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Fabien Szmytka, Eric Charkaluk, Andrei Constantinescu, Pierre Osmond. Probabilistic Low Cycle Fatigue criterion for nodular cast-irons. *International Journal of Fatigue*, 2020, 139, pp.105701. 10.1016/j.ijfatigue.2020.105701 . hal-03080253

HAL Id: hal-03080253

<https://hal.science/hal-03080253v1>

Submitted on 18 Dec 2020

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Probabilistic Low Cycle Fatigue criterion for nodular cast-irons

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Abstract

This paper proposes an original method for characterising the Low Cycle Fatigue (LCF) lifetime using probability density functions. The protocol is based on statistics of microstructure heterogeneities taken as damage initiation sites, a qualitative mechanical analysis of the heterogeneities harmfulness and the definition of a micro-crack growth law. The technique is here established and the associated model identified for a nodular cast iron where the graphite nodules are assumed to be the damage initiation zones. The LCF lifetime is characterised from both a large set of experimental test between 300 and 600 °C and damage observations at the micro-scale. Experimental post-mortem observation combined with a simple numerical study first enable to assume the harmfulness of nodules according to their size and their probable role in the damage process. A probability density function for the lifetime is then built from the following steps: (i) a quantitative analysis of the material micro-structure, which provides the probability density of nodules occurrence depending of their size (ii) an extreme value analysis using a Gumbel distribution and (iii) a micro-crack growth law associated with LCF conventional terms of energy densities. Its parameters are obtained using an optimisation process applied to laboratory fatigue experiment. The obtained probability function provides a good match for the lifetime and greatly improves results given by conventional criteria. It moreover provides a robust estimate of the lifetime scatter for different types of fatigue tests.

Keywords: High Temperature Low Cycle Fatigue, Thermomechanical fatigue,

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1 **1. Introduction**

2 Cast-iron has been a traditional engineering material in the past centuries. Nev-
3 ertheless, its castability, low price and mechanical resistance induces that it is still
4 an actual materials of choice for parts exposed to high temperatures such as auto-
5 motive exhaust manifolds or turbochargers. Nodular and compacted cast-irons are
6 indeed still a strong material option despite recent technology innovations, such as
7 cylinder heads with integrated exhaust manifolds [1]. Such structures operate at
8 high temperatures where the material behaviour becomes strongly non-linear and
9 exhibits viscoplastic effects. Moreover, exposed to large thermal gradients as well
10 as severe mechnaical boundary conditions, they often experience high tempera-
11 ture LCF or even thermo-mechanical fatigue (TMF), that lead to the appearance of
12 cracks during operation [2, 3, 4].

13
14 The state-of-art in design for these structures evolved in the last decades from
15 prototype testing to virtual design performed using mechanical analysis software
16 as proposed in a pioneering paper [5] for an automotive application. In order to
17 keep the computational costs under control, they proposed as a main assumption to
18 decouple material non-linear behaviour and damage provided the constitute model
19 to be precisely identified. The structure lifetime is then associated with the initi-
20 ation of a so-called macroscopic crack (of the order of a millimetre). The fatigue
21 criterion then defines a number of cycles which is related to a local dissipated en-
22 ergy density as proposed initially by [6, 7].

23
24 The method has since successfully been applied on various structures by sev-
25 eral authors, see for example [8, 9, 10, 11, 12, 13]. The fatigue criterion parameters
26 are usually identified from conventional LCF tests [14] at different temperatures.
27 The initial criterion has also been adapted to account for various effects, like mean
28 stress or multi-axial loading [15, 16, 17]. The dissipated energy LCF criterion has
29 thus been found to be rather robust[18]. Moreover, under LCF conditions, several
30 groups [19, 20, 14] demonstrated that the criterion can be physically justified when
31 damage and lifetime are mainly controlled mainly the propagation of micro-cracks
32 at the micro-structure scale.

33
34 A major drawback of this method, however, is that it does not allow the es-
35 timation of fatigue behaviour randomness. Material fatigue failure has indeed a
36 random aspect that can be observed through the scatter of the lifetimes for a given

37 load and the occurrence of various crack initiation zones observed during testing.
38 The randomness of material properties, defects or even manufacturing variation
39 cause the uncertainty of damage variable-lifetime relationship and seriously affect
40 the dispersion of fatigue life. The previously discussed criterion has a determin-
41 istic formalism that thus prevent a reliable analysis of lifetime scatter, although it
42 is highlighted in various studies especially when LCF is investigated over different
43 temperatures [21, 22, 23, 24]. Moreover, recent probabilistic TMF design proto-
44 cols for automotive parts relying on stress-strength interference analysis absolutely
45 need a reliable estimation of this scatter [25]. Randomness is however usually in-
46 troduced in the design process by a conservative strategy which induces most often
47 arbitrary safety factors.

48
49 The aim of this paper is to overcome this fundamental drawback by enriching
50 the criterion with a micro-mechanical analysis covering the random character of
51 the damage initiation sites and keeping the computational complexity and costs of
52 the design method at the same level as before. Several attempts have already been
53 proposed in the literature in this direction. For example, Doudard and Calloch [26]
54 included a Weibull distribution for the defects occurrence in order to explain the
55 evolution of self-heating measurements for high-cycle fatigue (HCF). In the LCF
56 domain, Maurel et al. [27] proposed a refined engineering fatigue model based on
57 the dissipated energy and micro-crack growth law based on microstructural char-
58 acteristics for stainless steels. A further step was proposed in Tabibian et al.[23]
59 where pores in a lost-foam cast aluminium alloy are considered a initiation site for
60 LCF micro-crack propagation. A statistical analysis of pores sizes is then com-
61 bined with a micro-crack basic growth law and a standard LCF criterion in order
62 to successfully predict the experimental scatter.

63
64 These previous attempts are then used as a basis to build a LCF fatigue criterion
65 which explicitly incorporates the statistics of micro-structural defects responsible
66 for the damage initiation. The material under scrutiny is a cast-iron for which a
67 fatigue criterion is sought over a wide temperature range (300 to 600 °C). Such
68 foundry alloys usual incorporate a large range of micro-structural heterogeneities
69 which can evolve into damage initiation sites. In the particular case of cast-irons,
70 carbides and graphite may appear in lamellar, compacted or spheroidal form and
71 coexist with the iron matrix. Casting of complex parts also generate differential
72 shrinkage and conduct to the formation of cavities or pores as documented in
73 [28, 29]. The different phases and/or pores are randomly distributed in the material
74 and often exhibit complex mythologies characterised by statistical distributions of
75 size and shape. The study of their role in fatigue damage is commonly highlighted
76 for high cycle fatigue[30] especially for aluminium alloys in [31, 32, 33, 34] and

77 for cast-iron in [35, 36, 37, 38, 39, 40].

78

79 In nodular cast-iron, "Void ratcheting" based on extended plasticity was iden-
80 tified as a potential mechanism leading to failure during experimental observations
81 [41] and numerical investigations [42]. In a theoretical problem setting, Mbiakop
82 et al. [43] showed, using finite element computations, that cyclic loading of pores
83 steer the local plastic field and the pores evolution shape indicating the onset of
84 cracks. Moreover, the study highlighted the importance of the local stress triaxial-
85 ity and that kinematic hardening decreases the rate of the evolution when compared
86 with isotropic hardening. In the other hand, the appearance of pores at the poles of
87 graphite nodules has been observed in the literature [44] and it seems a reasonable
88 assumption to consider them as potential initiation zones for fatigue micro-cracks.
89 As a consequence, it is therefore relevant to evaluate their role in the case of high
90 temperature Low-Cycle Fatigue. Let us further mention that under these loading
91 regimes, the material exhibits generalised viscoplasticity and that strong differ-
92 ences in material properties between the nodules and the matrix naturally induce
93 local strains and stresses concentrations, which may be accelerating factors in the
94 damage process.

95

96 This article starts with a the presentation of the considered nodular cast-iron
97 and the associated LCF tests and post-mortem damage analysis, which allows the
98 hypothesis of considering graphite nodules as initial defects. A quantitative anal-
99 ysis of the micro-structure is then performed and the morphology of graphite nod-
100 ules is examined by optical microscopy and digital post-processing. The observed
101 data, size and shape characteristics, is then represented using probability distribu-
102 tions. Hypotheses are then formulated regarding the evolution of fatigue damage
103 in the material. They are supported by the post-mortem tests analysis and by a
104 qualitative numerical model that allow them to be refined and to finally propose,
105 using the statistical theory of extreme values, the distribution of the largest nod-
106 ules inside a representative elementary volume. Under cyclic loading, nodules are
107 indeed considered as the origin of damage that propagates through micro-cracks
108 until failure. A fatigue model is then proposed: it is based on this distribution
109 and includes both micro-initiation and micro-propagation. As such, the path will
110 complete the idea proposed in [5], with the missing micro-structural information
111 and modelling while integrating ideas previously proposed by Seifert et al. [45].
112 As a consequence it offers both the standard deterministic estimate of lifetime and
113 an estimate of lifetime scatter. The parameters of the model are identified from a
114 series of LCF experiments using a complete optimisation process. The final discus-
115 sion exhibits their physical signification and examines the accuracy of the model to
116 describe lifetimes of structures under thermo-mechanical loading by applying the

117 model on an experimental database outside the identification sample.

118 2. Experimental Database

119 2.1. Material and LCF tests

120 SiMo nodular cast-iron is widely used by the automotive industry, especially
121 for exhaust systems as silicon and molybdenum enhance high temperature perfor-
122 mances. The specific material used for this study present a ferritic structure up to
123 850 °C. Its chemical composition is given in Table 1.

124

C	Si	Mo	Mn	S	P	Ni	Cr	Cu
3.2	3.92	0.58	0.13	0.01	0.01	0.062	0.03	0.016

Table 1: Chemical composition of SiMo cast-iron (% in weight)

125 A quick analysis of the alloy micro-structure reveals the presence of a ferritico-
126 pearlitic matrix but with a ferrite content between 85 to 90% depending on the ob-
127 served areas. As seen on Figure 1, The micro-structure is also characterised by the
128 presence of graphite nodules with a volume fraction of about 9%.

129

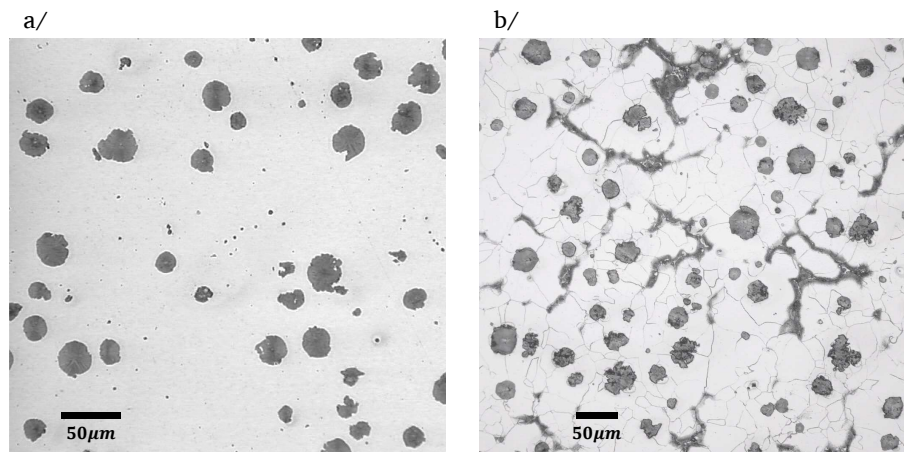


Figure 1: Microstructure of SiMo cast-iron without (a/) and with (b/) chemical etching

130 The study first focuses here on the Low Cycle Fatigue (LCF) of this cast-iron
131 between 300 and 600°C. These temperatures present indeed a good compromise

132 in terms of material behaviour with an equivalent influence of classical plasticity
 133 (mainly kinematic hardening) and viscosity as shown by Szmytka et al. [46]. With
 134 conventional LCF lifetimes [5], the damage produced by the cyclic viscoplasticity
 135 can then be analysed without risks of high interference with oxidation or effects
 136 induced by the temperature history, typical in the case of TMF [47].

137

138 A complete experimental database is carried out for this alloy on cylindrical
 139 specimens with micro-structures representative of industrial parts (exhaust mani-
 140 folds and turbochargers). This database is constituted of the tests performed by
 141 Constantinescu et al[5] at 350 and 600°C and completed with new tests at 300
 142 and 500°C. For the latter, 120s-long hold times in tension or compression have
 143 been added in order to evaluate the influence of the mean stress on the lifetime.
 144 The total strain rate was $10^{-3} s^{-1}$ and the proportion between extreme strains was
 145 $\varepsilon_{max}/\varepsilon_{min} = -1$. Different strain amplitude $\Delta\varepsilon = \varepsilon_{max} - \varepsilon_{min}$ were tested and the
 146 experiments are summarised on Table 2. These tests constitute the identification
 147 database for the proposed criterion.

148

Temperature (°C)	Number of tests	Dwell phase	$\Delta\varepsilon(\%)$	Lifetime
300	3	none	0.6-0.8	280-1100
300	2	tension	0.6-0.8	1050-1090
300	3	compression	0.6-0.8	260-1650
350	6	none	0.7-2.0	12-610
500	3	none	0.5-0.8	490-2620
500	5	tension	0.5-0.8	490-3700
500	2	compression	0.6-0.8	410-720
600	9	none	0.5-2.0	70-8190

Table 2: Low Cycle Fatigue tests condition summary

149 Observed lifetimes range between 100 and 8190 cycles and are therefore char-
 150 acteristic of LCF conditions. A single test has a very short lifetime that could be
 151 considered as outside the LCF range with 12 cycles. However, it was decided to
 152 keep this extreme point in the database, as the loading is still representative of the
 153 structures to be designed and provides useful information on the dispersion of the
 154 fatigue response.

155 2.2. Fracture surface analysis and damage mechanisms

156 Some specimens from the experimental basis were analysed to post-mortem
 157 quantify the influence of the micro-structure on the lifetime and on fatigue damage

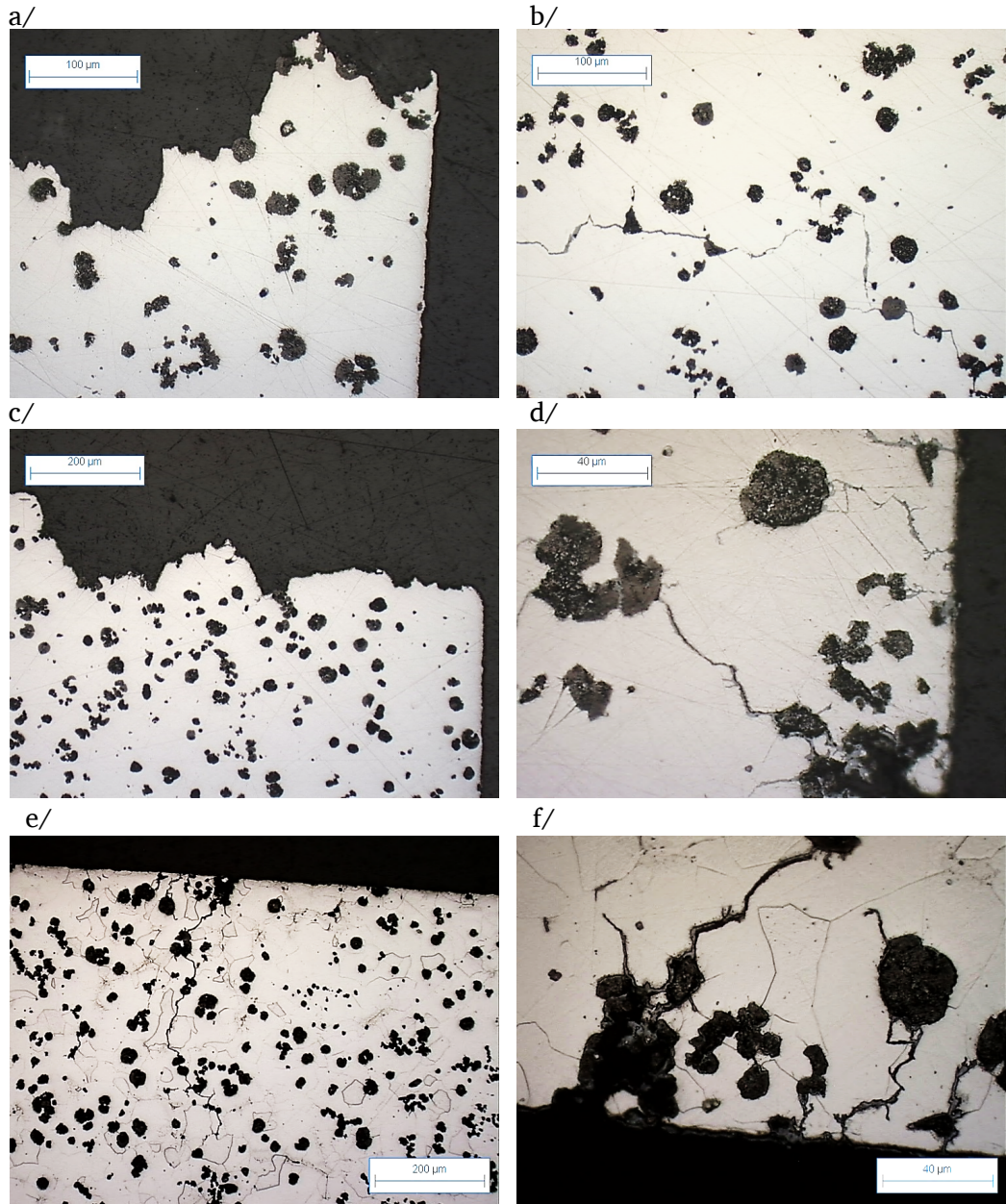


Figure 2: Fractography of LCF tested specimen at 300°C (a/ and b/) and at 500°C without (c/ and d/) and with (e/ and f/) chemical etching

158 mechanisms for this alloy. At 300°C, fatigue propagation area is fairly limited on
159 the specimen, the crack propagation is indeed brittle for 95% of the gauge sec-
160 tion. As shown in Figure 2.a and .b, the propagation occurs in the matrix while
161 the crack path seems highly oriented by graphite nodules distribution. At 500 °C,
162 fatigue propagation area is in the contrary fairly large (about 75% of the gauge sec-
163 tion) but the same conclusions could be drawn. Fracture surface are indeed poorly
164 oxidised and many secondary cracks are observed around the main crack, which
165 justifies neglecting oxidation interference with damage mechanisms.

166

167 The observations reveal the important role played by the graphite nodules in
168 the crack micro-propagation inside the ferritic matrix. Moreover, crack paths are
169 enhanced and concentrated around the larger modules, as reported by several anal-
170 yses in the literature [48, 41], The application of chemical etching on the observed
171 surfaces unveil a trans-granular crack propagation. Additionally, let us remark that
172 no pores or other casting defects have been observed on the studied surfaces. One
173 can therefore assume that the fatigue crack initiation occurs in the vicinity of the
174 graphite nodules and their size distribution is of prime importance in the micro-
175 crack propagation, defining as such the main factors affecting the fatigue damage
176 of this particular cast-iron.

177 *2.3. Thermal Fatigue test : validation database*

178 Thermal fatigue tests are also performed on a peculiar specimen by Constan-
179 tinescu et al. [5] to validate their model. The experimental set-up described in
180 their article is designed to induce very severe thermomechanical loads. In this ex-
181 perimental protocol, axial symmetric clamped specimens are heated by the Joule
182 effect. The heating procedure produces a thermal gradient along the specimen axis,
183 which also induces a variation of the mechanical fields in the same direction. This
184 experimental set-up is presented on Figure 3.

185

186 The set-up, the imposed thermal loads and the adjustable rig stiffness that gen-
187 erates thermomechanical strains and stresses in the specimen are detailed in [5].
188 Eight identical tests were performed on the same nodular cast-iron between 40 and
189 700 °C with a heating rate of 20 °C.s⁻¹. The maximum temperature is obtained
190 in a region of approximately 10mm in the centre of the specimen and with hold-
191 ing times between 30 and 900s at maximum temperature. The observed lifetimes
192 are respectively 270, 503, 511, 551, 969, 1194, 1256 and 1388 cycles, underlying
193 an important scatter in the fatigue behaviour. These tests will be our validation
194 sample.

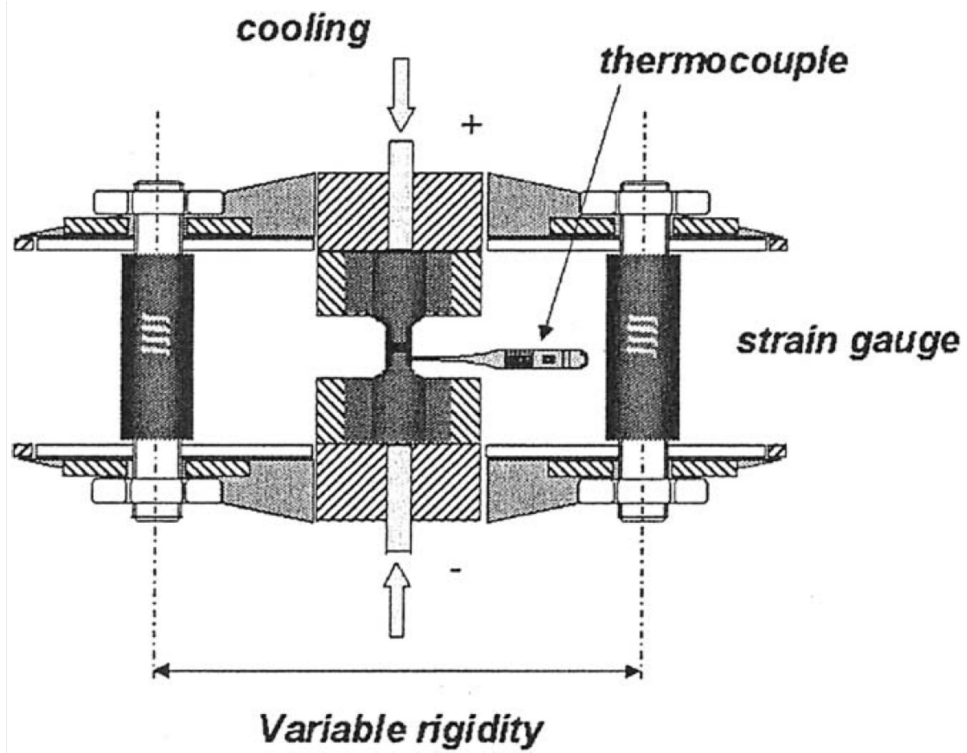


Figure 3: Thermal fatigue set-up from [5]

195 **3. Quantitative analysis of micro-structure**

196 *3.1. Nodule population analysis and distribution modelling*

197 Several samples from both fatigue specimens but also exhaust manifolds or
 198 turbochargers were analysed by optical microscopy. The obtained images were
 199 processed using Fiji software following a precise procedure for counting the num-
 200 ber and measuring the size and shape of the graphite nodules, inspired by the work
 201 of [23] and [49]. 35 images of identical size were investigated for a studied surface
 202 of about 110mm^2 . The studied regions correspond to those which appear as criti-
 203 cal regarding TMF (examples of exhaust structures and critical zones localisation
 204 could be found in [1]). All analyses were performed in two dimensions, for sake
 205 of simplicity for industrial parts and as it remains relevant in the global approach
 206 of a criterion based on micro-propagation.

Nodules Measure	$\tilde{\sigma}_1$	$\tilde{\sigma}_2$	$\tilde{\mu}_1$	$\tilde{\mu}_2$	Mean size - μm (for population 1)	Mean size - μm (for population 2)
Feret diameter	0.473	0.390	1.559	3.040	4.7	20.9
$\sqrt{\text{area}}$	0.444	0.402	1.229	2.757	3.4	15.8

Table 3: Statistical parameters for nodules size.

207 Statistical analysis of the size of graphite nodules was performed from the mea-
 208 surements in term of their Feret diameter and the square root of their surface, de-
 209 noted next as a and $\sqrt{\text{area}}$ respectively. Figure 4.a and .b show the histograms
 210 of the nodules population distributed to the classes of the two types of measures.
 211 14059 nodules were detected with Feret diameter, a , lying between 1.4 and $69\ \mu\text{m}$
 212 and the square root of their surface $\sqrt{\text{area}}$ varying between 0.83 and $53.8\ \mu\text{m}$.

213
 214 For both dimensional measures, a and $\sqrt{\text{area}}$, one can recognise two popu-
 215 lations of nodules: the first one consists in small nodules covering a majority of
 216 the total population while the second one is constituted of medium or large size
 217 nodules covering a minority of the total population. Similar observations have also
 218 been reported in [49] and accordingly we propose to represent the population by
 219 double log-normal distributions expressed as:

$$p_s = \frac{1}{s \sqrt{2\pi\tilde{\sigma}_1\tilde{\sigma}_2}} \left[\tilde{\sigma}_2 \exp\left(-\frac{1}{2} \left(\frac{\ln(s) - \tilde{\mu}_1}{\tilde{\sigma}_1}\right)^2\right) + \tilde{\sigma}_1 \exp\left(-\frac{1}{2} \left(\frac{\ln(s) - \tilde{\mu}_2}{\tilde{\sigma}_2}\right)^2\right) \right] \quad (1)$$

220 where s is the nodule size. $\tilde{\sigma}_{1,2}$ and $\tilde{\mu}_{1,2}$ denote the mean value and the standard
 221 deviation of each log-normal distribution. The optimal parameters identified from

222 the measured data are displayed in Table 3 and the comparison of the measured and
223 estimated histograms are displayed in Figure 4.c and .d. One can further remark an
224 excellent match between the measurement and the model.

225

226 The average sizes of the two populations of nodules are relatively close for both
227 measures as the Feret diameters and the \sqrt{area} with 20.9 and 4.7 μm and 15.8 and
228 3.4 μm respectively. There are small differences in the histograms, especially for
229 the population of small nodules which are more numerous and less dispersed for
230 their geometric size. Next, we will only consider the Feret diameter distribution for
231 the construction of the fatigue criterion which seems perfectly suited to describe
232 nodules size even if the square root of the surface \sqrt{area} is a recognised measure
233 of the default size of particles (in terms of results for the identification of fatigue
234 criteria and for all the conclusions of following sections, this choice has anyway
235 very little influence).

236 3.2. Shape of nodules

237 The particular shape of the nodules conducts to stresses and strains concentra-
238 tions due to the contrast of mechanical and thermal expansion material parameters
239 between the nodules and the ferritic matrix. Nodules can therefore act as onset of
240 damage zone for the considered material. It is well known from classical elasticity
241 theory that the concentration will be inverse proportional with the curvature of the
242 shape. However it is straightforward that the exact 3D shape of the nodules cannot
243 be recovered without tomography measurements that conducts to explosive costs
244 when applied to large population on industrial parts. Therefore, a single scalar, the
245 circularity c , is further assumed as the appropriate and statistically representative
246 parameter for representing the shapes of the population of nodules. Circularity c is
247 defined for a particle with an area of A and a perimeter p as :

$$c = 4\pi \frac{A}{p^2} \quad (2)$$

248 It reaches 0 for infinitely elongated polygon and 1 for perfect circles. Figure
249 5.a displays the distribution of nodules according to their Feret diameter and their
250 circularity. A quick analysis shows that the population of smallest nodules presents
251 a circularity essentially close to 1. This is undoubtedly due to the resolution of the
252 images used for the nodules statistical analysis: these very small elements can
253 be assimilated to areas mainly concentrated in squares of 2 pixels on each side.
254 The circularity of the small elements is therefore not necessarily relevant here.
255 However, as will be specified in the following sections, it is essentially the larger
256 nodules that are of interest in the formalism we develop for fatigue. On the other
257 hand, the largest nodules have a lower circularity, between 0.75 and 0.9. Their

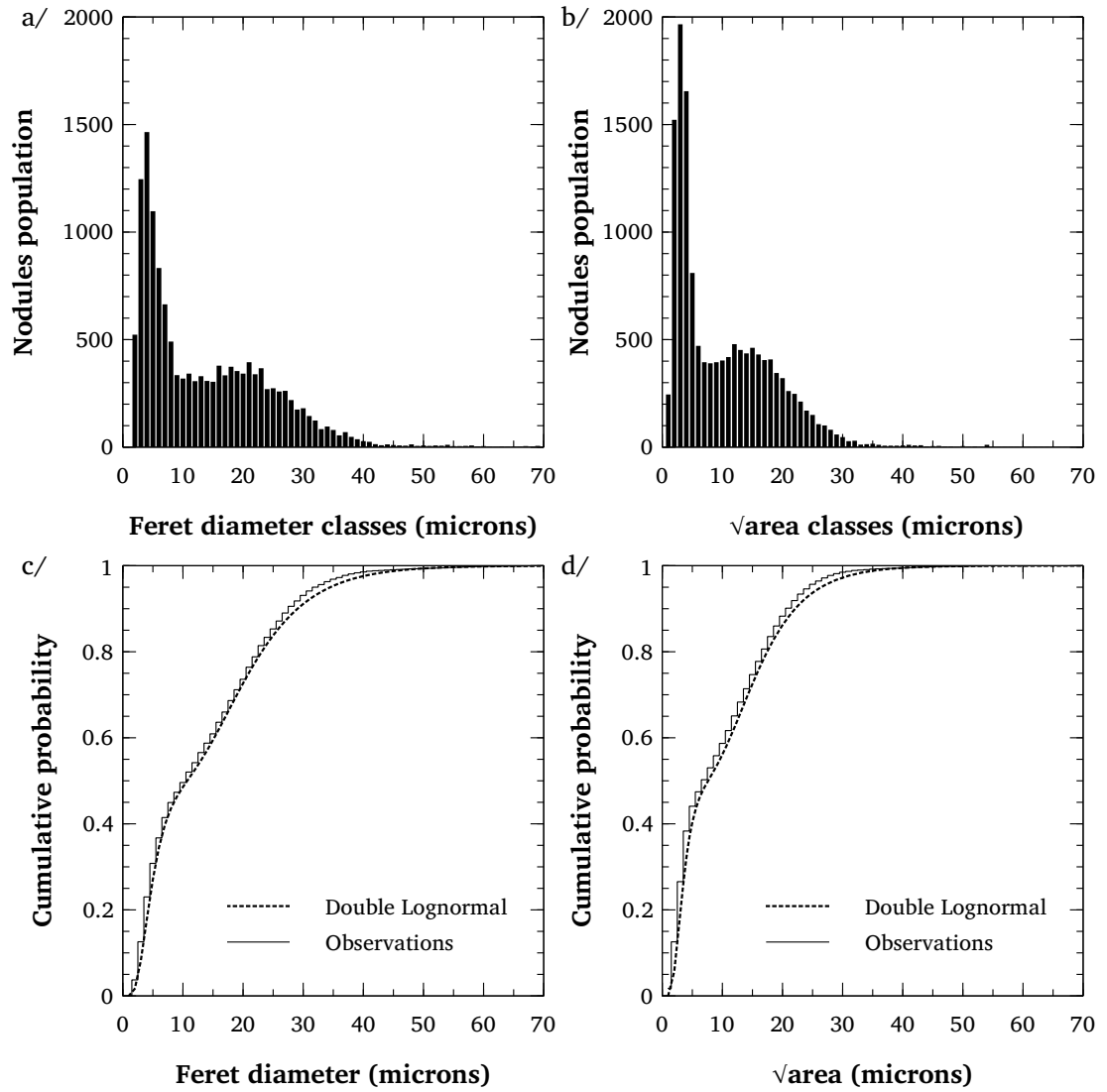
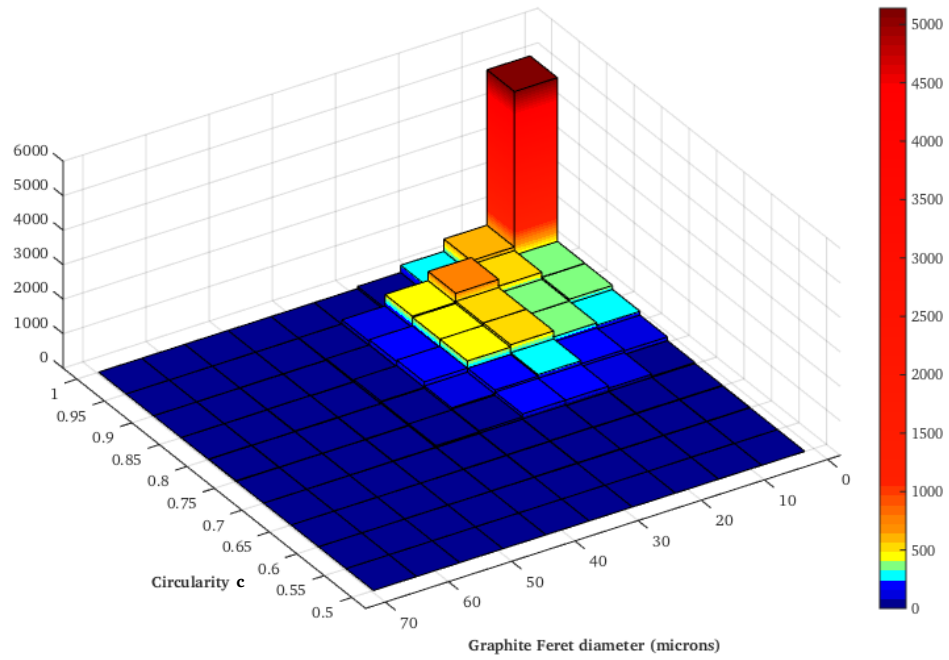
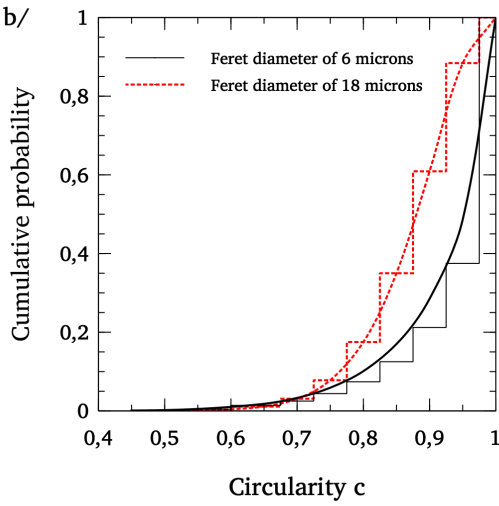


Figure 4: Histogram of nodules populations based on their Feret diameter (a/) and their $\sqrt{\text{area}}$ (b/). Nodule size probability estimated by a double Log-normal distribution for Feret diameter (c/) and $\sqrt{\text{area}}$ (b/)

a/



b/



c/

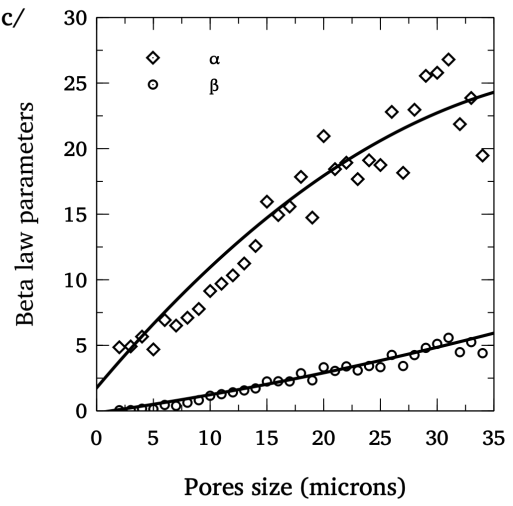


Figure 5: Graphite nodule : observed circularity distribution versus nodule size (a/) Beta distribution for different size (b/) and Beta law parameters (c/)

258 morphology, visualised here in 2D, presents a segmented border with punctually
259 zones with an important curvature. This can be explained by their rapid growth
260 during the alloy solidification and the loss of stability of the material interfaces.
261 Moreover, the nodule population with a circularity smaller than 0.75 is extremely
262 reduced. As a consequence, in each class of nodule size, the statistical distribution
263 of the circularity of the nodules can easily be represented by a beta law, expressed
264 as :

$$p_c = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} c^{\alpha-1} (1 - c)^{\beta-1} \quad (3)$$

265 with c , the circularity. α and β are the parameters of the distribution. The evo-
266 lution of these two parameters with the nodule size in microns are presented on
267 Figure 5.c. Their evolution can be represented by simple models, as for example
268 the second-order polynomial estimation used here. A good match of the circularity
269 distribution is displayed on Figure 5.b for 2 classes of nodule size.

270
271 With the size and circularity of nodules now fully available and formalised
272 using statistical distributions, the role of nodules in the damage process and the
273 integration of these distributions in a fatigue criterion can now be discussed.
274

275 4. From defects to lifetime

276 4.1. Damage initiation and evolution theory

277 From the engineering point of view, TMF or high-temperature LCF failure of a
278 part, at the macro-scale, is usually related to the apparition of an observable crack,
279 the length of which sufficient to break a material representative volume element
280 (RVE). This failure is the resulting action of the evolution of irreversible processes
281 that occur at the material micro-scale. For nodular cast-irons, macroscopic failure
282 can be linked to the progressive damage of each of its constituent phases as under-
283 lined in [28, 52]. From a meso-scale point of view, damage can occur as spheroid
284 rupture, ductile damage in the ferritic matrix or spheroid/matrix debonding. These
285 micro-mechanisms of failure are controlled by the macroscopic load imposed to a
286 RVE [53].

287
288 From an experimental point of view and in accordance with what was pre-
289 sented in the section detailing the material and the LCF tests, the important role
290 of graphite nodules in the damage cannot indeed be put aside. Several studies in
291 the literature in the context of low cycle fatigue at room temperature have high-
292 lighted this role and observed the classic damage mechanisms of cast iron in the

293 case where plasticity is generalised in the material. Thus, Harada et al.[54] then
294 Komotori et al.[55] have pointed out, on various loading regimes, that cracks oc-
295 ccurring for ferritic cast-irons result from the coalescence of micro-voids originated
296 from debonding of the matrix/graphite interface. Later, Bubenko et al. [56] showed
297 that the low-cycle fatigue cracks always start at the interface between the graphite
298 nodule and surrounding ferrite matrix, while graphite nodules remain generally un-
299 broken. Canzar et al.[57] proved that size, shape and distribution of the graphite
300 nodules has no significant influence on cyclic hardening of the material but that
301 they play a major role in the crack initiation and propagation process. They showed
302 that the larger irregularly shaped nodules tend to reduce fatigue strength. All these
303 observations therefore suggest that the damage mechanism corresponds essentially
304 to the location of plastic stresses and strains around the largest nodules which, after
305 decohesion of the matrix/nodule interface, act as pores where micro-cracks initiate.

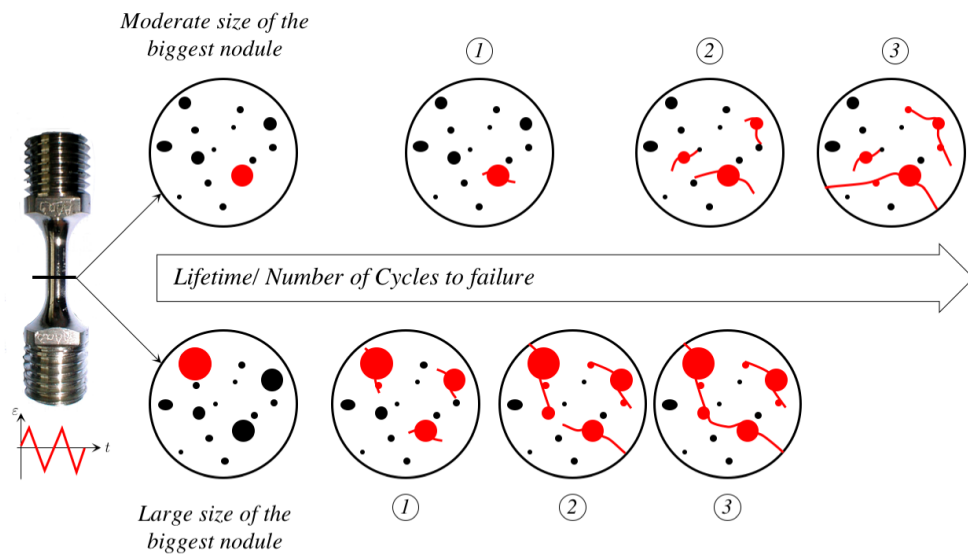
306

307 From a numerical point of view, nodular cast irons have given rise in the litera-
308 ture to numerous studies on plasticity and damage at the microstructural level, and
309 in particular on the behaviour of graphite nodules in a ductile ferritic matrix. Many
310 authors have thus carried out simulations of representative elementary volumes in
311 order to better understand the interactions between the matrix and the nodules and
312 to develop damage models based on the nucleation, growth and coalescence of cav-
313 ities in a solid fraction.[58, 59, 60, 61, 62, 63]. In the simulations, considering easy
314 decohesion of graphite nodule and ductile matrix, the graphite nodule is modelled
315 either as a void or as a rigid particle in the cell model [64]. The cyclic elastoplastic
316 computation of a representative element volume with a spherical pore in its center
317 discussed in [67] is thus an interesting starting point. Authors show that the
318 dissipative mechanism are maximal at the interface of the matrix with the pore.
319 Moreover the shape of the zone encompassing the maximal dissipation, which is
320 at the origin of damage are located in most of the cases around the equator of the
321 pore and provide an ideal initiation for the fatigue crack propagation. The fact
322 that the results in [67] have similar shapes for isotropic or kinematic hardening in-
323 dicates that the results obtained with the particularly chosen viscoplastic material
324 behaviour can be generalised to other configurations.

325

326 Considering all these observation, a high-temperature LCF damage initiation
327 and propagation process can be proposed. The mechanical behaviour of spheroidal
328 graphite cast irons is indeed clearly influenced by their generally heterogeneous mi-
329 crostructure (composed at first order of a ductile ferritic matrix and fragile graphite
330 spheroids). These latter naturally induce stress concentrations and quickly lead to
331 debonding and pore generation. It then seems realistic to consider that they will
332 be the sites of early LCF or TMF damage. The stress concentration is a function

333 of the size and the shape of the considered nodules ; the most significant damage
 334 shall thus appear first on the largest and least spherical particles. In the rest of this
 335 article, we will decide not to take into account either the influence of neighbouring
 336 nodules on the mechanical fields around a considered nodule, or the existence of
 337 possible aggregates of particles that could be the consequence of the solidification
 338 process and generate an increase in local mechanical fields. These two elements
 339 can legitimately play a role in the damage mechanisms and may be investigated
 340 later.



1. Micro-crack initiation 2. Micro-crack propagation 3. Coalescence and failure

Figure 6: Simple hypothesis of damage evolution as a function of the maximum nodule size within a volume element loaded by an identical load

341 Based on the preceding results, we can make the simplifying assumption that
 342 within a representative elementary volume containing a large family of graphite
 343 nodules in cast-iron, the damage will then localise around the largest graphite
 344 nodule with the lowest circularity. The localised dissipation mechanisms will initiate
 345 a micro-crack which will propagate from this nodule as illustrated on Figure 6.
 346 The crack initiation will relax the local load and with increasing number of cycles
 347 further micro-cracks will appear on smaller nodules. With further cycles micro-
 348 cracks will start to propagate until coalescence inducing the complete failure of the

349 volume element under consideration. The failure of the volume element represent
350 the initiation of a macroscopic crack and the onset of the structural failure.

351

352 We shall further infer that propagation is mainly driven by the material be-
353 haviour of the ferritic matrix. Comparable phenomena have been observed on
354 aluminium alloys under similar loading conditions [66]. One can conclude that
355 the lifetime of a representative volume element, i.e. number of cycle to the initia-
356 tion of a macroscopic crack is dependent on its the largest graphite nodule and the
357 properties of the matrix.

358 4.2. Extreme value statistics

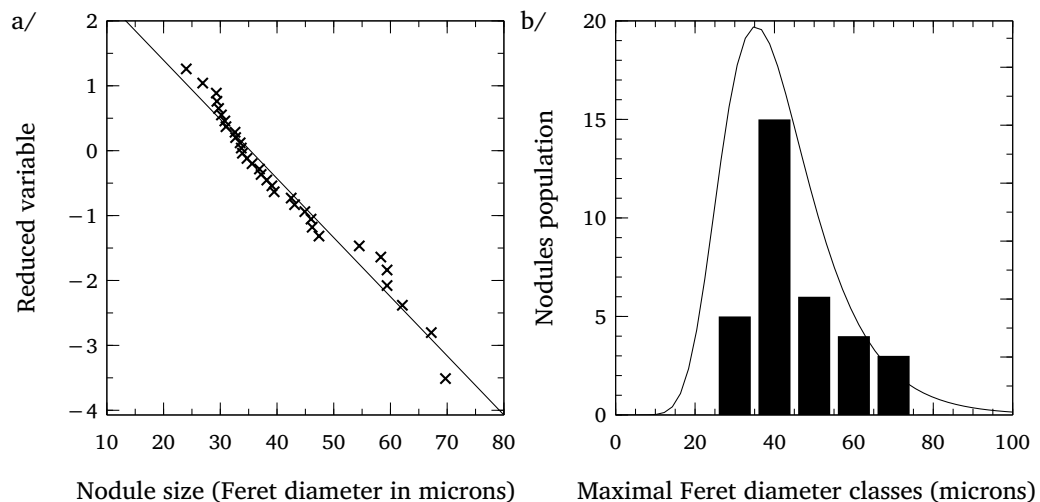


Figure 7: a/ Gumbel probability plot ; b/ Gumbel identified distribution for given classes of maximal nodule size for each sample

359 Once the damage mechanism are assumed, a discussion has to be conducted on
360 the type of distribution to be used to represent graphite nodules in a fatigue context.
361 Tabibian et al.[23] introduce a simple distribution of pore size into a micro-crack
362 propagation law for an aluminium alloy. This choice is simple and can be improved
363 as it is strongly related to the size of the zone used to identify the distribution with-
364 out taking into account of distribution tail estimation..

365

366 For high-cycle fatigue, the statistical theory of extreme values is very often
367 used to represent the distribution of the largest particles or defects observed on

368 a plane section of fixed area [30]. In our case, considering the total distribu-
 369 tion of nodules would be possible but certainly too simplistic. Indeed, the over-
 370 representation of small nodules would undoubtedly favour the calculation of very
 371 long lifetime, which is far to be conservative. Here, the statistical measurements of
 372 nodules size and circularity have first been carried out on surfaces with a constant
 373 area chosen to be representative both of critical areas observed on the parts to be
 374 designed and to specimens failure area (size equivalent of circle with 1mm radius).
 375 A sample of observations of maximum sizes of nodules for each investigated op-
 376 tical microscopy picture can then be obtained and serve as a basis for an extreme
 377 values distribution. As explained by Beretta et al.[65], extreme value theory allows
 378 us to know the asymptotic behaviour of the maximum taken by the values of identi-
 379 cally distributed and independent random variables. This law includes parameters
 380 that can be estimated on the basis of the extreme values taken in blocks of fixed
 381 size for the available data, which is the case here as detailed in section 3.1. Here,
 382 the extreme value distribution arises as the possible limit distributions for the max-
 383 imum value of nodule size considering a given area. For the 35 pictures analysed,
 384 we obtained the following sample

Picture	Max. size (μm)	Picture	Max. size (μm)	Picture	Max. size (μm)
1	37.2	13	44.9	25	54.5
2	30.2	14	47.4	26	39.5
3	33.9	15	38.2	27	46.0
4	42.5	16	43.1	28	30.8
5	58.3	17	35.6	29	59.4
6	69.7	18	33.7	30	67.2
7	69.8	19	39.1	31	29.7
8	69.5	20	29.3	32	59.4
9	46.2	21	29.4	33	62.1
10	32.6	22	34.7	34	37.6
11	32.7	23	26.9	35	31.2
12	31	24	24.0		

Table 4: Maximal nodule size value per investigated image.

385 The Gumbel plot assess the fit of data sample to the Gumbel distribution. It
 386 implies first that the maximum values to be arranged in increasing order. It induces
 387 a so-called reduced variable which is equal to $\ln(\ln(i/n + 1))$ with n the sample
 388 size and i , the i^{th} smallest maximum value. This variable is plotted versus the
 389 corresponding value of nodule maximal size. A straight line indicates a good fit to

390 the distribution which is here the case as seen on Figure 7.a. A Gumbel distribution
 391 is then identified for the sample and plotted on Figure 7.b. The probability that the
 392 biggest nodules on a critical zone has a size equal to a is expressed as

$$G(a) = \exp\left(-\exp\left(-\frac{a-\lambda}{\delta}\right)\right) \quad (4)$$

393 with λ and δ , two parameters respectively identified to 35.28 and 10.97 . This
 394 distribution will then be taken as the probability of finding maximum nodules ei-
 395 ther in the fatigue specimens or in the part critical zone[65]. Here, circularity is
 396 not directly taken into account. It simply allows us to consider the largest nodules
 397 as the most critical. A finer integration by considering, for example, a combination
 398 of the largest nodules with the lowest circularity could be envisaged but it would
 399 require a heavy additional work of fine analysis of the role of the nodules morphol-
 400 ogy in the fatigue process.

402 *4.3. A probabilistic fatigue criterion*

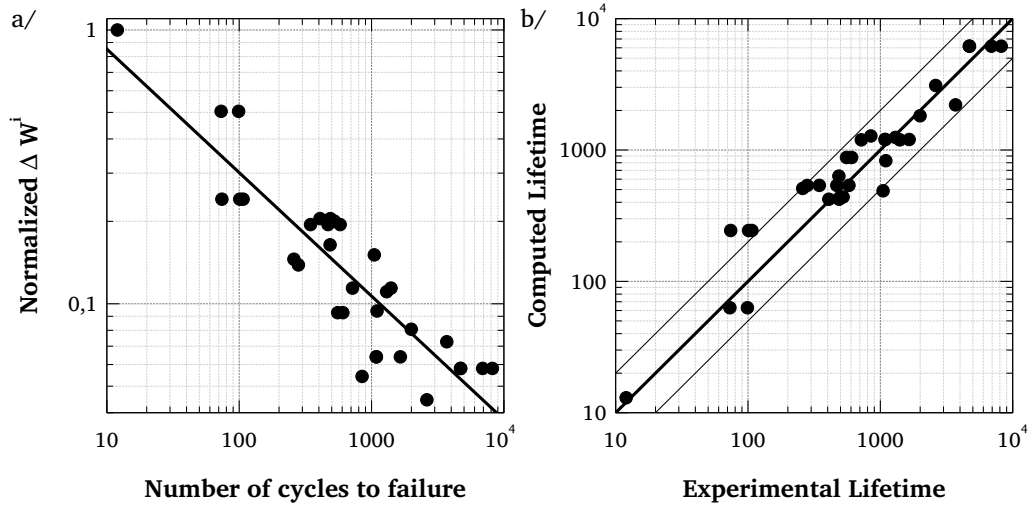


Figure 8: Deterministic fatigue criterion

403 The relation between viscoplastic dissipated energy over the stabilised cycle
 404 and the number of cycles to failure has proven to be a robust fatigue criterion
 405 starting with the first results by Skelton, the development including structural com-
 406 putational proposed in [5] and the evolution including mean stress in [16, 17]. If

407 the viscoplastic dissipated energy over the stabilised hysteresis loop is denoted W^p
 408 and the number of cycles to failure, N_f , the criterion takes the following form:

$$W^p N_f^b = \int_{\text{cycle}} \underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}} dt N_f^b = c \quad (5)$$

409 where b and c denote two material parameters identified using a database of
 410 isothermal LCF tests. The direct application of this fatigue criterion, proposed
 411 by [5] over the present cast-iron LCF database appears to be fairly robust as it
 412 revealed by the results in Figure 8. However, the scatter of the data characterised
 413 by the coefficient of determination R^2 equal to 0.80, stay important and cannot be
 414 precisely determined. As a partial conclusion, one can state that the energy density
 415 remains a reliable marker for fatigue damage, but can neither accurately estimate
 416 the variability of the fatigue response nor explain the role of the microstructure.

417
 418 As an outcome, we propose to combine the description of the microstructure
 419 variability proposed Charkaluk et al.[23] which will provide the mathematical form
 420 of the criterion with the micro-crack propagation law advanced in Maurel et al.[27]
 421 for ferritic stainless steels. As matter a fact, the failure probability p_f of an elemen-
 422 tary volume element can be described using the probability of finding a graphite
 423 nodule of maximal size a_0 within this RVE according to the formula:

$$p_f = \int_{a_0}^{a_f} \frac{da}{\left(\frac{aW^p}{\gamma_p}\right)^{m_p} + \left(\frac{aW^e}{\gamma_e}\right)^{m_e}} \quad (6)$$

$$W^e = \frac{1}{3} \int_{\text{cycle}} \text{tr}\left(\underline{\underline{\sigma}}\right) \text{tr}\left(d\underline{\underline{\varepsilon}}^e\right) \quad (7)$$

424 with $\underline{\underline{\varepsilon}}^e$, the elastic part of the strain tensor and $\gamma_p, \gamma_e, m_p, m_e$ are four parameters
 425 to be identified. a_0 is here a random variable representing the size of the largest
 426 nodule and taken as a initial defect size. a_f is the final length of micro-crack, prop-
 427 agated from the nodule. It is associated with a critical or unacceptable crack for the
 428 part or specimen under scrutiny. The size of a macroscopic crack on a laboratory
 429 fatigue test specimen that induces final failure is in mm size, while on a structure
 430 final failures are induced by cm or dm size crack. As already discussed in [23], it
 431 seems reasonable to choose a critical size $a_f = 2\text{mm}$.

432
 433 The final form of the criterion is however relatively simple. This is a direct
 434 consequence of a series of underlying hypothesis as well as the absence of possible
 435 interaction of the different micro-cracks, i.e. the one starting at large nodules with
 436 the ones starting at smaller ones. In order to be conservative, a shorter value of
 437 1mm is therefore used.

438 **5. Results and discussion**

439 **5.1. Criterion identification**

440 Let us first remark that the number of cycles to failure N_f are considered next
 441 as a random variable. Its probability distribution is directly linked with the previ-
 442 ously identified Gumbel distribution for maximal nodule size. Starting from this
 443 observation, one can identify the criterion fatigue parameters from the LCF exper-
 444 imental lifetime results, by matching the probability distribution for the number of
 445 cycles to failure and failure probability expressed in equation 6. The probability
 446 for the number of cycles to failure is directly dependant of the loading condition
 447 on a part or more easily of the load applied on a specimen. These conditions are
 448 represented by the couple (W^p, W^e) that can easily be computed from the moni-
 449 tored strains and stresses evolution. N_f is here considered to follow a log-normal
 450 distribution for the sake of simplicity:

$$p_{f,(W^p,W^e)}(N) = \frac{1}{2\pi N\sigma_{(W^p,W^e)}} \exp\left(-\frac{(\ln(N) - \mu_{(W^p,W^e)})^2}{2\sigma_{(W^p,W^e)}^2}\right) \quad (8)$$

451 The distribution parameters $\sigma_{(W^p,W^e)}$ and $\mu_{(W^p,W^e)}$ can be identified from the
 452 nodule size Gumbel distribution. A number of cycles to failure can indeed be com-
 453 puted for each class of nodule maximal size obtained from the Gumbel distribution
 454 (an example of classes is represented in Figure 7). For each class, the correspond-
 455 ing maximal value is introduced as a_0 in equation 6, introducing chosen values for
 456 γ_p, γ_e, m_p and m_e . This step leads, for a given loading condition (W^p, W^e) to a com-
 457 plete histogram for the number of cycles to failure and the log-normal parameters
 458 are then easily identified with the computation of the mean value and the standard
 459 deviation of the identified population.

460
 461 It is then easy to compare the mode of this distribution (depending on the 4
 462 parameters γ_p, γ_e, m_p and m_e of equation 6) with the actual result of the LCF test for
 463 which (W^p, W^e) were induced by the loading conditions on the specimen. A least-
 464 square cost function is then calculated and its minimisation allows us to obtain
 465 the optimal values for γ_p, γ_e, m_p and m_e . The results of this simple least-square
 466 optimisation are presented in Table 5.

Parameter	$\gamma_p(mJ.mm^{-2})$	$\gamma_e(mJ.mm^{-2})$	m_p	m_e
Optimized value	4.29	5.51	2.57	2.02

Table 5: Optimized criterion parameters

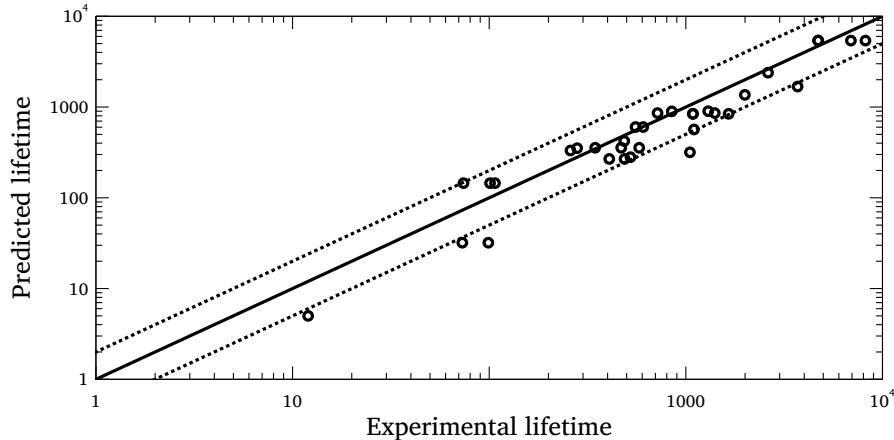


Figure 9: Experimental versus predicted lifetime with the proposed criterion

467 The comparison of these results with those obtained by Maurel et al.[27] on
 468 stainless steel only at 300°C is interesting. The LCF tests are indeed carried out
 469 under identical conditions and Maurel had chosen the grain size as the initial crack
 470 size for deterministic results. The parameter m_e has been set at 2 in the case of
 471 stainless steel, consistent with traditional databases, which is very close to the value
 472 we obtained through optimisation. This result, for two ferritic matrices, underlines
 473 the robustness of our identification process. Taking into account Maurel's formula-
 474 tion which introduces a correction related to grain size in mathematical formalism,
 475 we find very similar values also for γ_e . The contribution of the "elastic" part for
 476 two ferritic matrices therefore seems to be similar here. However, the viscoplastic
 477 part is much more pronounced in the context of the considered cast-iron: indeed,
 478 m_p is here higher (2.57 against 1.62), thus accentuating the viscoplastic contribu-
 479 tion to propagation, which also sees its attenuating factor γ_p decrease by a factor of
 480 10 (once again by taking into consideration the grain size correction from Maurel
 481 and his co-authors). These differences can be partly explained by the high vis-
 482 coplasticity of the considered cast-iron for high temperatures, much higher than
 483 that of stainless steel considered by Maurel and his co-authors at 300°C as pre-
 484 viously shown in a comparative study[47]. This high viscoplastic activity greatly
 485 reduces the observed stress levels and ultimately the W^p ones. To obtain a suffi-
 486 cient and significant driving force for the damage, the adjustment of the parameters
 487 of the propagation law appears logical. For a given pair (W^p , W^e), the distribution
 488 mode is furthermore given by:

$$N_{computed} = \exp\left(\mu_{(W^i, W^e)} - \sigma_{(W^i, W^e)}^2\right) \quad (9)$$

489 This value is compared to the experimental one on Figure 9 which underlines a
 490 very good matching between computed and experimental lifetime. The correlation
 491 coefficient R^2 is here equal to 0.92, reaching a higher value than the deterministic
 492 criterion detailed previously. The proposed method supplies therefore a lifetime
 493 estimation at least as effective as the deterministic criterion. However, additional
 494 information in the probability density functions of failure are an important im-
 495 provement as it is possible to now estimate the scattering of the fatigue strength for
 496 the material.

497 5.2. Criterion validation

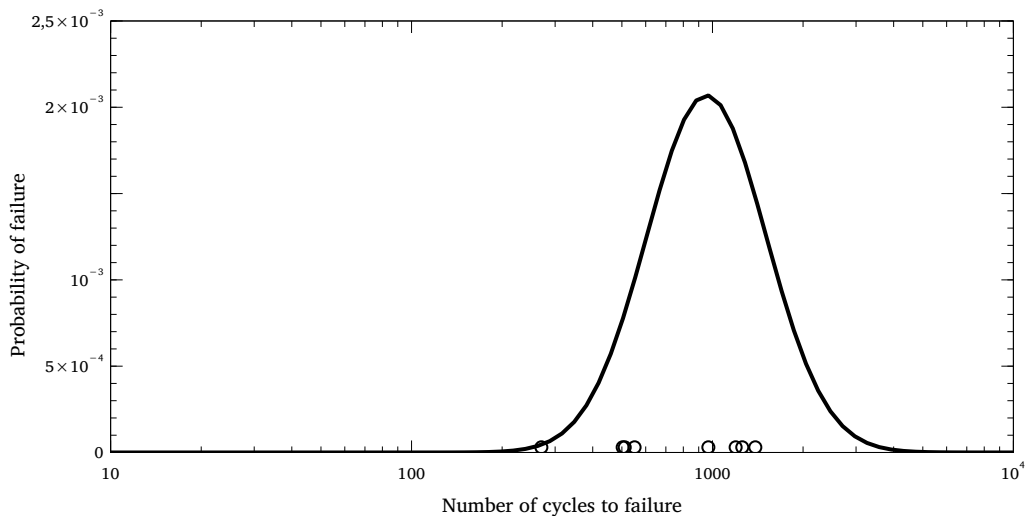


Figure 10: Probability of failure distribution for thermal-mechanical structural tests[5]. Circles represent experimental lifetimes.

498 The thermal fatigue tests presented in section 2.3 is here used as a validation
 499 sample for the proposed criterion. They are simulated in Abaqus using an elasto-
 500 viscoplastic behaviour detailed in [68]. The clamp value due to the variable stiff-
 501 ness (proportional to the flexural moment of the beams) is here fixed at 183 000
 502 N/mm and the dwell time at 700 °C at 60s. The thermal load of this experiment was
 503 numerically estimated by an electric and a thermal FEM computation on Abaqus.

504 The results of the thermal-electrical computation is then introduced in a mechanical
505 computation. The FE model is here constituted of 43934 nodes and 28805 C3D10
506 10-node quadratic tetrahedral elements. The numerical protocol that enables to de-
507 tect the most critical finite element for thermomechanical fatigue are explained in
508 [5]. For these tests, the obtained values of W^p and W^e for the most critical element
509 are respectively 1.44 and $0.44mJ.mm^{-3}$ and allow the identification of a number of
510 cycles to failure distribution using the parameter set identified on the LCF database
511 as shown on Figure 10 where the experimental lifetimes are figured by circles. The
512 Figure represents the probability density for the number of cycles to failure and
513 the considered load. An fair good representation of the variability of the fatigue
514 response is highlighted with most of the tests lifetimes in the high probability zone
515 and a very relevant estimate of the average lifetime (about 1150 cycles versus 850
516 experimentally). Thus, despite the limited temperature range for model identifica-
517 tion, satisfactory initial results in thermomechanical fatigue can be obtained, which
518 constitutes a first validation of the criterion.

519 6. Conclusions

520 This paper proposes a method for estimating the probability density distribu-
521 tions of lifetime in high temperatures LCF and by extension in TMF for nodu-
522 lar cast irons. It is based on the estimation of the probability density of nodule
523 sizes using optical microscope image analysis and identifying a double log-normal
524 distribution. After analysis of the fatigue test results supplemented by literature
525 analyses, nodules are identified as the preferred initiation zone. A probability den-
526 sity function over the lifetime is then proposed based on an extreme value Gumbel
527 distribution for nodule maximal size and a micro-crack propagation law integrat-
528 ing energy densities identified on the loading cycle. The four parameters of the
529 proposed criteria are optimised to have the most probable estimated lifetimes that
530 correspond to the experimental observations. The proposed criterion provides pre-
531 dictive results for the studied cast iron, while the lifetime probability is explic-
532 itly calculated and successfully estimates the results obtained with thermal fatigue
533 tests. The parameters identified for the model are also consistent with the literature
534 and physics of damage for such cast-iron.

535
536 The discussed model is based on a series of material assumptions, i.e. the type
537 and shape of the nodule, which must be adapted each time the method is extended
538 to a class of materials. The ability of such a model to correctly describe low cycle
539 fatigue behaviour at higher temperatures (presence of oxidation, decarburisation)
540 is not guaranteed and it goes without saying that it should be modified to be effec-
541 tive. The ability to predict lifetime in TMF should also be explored in more detail,

542 particularly for wider range of thermal load.

543 **References**

- 544 [1] F. Szymtka, P. Michaud, L. Rémy, A. Köster, Thermo-mechanical fatigue
545 resistance characterization and materials ranking from heat-flux-controlled
546 tests. Application to cast-irons for automotive exhaust part, International
547 Journal of Fatigue, Volume 55, 2013, Pages 136-146,
- 548 [2] A. Benoit, M.H. Maitournam, L. Rémy, F. Oger, Cyclic behaviour of struc-
549 tures under thermomechanical loadings: Application to exhaust manifolds,
550 International Journal of Fatigue, Volume 38, 2012, Pages 65-74
- 551 [3] S. Ghodrati, A.C. Riemsdijk, M. Janssen, J. Sietsma, L.A.I. Kestens, Mea-
552 surement and characterization of Thermo-Mechanical Fatigue in Compacted
553 Graphite Iron, International Journal of Fatigue, Volume 48, 2013, Pages 319-
554 329
- 555 [4] M. Ekström, S. Jonsson, High-temperature mechanical- and fatigue proper-
556 ties of cast alloys intended for use in exhaust manifolds, Materials Science
557 and Engineering: A, Volume 616, 2014, Pages 78-87
- 558 [5] A. Constantinescu, E. Charkaluk, G. Lederer, L. Verger, A computational ap-
559 proach to thermomechanical fatigue, International Journal of Fatigue, Volume
560 26, Issue 8, August 2004, Pages 805-818
- 561 [6] Garud, Y. A new approach to the evaluation of fatigue under multiaxial load-
562 ings (1981). Journal of Engineering Materials and Technology- Transactions
563 of the ASME , 103(2):118-125.
- 564 [7] Skelton, R. P. "Energy criterion for high temperature low cycle fatigue fail-
565 ure." Materials science and technology 7.5 (1991): 427-440.
- 566 [8] Thalmair, S., Thiele, J., Fischersworring-Bunk, A., Ehart, R., Guillou, M.,
567 Cylinder heads for high power gasoline engines - Thermomechanical fatigue
568 life prediction (2006) SAE Technical Papers, .
- 569 [9] Kharkhour, H., Morin, G. (2007). Thermal fatigue of exhaust manifolds:
570 Contribution of numerical simulation to field reliability assessment. Bulletin-
571 Cercle d'études des métaux, 17(17), 135.
- 572 [10] Gosar, A., Nagode, M., Energy dissipation under multiaxial thermomechanical
573 fatigue loading (2013) International Journal of Fatigue, 48, pp. 223-230.

- 574 [11] Roger, F., Chidley, A., Thermo-mechanical fatigue design of automotive heat
575 exchangers (2013) *European Journal of Computational Mechanics*, 22 (2-4),
576 pp. 228-235.
- 577 [12] Szmytka, F., Salem, M., Rézai-Aria, F., Oudin, A., Thermal fatigue analysis
578 of automotive Diesel piston: Experimental procedure and numerical protocol
579 (2015) *International Journal of Fatigue*, 73, pp. 48-57.
- 580 [13] Castro Güiza, G.M., Hormaza, W., Galvis E, A.R., Méndez Moreno, L.M.,
581 Bending overload and thermal fatigue fractures in a cast exhaust manifold
582 (2017) *Engineering Failure Analysis*, 82, pp. 138-148.
- 583 [14] L. Rémy. Thermal mechanical fatigue (including thermal shock). In Editors
584 in Chief: I. Milne, R. O. Ritchie, and B. Karihaloo, editors, *Comprehensive*
585 *Structural Integrity*, pages 113-199. Pergamon, Oxford, 2003.
- 586 [15] S.K. Koh, Fatigue damage evaluation of a high pressure tube steel using cyclic
587 strain energy density. *International Journal of Pressure Vessels and Piping*,
588 2002, vol. 79, no 12, p. 791-798.
- 589 [16] S. Amiable, S. Chapuliot, A. Constantinescu, A. Fissolo. A computational
590 lifetime prediction of a thermal shock experiment. Part II: discussion on dif-
591 ference fatigue criteria. *Fatigue & Fracture of Engineering Materials & Struc-*
592 *tures*, 2006, 29(3), 219-227.
- 593 [17] S. Tabibian, E. Charkaluk, A. Constantinescu, F. Szmytka, A. Oudin. TMF
594 criteria for Lost Foam Casting aluminum alloys. *Fatigue & Fracture of Engi-*
595 *neering Materials & Structures*, 2013, vol. 36, no 4, p. 349-360.
- 596 [18] Korsunsky, A. M., Dini, D., Dunne, F. P., Walsh, M. J. (2007). Compara-
597 tive assessment of dissipated energy and other fatigue criteria. *International*
598 *journal of fatigue*, 29(9-11), 1990-1995.
- 599 [19] Tomkins, B. (1968). Fatigue crack propagation?an analysis. *Philosophical*
600 *magazine*, 18(155), 1041-1066.
- 601 [20] Lamba, H. S. (1975). The J-integral applied to cyclic loading. *Engineering*
602 *Fracture Mechanics*, 7(4), 693-703.
- 603 [21] S.Y. Zamrik, An investigation of strain cycling behavior of 7075-T6 alu-
604 minium under combined state of strain, NASA CR-72843. Washington, DC:
605 National Aeronautics and Space Administration, 1972

- 606 [22] Taner Gocmez, Ali Awarke, Stefan Pischinger, A new low cycle fatigue crite-
607 rion for isothermal and out-of-phase thermomechanical loading, *International*
608 *Journal of Fatigue*, Volume 32, Issue 4, 2010, Pages 769-779,
- 609 [23] E. Charkaluk, A. Constantinescu, F. Szymtka, S; Tabibian. Probability den-
610 sity functions: From porosities to fatigue lifetime. *International Journal of*
611 *Fatigue*, 2014, vol. 63, p. 127-136.
- 612 [24] H. Gao, A. Wang, E. Zio, G. Bai, An integrated reliability approach with
613 improved importance sampling for low-cycle fatigue damage prediction of
614 turbine disks, *Reliability Engineering & System Safety*, 2020,
- 615 [25] Szymtka, F., Oudin, A. (2013). A reliability analysis method in thermome-
616 chanical fatigue design. *International Journal of Fatigue*, 53, 82-91.
- 617 [26] Doudard, Cédric, and Sylvain Calloch. "Influence of hardening type on self-
618 heating of metallic materials under cyclic loadings at low amplitude." *European*
619 *Journal of Mechanics-A/Solids* 28.2 (2009): 233-240.
- 620 [27] V. Maurel, L. Rémy, F.Dahmen, N. Haddar. An engineering model for low
621 cycle fatigue life based on a partition of energy and micro-crack growth. *In-*
622 *ternational Journal of Fatigue*, 31(5), pages 952-961, 2009.
- 623 [28] Bavard, K., Bernhart, G., Zhang, X. P. (2003). High temperature low cycle
624 fatigue of spheroidal graphite cast iron. *International Journal of cast Metals*
625 *research*, 16(1-3), 233-238.
- 626 [29] Limodin, Nathalie, et al. "Application of X-ray microtomography to study
627 the influence of the casting microstructure upon the tensile behaviour of an
628 Al-Si alloy." *Nuclear Instruments and Methods in Physics Research Section*
629 *B: Beam Interactions with Materials and Atoms* 324 (2014): 57-62.
- 630 [30] Murakami, Y., Endo, M. (1994). Effects of defects, inclusions and inhom-
631 ogeneities on fatigue strength. *International journal of fatigue*, 16(3), 163-182
- 632 [31] Couper, M. J., Neeson, A. E., Griffiths, J. R. (1990). Casting defects and
633 the fatigue behaviour of an aluminium casting alloy. *Fatigue & Fracture of*
634 *Engineering Materials & Structures*, 13(3), 213-227.
- 635 [32] Skallerud, B., Iveland, T., Härkegård, G. (1993). Fatigue life assessment
636 of aluminum alloys with casting defects. *Engineering Fracture Mechanics*,
637 44(6), 857-874.

- 638 [33] Dezecot, S., Brochu, M. (2015). Microstructural characterization and high
639 cycle fatigue behavior of investment cast A357 aluminum alloy. *International*
640 *Journal of Fatigue*, 77, 154-159.
- 641 [34] Viet-Duc Le, Franck Morel, Daniel Bellett, Nicolas Saintier, Pierre Osmond
642 (2016), Multiaxial high cycle fatigue damage mechanisms associated with the
643 different microstructural heterogeneities of cast aluminium alloys. *Materials*
644 *Science and Engineering: A*, 649, 426-440
- 645 [35] Endo, M. (1989). Effects of graphite shape, size and distribution on the fatigue
646 strength of spheroidal graphite cast irons. *J. Soc. Mater. Sci., Jpn.*, 38(433),
647 1139-1144.
- 648 [36] Yaacoub Agha, H., Béranger, A. S., Billardon, R., Hild, F. (1998). High cycle
649 fatigue behaviour of spheroidal graphite cast iron. *Fatigue & Fracture of En-*
650 *gineering Materials & Structures*, 21(3), 287-296.
- 651 [37] Nadot, Y., Mendez, J., Ranganathan, N. A., Beranger, A. S. (1999). Fatigue
652 life assessment of nodular cast iron containing casting defects. *Fatigue &*
653 *fracture of engineering materials & structures*, 22(4), 289-300.
- 654 [38] Marrow, T. J., Buffiere, J. Y., Withers, P. J., Johnson, G., Engelberg, D.
655 (2004). High resolution X-ray tomography of short fatigue crack nucleation
656 in austempered ductile cast iron. *International journal of fatigue*, 26(7), 717-
657 725.
- 658 [39] Nadot, Y., Mendez, J., Ranganathan, N. (2004). Influence of casting defects
659 on the fatigue limit of nodular cast iron. *International Journal of Fatigue*,
660 26(3), 311-319.
- 661 [40] Costa, N., Machado, N., Silva, F. S. (2010). A new method for prediction of
662 nodular cast iron fatigue limit. *International Journal of Fatigue*, 32(7), 988-
663 995.
- 664 [41] Huetter, G., Zybelle, L., Kuna, M. (2015). Micromechanisms of fracture in
665 nodular cast iron: From experimental findings towards modeling strategies? A
666 review. *Engineering Fracture Mechanics*, 144, 118-141.
- 667 [42] Lukhi, M., Kuna, M., Hütter, G. (2018). Numerical investigation of low cycle
668 fatigue mechanism in nodular cast iron. *International Journal of Fatigue*, 113,
669 290-298.

- 670 [43] Mbiakop, A and Constantinescu, A and Danas, K, An analytical model for
671 porous single crystals with ellipsoidal voids, *Journal of the Mechanics and*
672 *Physics of Solid*, 84, 436-467, 2015=Pergamon
- 673 [44] M.J. Dong, C. Prioul, D. François. Damage effect on the fracture toughness
674 of nodular cast iron: part I. Damage characterization and plastic flow stress
675 modeling. *Metallurgical and Materials Transactions A*, 28(11), 2245-2254,
676 1997
- 677 [45] Seifert, T., Riedel, H. (2010). Mechanism-based thermomechanical fatigue
678 life prediction of cast iron. Part I: Models. *International Journal of Fatigue*,
679 32(8), 1358-1367.
- 680 [46] F. Szymtka, L. Rémy, H. Maitournam, A. Köster, M.Bourgeois, New flow
681 rules in elasto-viscoplastic constitutive models for spheroidal graphite cast-
682 iron, *International Journal of Plasticity*, Volume 26, Issue 6, June 2010, Pages
683 905-924
- 684 [47] L. Rémy, F. Szymtka, L. Bucher, Constitutive models for bcc engineering
685 iron alloys exposed to thermal-mechanical fatigue, *International Journal of*
686 *Fatigue*, Volume 53, August 2013, Pages 2-14
- 687 [48] I. Hervas, M.B. Bettaieb, E. Hug. Damage mechanisms evolution of ductile
688 cast irons under thermomechanical loadings. *International Journal of Materi-*
689 *als and Product Technology*, 2013, vol. 47, no 1, p. 23-32.
- 690 [49] K. M. Pedersen, N. S. Tiedje, Graphite nodule count and size distribution in
691 thin-walled ductile cast iron, *Materials Characterization*, Volume 59, Issue 8,
692 August 2008, Pages 1111-1121, ISSN 1044-5803
- 693 [50] Takahashi, R., Sibuya, M. (2002). Metal fatigue, Wicksell transform and ex-
694 treme values. *Applied Stochastic Models in Business and Industry*, 18(3),
695 301-312.
- 696 [51] Fullman, RL and others, Measurement of particle sizes in opaque bodie,
697 1953, General Electric Research Laboratory
- 698 [52] C. Verdu, J. Adrien, J.Y. Buffière, Three-dimensional shape of the early stages
699 of fatigue cracks nucleated in nodular cast iron, *Materials Science and Engi-*
700 *neering: A*, Volumes 483-484, 15 June 2008, Pages 402-405
- 701 [53] N. Bonora, A. Ruggiero, Micromechanical modeling of ductile cast iron in-
702 corporating damage. Part I: Ferritic ductile cast iron, *International Journal of*
703 *Solids and Structures*, Volume 42, Issues 5-6, March 2005, Pages 1401-1424

- 704 [54] Harada S., Akiniwa Y., Ueda T., 1992. The effect of microstructure on the
705 low-cycle fatigue behavior of ductile cast iron. In K.-T. Rie et al. (eds.), *Low*
706 *Cycle Fatigue and Elasto-Plastic Behaviour of Materials*–3
- 707 [55] Komotori J., Shimizu M., 1998. Fracture mechanism of ferritic ductile cast
708 iron in extremely low cycle fatigue. In: Rie K-T, Portella PD, editors. *Low*
709 *cycle fatigue and elastoplastic behaviour of materials*. Netherlands: Springer,
710 pp. 39–44
- 711 [56] Bubenko, L., Konecna, R., Nicoletto, G. (2009). Observation of fatigue crack
712 paths in nodular cast iron and ADI microstructures. *Materials Engineering*,
713 16(3), 13.
- 714 [57] Canzar P., Tonkovic Z., Kodvanj J., 2012. Microstructure influence on fatigue
715 behaviour of nodular cast iron. *Materials Science and Engineering A556*,
716 88–99
- 717 [58] Needleman, A., A Continuum Model for Void Nucleation by Inclusion
718 Debonding., *ASME. J. Appl. Mech.* September 1987; 54(3): 525–531.
- 719 [59] Brocks, W., Hao, S., Steglich, D., 1996. Micromechanical modeling of the
720 damage and toughnessbehavior of nodular cast iron materials. *J. de Physique*
721 *IV 6 C6*, 43–52.
- 722 [60] Kuna, M., Sun, D.-Z., 1996. Analyses of void growth and coalescence in cast
723 iron by cell models.*J. de Physique IV 6 C6*, 113–122.
- 724 [61] D. Steglich, W. Brocks, Micromechanical modelling of the behaviour of duc-
725 tile materials including particles, *Computational Materials Science*, Volume
726 9, Issues 1–2, 1997, Pages 7-17,
- 727 [62] K.S. Zhang, J.B. Bai, D. François, Ductile fracture of materials with high
728 void volume fraction, *International Journal of Solids and Structures*, Volume
729 36, Issue 23, 1999, Pages 3407-3425,
- 730 [63] C. Berdin, M.J. Dong, C. Prioul, Local approach of damage and fracture
731 toughness for nodular cast iron, *Engineering Fracture Mechanics*, Volume
732 68, Issue 9, 2001, Pages 1107-1117,
- 733 [64] Lukhi M., Kuna M., Hütter G., 2018. Numerical investigation of low cycle
734 fatigue mechanism in nodular cast iron. *International Journal of Fatigue* 113,
735 290–298

- 736 [65] Beretta, S. , Anderson, C and others, Extreme value statistics in metal fatigue,
737 Riunione Scientifica della Società Italiana di Statistica, 251-260, 2002
- 738 [66] Dezecot, S., Maurel, V., Buffière, J. Y., Szmytka, F., Köster, A. (2017). 3D
739 characterization and modeling of low cycle fatigue damage mechanisms at
740 high temperature in a cast aluminum alloy. *Acta Materialia*, 123, 24-34.
- 741 [67] Mbiakop, A., Constantinescu, A. and Danas, K. , On void shape effects of
742 periodic elasto-plastic materials subjected to cyclic loading, *European Journal*
743 *of Mechanics - A Solids*, pages 481-499, 49, 2015
- 744 [68] F. Szmytka, L. Rémy, H.M. Maitournam, A. Köster, M. Bourgeois. New flow
745 rules in elasto-viscoplastic constitutive models for spheroidal graphite cast-
746 iron. *International Journal of Plasticity*, 26(6), 905-924, 2010
- 747 [69] Szmytka, F., Forré, A., Augustins, L. (2015). A time increment control for
748 return mapping algorithm applied to cyclic viscoplastic constitutive models.
749 *Finite Elements in Analysis and Design*, 102, 19-28.