the slope and ray direction. For some
 235 directions, the shape of spread images can be very complex polygons, possibly much larger
 236 than the scene horizontal section. The impossibility to accurately define their shapes for any
 237 configuration can introduce two other drawbacks. (1) Difficulty to connect the polygon-shape
 238 spread images to generate parallelogram shape image products. (2) Impossibility to know if
 239 pixels with zero value in the image plane corresponds to the spread image (*i.e.*, the observed
 240 scene), which can lead to images with overestimated mean radiance if totally shadowed zones
 241 are observed (*i.e.*, apparent reflectance equal to zero).

242



243

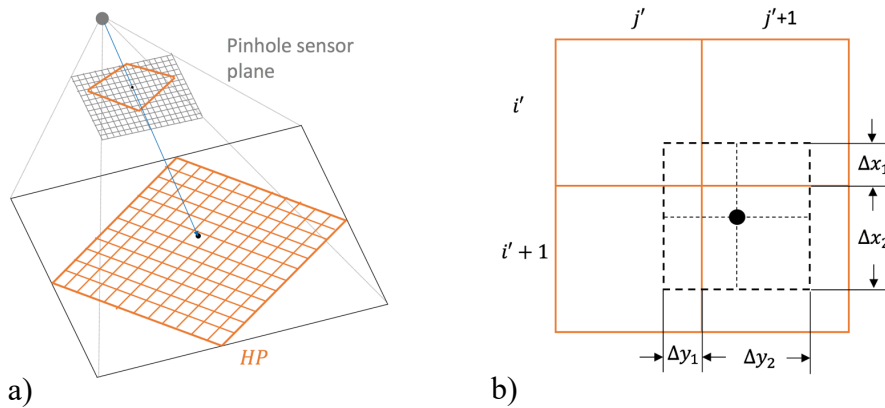
244 Figure 5. 2D view of the infinite slope mode RT modelling of initial DART. a) Illumination stage. b)
 245 Spread image generation stage. The scene window for direction Ω_n is the dark blue region at
 246 the scene vertical face. Its shape depends on the DEM and the illumination direction. Red
 247 arrows are rays in the paper plane. Illumination ray sources start from the scene top and
 248 window. Rays that cross scene top/window never re-enter the scene, conversely to rays that
 249 cross other parts of the scene vertical face. HP is the horizontal plane that stores the spread
 250 images.

251 Atmosphere backscattered energy and RS images are derived from spread images on HP. In the
 252 Earth-Atmosphere coupling stage, upward rays are assumed to come from the rectangular scene
 253 top, since the shape of spread images is too complex to manage. In the RS image generation

254 stage, the centre of each sensor pixel (i, j) is projected onto HP along its viewing direction,
 255 Figure 6.a illustrates a pinhole camera image generation process. Then, a bi-linear interpolation
 256 method derives the pixel value $X_{\text{sensor}}(i, j)$ from the values of four pixels $(X_{\text{HP}}(i', j')$,
 257 $X_{\text{HP}}(i' + 1, j')$, $X_{\text{HP}}(i', j' + 1)$, $X_{\text{HP}}(i' + 1, j' + 1)$) on HP that surround the projected point
 258 (Eq. (2)).

$$\begin{aligned}
 X_{\text{sensor}}(i, j) = & (X_{\text{HP}}(i', j') \cdot \Delta x_1 + X_{\text{HP}}(i' + 1, j') \cdot \Delta x_2) \cdot \Delta y_1 \\
 & + (X_{\text{HP}}(i', j' + 1) \cdot \Delta x_1 + X_{\text{HP}}(i' + 1, j' + 1) \cdot \Delta x_2) \cdot \Delta y_2
 \end{aligned}
 \tag{2}$$

259 where Δx and Δy are the scale coefficients between 0 and 1, they are shown in Figure 6.b.



260
 261 Figure 6. Pinhole camera image generation using an interpolation method. a) Projection onto HP of
 262 the centre of a pixel in the sensor image. b) Nadir view of the projected point. Δx and Δy are
 263 the scale coefficients that vary from 0 to 1.

264

265 3.2 Results and problems

266

267 This section illustrates issues of the initial infinite slope mode through visual and statistical
 268 analysis of DART BRF profile, and RB and RS images (Figure 7 and Figure 8). To facilitate
 269 the analysis, two solo slope simulations are analysed, first a slope with a Lambertian bare
 270 ground (Figure 7.a), and then a slope with a tree (Figure 8.a). Table 2 lists DART input
 271 parameters. The solo slope is created by the DART DEM generator. Figure 7.b shows its energy

272 map (*i.e.*, intercepted flux $\sum_z W(i, j, z)$ ($W/\mu\text{m}$) per pixel (i, j)) with regular microgrooves and
273 polygon-shapes. The intercepted energy flux has a relative variability $(W_{\text{max}}-W_{\text{min}})/W_{\text{mean}}$ up
274 to 25% although it should be zero. It is due to a mismanagement of illumination and coupling
275 energy. Figure 7.c shows the scene reflectance in the solar plane, the BRF varies up to 11%
276 relative to nadir reflectance whereas it should be a constant. Its dependence on the viewing
277 zenith angle θ_v , is due to the boarder effect when connecting polygon-shape spread images. It
278 is all the more serious if the viewing direction is close to the slope (viewing zenith angle $\theta_v < -$
279 54° in Figure 7.c).

280

281 The simulation of a cherry tree on the solo slope illustrates other issues. Tree trunk, branches
282 and leaves are Lambertian surfaces. Figure 8.b-f shows DART RS images: satellite
283 (orthographic) camera for viewing directions equal to $(\theta_{\text{sensor}}=41^\circ, \varphi_{\text{sensor}}=194^\circ)$ (b) and
284 $(\theta_{\text{sensor}}=80^\circ, \varphi_{\text{sensor}}=355^\circ)$ (c), fisheye camera (d), pinhole camera (e) and pushbroom (f).
285 Several issues affect image quality: (1) microgrooves, (2) border effect, and (3) erroneous areas.
286 Microgrooves arise because the ray illumination mesh on the object surface is coarser than
287 DART cell dimensions, for some directions. As a result, triangles between two rays are less
288 illuminated. If the image resolution is high enough, these less illuminated triangles can lead to
289 lines of darker pixels. Because of the microgrooves, the reflectance of ground (averagely 0.29)
290 in Figure 8.b has maximum difference 0.04 with standard deviation 0.004. The initial modelling
291 strategy did not manage the spatial variability of the density of illumination rays on local
292 topography. Hence, the density of illumination rays varies with the local slopes, although it is
293 spatially uniform on the BOA horizontal plane. The second and third issues arise with the
294 procedure that connects the spread images. In the initial strategy, the polygon-shape spread
295 images are juxtaposed in order to obtain a parallelogram RS image. According to the definition
296 of the infinite slope, the opposite sides of any spread image should be continuous. However,

297 this is not always the case if the slope or the viewing direction is very oblique. Also,
 298 discretization explains that pixel values on the borders of a spread image can be over or
 299 underestimated. It results that the connected borders of the spread image appear with darkish
 300 and whitish tones in the final image (Figure 8.b). The complex shape of the spread images also
 301 gives rise to zones without signal (*i.e.*, yellow zones) or with overestimated neighbour zones
 302 (*i.e.*, whitish zones) (Figure 8.c). In addition, the yellow zone on the cherry tree in Figure 8.c
 303 has zero reflectance. These zones are discarded since the initial algorithm cannot distinguish
 304 zero radiance and no-signal pixel.

305

306 In short, Figure 7 and Figure 8 stress two major sources of inaccuracy of the initial infinite
 307 slope mode: scene illumination and polygon-shape spread image management. An important
 308 technical drawback must be noted as well: the required RAM to store images greatly increases
 309 with the obliquity of viewing directions. For example, in the above schematic example, the
 310 infinite slope mode requires more than 8 times RAM than the repetitive scene mode. The next
 311 section presents a new modelling strategy that was developed to improve this situation.

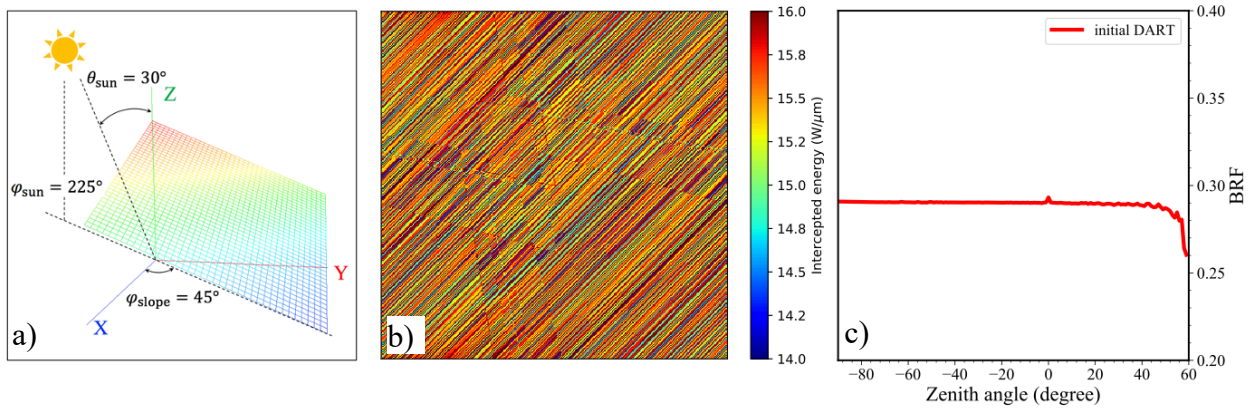
312

313 Table 2. DART input parameters. θ is a zenith angle, φ is an azimuth angle oriented anticlockwise
 314 from South ($\varphi_{\text{South}} = 0^\circ$, $\varphi_{\text{East}} = 90^\circ$, $\varphi_{\text{North}} = 180^\circ$ and $\varphi_{\text{West}} = 270^\circ$). ρ is a reflectance.
 315 Dash symbol stands for “no-value”. USSTD76 atmospheric constituent profile and RURAL
 316 aerosol model are defined in Anderson et al., 1986 and Shettle and Fenn, 1979, respectively.

Parameters	Solo slope	Solo slope + Cherry tree
Spectral band	0.56 μm	0.56 μm
Sun direction	$\theta_{\text{sun}}=30^\circ, \varphi_{\text{sun}}=225^\circ$	$\theta_{\text{sun}}=30^\circ, \varphi_{\text{sun}}=225^\circ$
Atmosphere model	USSTD76	USSTD76

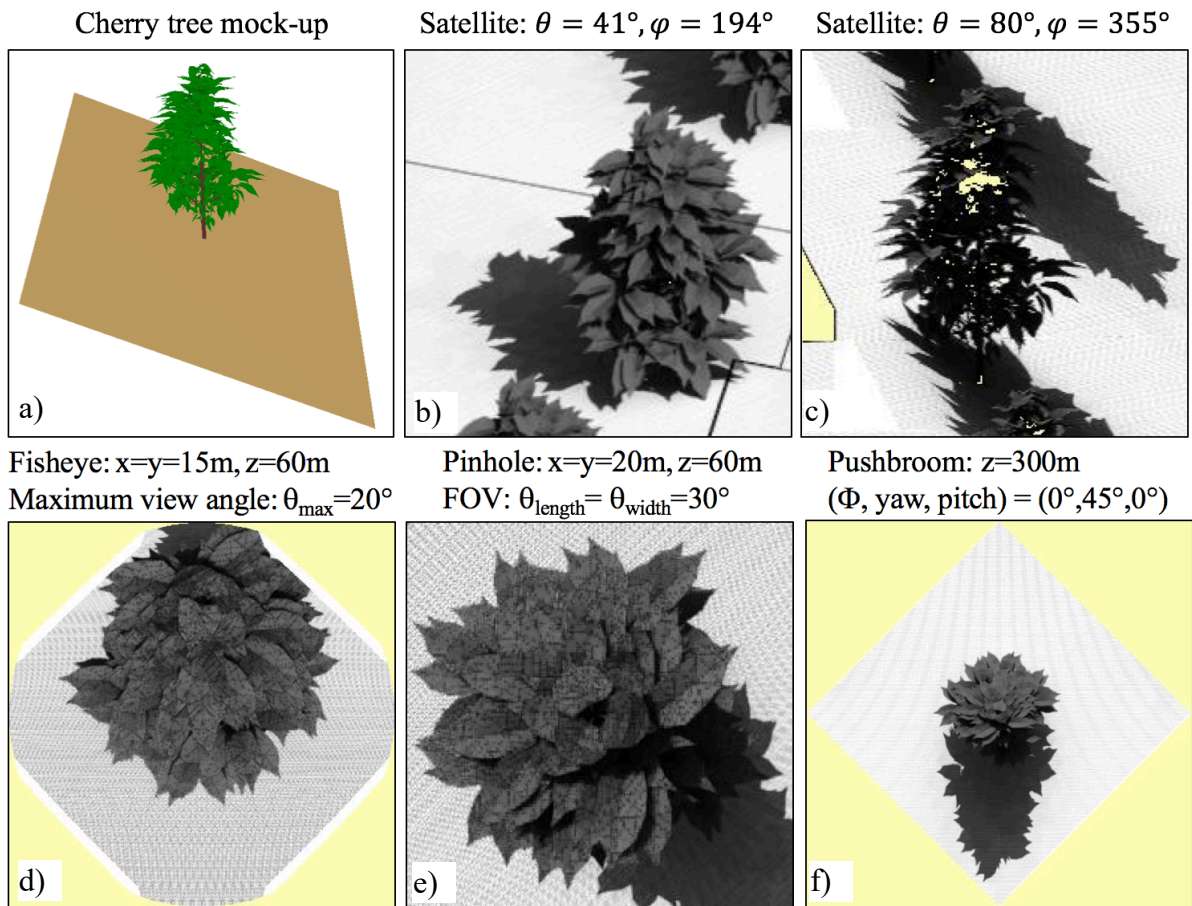
Aerosol model	RURAL_V23KM	RURAL_V23KM
Scene dimension	$X = Y = 40$ m	$X = Y = 40$ m
Spatial resolution	0.125 m	0.125 m
DEM	30° solo slope ($\varphi_{\text{slope}}=45^\circ$)	30° solo slope ($\varphi_{\text{slope}}=45^\circ$)
ρ_{ground}	0.5	0.5
Tree	-	Cherry tree
Tree position	-	$x = y = 20$ m
ρ_{leaf}	-	0.11
τ_{leaf}	-	0.1
ρ_{trunk}	-	0.075

317



318

319 Figure 7. Simulation of a solo slope, using the initial infinite slope mode. DEM mesh with indication
 320 of slope azimuth angle φ_{slope} , and zenith and azimuth angles (θ_{sun} , φ_{sun}) of the direction in
 321 which the sun is seen from the Earth surface (a), φ is anticlockwise from South. Solo slope:
 322 spectral energy flux map (b) and angular reflectance in the solar plane (c).



323

324

325

326

327

328 4 The new infinite slope mode

329 4.1 Introduction of oblique planes OP

330

331 The new modelling strategy of the infinite slope mode aims to get more accurate and more

332 systematic modelling of the scene illumination, the Earth-atmosphere radiative coupling, and

333 the image generation, for sloping landscapes. The innovation is to introduce an auxiliary

334 oblique plane OP that depends only on topography to store spread images and to simulate scene

335 illumination, instead of HP. OP ensures that the spread images have the same shape and size,
 336 and that the RAM demand is independent of topography. Geometrically, OP is a parallelogram,
 337 the four sides of which belong to the four vertical faces of the scene (Figure 9.b). The opposite
 338 sides of OP have altitude differences ΔZ_X along axis X and ΔZ_Y along axis Y equal to the
 339 altitude differences between the duplicated scenes (Figure 4, Figure 9.b). The vertical projection
 340 of any OP pixel on HP has the same size and shape as DART scene cells.

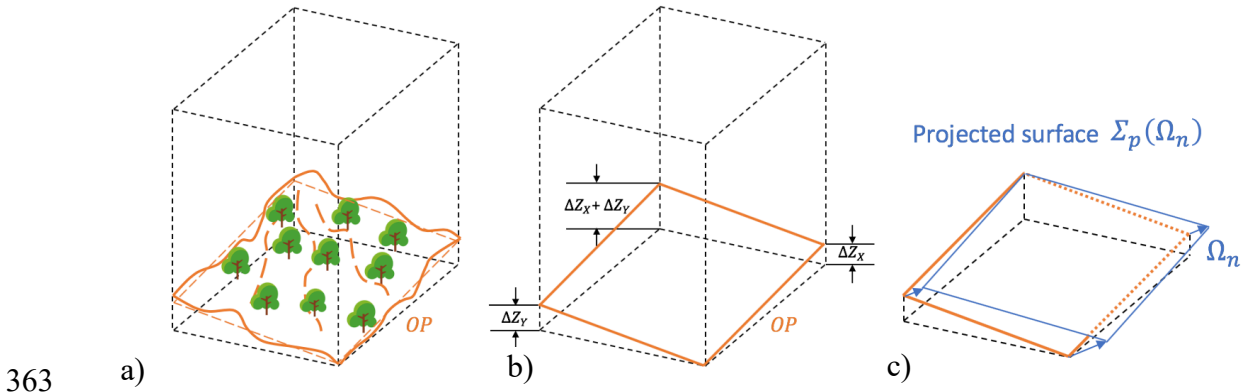
341
 342 DART atmospheric RT and Earth surface RT modelling are sequential. In the scene
 343 illumination and Earth-atmosphere radiative coupling stages, energy conservation is ensured.
 344 To quantify downward radiation from the horizontal atmosphere to OP and upward radiation
 345 from OP to the horizontal atmosphere, the parallel projection of OP along Ω_n onto a horizontal
 346 plane is computed per direction Ω_n . Its projected surface $\Sigma_p(\Omega_n)$ (Figure 9.c) is a parallelogram.
 347 The four corner coordinates (x_p, y_p, z_p) of $\Sigma_p(\Omega_n)$ depend on the direction vector Ω_n and the
 348 coordinates (x_o, y_o, z_o) of the four corners of OP. Ω_n is characterized by its zenith angle θ and
 349 azimuth angle φ . We have $\theta = 0^\circ$ for nadir direction, $(\theta = 90^\circ, \varphi = 0^\circ)$ for a direction parallel
 350 to the x-axis, and $(\theta = 90^\circ, \varphi = 90^\circ)$ for a direction parallel to the y-axis.

$$\begin{cases} z_p - z_o = d \cos \theta \\ x_p - x_o = d \sin \theta \cos \varphi \\ y_p - y_o = d \sin \theta \sin \varphi \end{cases} \quad (3)$$

351 where d is the distance between OP corner (x_o, y_o, z_o) and the projected point (x_p, y_p, z_p) . The
 352 norm of the cross product of any two adjacent sides of the projected parallelogram $\Sigma_p(\Omega_n)$ is
 353 its area $S_p(\Omega_n)$. Note that the parallel projection of OP on any horizontal plane, including the
 354 horizontal atmosphere, gives the same projected area $S_p(\Omega_n)$.

355

356 In presence of slope, if the viewing direction is more inclined than the slope (*i.e.*, $\Omega_n^\uparrow \cdot \mathbf{n} < 0$,
 357 with \mathbf{n} the normal of OP), no signals can be observed along these upward directions Ω_n^\uparrow . In
 358 addition, if the illumination direction is more inclined than the slope (*i.e.*, $\Omega_n^\downarrow \cdot \mathbf{n} > 0$), no light
 359 along these downward directions Ω_n^\downarrow can reach the scene. Therefore, these directions are set as
 360 inactive and called inactive directions $\Omega_{n,\text{inactive}}$. Inactive downward directions $\Omega_{n,\text{inactive}}^\downarrow$ are
 361 eliminated during the illumination stage, and inactive upward directions $\Omega_{n,\text{inactive}}^\uparrow$ are not
 362 considered to compute atmosphere backscattered energies.



363 a)
 364 Figure 9. DART infinite slope mode. a) Earth scene and its fictive oblique plane OP at the scene bottom.
 365 b) DEM altitude differences are ΔZ_X along axis X and ΔZ_Y along axis Y axis. c) Projected
 366 surface (blue parallelogram) $\Sigma_p(\Omega_n)$ of OP along direction Ω_n onto the horizontal plane that
 367 contains the highest point of OP, it has an area equal to $S_p(\Omega_n)$.

368

369 4.2 Illumination of the Earth scene

370

371 Atmospheric RT modelling, including sun radiation and atmosphere thermal emission and
 372 scattering, gives a spectral energy flux $W_h(\Omega_n)$ [$\text{W}/\mu\text{m}$] per direction $(\Omega_n, \Delta\Omega_n)$ per spectral
 373 band in three fictive layers (Gastellu-Etchegorry et al., 1996). (1) BOA: $W_{\text{BOA}}(\Omega_n^\downarrow)$ [$\text{W}/\mu\text{m}$]

374 per downward direction Ω_n^\downarrow including solar direction Ω_s . (2) TOA: $W_{\text{TOA}}(\Omega_n^\uparrow)$ [W/ μm] per
 375 upward direction Ω_n^\uparrow . (3) Sensor layer at any altitude h : $W_{\text{sensor}}(\Omega_n)$ [W/ μm] per upward and
 376 downward direction Ω_n . Equation (4) gives the irradiance of each fictive layer.

$$\begin{aligned}
 E_{\text{BOA}}^{\text{dir}} &= \frac{W_{\text{BOA}}(\Omega_s)}{S_{\text{scene}}}, & E_{\text{BOA}}^{\text{diff}} &= \frac{\sum_{\Omega_n^\downarrow} W_{\text{BOA}}(\Omega_n^\downarrow)}{S_{\text{scene}}} \\
 E_{\text{sensor}}^{\text{dir}} &= \frac{W_{\text{sensor}}(\Omega_s)}{S_{\text{scene}}}, & E_{\text{sensor}}^{\text{diff}} &= \frac{\sum_{\Omega_n^\downarrow} W_{\text{sensor}}(\Omega_n^\downarrow)}{S_{\text{scene}}}
 \end{aligned} \tag{4}$$

377 with S_{scene} the area of scene horizontal section, E^{dir} the direct irradiance and E^{diff} the diffused
 378 irradiance.

379

380 The downward flux incident along direction Ω_n^\downarrow on a pixel (i, j) at BOA level, knowing that the
 381 BOA interface has $I_{\text{BOA}} \cdot J_{\text{BOA}}$ pixels, is

$$W_{\text{BOA}}(i, j, \Omega_n^\downarrow) = \frac{W_{\text{BOA}}(\Omega_n^\downarrow)}{I_{\text{BOA}} \cdot J_{\text{BOA}}} \tag{5}$$

382 Using the law of conservation of energy, the incident energy flux per direction Ω_n^\downarrow on pixel
 383 (i, j) of OP (*i.e.*, I_{BOA} lines and J_{BOA} columns) is

$$W_{\text{OP}}(i, j, \Omega_n^\downarrow) = W_{\text{BOA}}(i, j, \Omega_n^\downarrow) \cdot \frac{S_p(\Omega_n^\downarrow)}{S_{\text{scene}}} \tag{6}$$

384 with $S_p(\Omega_n^\downarrow)$ the projected surface of OP onto the horizontal atmosphere along direction Ω_n^\downarrow .

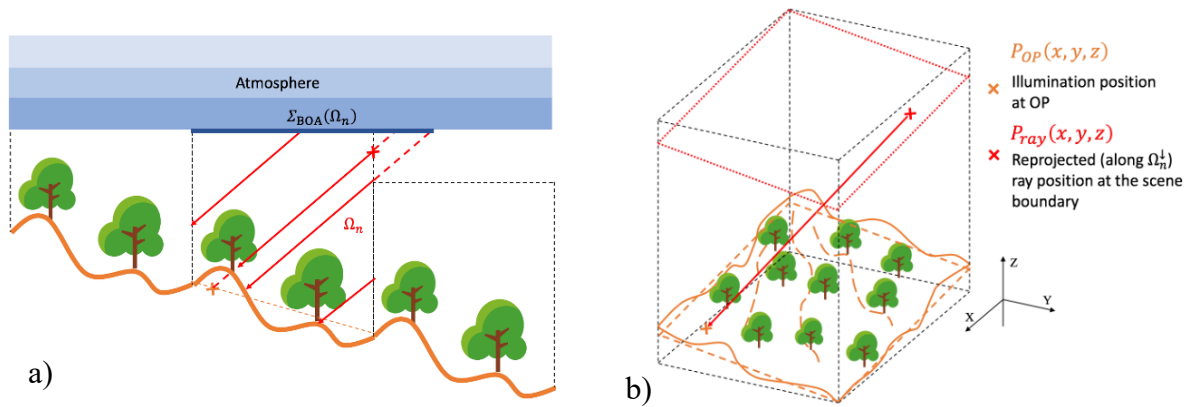
385

386 In the illumination stage, M equal energy rays are sent per OP pixel per downward direction to
 387 illuminate the Earth scene. To uniformly illuminate sloping terrains (*i.e.*, same user-defined ray
 388 density M per direction), the illumination position $P_{\text{OP}}(x, y, z)$ on OP is first sampled using a
 389 semi-random method per pixel (i, j) . Note a larger M improves the accuracy of results but
 390 increases computation time. Default $M = 100$ is a compromise between accuracy and time cost.

391 Then, $P_{OP}(x, y, z)$ is projected to the scene top and scene window per downward direction Ω_n^\downarrow
 392 with a backward ray tracing method to set the ray source position $P_{ray}(x, y, z)$ (Figure 10) at
 393 scene top or scene window. Ray energy flux $W_{ray}(i, j, \Omega_n^\downarrow, P_{ray})$ is computed only for active
 394 directions.

$$W_{ray}(i, j, \Omega_n^\downarrow, P_{ray}) = \frac{W_{OP}(i, j, \Omega_n^\downarrow)}{M} \quad (7)$$

395



396

397 Figure 10. 2D (a) and 3D (b) views of the scene illumination stage. $\Sigma_{BOA}(\Omega_n)$ is the projected surface
 398 of OP onto the horizontal BOA plane with backward ray tracing method; its area is $S_p(\Omega_n)$.
 399 After atmospheric RT modelling, rays through $\Sigma_{BOA}(\Omega_n)$ along direction Ω_n illuminates the
 400 scene. $P_{ray}(x, y, z)$ (x) is the source point of an incident ray, on the scene top or scene window.
 401 $P_{OP}(x, y, z)$ (x) is the projection on OP of $P_{ray}(x, y, z)$ along incident direction Ω_n .

402

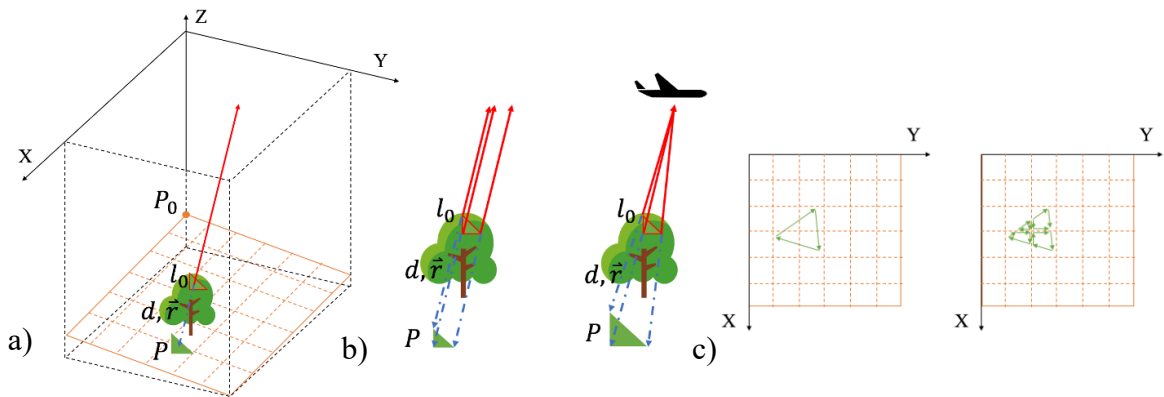
403 4.3 Spread of ray energy on OP during Earth surface RT modelling

404

405 After the illumination and landscape thermal emission stages, the scene surface and volume
 406 elements iteratively scatter all or part of the intercepted radiation, *i.e.*, radiation intercepted at
 407 iteration n is scattered at iteration $n + 1$ (*i.e.*, step 2 in Figure 2). The number of iterations must

408 be large enough in order to get accurate results. It depends on the spectral domain and on
 409 landscape characteristics. Generally, the energy flux cumulated on the spread images follows
 410 an exponential relation with iteration number n (Kallel Abdelaziz, 2018). Hence, at the end of
 411 the Earth surface RT process, the energy fluxes per pixel per direction $W^{\text{iter } n}(i, j, \Omega_n^\uparrow)$ at
 412 different iteration orders n are extrapolated to get $W^{\text{iter } \infty}(i, j, \Omega_n^\uparrow)$ at an infinite iteration order.
 413 Upward radiation that exits the scene top/window is projected onto OP (Figure 11.a) at the
 414 scene bottom along its propagation direction instead of HP at scene bottom (Figure 5.b) as in
 415 the original DART RT modelling in Gastellu-Etchegorry et al., 1996. Storing ray energy in a
 416 plane at the bottom of the scene minimizes the shift of Earth objects in directional images and
 417 simplifies the generation of RS images (Yin et al., 2015).

418



419

420 Figure 11. Ray projection on OP. a) The red triangle gives rise to a ray along direction \mathbf{r} (red arrow)
 421 that reaches the scene top. The green triangle is its projection onto OP. P is the projection on
 422 OP of a corner l_0 of the red triangle. d is the distance $\|P - l_0\|$. P_0 is the highest point of OP.
 423 b) Parallel projection for the orthographic camera and perspective projection for the
 424 perspective camera. c) Nadir view of the projected triangle on OP and its break-down into four
 425 polygons.

426 The projection process is presented here. Let a facet Σ in the 3D scene gives rise to an emitted
 427 and/or scattered energy flux along direction \mathbf{r} that exits the scene top or scene window. In that
 428 case, Σ is projected along \mathbf{r} onto OP where the energy flux is stored per pixel in a spread image.
 429 Its projected surface is Σ_{OP} (Figure 11.a). Let a corner P of Σ_{OP} be the projection of a corner l_0
 430 of Σ along direction \mathbf{r} onto OP (normal \mathbf{n}) (Figure 11.a), P_0 the highest point of OP, and d the
 431 distance $\|P - l_0\|$. The coordinate of P is computed with

$$\begin{cases} (P_0 - P) \cdot \mathbf{n} = 0 \\ P = l_0 + d \cdot \mathbf{r} \end{cases} \Rightarrow \begin{cases} d = \frac{(P_0 - l_0) \cdot \mathbf{n}}{\mathbf{r} \cdot \mathbf{n}} \\ P = l_0 + d \cdot \mathbf{r} \end{cases} \quad (8)$$

432 In Figure 11.c, the projected surface Σ_{OP} is at the intersection of four pixels. Hence, it is
 433 partitioned into four polygons with one polygon Σ_{OP}^k per pixel k . The energy flux distributed
 434 per pixel k is proportional to the surface ratio $\Sigma_{OP}^k/\Sigma_{OP}$.

435

436 After the Earth surface RT stage, DART products (reflectance $\rho^{\text{iterX}}(i, j, \Omega_n^\uparrow)$, radiance
 437 $L^{\text{iterX}}(i, j, \Omega_n^\uparrow)$, brightness temperature $T_B^{\text{iterX}}(i, j, \Omega_n^\uparrow)$, etc.) and their mean values are derived
 438 (Eq. (9)) from the energy fluxes $W_{OP}^{\text{iterX}}(i, j, \Omega_n^\uparrow)$ per pixel per direction on the spread images.

439

$$\begin{aligned} L^{\text{iterX}}(i, j, \Omega_n^\uparrow) &= \frac{W_{OP}^{\text{iterX}}(i, j, \Omega_n^\uparrow)}{S_p(i, j, \Omega_n^\uparrow) \cdot \cos \theta_n \cdot \Delta \Omega_n^\uparrow}, \quad \rho^{\text{iterX}}(i, j, \Omega_n^\uparrow) = \frac{\pi \cdot L^{\text{iterX}}(i, j, \Omega_n^\uparrow)}{E_{BOA}^{\text{dir}} + E_{BOA}^{\text{diff}}}, \\ \bar{L}^{\text{iterX}}(\Omega_n^\uparrow) &= \frac{\sum_i \sum_j L^{\text{iterX}}(i, j, \Omega_n^\uparrow)}{I_{BOA} / J_{BOA}}, \quad \omega_{\text{scene}}^{\text{iterX}} = \frac{\sum_{\Omega_n^\uparrow} \bar{L}^{\text{iterX}}(\Omega_n^\uparrow) \cos \theta_n \Delta \Omega_n^\uparrow}{E_{BOA}^{\text{dir}} + E_{BOA}^{\text{diff}}} \end{aligned} \quad (9)$$

440

441 with $S_p(i, j, \Omega_n^\uparrow)$ the projected surface of OP pixel (i, j) onto HP along direction Ω_n^\uparrow , $\Delta \Omega_n^\uparrow$ the
 442 solid angle of direction Ω_n^\uparrow , and θ_n the zenith angle of direction Ω_n^\uparrow . $\bar{L}(\Omega_n^\uparrow)$ is the average
 443 exitant radiance along direction Ω_n^\uparrow and ω_{scene} is the scene albedo.

444

445 4.4 Earth surface RT of the atmosphere backscattered radiation

446

447 After the Earth surface RT stage (*i.e.*, step 2 in Figure 2), part of the upward flux
 448 $W_{OP}^{iter\ X}(i, j, \Omega_n^\uparrow)$ that exits the scene is backscattered by the atmosphere to the Earth surface (*i.e.*,
 449 step 3 in Figure 2). The backscattered flux is computed by convolving upward radiation from
 450 the Earth scene with transfer function $TF_{BA-BA}(\Omega_n^\uparrow, \Omega_n^\downarrow)$ (Grau and Gatellu-Etchegorry, 2013)

$$W_{BOA}^{backscat}(\Omega_n^\downarrow) = \frac{1}{1 - \omega_{scene} \cdot \bar{S}} \sum_{\Omega_n^\uparrow} W_{BOA}^{iter\ X}(\Omega_n^\uparrow) \cdot TF_{BA-BA}(\Omega_n^\uparrow, \Omega_n^\downarrow) \quad (10)$$

451 with \bar{S} the average atmosphere albedo and $W_{BOA}^{iter\ X}(\Omega_n^\uparrow) = \sum_i \sum_j W_{OP}^{iter\ X}(i, j, \Omega_n^\uparrow) \cdot \frac{S_{scene}}{S_p(\Omega_n^\uparrow)}$ the

452 upward energy flux at BOA along direction Ω_n^\uparrow . $W_{BOA}^{backscat}(\Omega_n^\downarrow)$ is used to get backscattered

453 energy flux per pixel per direction at BOA $W_{BOA}^{backscat}(i, j, \Omega_n^\downarrow) = \frac{W_{BOA}^{backscat}(\Omega_n^\downarrow)}{I_{BOA} \cdot J_{BOA}}$ and the

454 atmosphere backscattered irradiance $E_{BOA}^{backscat} = \frac{\sum_{\Omega_n^\downarrow} W_{BOA}^{backscat}(\Omega_n^\downarrow)}{S_{scene}}$. More Details are given in

455 appendix A.

456

457 Atmosphere backscattering gives a new Earth scene illumination (*i.e.*, step 4 in Figure 2). The

458 resulting backscattered flux per OP pixel (Eq. (11)) ensures the conservation of energy (Eq. (6))

$$W_{OP}^{backscat}(i, j, \Omega_n^\downarrow) = W_{BOA}^{backscat}(i, j, \Omega_n^\downarrow) \cdot \frac{S_p(\Omega_n^\downarrow)}{S_{scene}} \quad (11)$$

459 Then, the Earth surface RT is simulated again, and upward energy $W_{OP}(i, j, \Omega_n^\uparrow)$ per direction

460 is cumulated per OP pixel per direction. The resulting upward flux $W_{OP}^{coupl}(i, j, \Omega_n^\uparrow)$ is used to

461 compute pixel and scene radiance and reflectance products (marked as coupl in equation (12)).

$$L^{coupl}(i, j, \Omega_n^\uparrow) = \frac{W_{OP}^{coupl}(i, j, \Omega_n^\uparrow)}{S_p(i, j, \Omega_n^\uparrow) \cdot \cos \theta_n \cdot \Delta \Omega_n^\uparrow}, \rho^{coupl}(i, j, \Omega_n^\uparrow) = \frac{\pi \cdot L^{coupl}(i, j, \Omega_n^\uparrow)}{E_{BOA}^{dir} + E_{BOA}^{diff} + E_{BOA}^{backscat}} \quad (12)$$

$$\bar{L}^{\text{coupl}}(\Omega_n^\uparrow) = \frac{\sum_i \sum_j L^{\text{coupl}}(i, j, \Omega_n^\uparrow)}{I_{\text{BOA}} \cdot J_{\text{BOA}}}, \quad \omega_{\text{scene}}^{\text{coupl}} = \frac{\sum_{\Omega_n^\uparrow} \bar{L}^{\text{coupl}}(\Omega_n^\uparrow) \cos \theta_n \Delta \Omega_n^\uparrow}{E_{\text{BOA}}^{\text{dir}} + E_{\text{BOA}}^{\text{diff}} + E_{\text{BOA}}^{\text{backscat}}}$$

462 Fluxes $W_{\text{OP}}^{\text{coupl}}(i, j, \Omega_n^\uparrow)$ are tracked in the atmosphere and then cumulated at TOA and sensor
 463 levels (*i.e.*, step 5 in Figure 2) (Grau and Gatellu-Etchegorry, 2013). More details are given in
 464 appendix B.

465

466 **4.5 Image generation**

467

468 Once the radiometric products are computed as spread images, they are projected to the sensor
 469 plane. Here, the initial algorithm (Gastellu-Etchegorry et al., 2008; Yin et al., 2015) is improved
 470 to get more accurate sensor images: sensor image pixel value is computed with a precise
 471 projection method instead of the bi-linear interpolation method. In addition, the lambert
 472 azimuthal equal-area projection is introduced to generate fisheye images.

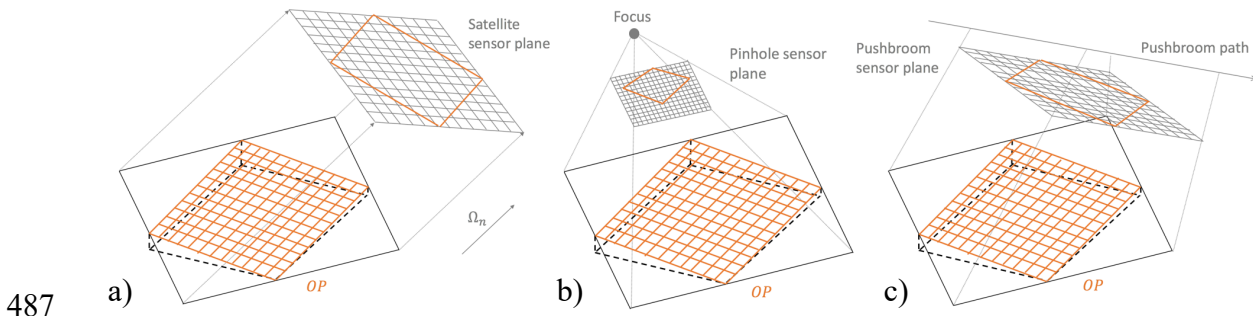
473

474 Although the interpolation method is simple to implement and computer efficient, it degrades
 475 images, and the associated error increases for sloping landscapes and oblique viewing directions.
 476 Hence, we implemented a precise projection method that costs reasonable calculation time. For
 477 satellite (orthographic) camera, spread image pixels are parallelly projected to sensor plane
 478 along the viewing direction Ω_n (Figure 12.a). For pinhole and pushbroom camera, spread image
 479 pixels are perspectively projected to sensor plane along the local viewing direction Ω_n to
 480 camera focus (Figure 12.b, c). For fisheye camera, pixels are projected to a unit sphere along
 481 its local viewing direction to the camera focus and then this pixel on the unit sphere is spread
 482 on the image plane at the bottom of the unit sphere using the Lambert azimuthal equal-area
 483 projection equation (Eq. (13)) that projects any point (x, y, z) on the unit sphere onto its bottom
 484 plane at $(X, Y, -1)$ (Figure 13).

485

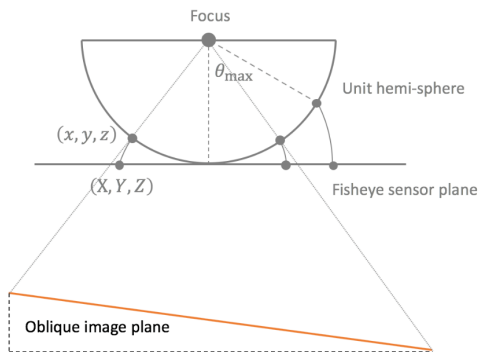
$$(X, Y, Z) = \left(\sqrt{\frac{2}{1-z}} x, \sqrt{\frac{2}{1-z}} y, -1 \right) \quad (13)$$

486



487

488 Figure 12. Projection processes for simulating images. Parallel projection along direction Ω_n for
 489 satellite (orthographic) camera (a), and perspective projection along local viewing direction
 490 for pinhole camera (b) and pushbroom camera (c).



491

492 Figure 13. Projection processes for simulating fisheye images. First, pixel on OP is projected onto a
 493 unit sphere. Then, it is spread on the sensor plane with Lambert azimuthal equal-area
 494 projection method that projects a point (x, y, z) of the unit sphere onto the sensor plane at
 495 point (X, Y, Z) . θ_{\max} is the maximum viewing zenith angle.

496

497 Image dimensions, resolutions and field of view (FOV) are user-defined for perspective and
 498 fisheye cameras whereas image resolution is that of DART cell dimensions for satellite
 499 (orthographic) cameras. Projected images are further rasterized by the pixel grid in the sensor
 500 plane. For that, the four corners of each pixel (i', j') on OP are projected to the sensor plane
 501 along the viewing direction Ω_n^\uparrow , where they make up a new quadrilateral $\Sigma_{i', j'}^{\text{sensor}}$. As in section
 502 4.3, $\Sigma_{i', j'}^{\text{sensor}}$ is spread over sensor pixels, and $f(i, j, i', j', \Omega_n^\uparrow)$ stores the part of $\Sigma_{i', j'}^{\text{sensor}}$ in a
 503 sensor pixel (i, j) . For example, $f(i, j, i', j', \Omega_n^\uparrow) = 0.5$ if $\Sigma_{i', j'}^{\text{sensor}}$ occupies 50% surface of a
 504 sensor pixel (i, j) . For any radiometric product, sensor pixel values $X_{\text{sensor}}(i, j, \Omega_n^\uparrow)$ are
 505 resampled (Eq. (14)) using the values $X_{\text{OP}}(i', j', \Omega_n^\uparrow)$ of the projected OP pixels.

$$X_{\text{sensor}}(i, j, \Omega_n^\uparrow) = \sum_{i'} \sum_{j'} X_{\text{OP}}(i', j', \Omega_n^\uparrow) \cdot f(i, j, i', j', \Omega_n^\uparrow) \quad (14)$$

506 Coordinates $(x_{\text{DART}}, y_{\text{DART}}, z_{\text{DART}})$ in the scene reference are transformed into coordinates
 507 $(x_{\text{sensor}}, y_{\text{sensor}}, z_{\text{sensor}})$ in the sensor reference with a transfer matrix A (Eq. (15)), where
 508 $(\theta, \varphi) = \text{sensor}(\text{zenith}, \text{azimuth})$ angles in the DART scene coordinate reference.

$$\begin{bmatrix} x_{\text{sensor}} \\ y_{\text{sensor}} \\ z_{\text{sensor}} \end{bmatrix} = A \begin{bmatrix} x_{\text{DART}} \\ y_{\text{DART}} \\ z_{\text{DART}} \end{bmatrix} \quad \text{with } A = \begin{bmatrix} \cos \theta \cos \varphi & \cos \theta \sin \varphi & -\sin \theta \\ -\sin \varphi & \cos \varphi & 0 \\ \sin \theta \cos \varphi & \sin \theta \sin \varphi & \cos \theta \end{bmatrix} \quad (15)$$

509 Due to the discretization, some pixels on the borders of the sensor image cannot be entirely
 510 covered by the projected pixels of the spread image. Therefore, this leads to underestimated
 511 pixel values. To manage this problem, we eliminate the pixels in the sides of the sensor image
 512 that are not entirely filled by the projected pixels of the spread image.

513

514 **5 Results and validation**

515

516 5.1 Results and improvements

517

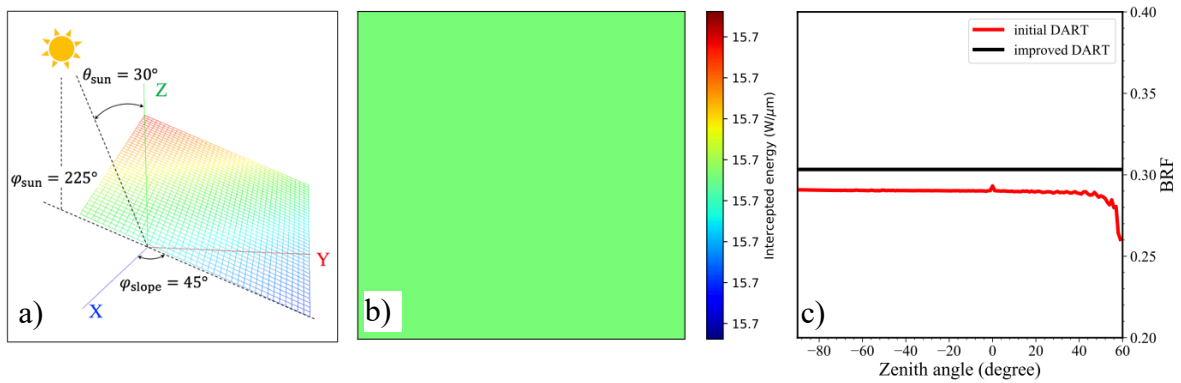
518 With the new modelling strategy, conversely to the initial infinite slope mode, scene irradiance
519 is spatially uniform, spread images have the same shape and dimension for any viewing
520 direction, and energy conservation is ensured during the Earth-Atmosphere radiative coupling.
521 These improvements allow DART to simulate accurate RS signals of sloping landscapes. In
522 addition, image accuracy is further improved thanks to the new projection method instead of
523 the previous bi-linear interpolation method. Here, these results are illustrated with DART
524 simulations of the same scenes as in section 3.2 (Figure 7, Figure 8). As expected, the issues
525 previously noted are corrected. For the solo slope scene, the intercepted energy flux is spatially
526 constant (Figure 14.b) and reflectance in the solar plane is constant (Figure 14.c). Also, all
527 sensor images have a correct visual aspect (Figure 15.b-f): no more microgrooves, border effect,
528 and over or underestimated radiance values in parts of the images. Improvements on the
529 intercepted energy flux $\sum_z W(i, j, z)$ ($W/\mu\text{m}$) per pixel (i, j) and the scene average reflectance
530 $\rho(\theta_v)$ in the solar plane $\varphi_v = 225^\circ$ per zenith angle, with a step $\Delta\theta_v = 1^\circ$, are quantified using
531 two quantities (*i.e.*, absolute relative error $\varepsilon_X(q)$ and mean absolute relative error $\bar{\varepsilon}_X$) defined
532 in equation (16).

$$\varepsilon_X(q) = \frac{|X_{\text{ref}}(q) - X(q)|}{X_{\text{ref}}(q)} \times 100\% \quad \bar{\varepsilon}_X = \frac{1}{N} \sum_{q=1}^N \varepsilon_X(q) \quad (16)$$

533 where X indicates a type of DART product ($X = \rho$ for reflectance; $X = w$ for intercepted
534 energy flux), $X(q)$ is an element q in X product. Subscript ref is the reference value under the
535 same simulation condition. $\bar{\varepsilon}_\rho$ is the average value of $\varepsilon_\rho(\theta_v)$ for each zenith angle θ_v in the
536 solar plane, and $\bar{\varepsilon}_w$ is the average value of $\varepsilon_w(i, j)$ per pixel (i, j) .

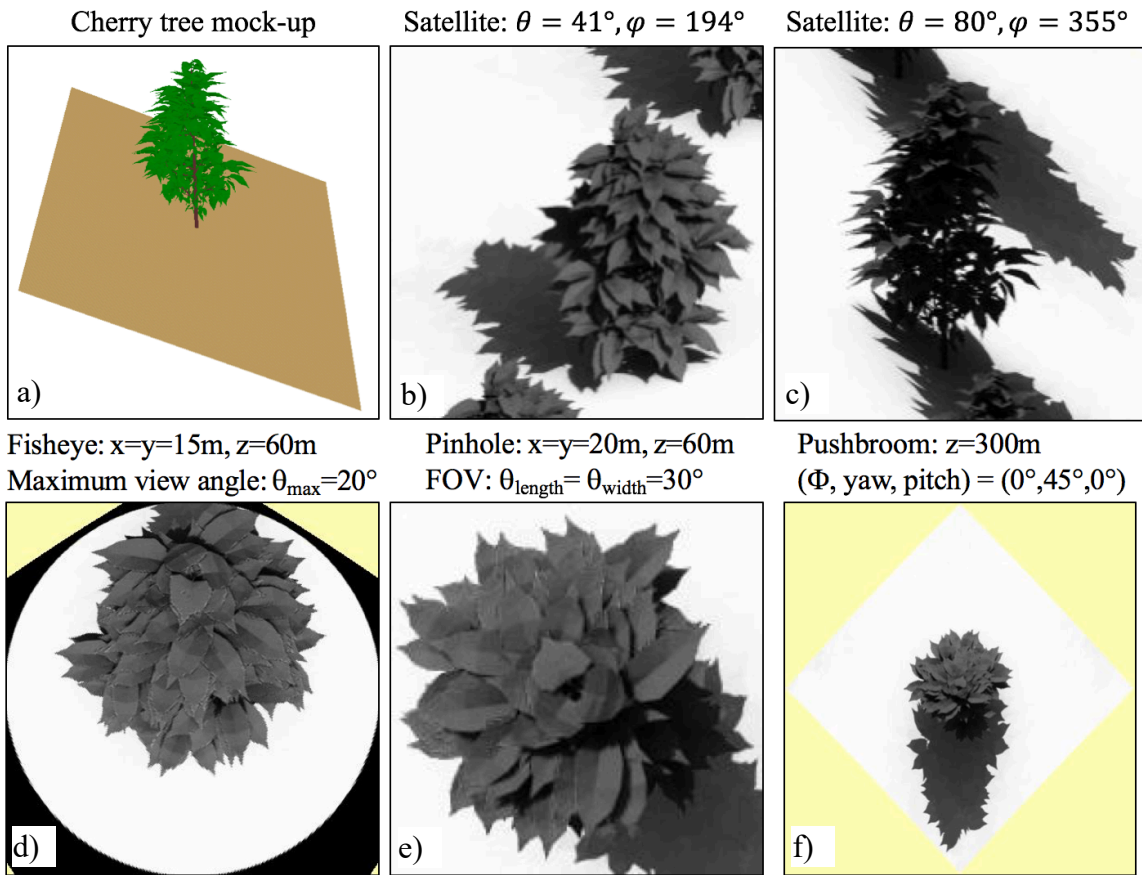
537

538 The new infinite slope mode is assumed to be the reference. This is fully justified for the solo
539 slope for which results have been computed analytically. For example, the directional
540 reflectance factor of {solo slope with slope normal \mathbf{n} and Lambertian reflectance ρ_{ground} } for
541 the sun direction Ω_{sun} (zenith angle θ_{sun}) is $\rho_{\text{ground}} \cdot \frac{|\Omega_{\text{sun}} \cdot \mathbf{n}|}{\cos \theta_{\text{sun}}}$ for any viewing direction. Figure
542 14.c, Figure 16 and Table 3 show the errors in the initial infinite slope mode. For the solo slope
543 scene, $\bar{\epsilon}_{\rho} \approx 4.6\%$ and $\bar{\epsilon}_w \approx 2.8\%$. For the scene of solo slope with a cherry tree, $\bar{\epsilon}_{\rho} \approx 4.0\%$ and
544 $\bar{\epsilon}_w \approx 3.1\%$. Two technical improvements must be noted. (1) The new Lambert azimuthal equal-
545 area projection improves the realism of fisheye image (see Figure 15.d and Figure 8.d). (2)
546 RAM demand and computer time are greatly decreased for oblique slopes. For example, for the
547 “Solo slope + Tree” simulation, RAM demand is reduced by 90% and simulation time is
548 reduced by 10%.



549

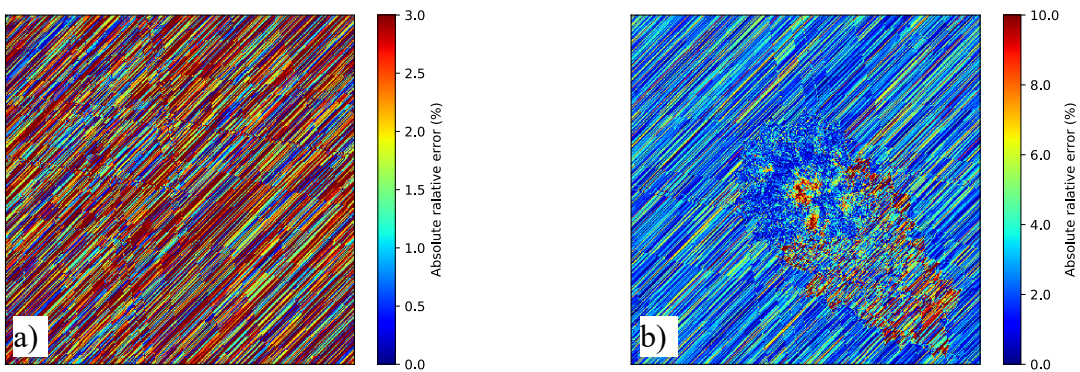
550 Figure 14. DART simulations with the new infinite slope mode. Solo slope. Same legend as Figure 7.



551

552 Figure 15. DART simulations with the new infinite slope mode. Solo slope plus a cherry tree. Same

553 legend as Figure 8.



554 Figure 16. Absolute relative error of DART energy flux images (*i.e.*, $\sum_z W(i, j, z)$ (W) per pixel (i, j))

555 simulated with the initial infinite slope mode. a) Solo slope. b) Solo slope with a cherry tree.

556 Table 3. Summary of the relative error of the initial infinite slope mode for a solo slope and a solo
 557 slope with a cherry tree, the results simulated with the new infinite slope are used as reference.

Error	Solo slope	Solo slope + Cherry tree
$\bar{\varepsilon}_\rho$	4.6%	4.0%
$\max(\varepsilon_\rho)$	14.2%	7.0%
$\bar{\varepsilon}_w$	2.8%	3.1%
$\max(\varepsilon_w)$	14.5%	29.9%

558

559 5.2 Validation

560

561 Ideally, the validation of a modelling approach should be performed with actual measurements.
 562 As a first step, here we validate the new modelling strategy of the infinite slope mode using a
 563 purely modelling approach. The new DART infinite slope mode is tested with a rugged slope
 564 and a rugged slope with realistic forest. For that, two scenes are simulated:

565 1) Experimental scene: it was simulated with the initial (index init) and new (index new)
 566 infinite slope modes.

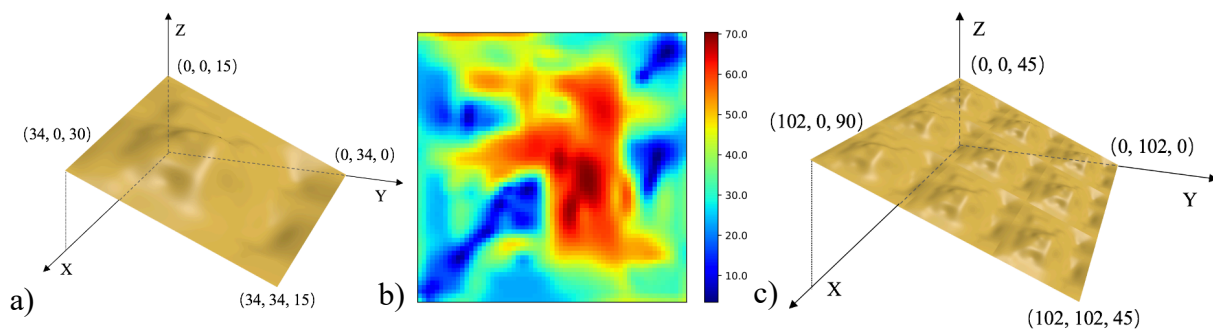
567 2) Reference scene: nine duplications of the experimental scene are juxtaposed with respect to
 568 the continuity of slopes and altitudes. Its central sub-zone corresponds to the experimental scene,
 569 which is called “reference experimental scene” (index ref). Although simulated with the new
 570 infinite slope mode, there is no fictive ray tracing (*i.e.*, application of the infinite slope mode)
 571 at the borders of the “reference experimental scene”. Therefore, the “reference experimental
 572 scene” is used as the reference.

573

574 **5.2.1 Rugged slope**

575

576 The rugged slope (Figure 17.a) in the experimental scene (34 m × 34 m) is created by Blender
577 software (www.blender.org). It has relatively large mean altitude shifts: $\Delta Z_X = 15$ m and
578 $\Delta Z_Y = 15$ m along the axes X and Y, respectively. Figure 17.b shows its slope map. The rugged
579 slope has an average slope 38° and a maximum slope 70° . By repeating 9 times the experimental
580 scene with respect to the continuity of slope and altitude, the reference scene (102 m × 102 m)
581 (Figure 17.c) is created, it has mean altitude shifts $\Delta Z_X = 45$ m and $\Delta Z_Y = 45$ m along the
582 axes X and Y, respectively. DART simulations are for a direct sun illumination ($\theta_{\text{sun}} =$
583 $40^\circ, \varphi_{\text{sun}} = 300^\circ$), a 0.125 m spatial resolution and a NIR (near infrared: $0.87 \mu\text{m}$) spectral
584 band. Ground optical properties are derived from the DART optical properties database. In
585 addition, the roughness of the slope is adjusted by a z-scale factor F_z . For example, $F_z = 2$
586 means all altitude values in the 3D slope are multiplied by 2. In this study, we investigated 3
587 rugged slopes with different roughness ($F_z = 0.8, 1.0$ and 1.2) to assess the robustness of the
588 new method.

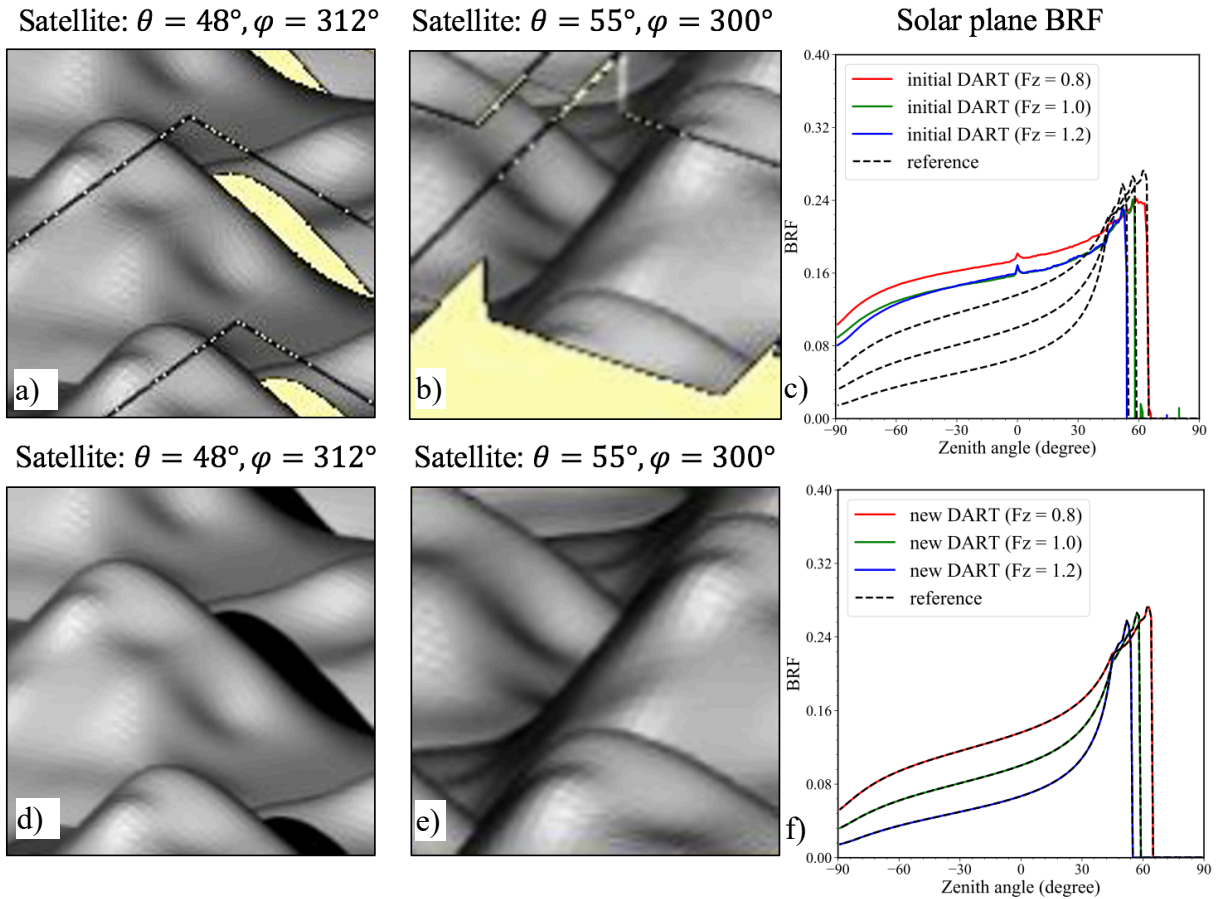


589

590 Figure 17. The rugged slope. a) 3D view. b) slope map (unit $^\circ$). c) 9 duplications of the rugged slope
591 (a) with respect to the continuity of slopes and altitudes.

592 Figure 18.a, b shows two satellite images simulated with initial infinite slope mode ($F_z = 1.0$),
593 the boarder effect and erroneous areas are obvious. These defects do not appear in the images
32

594 simulated by the new infinite slope mode. The microgrooves do not appear in Figure 18.a, b
595 because the variation of pixel reflectance masks the minor reflectance change due to the non-
596 uniform illumination. Figure 18.c, f show the BRFs (*i.e.*, $\rho_{\text{ref}}(\theta_v)$, $\rho_{\text{init}}(\theta_v)$ and $\rho_{\text{new}}(\theta_v)$) in
597 the solar plane (*i.e.* $\varphi_v = 300^\circ$) with zenith angle step $\Delta\theta_v = 2^\circ$ for 3 different z-scale factors
598 ($F_z = 0.8, 1.0$ and 1.2). They illustrate that the roughness can greatly impact the results of the
599 initial infinite slope mode ($\bar{\varepsilon}_{\rho_{\text{init}}} \approx 33.3\%$, 69.0% and 174.8% for $F_z = 0.8, 1.0$ and 1.2) whereas
600 it has no impact on the new infinite slope mode ($\bar{\varepsilon}_{\rho_{\text{new}}} < 10^{-4}$ for $F_z = 0.8, 1.0$ and 1.2). As
601 explained in section 3.1, the BRF in $[-90^\circ, \sim 50^\circ]$ region is overestimated because the initial
602 method discards total shadow (*i.e.*, apparent reflectance equal to zero) when computing the
603 scene average reflectance (yellow zone in Figure 18.a). For images in the $[\sim 50^\circ, 90^\circ]$ region,
604 nearly no shadows are observed. Then, the underestimated BRF is due to the presence of black
605 borders (black lines in Figure 18.b), and mismanagement of image connections (yellow zone in
606 Figure 18.b).



607

608 Figure 18. DART simulated satellite images of the rugged slope (experimental scene) with the initial
 609 (a, b) and new (d, e) infinite slope modes. The reflectance profile of near infrared band (0.87
 610 μm) in the solar plane are computed for 3 different z-scale factors ($F_z = 0.8, 1.0$ and 1.2).

611

612 5.2.2 Rugged slope with a realistic forest

613

614 The scene is a 3D forest over a rugged slope (Figure 17.a). The forest stand is the central part
 615 of the Järvelja Birch stand (HET09_JBS_SUM) in RAMI experiment ([https://rami-
 616 benchmark.jrc.ec.europa.eu](https://rami-benchmark.jrc.ec.europa.eu)). It contains 143 trees with 67 birch trees, 48 linden trees, 13 aspen
 617 trees, 5 common alder trees, 5 spruce trees, and 5 ash and maple trees (Figure 19.a). After the
 618 duplication of the experimental scene, the reference scene has 1287 trees in total. Figure 19.b

619 shows the tree spatial distribution in the reference scene. The repetitive pattern of trees can be
620 noted. DEM in the reference scene is the same as Figure 17.c.

621

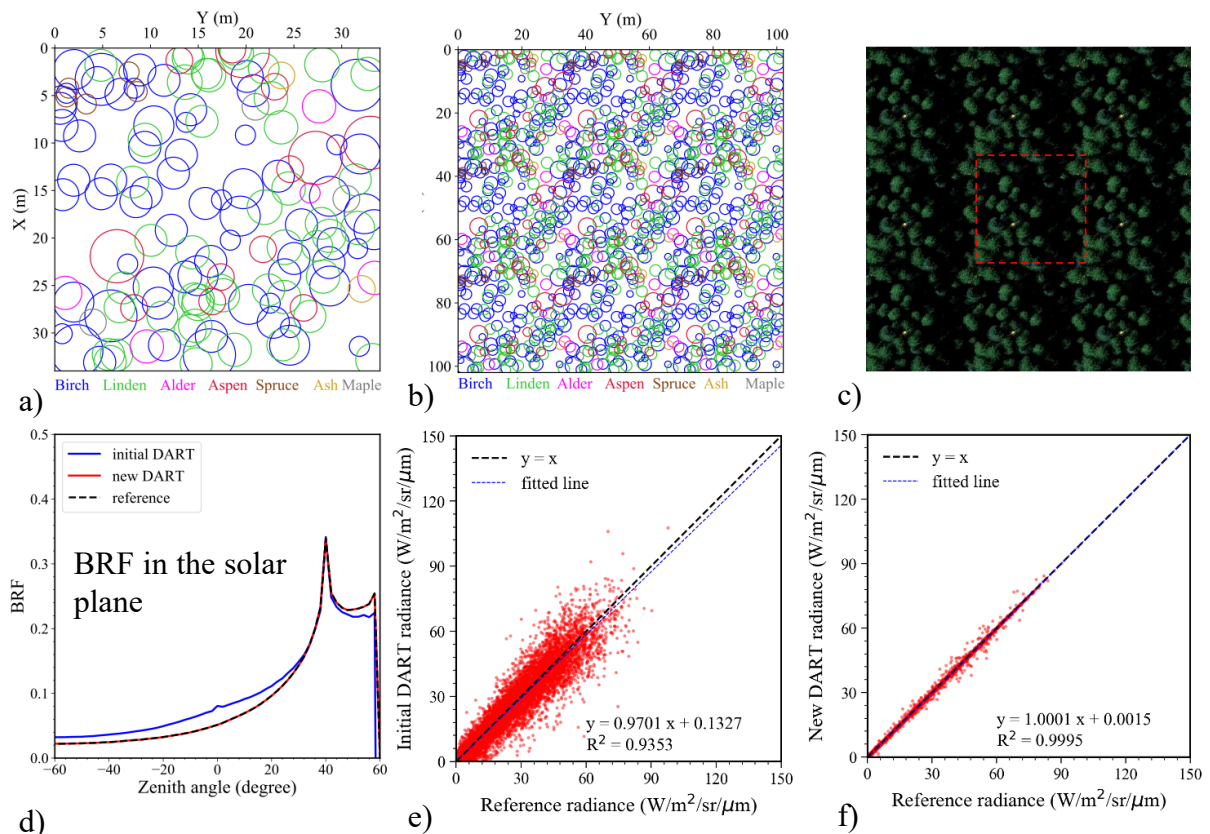
622 DART simulations are conducted with direct sun illumination ($\theta_{\text{sun}} = 40^\circ, \varphi_{\text{sun}} = 300^\circ$), a
623 0.25 m spatial resolution and 4 spectral bands at NIR (near infrared: $0.87 \mu\text{m}$), R (red: $0.66 \mu\text{m}$),
624 G (green: $0.57 \mu\text{m}$), B (blue: $0.44 \mu\text{m}$). Specific optical properties were assigned per tree
625 species. They were simulated by the PROSPECT model using leaf biochemistry parameters
626 obtained by inversion of the optical properties provided by the RAMI experiment ([https://rami-
627 benchmark.jrc.ec.europa.eu](https://rami-benchmark.jrc.ec.europa.eu)) (Jacquemoud et al., 1990, Feret et al., 2017). Figure 19.c shows
628 the nadir colour satellite image of the reference scene. A red square outlines the reference
629 experimental scene in the centre.

630

631 Figure 19.d shows the BRFs (*i.e.*, $\rho_{\text{ref}}(\theta_v)$, $\rho_{\text{init}}(\theta_v)$ and $\rho_{\text{new}}(\theta_v)$) of the three simulations in
632 the solar plane (*i.e.* $\varphi_v = 300^\circ$) with zenith angle step $\Delta\theta_v = 2^\circ$. Compared to the reference
633 ρ_{ref} , the average absolute relative error of the initial infinite slope mode ρ_{init} is $\bar{\varepsilon}_{\rho_{\text{init}}} \approx 35.4\%$,
634 with maximum absolute relative error $\max(\varepsilon_{\rho_{\text{init}}})$ up to 65.6%. These large error values are
635 mostly due to inaccurate illumination with the strong topography, the mismanagement of image
636 connection and also the fact that pixels in total shadow are discarded when computing the scene
637 average radiance (cf. section 3.1). As expected, the new infinite slope mode greatly improves
638 the situation: $\rho_{\text{new}}(\theta_v) \approx \rho_{\text{ref}}(\theta_v)$ with $\bar{\varepsilon}_{\rho_{\text{new}}} \approx 0.01\%$. Pixel-wise comparison of the nadir
639 radiance images $L(i, j)$ at NIR band between the experimental scene and the reference
640 experimental scene is shown in Figure 19.e and Figure 19.f, respectively for scatter plot
641 “ $L_{\text{ref}}(i, j), L_{\text{init}}(i, j)$ ” and scatter plot “ $L_{\text{ref}}(i, j), L_{\text{new}}(i, j)$ ”. Radiance values “ $L_{\text{ref}}(i, j),$
642 $L_{\text{init}}(i, j)$ ” are very scattered in Figure 19.e. It is consistent with the relatively low coefficient

643 of determination $R^2 \approx 0.935$. The radiance intercept 0.13 and slope 0.97 indicates the tendency
 644 to overestimate radiance in shadow area and to underestimate radiance in brilliant area due to
 645 non-uniform scene illumination in presence of strong and asymmetric topography. Figure 19.f
 646 shows $R^2 > 0.999$, a slope 1.0001 and an intercept as low as 0.0015. The slim difference
 647 between the new infinite slope mode and the reference is probably due to DART discretisation
 648 (*i.e.* illumination mesh) combined with the limit of double precision (*i.e.* 16 decimal digits).
 649 For example, altitude shift ΔZ of 15 m or 45 m can have a double coding inaccuracy of
 650 magnitude 10^{-14} in both the experimental scene and the reference scene, which implies slightly
 651 different starting points for the illumination rays. As a result, in the mountain forest scene that
 652 is simulated with tens of millions of facets, a ray intercepted by a facet edge in the experimental
 653 scene could pass through in the reference scene, which leads to very minor difference between
 654 few pixels of the experimental and reference scene.

655



656

657 Figure 19. Tree spatial distribution in the experimental (a) and reference (b) scene. c) Nadir colour
658 composite satellite image of the reference scene simulated with the new infinite slope mode.
659 Its central red square outlines the “reference experimental scene”. d) NIR BRF in the solar
660 plane for “experimental scene + initial infinite slope mode” (–), “experimental scene + new
661 infinite slope mode” (–) and “reference experimental scene + new infinite slope mode” (- -).
662 Scatter plots of pixel NIR nadir radiance for “Initial - Reference” (e) and “New - Reference”
663 (f).

664

665 **6 Concluding remarks**

666

667 This paper presents a new modelling strategy for DART infinite slope mode. This strategy leads
668 to accurate and efficient simulations of RS signals and radiative budget for sloping landscapes.
669 Five key points must be noted: (1) The horizontal atmosphere surface crossed by the upward
670 flux from the landscape or the downward flux from atmosphere is computed per direction in
671 order to ensure energy conservation during the Earth-Atmosphere coupling. (2) The
672 illumination ray-mesh is computed per direction in order to ensure uniform scene illumination
673 for any sloping landscape. (3) The spread images are stored on an oblique plane OP that is
674 topography-adapted instead of the previous horizontal plane HP. It allows DART to generate
675 accurate and high-quality RS images with less RAM demand. (4) Pixels on the spread image
676 are projected on the sensor plane with a precise projection method instead of the bi-linear
677 interpolation method, which further improves the accuracy of the simulated RS image. (5) The
678 Lambert azimuthal equal-area projection is applied to fisheye cameras, which increases the
679 realism of the simulated fisheye images. We simulated four scenarios to assess this
680 improvement:

681

682 - Solo slope: the initial infinite slope mode gives the intercepted energy flux map that has
683 a spatial relative variability $(W_{\max}-W_{\min})/W_{\text{mean}}$ up to 25% and a reflectance relative
684 difference up to 11%. In addition, it underestimates the reflectance in the solar plane by
685 4.6%. The new infinite slope mode simulates uniform and accurate intercepted energy
686 flux map and BRF.

687

688 - Solo slope with a cherry tree: the initial infinite slope mode leads to images with
689 microgrooves, border effects, and erroneous zones. The BRF in the solar plane has an
690 average relative error 4.0% as well. The new infinite slope mode simulates accurate and
691 high-quality RS images without the defects of the initial method.

692

693 - Rugged slope: the initial infinite slope mode gives reflectance in the solar plane that has
694 an average relative error up to 69.0%. Indeed, border effects and erroneous zones appear
695 in the simulated images. These errors and defects increase with the slope roughness and
696 obliquity. The new infinite slope improves the accuracy of results with relative error
697 less than 10^{-4} whatever the slope roughness and obliquity. It also simulates accurate
698 and high-quality RS images without defects.

699

700 - Rugged slope with a realistic forest: the initial infinite slope mode gives reflectance in
701 the solar plane that has an average relative error up to 35.4%. In addition, the pixel-wise
702 comparison between the reference experimental scene and the experimental scene gives
703 a coefficient of determination R^2 as low as 0.935. Conversely, the new infinite slope
704 mode gives accurate BRF profiles in the solar plane, and a coefficient of determination
705 R^2 larger than 0.999 in the pixel-wise comparison.

706

707 In short, these results stress the great improvements of DART infinite slope mode. One should
708 note that the four considered scenarios are a bit extreme because the slopes are relatively large.
709 With less oblique slope, the initial infinite slope mode is less inaccurate.

710

711 Current DART infinite slope mode is only available for stratified homogeneous atmosphere. In
712 order to simulate adjacent effect risen by the heterogeneous backscattering, multi-cell in each
713 atmosphere layer should be considered in the future work. Also, the accuracy of this modelling
714 strategy should be assessed through the comparison of DART and satellite images, using field
715 measurements in order to ensure an accurate modelling of the scene.

716

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718

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720 very thankful to computer scientists Jordan Guilleux and Eric Chavanon in DART team.

721

722 **Appendix A. Atmosphere backscattering**

723

724 The backscattered flux is computed by convolving upward radiation from the Earth scene with
725 transfer function TF_{BA-BA} , at interface BOA (Grau and Gatellu-Etchegorry, 2013).
726 $TF_{BA-BA}(\Delta i, \Delta j, \Omega^\uparrow, \Omega'^\downarrow)$, with $\Delta i = i - i'$ and $\Delta j = j - j'$, gives the flux $W_{BOA}(i', j', \Omega'^\downarrow)$ that the
727 atmosphere backscatters to pixel (i', j') onto BOA interface for any downward direction Ω'^\downarrow ,
728 due to unit upward flux $W_{BOA}(i, j, \Omega^\uparrow) = 1$ from pixel (i, j) on BOA interface, along upward

729 direction Ω^\uparrow . With an atmosphere spherical albedo \bar{S} and scene albedo ω_{scene} , backscattered
 730 energy of infinite coupling order is assumed to be

$$W_{\text{BOA}}^{\text{backscat}}(i', j', \Omega'^\downarrow) = \frac{1}{1 - \omega_{\text{scene}} \cdot \bar{S}} \sum_i \sum_j \sum_{\Omega^\uparrow} W_{\text{BOA}}^{\text{iterX}}(i, j, \Omega^\uparrow) \cdot \text{TF}_{\text{BA-BA}}(\Delta i, \Delta j, \Omega^\uparrow, \Omega'^\downarrow)$$

731 Transfer functions $\text{TF}_{\text{BA-BA}}$, $\text{TF}_{\text{BA-sensor}}$ and $\text{TF}_{\text{BA-TA}}$ are computed in the atmospheric RT
 732 stage. Their use greatly decreases computer time since it avoids repeated RT calculation per
 733 pixel. The new infinite slope mode is adapted to the case where each atmosphere MA layer
 734 consists of a single cell. Then, pixels (i, j) cannot be distinguished in the backscattering process,
 735 and $\text{TF}_{\text{BA-BA}}(\Delta i, \Delta j, \Omega^\uparrow, \Omega'^\downarrow)$ is simplified as $\text{TF}_{\text{BA-BA}}(\Omega^\uparrow, \Omega'^\downarrow)$. The flux $W_{\text{BOA}}^{\text{iterX}}(\Omega_n^\uparrow)$ at BOA
 736 level is calculated by subsampling the flux $W_{\text{OP}}^{\text{iterX}}(i, j, \Omega_n^\uparrow)$ per pixel at OP

$$W_{\text{BOA}}^{\text{iterX}}(\Omega_n^\uparrow) = \sum_i \sum_j W_{\text{OP}}^{\text{iterX}}(i, j, \Omega_n^\uparrow) \cdot \frac{S_{\text{scene}}}{S_p(\Omega_n^\uparrow)}$$

737 with S_{scene} the area of scene horizontal section, $S_p(\Omega_n^\uparrow)$ the projected surface of OP onto the
 738 horizontal atmosphere along direction Ω_n^\uparrow .

739

740 It leads to backscattered flux $W_{\text{BOA}}^{\text{backscat}}(\Omega_n^\downarrow)$ and backscattered flux per pixel ($I_{\text{BOA}} \cdot J_{\text{BOA}}$
 741 pixels) at BOA

$$W_{\text{BOA}}^{\text{backscat}}(\Omega_n^\downarrow) = \frac{1}{1 - \omega_{\text{scene}} \cdot \bar{S}} \sum_{\Omega_n^\uparrow} W_{\text{BOA}}^{\text{iterX}}(\Omega_n^\uparrow) \cdot \text{TF}_{\text{BA-BA}}(\Omega_n^\uparrow, \Omega_n^\downarrow)$$

$$W_{\text{BOA}}^{\text{backscat}}(i, j, \Omega_n^\downarrow) = \frac{W_{\text{BOA}}^{\text{backscat}}(\Omega_n^\downarrow)}{I_{\text{BOA}} \cdot J_{\text{BOA}}}$$

742 The atmosphere albedo \bar{S} and backscattered irradiance $E_{\text{BOA}}^{\text{backscat}}$ are calculated as

$$\bar{S} = \frac{\sum_{\Omega_n^\downarrow} \sum_{\Omega_n^\uparrow} W_{\text{BOA}}^{\text{iterX}}(\Omega_n^\uparrow) \cdot \text{TF}_{\text{BA-BA}}(\Omega_n^\uparrow, \Omega_n^\downarrow)}{\sum_{\Omega_n^\uparrow} W_{\text{BOA}}^{\text{iterX}}(\Omega_n^\uparrow)}, E_{\text{BOA}}^{\text{backscat}} = \frac{\sum_{\Omega_n^\downarrow} W_{\text{BOA}}^{\text{backscat}}(\Omega_n^\downarrow)}{S_{\text{scene}}}$$

743

744 **Appendix B. Simulation of DART products at sensor level and TOA**

745

746 The Earth surface RT of atmosphere backscattered radiance (step 4 in Figure 2) gives

747 $W_{OP}^{coupl}(i, j, \Omega_n^\uparrow)$. Then (step 5 in Figure 2), energies at BOA are transferred to TOA and sensor

748 levels. Fluxes at these levels are calculated using $W_{OP}^{coupl}(i, j, \Omega_n^\uparrow)$, direct transmittances

749 $\{T_{BA-TOA}, T_{BA-sensor}\}$ and transfer functions $\{TF_{BA-TOA}(\Omega_n^\uparrow, \Omega_{n'}^\uparrow), TF_{BA-sensor}(\Omega_n^\uparrow, \Omega_{n'}^\uparrow)\}$

750 (Grau and Gatellu-Etchegorry, 2013) (*i.e.*, step 5 in Figure 2). $TF_{BA-TOA(sensor)}(\Omega_n^\uparrow, \Omega_{n'}^\uparrow)$ gives

751 the upward flux $W_{TOA(sensor)}(\Omega_{n'}^\uparrow)$ per upward direction $\Omega_{n'}^\uparrow$ at TOA (sensor) level, due to a

752 unit flux $W_{BOA}(\Omega_n^\uparrow) = 1$ from BOA level along the upward direction Ω_n^\uparrow . The energy flux

753 $W_{TOA}(i, j, \Omega_n^\uparrow)$ per pixel (i, j) , in the images ($I_{BOA} \cdot J_{BOA}$ pixels) per viewing direction Ω_n^\uparrow at

754 TOA and sensor levels is computed as:

755

$$\begin{aligned}
 & W_{TOA}(i, j, \Omega_n^\uparrow) \\
 &= W_{OP}^{coupl}(i, j, \Omega_n^\uparrow) \cdot T_{BA-TOA} \\
 &+ \frac{W_{TOA}(\Omega_n^\uparrow) \cdot \frac{S_p(\Omega_n^\uparrow)}{S_{scene}} + \sum_{\Omega_{n'}^\uparrow} \sum_i \sum_j W_{OP}^{coupl}(i, j, \Omega_n^\uparrow) \cdot TF_{BA-TOA}(\Omega_n^\uparrow, \Omega_{n'}^\uparrow)}{I_{BOA} \cdot J_{BOA}}
 \end{aligned}$$

$$\begin{aligned}
 & W_{sensor}(i, j, \Omega_n^\uparrow) \\
 &= W_{OP}^{coupl}(i, j, \Omega_n^\uparrow) \cdot T_{BA-sensor} \\
 &+ \frac{W_{sensor}(\Omega_n^\uparrow) \cdot \frac{S_p(\Omega_n^\uparrow)}{S_{scene}} + \sum_{\Omega_{n'}^\uparrow} \sum_i \sum_j W_{OP}^{coupl}(i, j, \Omega_n^\uparrow) \cdot TF_{BA-sensor}(\Omega_n^\uparrow, \Omega_{n'}^\uparrow)}{I_{BOA} \cdot J_{BOA}}
 \end{aligned}$$

756 with S_{scene} the area of scene horizontal section, $S_p(\Omega_n^\uparrow)$ the projected surface of OP onto the

757 horizontal atmosphere along direction Ω_n^\uparrow .

758

759 The backscattered irradiance $E_{\text{sens}}^{\text{backscat}}$ at sensor level is calculated as

$$E_{\text{sens}}^{\text{backscat}} = \sum_{\Omega_n^\downarrow} \sum_{\Omega_n^\uparrow} \frac{\sum_i \sum_j W_{\text{OP}}^{\text{coupl}}(i, j, \Omega_n^\uparrow)}{S_p(\Omega_n^\uparrow)} \cdot \text{TF}_{\text{BA-sensor}}(\Omega_n^\uparrow, \Omega_n^\downarrow)$$

760 Associated radiometric products are also computed

$$L_{\text{sensor}}^{\text{coupl}}(i, j, \Omega_n^\uparrow) = \frac{W_{\text{sensor}}(i, j, \Omega_n^\uparrow)}{S_p(i, j, \Omega_n^\uparrow) \cdot \cos \theta_n \cdot \Delta \Omega_n^\uparrow}, \rho_{\text{sensor}}^{\text{coupl}}(i, j, \Omega_n^\uparrow) = \frac{\pi \cdot L_{\text{sensor}}^{\text{coupl}}(i, j, \Omega_n^\uparrow)}{E_{\text{sensor}}^{\text{dir}} + E_{\text{sensor}}^{\text{diff}} + E_{\text{sensor}}^{\text{backscat}}}$$

$$L_{\text{TOA}}^{\text{coupl}}(i, j, \Omega_n^\uparrow) = \frac{W_{\text{TOA}}(i, j, \Omega_n^\uparrow)}{S_p(i, j, \Omega_n^\uparrow) \cdot \cos \theta_n \cdot \Delta \Omega_n^\uparrow}, \rho_{\text{TOA}}^{\text{coupl}}(i, j, \Omega_n^\uparrow) = \frac{\pi \cdot L_{\text{TOA}}^{\text{coupl}}(i, j, \Omega_n^\uparrow)}{E_{\text{TOA}}^{\text{dir}}}$$

761 with $L_{\text{sensor}}^{\text{coupl}}$ and $\rho_{\text{sensor}}^{\text{coupl}}$ the radiance and reflectance at sensor level, $L_{\text{TOA}}^{\text{coupl}}$ and $\rho_{\text{TOA}}^{\text{coupl}}$ the
762 radiance and reflectance at TOA. $E_{\text{TOA}}^{\text{dir}}$ is the direct irradiance at TOA. $\Delta \Omega_n^\uparrow$ is the solid angle
763 of direction Ω_n^\uparrow and θ_n is the zenith angle of direction Ω_n^\uparrow . Superscript “coupl” refers to DART
764 products after the radiative coupling with the atmosphere.

765

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913 **List of Figure Captions**

914

915 Figure 1. DART 3D discrete mock-up. Landscape elements are made of triangles, and/or fluid
916 and turbid vegetation cells. The landscape itself is a cell matrix in order to ease ray tracing. The
917 atmosphere has three levels: upper level made of layers, middle level made of cells of any size,
918 and lower level in the landscape.

919

920 Figure 2. Major steps for modelling RT in the Earth-Atmosphere system. Thermal emission in
921 mode (T) is shown by the red symbol (steps 1 and 2). Arrows indicate directly transmitted solar
922 (orange) and thermal (red) radiation, and scattered radiation (yellow), either thermal or solar.

923

924 Figure 3. The three DART modes for simulating RT in 3D scenes. a) Isolated scene mode. b)
925 Repetitive scene mode. c) Infinite slope mode. The actually simulated scene is framed by a
926 black box. The dotted box frames one of the fictive neighbourhoods of the actually simulated
927 scene.

928

929 Figure 4. DART scene in infinite slope mode with its infinite duplications. Three duplications
930 are shown here. The mean altitude differences between neighbour Earth scenes are indicated
931 (ΔZ_X for axis X and ΔZ_Y for axis Y). DEM Edges are smoothly connected with Bezier triangles.

932

933 Figure 5. 2D view of the infinite slope mode RT modelling of initial DART. a) Illumination
934 stage. b) Spread image generation. The scene window for direction Ω_n is the dark blue region
935 at the scene vertical face. Its shape depends on the DEM and the illumination direction. Red
936 arrows are rays in the paper plane. Illumination ray sources start from the scene top and window.
937 Rays that cross scene top / window never re-enter the scene, conversely to rays that cross other
938 parts of the scene vertical face. HP is the horizontal plane that stores the spread images.

939

940 Figure 6. Pinhole camera image generation using an interpolation method. a) Projection onto
941 HP of the centre of a pixel in the sensor image. b) Nadir view of the projected point. Δx and Δy
942 are the scale coefficients that vary from 0 to 1.

943

944 Figure 7. Simulation of a solo slope, using the initial infinite slope mode. DEM mesh with
 945 indication of slope azimuth angle φ_{slope} , and zenith and azimuth angles ($\theta_{\text{sun}}, \varphi_{\text{sun}}$) of the
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 947 Solo slope: spectral energy flux map (b) and angular reflectance in the solar plane (c).

948

949 Figure 8. Simulation of a cherry tree on a solo slope, using the initial infinite slope mode.
 950 Satellite (orthographic) camera images (b, c) with 2 viewing directions, fisheye camera image
 951 (d), pinhole camera image (e) and pushbroom image (f).

952

953 Figure 9. DART infinite slope mode. a) Earth scene and its fictive oblique plane OP at the scene
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957

958 Figure 10. 2D (a) and 3D (b) views of the scene illumination stage. $\Sigma_{\text{BOA}}(\Omega_n)$ is the projected
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 960 $S_p(\Omega_n)$. After atmospheric RT modelling, rays through $\Sigma_{\text{BOA}}(\Omega_n)$ along direction Ω_n
 961 illuminates the scene. $P_{\text{ray}}(x, y, z)$ (**x**) is the source point of an incident ray, on the scene top or
 962 scene window. $P_{\text{OP}}(x, y, z)$ (**x**) is the projection on OP of $P_{\text{ray}}(x, y, z)$ along incident direction
 963 Ω_n .

964

965 Figure 11. Ray projection on OP. a) The red triangle gives rise to a ray along direction \mathbf{r} (red
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970 four polygons.

971

972 Figure 12. Projection processes for simulating images. Parallel projection along direction Ω_n
973 for satellite (orthographic) camera (a), and perspective projection along local viewing direction
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975

976 Figure 13. Projection processes for simulating fisheye images. First, pixel on OP is projected
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979 (X, Y, Z) . θ_{\max} is the maximum viewing zenith angle.

980

981 Figure 14. DART simulations with the new infinite slope mode. Solo slope. Same legend as
982 Figure 7.

983

984 Figure 15. DART simulations with the new infinite slope mode. Solo slope plus a cherry tree.
985 Same legend as Figure 8.

986

987 Figure 16. Absolute relative error of DART energy flux images (*i.e.*, $\sum_z W(i, j, z)$ (W) per pixel
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990

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993

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997

998 Figure 19. Tree spatial distribution in the experimental (a) and reference (b) scene. c) Nadir
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1002 new infinite slope mode” (–) and “reference experimental scene + new infinite slope mode” (–
1003 –). Scatter plots of pixel NIR nadir radiance for “Initial - Reference” (e) and “New - Reference”
1004 (f).