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Discomfort Glare from Several Sources: A Formula for Outdoor Lighting

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ABSTRACT

Many models have been developed to predict the discomfort glare generated by one light source in the visual field. Some of them were generalized to predict discomfort glare from several sources, but they were not directly built from multi-source data. In their generalization, they do not consider the potential interactions due to the fact that the light sources are switched on together. In the present work, a psycho-visual experiment was conducted for a better understanding of the glare produced by several sources using a matching task at the Borderline between Comfort and Discomfort (BCD). A formula is proposed to compute the photometric and geometric characteristics of an equivalent source which generates the same level of discomfort glare as several sources switched on together. This formula is the first step towards a multi-source model of discomfort glare. The practical implications are discussed.

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Discomfort glare; LED; multi-source; psychophysics; outdoor lighting

1. Introduction

LED technology is nowadays employed in most lighting applications. This type of light source has many advantages over previous technologies, such as high luminous efficiency, great flexibility for the design of lighting installations, and a longer operational life. It also has some drawbacks, which need to be under control. For instance, LED lamps are very small (compared to previous technologies), which leads to higher levels of luminance and to more discomfort glare (Van Bommel 2015, pp.56–57).

Discomfort glare is defined by the CIE (2013a) as “the glare that causes discomfort without necessarily impairing the vision of objects”. It occurs when the luminance or the luminance contrast between the light source and the background, is too high. Many computational models have been proposed in order to predict a mean level of discomfort glare on a rating scale from the photometric and geometric characteristics of the visual scene (e.g. de Boer and Schreuder (1967); Einhorn (1969); Hopkinson (1972); Einhorn (1979); Wienold and Christoffersen (2006); Bullough et al. (2008)). Most models were first developed to predict the

discomfort glare caused by a single light source in the visual field (CIE 1995; Hopkinson 1940; Lin et al. 2015, 2014; Luckiesh and Guth 1949; Petherbridge and Hopkinson 1950; Schmidt-Clausen and Bindels 1974). They share a general form as follows:

$$R = a \log \left(\frac{L_s^\alpha \omega_s^\beta}{L_b^\gamma p_s^\delta} \right) + b \quad (1)$$

where:

- R is the mean level of discomfort glare (based on subjective ratings collected from a panel of observers) on a rating scale;
- L_s is the luminance of the source (cd/m^2 or fL);
- L_b is the background luminance (cd/m^2 or fL);
- ω_s is the solid angle of the source (sr);
- p_s is the position of the source in the visual field, either described by its eccentricity in degrees or arc minutes (Hopkinson 1940; Lin et al. 2015, 2014; Schmidt-Clausen and Bindels 1974) or by the Guth position index (CIE 1995; Einhorn 1979; Luckiesh and Guth 1949);
- a , b , α , β , γ and δ depend on each particular model (see Appendix A for a full index of notations).

Some authors have investigated the effect of more than one source on discomfort glare. Hopkinson (1940) introduced the concept of adaptation additivity. In his experiment, several light sources were switched on together, and the background luminance was adjusted by the participants so as to feel a given degree of discomfort glare. Hopkinson found that this background luminance was equal to the sum of the background luminance values set for each light source switched on alone at the same level of discomfort. Based on this observation, some authors have generalized their single-source model to a multi-source model by assuming the additivity of the contribution G_i of each light source i to the discomfort glare (Einhorn 1979; Hopkinson 1940; Schmidt-Clausen and Bindels 1974) leading to:

$$\left\{ \begin{array}{l} R = a \log \frac{1}{L_b^y} \sum_{i=1}^n G_i + b \\ \text{with } G_i = \frac{L_{si}^\alpha \omega_{si}^\beta}{p_{si}^\delta} \end{array} \right. \quad (2)$$

In their study, Luckiesh and Guth (1949) proposed the concept of an “equivalent source”, i.e. a single source which generates the same degree of discomfort glare as several sources switched on together. They computed a model fitted with one-source data, and speculated that the discomfort glare with several sources would result from the addition of the individual contribution to glare from each source (as in Eq. 2). They tested the idea with multi-source data, but it did not work with their model.

Apart from the Glare Control Mark (GCM) (CIE 1976; de Boer and Schreuder 1967), none of the available multi-source models were directly developed from multi-source data. The present work consists in developing a formula to describe the photometric and geometric characteristics of an equivalent light source which generates the same level of discomfort glare as several sources switched on together, based on data collected on multi-source stimuli. For some reasons explained below, this was made at the Borderline between Comfort and Discomfort (the BCD).

This work is agnostic with respect to any model predicting the mean level of discomfort glare. We were not interested in absolute thresholds, which

are subject to various experimental biases (Poulton 1989, pp. 21–59). Instead, we have focused on a formula that would help simplifying a complex visual scene into a simpler one, from several to one equivalent light source. Both indoor and outdoor lighting design may benefit from such a simplification.

2. State of the art

The additivity hypothesis of previous multi-source models does not consider the possible effect of spatial interactions between light sources on discomfort glare. Especially, their spatial arrangement (which can be quantified with the angular distance between the sources) may have an effect. This is notably suggested by the recent literature on source heterogeneity (Donners et al. 2015; Funke and Schierz 2015; Kasahara et al. 2006; Tashiro et al. 2015) and by Wilkins’ seminal work on visual discomfort (Boyce and Wilkins 2018; Wilkins 1986; Wilkins et al. 1984).

Indeed, some authors, focusing on non-uniform LED luminaires (in terms of luminance distribution) in indoor lighting, have investigated the impact of the spatial arrangement on discomfort glare. Different types of LED arrays have been studied, and three parameters have emerged to describe the spatial arrangement: the pitch (i.e. the distance between two consecutive LEDs), the area of the array, and the number of LED elements in the array. The findings differ depending on the eccentricity of the source: at a given illuminance, for eccentricities below 10° , a non-uniform luminaire was found to cause more discomfort than a uniform one (Donners et al. 2015; Tashiro et al. 2015). However, no consensus was established beyond 10° (CIE 2013b): Waters et al. (1995) found that non-uniform light sources produce less discomfort glare than uniform ones with the same average luminance. On the other hand, Donners et al. (2015) found no influence of the luminance pattern of LED arrays on discomfort glare for eccentricities beyond 10° .

From this literature, these two ranges of source eccentricity should be considered separately. Up to 10° of eccentricity, increasing the LED pitch causes an increase in discomfort glare when the pitch is lower than 0.2° (Donners et al. 2015; Kasahara

et al. 2006). This effect is not found for LED pitches above 0.2° (at least up to 5°), in both eccentricity ranges below and beyond 10° (Funke and Schierz 2015; Tashiro et al. 2015). Considering non-uniform sources as a limit case, it is clear that an effect is present in the central vision for very small inter-distances.

These results have been obtained by varying either the area of the array or the number of LEDs. Unfortunately, these studies focused on luminaire design and did not investigate the effect of the pitch independently of other factors (number of sources and luminaire size). Moreover, these studies are mainly carried out in photopic lighting conditions with background luminance values above 50 cd/m^2 , which is typical of indoor lighting. Very few studies addressed the issue of spatial arrangement in mesopic or scotopic lighting conditions (i.e. for background luminance below 10 cd/m^2) which usually qualify outdoor lighting (Tashiro et al. 2015).

When comparing discomfort glare in indoor and outdoor lighting, the ranges of the main parameters differs (background luminance, size, number and eccentricity of the sources). In an indoor environment, there is a large variety of light sources (daylighting, artificial lighting on ceilings, task lights) with various size and spectrum, which influence the magnitude of discomfort glare (CIE 2002; Wienold and Christoffersen 2006). The CIE (1995) introduced the Unified Glare Rating (UGR) model to predict the discomfort glare from indoor artificial lighting sources. This model received some modifications, notably for “small light sources” (CIE 2002) and for non-uniform light sources in terms of luminance distribution (Funke and Schierz 2015; Tashiro et al. 2015); see Sec. 7.2 for a discussion of the “small sources” concept. Others models have been developed to predict the discomfort glare due to daylighting (Wienold and Christoffersen 2006).

In outdoor lighting, the situation is somehow different. There is a large number of similar light sources, all with a small apparent size, including alignments of luminaires and vehicle headlamps. Several-specific models have been proposed. The Glare Control Mark (GCM), recommended by CIE for road lighting (CIE 1976; de Boer and Schreuder 1967) is the only model established

directly from multi-source data. The GCM selects specific angles (80° and 88°) as features which are supposed to sum up the complete photometry of the luminaires. However, these two sample angles are not representative of all luminaires. For instance, some luminaires send no light at all at 88° : in this case, the GCM computation does not make any sense. Moreover, the GCM model cannot apply to automotive lighting.

The present work will focus on outdoor lighting conditions, where a multi-source formula seems both more needed and easier to build. On the road at night, the drivers’ gaze mainly focuses towards their trajectory (Lappi 2014; Winter et al. 2017) and they may be disturbed by the numerous light sources in their field of view, either from headlamps or luminaires. Thus, they may feel some difficulties to achieve their driving task, which may lead to road safety problems. In addition, the discomfort glare is known to discourage some users from driving in dark conditions, notably older drivers and those with a cataract (Keay et al. 2016; Rubin et al. 1994), which impairs their mobility.

3. Objective of the work

Several steps are required to develop a model predicting the level of discomfort glare due to multiple sources. This paper presents the first step towards such a model, through the identification of the parameter values associated with the sources. It focuses on the specificity of a complex visual scene with multiple sources, especially on the potential interactions between the sources. Therefore, we propose a multi-source formula at a constant level of discomfort glare, where several sources can be replaced by an equivalent source which produces the same level of discomfort glare. Such formula is the main component of a future model because it quantifies the contribution G of each source to the glare, and it is directly built from multi-source stimuli (unlike most of the previous work). The background luminance is fixed in this first step because it is not involved in the multi-source interactions.

Our formula will be computed from experimental data in the range of outdoor lighting, but the

proposed methodology, based on an equivalent source, can be extended to indoor lighting. Based on previous literature, the contribution to discomfort glare of an equivalent source is defined as:

$$G_{eq} = \frac{L_{eq}^{\alpha} \omega_{eq}^{\beta}}{p_{eq}^{\delta}} \quad (3)$$

where:

- L_{eq} is the luminance of the equivalent source (cd/m^2);
- ω_{eq} is the solid angle of the equivalent source (sr);
- p_{eq} is the position of the equivalent source in the visual field.

We have considered a formula with the following mathematical form ($n > 1$):

$$\begin{cases} G_{eq} = \sum_{i=1}^n G_i D_i \\ D_i = \frac{1}{n-1} \sum_{i=1}^n \phi(d_{ij}) \end{cases} \quad (4)$$

Where:

- G_{eq} is the contribution of the equivalent light source to the discomfort glare;
- G_i is the contribution of each light source i to the discomfort glare;
- d_{ij} is the distance between sources i and j ;
- D_i and ϕ are two unknown functions.

This formula means that a single light source (the equivalent source) produces the same level of discomfort as n light sources.

The present study will test the hypothesis that the inter-distance between sources have an effect on discomfort glare. From this experiment, we will be able to propose a function ϕ in Eq. 4 relevant for outdoor lighting, and to estimate the parameters α and δ in Eq. 3.

The parameter β in Eq. 3 quantifies the influence of the source's solid angle on discomfort glare. In order to ensure the mathematical consistency of the formula, several authors have emphasized that β should be equal to 1 (Clear 2013; Eble-Hankins and Waters 2005; Einhorn 1979, 1998; Poulton 1979): halving a light source into two parts should not change the

glare level predicted by Eq. 4. Although some authors use other values (see Table 4 in the Discussion part in Section 7 of this paper), we have followed this proposal, and in our formula, we use $\beta = 1$.

The methodology of this experiment was to collect data at a constant level of discomfort glare using a matching task, in which observers focused on equalization of sensations, which we expect to reduce range biases (Fotios and Cheal 2010; Kent et al. 2019). All stimuli were tuned by the participants so as to feel the same sensation as the one caused by a reference source. This reference source is set at a fixed level of discomfort glare. The Borderline between Comfort and Discomfort (BCD, Luckiesh and Guth (1949)) was selected for this purpose because it is relevant for lighting design and allows the participants not to experience too strong glare levels.

4. Material and method

4.1. Participants

Thirty-six participants took part in the experiment (23 men and 13 women). They were aged between 21 and 58 years ($M = 34.5$, $SD = 10.4$). They were recruited from the IFSTTAR institute; they were naive to the purpose of the experiment and did not work in the field of lighting. They wore their usual ocular corrections (glasses or lenses) if any. A visual acuity test (ErgoVision, Essilor) was conducted before the experiment: all participants had an acuity above 6/10, which is the French standard for driving. After removing the outliers, the final sample size was 31 for two-source stimuli (20 men and 11 women) aged between 21 and 58 years ($M = 33.8$, $SD = 9.8$) and 30 for three-source stimuli (19 men and 11 women) also aged between 21 and 58 years ($M = 34.0$, $SD = 9.8$).

4.2. Experimental settings

The experiment was carried out in a dark room (no window and no light source other than those described in the experiment). A digital RGB LED strip was fixed horizontally on a vertical screen (see Fig. 1, left). We used WS2812B RGB LEDs, which can be individually controlled with an Arduino microcontroller. R, G, and B values

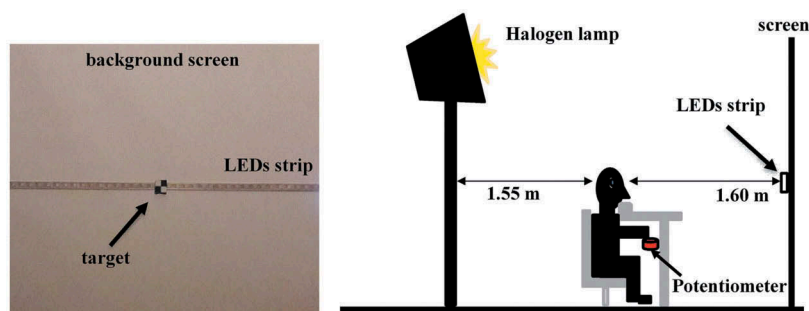


Fig. 1. Left: Photograph of the screen with the target and the LEDs strip. Right: Framework of the side view.

were always identical so that the LED spectral power distribution was always the same. The LED intensity could be tuned by changing the RGB values altogether.

Participants sat at a distance of 1.60 m from the screen in order to see the LEDs with a solid angle of about 5×10^{-6} sr (see Fig. 1, right). During the whole experiment, they were instructed to set their chin on a chin rest in order to ensure a reproducible line of sight and to stare at a target positioned in front of them in the middle of the LEDs strip. The eyes of the participants were at the same height as the light sources, and the target was designed to be easily visible in the dark. A halogen lamp was located behind the participants in order to create a homogeneous background luminance of 1 cd/m^2 , a recommended value for street lighting by the European standard (CEN 2003).

4.3. Experimental design

This experiment aimed at testing the influence of the inter-distance between two or three sources on the discomfort glare. To that purpose, different multi-source stimuli, with two or three sources and different inter-distances were presented to the participants (Fig. 2). A multi-source stimulus was composed of:

- One or two sources of fixed luminance: their luminance value L_F was fixed during each trial for each participant;
- One variable source (i.e. the test source): its luminance L_V^* was adjusted by the participant in such a way that the multi-source stimulus (i.e. when all the sources were switched on together) caused a sensation of glare at a specified level of discomfort (BCD in this study).

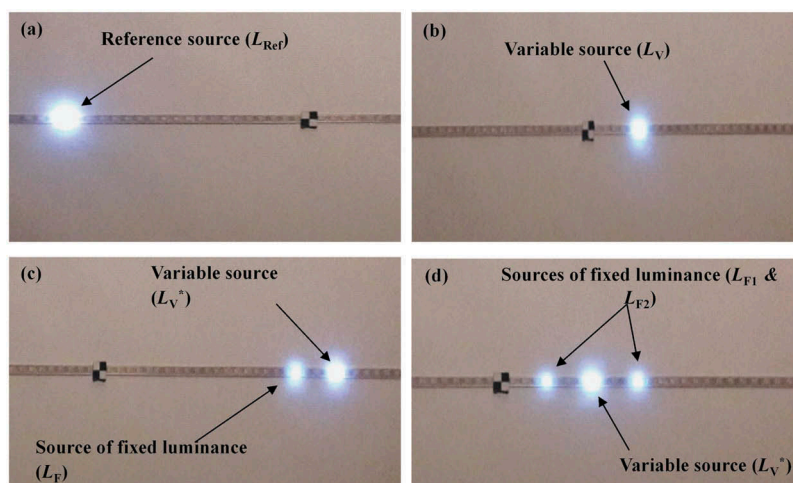


Fig. 2. Photographs of the experimental settings: (a) the Reference source; (b) example of a single-source stimulus; (c) example of a two-source stimulus; (d) example of a three-source stimulus.

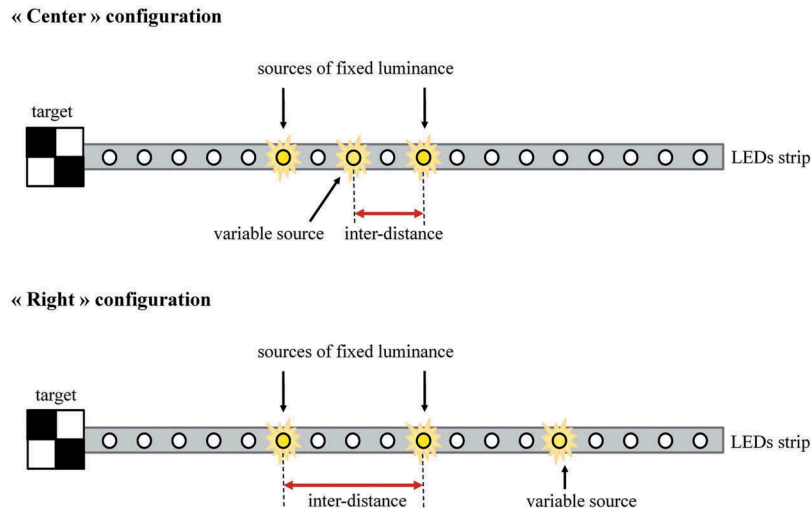


Fig. 3. Example of a DUO: 2 three-source stimuli with the same sources of fixed luminance. Top: Center configuration, where the variable source is located between the sources of fixed luminance. Bottom: Right configuration, where the variable source is located on the right side of the two sources of fixed luminance.

The position of the variable source was varied in a way which allows to present different inter-distances between the sources (see Fig. 3 in the case of three-sources stimuli). As the eccentricity of the source is known to impact the discomfort glare (a smaller eccentricity increases the discomfort), the luminance of the variable source at BCD is expected to change with the inter-distance. This is why the BCD value (i.e. the luminance of the variable source at the BCD) was not considered a relevant dependent variable. We have considered instead as our dependent variable the ratio of the variable source luminance when it is switched on with another source(s) at BCD and the luminance of this same source alone at BCD. This ratio does not depend on the source position, only on the inter-distance.

To do that, the luminance L_{Vij} of each variable source i switched on alone was preliminarily tuned at the BCD by each participant j (see Fig. 2(b)). This allowed to use the Luminance Ratio r_L as our dependent variable:

$$r_{Lij} = \frac{L_{Vij}^*}{L_{Vij}} \quad (5)$$

Where:

- L_{Vij}^* is the luminance of the variable source i (cd/m^2) that allows the participant j to feel at the BCD when the variable source is

simultaneously switched on with the light source(s) of fixed luminance (see Fig. 2(c) and (d));

- L_{Vij} is the luminance of the same variable source i (cd/m^2) at the BCD for the participant j when it is switched on alone (see Fig. 2 (b)).

To study the effect of the distance between light sources on the discomfort glare, we have compared r_L -values for various inter-distances.

The luminance value of each source of fixed luminance was chosen in order to generate alone a level of discomfort lower than the participant's BCD. Its intensity was arbitrarily selected from classical models of discomfort glare (Bennett 1977; Luckiesh and Guth 1949; Schmidt-Clausen and Bindels 1974), where the contribution to discomfort glare is proportional to the source luminance (i.e. $\alpha=1$ in Eq. 1). Using such a formula, for a two-source stimulus, the luminance value of the source of fixed luminance was set for each participant j to half the luminance value at BCD for the same source i alone:

$$L_{Fij} = \frac{L_{Vij}}{2} \quad (6)$$

For three-source stimuli, the same formula led to one-third of the single-source luminance for each of the sources of fixed luminance:

$$L_{Fij} = \frac{L_{Vij}}{3} \quad (7)$$

This does not mean that $\alpha = 1$. On the contrary, the experiment is built in order to estimate the value of α . At this stage, we had no hypothesis about the value of α . We just needed to provide a luminance to the source(s) of fixed luminance which generates less than each participant's BCD. This choice does not affect the value of α estimated from the luminance ratios.

To remove potential co-variables, various inter-distances have been tested with the same sources of fixed luminance. Therefore, the experimental design was different for two and three-source stimuli. For two-source stimuli, the experimental design included two experimental factors:

- The distance between the two light sources, with four values: 0.6°; 1.2°; 2.3° and 4.6°;
- The eccentricity range, with two modalities: < 10° and > 10°.

The characteristics of the two-source stimuli are provided in Table 1. All luminance values were computed from vertical illuminance measurements collected with a LMT B520 luxmeter situated at the position of participant's eyes facing the target (a chin rest was employed to ensure all participants had the same eye position). For each LED involved in the experiment, seven values of the LED intensity were sampled between the minimum and the maximum available intensity. The illuminance was found linear with respect to the LED intensity, and a regression allowed estimating illuminance for each intensity value. From this, the source luminance could be computed.

Table 1. Geometric characteristics of the two-source stimuli.

Eccentricity of the source of fixed luminance	Eccentricity of the variable source	Distance between sources
2.3°	2.9°	0.6°
2.3°	3.5°	1.2°
2.3°	4.6°	2.3°
2.3°	6.9°	4.6°
11.3°	11.9°	0.6°
11.3°	12.5°	1.2°
11.3°	13.6°	2.3°
11.3°	15.9°	4.6°

Table 2. Geometric characteristics of the three-source stimuli.

	Config. (see Fig. 3)	Eccentricity of the 1st fixed source	Eccentricity of the 2nd fixed source	Eccentricity of the variable source	Inter-distance
DUO 1	Center	2.3°	3.5°	2.9°	0.6°
	Right	2.3°	3.5°	4.6°	1.2°
DUO 2	Center	2.3°	4.6°	3.5°	1.2°
	Right	2.3°	4.6°	6.9°	2.3°
DUO 3	Center	2.3°	6.9°	4.6°	2.3°
	Right	2.3°	6.9°	11.3°	4.6°
DUO 4	Center	11.3°	12.5°	11.9°	0.6°
	Right	11.3°	12.5°	13.6°	1.2°
DUO 5	Center	11.3°	13.6°	12.5°	1.2°
	Right	11.3°	13.6°	15.9°	2.3°
DUO 6	Center	11.3°	15.9°	13.6°	2.3°
	Right	11.3°	15.9°	20.5°	4.6°

The three-source stimuli have been compared by pairs (labeled as "DUO"). In each DUO, the sources of fixed luminance were the same in the two configurations, and the variable source was different, resulting in different inter-distance (see Fig. 3). The fact that we computed luminance ratios ensures that the comparison in each DUO only tested a potential effect of the source inter-distance. Six DUOs were considered, the characteristics of which are given in Table 2.

4.4. Experimental protocol

The experiment was divided into four successive sessions: BCD setting, single-source stimuli, two-source stimuli, and three-source stimuli. For each session, the participants were constantly reminded by the experimenter to keep their gaze on the target. In all trials, the initial value of the variable source was set to the minimal possible intensity of the LED.

In a first session, the participants were asked to adjust the brightness of the Reference source with a potentiometer until they felt at their own BCD level. This source was the same for all participants and was located 11.3° left from the target (see Fig. 2(a)). This tuning was repeated six times. The mean luminance was computed for each participant j and the Reference source was set to this luminance L_{Refj} for the rest of the experiment. It is employed as the reference to equalize the sensation (in the matching tasks) in the next three sessions.

During the second session, participants were presented with single LEDs at different eccentricities, switched on alternatively with the Reference source (10 alternations of 1-s periods for each side). Three seconds were left between each trial. This protocol is inspired from Luckiesh and Guth's seminal paper (Luckiesh and Guth 1949). The 1-s duration was chosen based on findings from Ahmed and Bennett (1978). The participants had to adjust the luminance L_V^* of the variable source with the potentiometer until they felt the same sensation of discomfort glare as the one generated by the Reference source. This tuning was repeated three times.

The third and fourth sessions of the experiment were, respectively, composed of 8 two-source (see Table 1) and 12 three-source stimuli (Table 2), each presented with three repetitions. The participants were asked to adjust the luminance L_V^* of the variable source so that the degree of discomfort glare generated by the whole multi-source stimulus on the right side was the same as the one from the Reference source on the left side. Each stimulus lasted 14 s (i.e. seven alternations of 1-s periods for each side). Three seconds were left between each trial.

In sessions 2 and 3, the stimuli were presented in a random order for each participant. In session 4, the DUOs were presented in a random order for each participant with a counterbalanced presentation of the stimuli of each DUOs among the participants.

Before each session, the participants received a training in order to get accustomed to the task. The training was intended to get the participants used to all aspects of the experiment (stimuli, chin rest, target, potentiometer, etc.).

5. Results

5.1. Two-source stimuli

Five participants were detected as outliers by a Hierarchical Ascendant Clustering (Euclidean distance and average linkage) and were removed from the data.

Figure 4 shows the luminance ratios for the two-source stimuli (r_{L2S}) with 95% confidence intervals for the different inter-distances and for

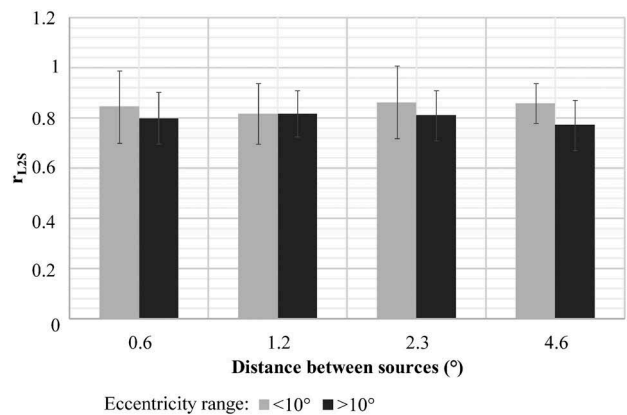


Fig. 4. Luminance ratios r_{L2S} for the two-source stimuli as a function of the inter-distance (degree) for eccentricity ranges $<10^\circ$ (light grey) and $>10^\circ$ (dark grey) with 95% confidence intervals.

each range of eccentricity. Mean r_{L2S} values are almost constant whatever the distance between the sources and the eccentricity range: the mean r_{L2S} is 0.82, and the intersection of all confidence intervals is [0.78; 0.87]. The mean BCD and mean ratios are given in Appendix B.

The luminance value of each source of fixed luminance was set to $L_F = L_V/2$ (see Eq. 6). However, the participants adjusted L_V^* to around 80% of L_V , meaning that the source of fixed luminance only contributed to 20% of the total discomfort glare. This result shows that there is no direct proportionality between the luminance of the source and the perception of discomfort glare. Therefore, α should differ from 1 in our formula (Eq. 3).

A repeated measures ANOVA was performed with two intra-subject factors: the sources inter-distance (with four values: 0.6°; 1.2°; 2.3° and 4.6°) and the eccentricity range (with two modalities: $\theta_{LED} < 10^\circ$ and $\theta_{LED} > 10^\circ$). Normality was satisfied according to the Kolmogorov–Smirnov test. The Mauchly test was employed to test for sphericity and the Greenhouse-Geisser correction was employed when sphericity was not satisfied. The analysis did not yield a significant effect of the inter-distance ($F(2.20, 66.14) = 0.07, p = .95$) or of the eccentricity range ($F(1, 30) = 1.87, p = .18$). There was no interaction either ($F(3, 90) = 0.42, p = .74$).

5.2. Three-source stimuli

Based on another Hierarchical Ascendant Clustering (Euclidean distance and average linkage), six outliers (the same as before + 1) were removed from the data. Figure 5 shows luminance ratios for the three-source stimuli (r_{L3S}) for each DUO, with 95% confidence intervals. Similar to Fig. 4, the visual inspection of the plot leads to postulate that these mean values are almost constant. The mean ratio for three-source stimuli is $r_{L3S} = 0.81$, and the intersection of all confidence intervals is [0.79; 0.87].

The luminance of the variable source L_V^* was adjusted by the participants around 80% of L_V . Thus, the two sources of fixed luminance contributed together to 20% of the total discomfort glare. Again, this is not consistent with the hypothesis that $\alpha = 1$, which would have resulted in each source contributing to one-third of the total discomfort glare.

A paired t-test was carried out for each of the six DUOS, to test for a possible effect of the sources inter-distance. Normality was satisfied according to Kolmogorov–Smirnov test. Homogeneity of variances was checked with the Levene test. As multiple t-tests were performed, we used the Holm Bonferroni Method in order to adjust the probability of making a type I error. Analysis did not yield any significant effect as shown in Table 3. Moreover, the confidence intervals of all DUOS intersect (see Fig. 5). Therefore, the inter-distance does not impact the discomfort

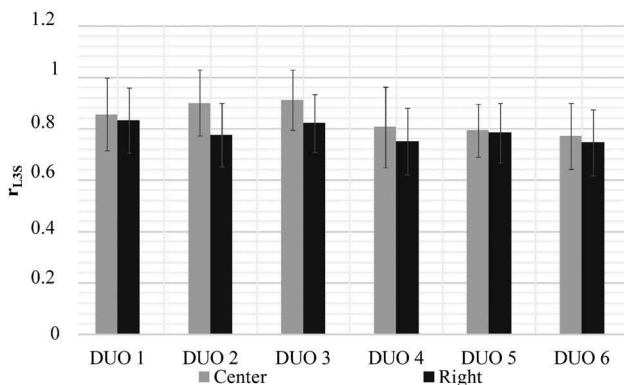


Fig. 5. Luminance ratios r_{L3S} for each DUO (see Table 2 and Fig. 3), with 95% confidence intervals. Data in light grey correspond to the “center” configuration. Those in dark grey correspond to the “right” configuration.

Table 3. Student’s t , p -value and the adjusted probability of Type I error for each DUO: all p -values are greater than the corresponding adjusted probability threshold.

DUO	Student’s t	p -value	adjusted probability of Type I error
DUO 1	0.4110	0.6841	0.0250
DUO 2	2.5284	0.0172	0.0083
DUO 3	2.0930	0.0452	0.0100
DUO 4	1.0659	0.2952	0.0125
DUO 5	0.2220	0.8259	0.0500
DUO 6	0.4586	0.6499	0.0167

glare when three sources are simultaneously switched on in the range of inter-distance of this study.

Given that all twelve 95% confidence intervals intersect, we can be confident that the ratio is constant across experimental conditions. To be sure, a one-factor ANOVA was conducted on these ratios, and no effect was found ($F(6.66, 193.28) = 1.69, p = .11$). In the following, we have considered that the ratios were the same, and we used the mean value for the computation of α .

5.3. Conclusion

Either with two-source or three-source stimuli, no effect of the inter-distance on the discomfort glare was found. Therefore, the spatial arrangement of the sources does not impact the discomfort glare, in the inter-distance range from 0.6° to 4.6° . This result was found for both eccentricity ranges below and above 10° . This also extends previous findings on non-uniformity in the mesopic range.

In the next section, we use these results to build a quantitative formula in order to link the discomfort glare from several light sources to the discomfort glare from a single source at the BCD level. Considering the parameters ranges in our experiments, this formula will be restricted to outdoor lighting at night (with a fixed background luminance $L_b = 1 \text{ cd/m}^2$).

6. Analytical formula

The results of our experiment allow simplifying the formula described in Eq. 4. According to the experimental data, there is no effect of the inter-distance between two or three sources on discomfort glare. So, the function ϕ in Eq. 4 is unnecessary. Considering that β was set to 1 for

mathematical consistency (Clear 2013), our speculative initial formula is changed to:

$$G_{eq} = \frac{L_{eq}^{\alpha} \omega_{eq}}{p_{eq}^{\delta}} = \sum_{i=1}^n \frac{L_i^{\alpha} \omega_i}{p_i^{\delta}} \quad (8)$$

where eq denotes the properties of the equivalent source.

6.1. Estimation of α

The parameter α quantifies the influence of the source luminance on the discomfort glare. With data collected from two-source stimuli, Eq. 8 becomes:

$$G_{eq} = \frac{L_F^{\alpha} \omega_F}{p_F^{\delta}} + \frac{(L_V^*)^{\alpha} \omega_V}{p_V^{\delta}} \quad (9)$$

During this experiment, the solid angle of the sources was constant: $\omega_F = \omega_V = \omega$. Moreover, all data were collected at the BCD. This leads to:

$$G_{BCD} = \frac{L_F^{\alpha} \omega}{p_F^{\delta}} + \frac{(L_V^*)^{\alpha} \omega}{p_V^{\delta}} \quad (10)$$

Using the data collected at BCD from single-source stimuli, we have:

$$G_{BCD} = \frac{L_V^{\alpha} \omega}{p_V^{\delta}} \quad (11)$$

Given that $r_{L2S} = L_V^*/L_V$ (see Eq. 5) and $L_F = L_V/2$ (see Eq. 6), we can write from Eq 10 and 11:

$$\begin{cases} \frac{L_F^{\alpha} \omega}{p_F^{\delta}} = \frac{(L_V/2)^{\alpha} \omega}{p_F^{\delta}} = \frac{G_{BCD}}{2^{\alpha}} \\ \frac{(L_V^*)^{\alpha} \omega}{p_V^{\delta}} = \frac{(r_{L2S} L_V)^{\alpha} \omega}{p_V^{\delta}} = r_{L2S}^{\alpha} \times G_{BCD} \end{cases} \quad (12)$$

It follows that:

$$G_{BCD} = \frac{G_{BCD}}{2^{\alpha}} + r_{L2S}^{\alpha} G_{BCD} \quad (13)$$

and then:

$$r_{L2S} = \left(1 - \frac{1}{2^{\alpha}}\right)^{1/\alpha} \quad (14)$$

The parameter α can thus be estimated from r_{L2S} (Eq. 14 would be identical if $\beta \neq 1$).

The same reasoning can be applied when three sources are simultaneously switched on. This leads to:

$$r_{L3S} = \left(1 - \frac{2}{3^{\alpha}}\right)^{1/\alpha} \quad (15)$$

Table 4. Values of α , β and δ from various models of discomfort glare (Bennett 1977; CIE 1995; Hopkinson 1940; Lin et al. 2015, 2014; Luckiesh and Guth 1949; Schmidt-Clausen and Bindels 1974).

Model	α	β	δ	position parameter
Hopkinson	1.30	1.00	0.75	eccentricity (degree)
Luckiesh & Guth	1.00	0.21	1.00	Guth position index
Schmidt-Clausen	1.00	1.00	0.46	eccentricity (arc min)
Bennett	1.00	0.60	$e^{0.05\theta}$	eccentricity (degree)
UGR	2.00	1.00	2.00	Guth position index
Lin	2.21	2.21	1.62	eccentricity (degree)

Again, α can be estimated from r_{L3S} .

From Eq. 15, the ratio r_{L3S} is expected to be the same for all three-source conditions. This is not a theoretical constraint but derives from the shape of our model (see Eq. 8), and it is also expected in most models from the literature based on similar formulas. Based on our results in Section 5.2, we can use the mean value of the ratio for the computation of α .

The mean value of r_{L2S} obtained with our data was 0.82 (CI95% = [0.78; 0.86]). According to Eq. 14, the corresponding value of α is 1.77 (CI95% = [1.62; 2.00]). The mean value of r_{L3S} obtained with our data is 0.81 (CI95% = [0.78; 0.85]). According to Eq. 15, the corresponding value of α is 1.71 (CI95% = [1.62; 1.84]).

α_{2S} and α_{3S} are two estimates of the same parameter α , based on two different sets of data. Then, the best guess for α is the mean value of the two estimates α_{2S} and α_{3S} : $\alpha=1.74$. The true value of α should be included in both confidence intervals, for α_{2S} and α_{3S} . The intersection of these two confidence intervals is [1.62; 1.84], which is consistent with the proposed value. This value of α is also included in the range [1; 2.21] found in the literature (see Table 4).

6.2. Estimation of δ

The parameter δ quantifies the influence of the position of the source on discomfort glare. In outdoor lighting, most models describe the position of the light sources in terms of eccentricity (Bennett 1977; Hopkinson 1940; Lin et al. 2015, 2014; Schmidt-Clausen and Bindels 1974) see Table 4.

To estimate δ in our formula, the data from the single-source stimuli were used (ranging from 2.3° to 15.9°). Data were collected at BCD level and consequently, Eq. 8 can be re-written as follows:

$$L_{eq} = \left(\frac{G_{BCD}}{\omega_{eq}} \right)^{1/\alpha} \times p_{eq}^{\delta/\alpha} \quad (16)$$

Here p_{eq} corresponds to the eccentricity θ_s of the source (in degree). A regression was conducted on our data, leading to $\delta=0.42$ (IC95% = [0.39; 0.45]) and $R^2=0.90$, which is close to the value in Schmidt-Clausen's model (with $\delta=0.46$, see Table 4).

7. Discussion

7.1. A multi-source formula of discomfort glare

We propose a formula which computes the characteristics of a single source which generates the same level of discomfort glare as several sources. This formula is built from multi-source data representative of outdoor lighting conditions ($L_b=1$ cd/m²). It can be expressed as follows:

$$G_{eq} = \frac{L_{eq}^{1.74} \omega_{eq}}{\theta_{eq}^{0.42}} = \sum_{i=1}^n \frac{L_i^{1.74} \omega_i}{\theta_i^{0.42}} \quad (17)$$

This formula has been derived from data at the BCD: all participants were at their own BCD level. We can reasonably assume that this formula is valid for other levels of discomfort glare because, in most models, the shape and parameters values of the model do not depend on the glare level. Further work is however needed: the same experiment should be reproduced at various levels of discomfort glare (i.e. other levels than the BCD), firstly to assess the value of our formula at another discomfort level, secondly in order to build a general model for multi-source glare out of this formula.

This formula allows replacing several sources in a scene with an equivalent source which produces the same level of discomfort glare. This equivalent source is described by three parameters (L_{eq} , ω_{eq} , θ_{eq}) which means that there is a set of possible equivalent sources. For example, we can choose one such equivalent source by fixing two parameters, and the model would compute the third one. The most straightforward approach is to set ω_{eq} as the average solid angle of the sources and θ_{eq} as their average eccentricities, and then to use the model to compute the equivalent luminance L_{eq} .

In order to calculate the value of G_{eq} at BCD in Eq. 17, the formula was applied to our data. Table 5 shows the predicted contribution to the discomfort glare G_{eq} at BCD: it is roughly constant (around $G_{eq}=25$) whatever the number of sources, which shows that the formula consistently represents the glare level from one to three sources. However, it would be hazardous to use this value as a threshold for visual comfort in outdoor places. Firstly, it has been calculated for white LEDs only, and secondly, although our stimulus range was included in the wide range of what happens in outdoor lighting, G_{eq} value was calculated from the limited stimulus range generated in laboratory conditions.

Importantly, Eq. 17 is a formula, not a model of discomfort glare. Indeed, it is not possible to directly match the computed G_{eq} value to a rating scale of discomfort glare: our experiments only addressed one level of discomfort glare. Consequently, our formula is not informative about the degree of discomfort. It just classifies a lighting environment into one of two distinct classes: the “comfort area” and the “discomfort area”. Further work would be needed at other levels of discomfort in order to predict the glare level of multi-source stimuli on a discomfort scale, that is, to make a multi-source glare model out of this formula.

The parameter β quantifies the influence of the source's solid angle on discomfort glare. Some authors (Clear 2013; Eble-Hankins and Waters 2005; Einhorn 1998; Poulton 1979) have argued that in order to ensure the mathematical consistency, β should be equal to 1: halving a light source into two parts should not change the glare level predicted by Eq. 2. We have collected more data in an additional experiment which is presented in the next section; these data confirms that $\beta = 1$ is a consistent value so that we use it in our formula.

Table 5. Contribution to the discomfort glare at BCD, from Eq. 17.

Type of stimuli	Mean G_{eq} (M)	SD (σ)	σ/M
Single source	25.8	3.0	0.12
Two sources	25.3	1.6	0.06
Three sources	23.6	2.4	0.10
All stimuli	24.8	2.6	0.10

The proposed formula was built at a specific background luminance ($L_b = 1 \text{ cd/m}^2$), while an increase of the background luminance is known to decrease the discomfort. In previous models predicting a de Boer glare rating, multiplying the background luminance by 10 results in an increase around +1 on the de Boer scale (Schmidt-Clausen: +0.9; GCM: +0.97; Lin et al.: +1.02).

The parameter γ quantifies the influence of the background luminance on discomfort glare in Eq. 1. Values of γ from previous models are provided in Table 6 (Bennett 1977; CIE 1995; Hopkinson 1940; Lin et al. 2015, 2014; Luckiesh and Guth 1949; Schmidt-Clausen and Bindels 1974). From this Table, the ratio α/γ is often close to 2. Based on this consideration, our γ -value should be 0.87 to have a ratio $\alpha/\gamma=2$. Further work is however needed to test this speculation.

7.2. Source size

The experiment was conducted with a fixed solid angle. But according to previous work, for “small” sources (below a size threshold), the discomfort glare only depends on the vertical illuminance at the observer’s eyes, which would lead to $\alpha = \beta$ in our formula (Eq. 2). In the sparse literature investigating discomfort glare due to small sources, various threshold values have been proposed on either the solid angle ω ($3 \times 10^{-4} \text{ sr}$ (Einhorn 1991)), the angular size (0.3° (Bullough and Hickcox 2012)), or the effective area (0.005 m^2 (Paul and Einhorn 1999)). Moreover, in the 1991 Glare Symposium, Einhorn (1991) reported the work of Benz’s thesis (in German) which showed that eccentricity also comes into play: for solid angle between 10^{-6} sr and 10^{-4} sr , at 0°

eccentricity, the discomfort glare mainly depends on the source luminance, while at 10° eccentricity, it depends on the vertical illuminance at the observer’s eyes alone. As no consensus emerged so far, an additional experiment was conducted in the range of solid angles and eccentricities relevant for outdoor lighting.

Experimental data were collected on 16 participants (4 women and 12 men). They were asked to adjust the luminance of a LED presented with various solid angles (5×10^{-5} , 10^{-5} , 5×10^{-6} and 10^{-6} sr) and eccentricities (2.5° and 10°) using a matching task with a reference LED previously set to the individual BCD. We used the same LEDs as in the main experiment, also with $L_b = 1 \text{ cd/m}^2$.

A Friedman test conducted on the four paired samples associated with the solid angle factor yielded a significant effect ($F(n = 15, dl = 3) = 69.72, p < 0.05$). Non parametric post-hoc test reveals three groups for which the difference is statistically significant: $10^{-6} \text{ sr}; 5 \times 10^{-6} \text{ sr}$ and $10^{-5} \text{ sr}; 5 \times 10^{-5} \text{ sr}$. Therefore, our results do not confirm previous results from the literature about “small sources”: for sources ranging from 10^{-6} to $5 \times 10^{-5} \text{ sr}$ (i.e. from about 4 to 27 arc-minutes in size) and eccentricity ranging from 2.5° to 10° , we found that both the luminance and the solid angle of the source have an impact on the discomfort glare, not only the illuminance at the observer’s eyes.

From these new data, the value of β could be estimated from linear regressions. Two regressions on the experimental data led to $\beta = 1.16$ at 2.5° ($R^2 = 0.99$) and $\beta = 1.15$ at 10° ($R^2 = 0.97$). Linear regressions on the same data with $\beta = 1$ led to $R^2=0.99$ at 2.5° and $R^2=0.98$ at 10° . This result supports the hypothesis mentioned above that $\beta = 1$, which is used in our formula.

7.3. Source position

7.3.1. Guth’s position index

For a source with a given eccentricity, the sensation of discomfort glare is not the same if the source is situated on the horizontal, diagonal or vertical axis (Kim et al. 2009; Kim and Kim 2010; Luckiesh and Guth 1949). This is why Luckiesh

Table 6. Values of γ from various models of discomfort glare (Bennett 1977; CIE 1995; Hopkinson 1940; Lin et al. 2015, 2014; Luckiesh and Guth 1949; Schmidt-Clausen and Bindels 1974).

Model	Value of γ	Ratio α/γ
Hopkinson	1.00	1.30
Luckiesh	0.44	2.27
Schmidt-Clausen	0.50	2.00
Bennett	0.30	3.33
UGR	1.00	2.00
Lin	1.02	2.17

and Guth (1949) developed the position index p_{Guth} , for which an analytical form was proposed by Levin (1975). The position index is defined as the ratio between the BCD luminance for a given source position and the BCD luminance for $\theta=0^\circ$. It was adopted by the CIE (1995) in the Unified Glare Rating to predict the discomfort glare in indoor lighting. In outdoor lighting, p_{Guth} was mostly ignored up to now.

We have considered using p_{Guth} to describe the position of the source in the visual field because it is more consistent, both with respect to azimuth and near $\theta=0^\circ$. To estimate δ_{Guth} , a regression was conducted, considering p_{Guth} in Eq. 16. From our data, this led to $\delta_{Guth}=3.87$ (IC95% = [3.61; 4.10]), but with $R^2=0.67$, it hints at a poor fit. Moreover, unlike the UGR and Luckiesh's model, we found $\alpha \neq \delta_{Guth}$. Thus, we have conducted another regression to test the consistency of value $\delta_{Guth} = \alpha$ (i.e. $\delta_{Guth}=1.74$). This led to $R^2=0.46$, which is an even poorer correlation. This is why we have used the source eccentricity as the relevant parameter to estimate a multi-source discomfort glare. From these preliminary results, it is difficult to conclude about the use of this position index, which would deserve further investigations in the luminance and angular range relevant to outdoor lighting.

7.3.2. Comparison with previous studies

As a tentative, we have compared our data with the ones from previous studies at the BCD (Bennett 1977; Luckiesh and Guth 1949). Given that the three studies do not consider the same ranges of parameters, we have considered a BCD luminance ratio in order to normalize the data. First, a common eccentricity was chosen as a reference ($\theta=5^\circ$, leading to $p_{Guth}=1.035$), because it was present in all three studies. Then, the ratio $L_{BCD}(p_{Guth})/L_{BCD}(1.035)$ was computed with the available data in the range $[0^\circ; 20^\circ]$ (see Fig. 6).

Our data are consistent with Bennett (1977) up to 12° (see Fig. 6), after which it seems to reach a plateau in our data. Such a plateau is not entirely unexpected, as it was found in a previous study on peripheral vision up to 20° (Havey and Pöppel 1972). The data from Luckiesh and Guth (1949)

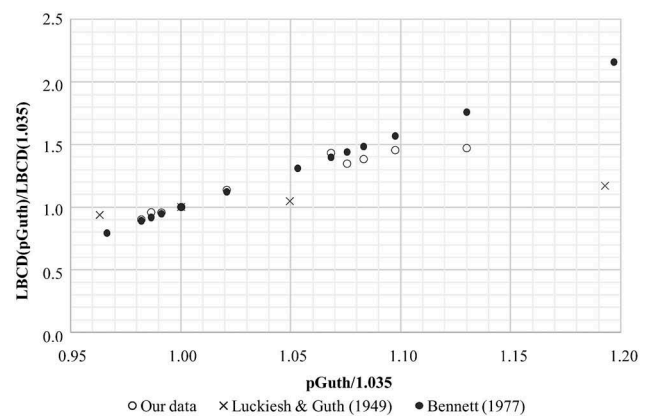


Fig. 6. Comparison of three available studies at the BCD (Luckiesh and Guth (1949), Bennett (1977), and this study).

have a different pattern, but a full comparison is uneasy because their eccentricity sampling is sparse (only three eccentricity values for this range). Moreover, their study included more spatial arrangements of the sources (on the horizontal, vertical and diagonal axis).

7.4. Methodological issues

Various sources of variability were involved in our data. During the experiments, the experimenter constantly reminded the participants to focus on the target. The best target for a fixation task has been discussed in previous studies in the context of photopic vision (Hirasawa et al. 2016; Thaler et al. 2013). The current experiments were in mesopic vision, and it was decided to have the highest possible contrast and a large enough target, in order for the fixation task to be as easy as possible. It has been proposed in the literature to use a dual-task paradigm, where the main task is a visual performance in central vision (e.g. a Landolt ring in Bargary's study (Bargary et al. 2015)), and the secondary task is the discomfort judgment. This allows filtering out the trials where the participants fail in the foveal task (presumably because they did not fixate the target). The unwanted feature of this approach is the possible impact on the discomfort itself (Sivak et al. 1991). Using an eye tracker would probably have led to more consistent data, removing the trials where the participants did not look at the target without this task effect. But if we consider that the unintended eye movements are randomly distributed, they should

have affected both the estimations L_{Vij}^* and L_{Vij} leaving the ratio r_{Lij} mostly unaffected. Consequently, the estimation of the parameters would be roughly the same (fitting with the ratios would result in the same estimation of δ).

In Luckiesh and Guth's work, in order to identify equivalent sources, the Reference source was set to the same luminance for all participants: thus, it generated various levels of discomfort glare depending on the participant's sensitivity to glare. Conversely, in our study, the Reference source was set at each participant's BCD. The advantage is that our data truly described the equivalent source at a constant level of discomfort glare. This protocol may have introduced some noise in the data because each participant's BCD luminance was estimated with some level of uncertainty. However, in our study, the dependent variable was a ratio of two settings own to the participants. This allowed to remove the large inter-individual variability of discomfort sensitivity. Some variability remains (inter- and intra-participants), mostly due to the accuracy in the adjustment matching task.

We have considered a constant solid angle equal to 5×10^{-6} sr. But as the LED strip was stuck on a vertical screen, the solid angle actually decreased slightly as the eccentricity of the stimuli increased (for instance, stimulus at $\theta = 15.9^\circ$ was seen with a solid angle $\omega = 4.3 \times 10^{-6}$ sr). Also, the three test sessions were carried out in strict order (single, two and three sources). We needed the results of the first session (one-LED) in order to tune the two other sessions, but we could have counterbalanced the 2-LEDs and 3-LEDs session. *A posteriori*, no effect of inter-distance was found neither for 2-LED nor for 3-LED stimuli and both sets of data lead to the same value of α . This suggests that no learning or fatigue effect occurred in the experiment.

The reference LED was situated on the left-hand side; therefore data may be affected by an experimental bias (Poulton 1989). This can be checked with a null condition, comparing this Reference LED with the single-source stimulus at the same eccentricity on the right-hand side (see Fig. 2(a,b)). The mean luminance ratio (test LED/Reference LED) of this null condition is 0.80 (SD=0.22, N=31), and a sample two-

tailed t-test reveals a significant departure from unity ($p < 0.0001$). However, correcting the data would have led to the same results, as the ratio r_L is the same after correction.

7.5. Limits and perspectives

Our formula is only valid so far for the specific conditions of our experiment: for instance, a maximum number of three sources and a spatial arrangement of the sources on the horizontal axis. Thus, this formula deserves to be tested in more experimental conditions, including different spatial arrangements, more sources, other inter-distances, and background luminance values.

We have focused on the relative positions of light sources with the same spectral distribution. In future work, it would be important to investigate spectral heterogeneity, in order to also predict the discomfort from more realistic visual environments.

Our work is the first step towards a model of the discomfort glare due to multiple sources. The proposed formula allows to quantify the contribution G of each source to the glare and is directly build from multi-source stimuli (unlike most of the previous work). It is the main component of a future multi-source discomfort glare model. In the next step, the background luminance and the logarithm transformation of the individual source contribution G will be investigated to determine a full model, predicting the level of discomfort, based on our formula.

We have adopted the traditional point of view on discomfort glare, where each light source is described individually in term of spatial and photometric characteristics. The CIE (2013b) has recently suggested that image-based approaches are an alternative to this source-based approach. Some promising work has already been proposed, based on human vision models (Donners et al. 2015; Safdar et al. 2018; Scheir et al. 2018), but such image processing cannot yet predict discomfort glare, they are limited to correlation studies. In some sense, our multi-source formula may be seen as a first step to bridge the gap between pure

single-source models and the future image-based models.

8. Conclusion

A formula which predicts the contribution to discomfort glare of more than one source in the visual field was introduced in this paper. To that purpose, a psychovisual experiment has been conducted. No effect of the spatial arrangement of the sources was found for inter-distances ranging from 0.6° to 4.6° , neither below nor above 10° of eccentricity.

The parameters of the proposed formula have been estimated from the collected data. This formula allows to predict the characteristics of an equivalent source which produces the same level of discomfort glare (BCD) than several sources switched on together (with the same spectrum). It is only relevant for outdoor lighting with a background luminance of 1 cd/m^2 . However, the methodology proposed in this paper can be extended to others situations, including indoor lighting. Especially, contrary to the common approach in the previous work, the formula was directly derived from data collected on multi-source scenes. To complete this work and develop a multi-source discomfort glare model for outdoor lighting, further data at other levels of discomfort glare is needed.

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References

Ahmed I, Bennett CA. 1978. Discomfort glare: duration-intensity relationship. *J Illum Eng Soc.* 8(1):36–39.

- Bargary G, Jia Y, Barbur JL. 2015. Mechanisms for discomfort glare in central vision. *Invest Ophthalmol Vis Sci.* 56:464–471.
- Bennett CA. 1977. Discomfort glare: concentrated source-parametric study of angularly small sources. *J Ies.* 7(1):2–15.
- Boyce PR, Wilkins AJ. 2018. Visual discomfort indoors. *Lighting Res Technol.* 50(1):98–114.
- Bullough JD, Brons JA, Rea MS. 2008. Predicting discomfort glare from outdoor lighting installations. *Lighting Res Technol.* 40(3):225–242.
- Bullough JD, Hickcox KS. 2012. Interactions among light source luminance, illuminance and size on discomfort glare. *SAE Int J Passeng Cars Mech Syst.* 5(1):199–201.
- CEN. 2003. EN 13201-2 European standard: road lighting part 2 – performance requirements. Brussels (Belgium): European Committee for Standardization. Technical report.
- CIE. 1976. Glare and uniformity in road lighting installations. Paris: Commission Internationale de l'Éclairage. Technical Report 31. Vienna, Austria.
- CIE. 1995. Discomfort glare in interior lighting. Vienna: Commission Internationale de l'Éclairage. Technical Report 117. Vienna, Austria.
- CIE. 2002. Glare from small, large and complex sources. Vienna; Commission Internationale de l'Éclairage. Technical Report 147. Vienna, Austria.
- CIE. 2013a. International lighting vocabulary. Vienne: Commission Internationale de l'Éclairage. Technical Report DIS 017/E. Vienna, Austria.
- CIE. 2013b. Review of lighting quality measures for interior lighting with LED lighting systems, Vienne: Commission Internationale de l'Éclairage. Technical Report 205. Vienna, Austria.
- Clear RD. 2013. Discomfort glare: what do we know? *Lighting Res Technol.* 45:141–158.
- de Boer JB, Schreuder DA. 1967. Glare as a criterion for quality in street lighting. *Lighting Res Technol.* 32(2):117–137.
- Donners MAH, Vissenberg MCJM, Geerdinck LM, van den Broeck-Cools JHF. 2015. A psychophysical model of discomfort glare in both outdoor and indoor applications. In: 28th session of the Commission Internationale de l'Éclairage, Manchester, UK.
- Eble-Hankins ML, Waters CE. 2005. VCP and UGR glare evaluation systems: a look back and a way forward. *Leukos.* 1(2):7–38.
- Einhorn HD. 1969. A new method for the assessment of discomfort glare. *Lighting Res Technol.* 1(4):235–247.
- Einhorn HD. 1979. Discomfort glare: a formula to bridge differences. *Lighting Res Technol.* 11(2):90–94.
- Einhorn HD. 1991. Characteristics of discomfort glare from small and large sources. In: First international symposium on Glare, Orlando, FL. p. 101–114.
- Einhorn HD. 1998. UGR: merits and application to multiple sources. *Lighting Res Technol.* 30(2):89–93.
- Fotios S, Cheal C. 2010. Stimulus range bias explains the outcome of preferred-luminance adjustments. *Lighting Res Technol.* 42(4):433–447.

- Funke C, Schierz C. 2015. Extension of the unified glare rating formula for non-uniform LED luminaires. In: Proc. of the 28th session of the CIE, Manchester, UK.
- Havey LO, Pöppel E. 1972. Contrast sensitivity of the human retina. *Am J Optom.* 49:748–753.
- Hirasawa K, Okano K, Koshiji R, Funaki W, Shoji N. 2016. Smaller fixation target size is associated with more stable fixation and less variance in threshold sensitivity. *PLoS One.* 11:11.
- Hopkinson RG. 1940. Discomfort glare in lighted streets. *Trans IES.* 5:1–24.
- Hopkinson RG. 1972. Glare from daylighting in buildings. *Appl Ergon.* 3(4):206–215.
- Kasahara T, Aizawa D, Irikura T, Moriyama M, Toda M, Iwamoto M. 2006. Discomfort glare caused by white LED light sources. *J Light Vis Environ.* 30(2):95–103.
- Keay L, Palagyi A, Do V, White A, Lamoureux E, Ivers RQ, Pesudovs K, Stapleton F, Boufous S, McCluskey P. 2016. Vision and driving status of older australians with cataract: an investigation of public hospital waiting lists. *Clin Exp Optom.* 99:449–455.
- Kent MG, Fotios S, Altomonte S. 2019. Discomfort glare evaluation: The influence of anchor bias in luminance adjustments. *Lighting Res Technol.* 51:131–146.
- Kim W, Han H, Kim JT. 2009. The position index of a glare source at the borderline between comfort and discomfort (BCD) in the whole visual field. *Build Environ.* 44:1017–1023.
- Kim W, Kim JT. 2010. A position index formula for evaluation of glare source in the visual field. *Indoor Built Environ.* 20(1):47–53.
- Lappi O. 2014. Future path and tangent point models in the visual control of locomotion in curve driving. *J Vis.* 14(12):1–22.
- Levin RE. 1975. Position index in VCP calculations. *J Illum Eng Soc.* 4(2):99–105.
- Lin Y, Fotios S, Wei M, Liu Y, Guo W, Sun Y. 2015. Eye movements and pupil size constriction under discomfort glare. *Invest Ophthalmol Vis Sci.* 56:1649–1656.
- Lin Y, Liu Y, Sun Y, Zhu X, Lai J, Heynderickx I. 2014. Model predicting discomfort glare caused by led road lights. *Opt Express.* 22(15):18056–18071.
- Luckiesh M, Guth SK. 1949. Brightness in visual field at borderline between comfort and discomfort (BCD). *Illum Eng.* 44:650–670.
- Paul B, Einhorn H. 1999. Discomfort glare from small light sources. *Int J Light Res Technol.* 31(4):139–144.
- Petherbridge P, Hopkinson RG. 1950. Discomfort glare and the lighting of buildings. *Illum Eng.* 15(39):39–79.
- Poulton EC. 1989. Bias in quantifying judgments. Hove (UK): LEA.
- Poulton K. 1979. Discomfort glare. In: *Proc. of the 19th session of the CIE*, Kyoto, Japan.
- Rubin GS, Roche KB, Prasada-Rao P, Fried LP. 1994. Visual impairment and disability in older adults. *Optom Vis Sci.* 71(12):750–760.
- Safdar M, Luo MR, Mughal MF, Kuai S, Yang Y, Fu L, Zhu X. 2018. A neural response-based model to predict discomfort glare from luminance image. *Lighting Res Technol.* 50:416–428.
- Scheir GH, Donners M, Geerdinck LM, Vissenberg M, Hanselaer P, Ryckaert WR. 2018. A psychophysical model for visual discomfort based on receptive fields. *Lighting Res Technol.* 50:205–217.
- Schmidt-Clausen H-J, Bindels J. 1974. Assessment of discomfort glare in motor vehicle lighting. *Lighting Res Technol.* 6(2):79–88.
- Sivak M, Flannagan M, Ensing M, Simmons CJ. 1991. Discomfort glare is task dependent. *Int J Vehicle Des.* 12(2):152–159.
- Tashiro T, Kawanobe S, Kimura-Minoda T, Kohko S, Ayama M. 2015. Discomfort glare for white LED light sources with different spatial arrangements. *Lighting Res Technol.* 47:316–337.
- Thaler L, Schültz AC, Goodale MA, Gegenfurtner KR. 2013. What is the best fixation target? The effect of target shape on fixational eye movements. *Vision Res.* 76:31–42.
- Van Bommel WJM. 2015. Road Lighting: fundamentals, technology and applications. London (UK): Springer.
- Waters CE, Mistrick RG, Bernecker CA. 1995. Discomfort glare from sources of non-uniform luminance. *J Illum Eng Soc.* 24:73–85.
- Wienold J, Christoffersen J. 2006. Evaluation methods and development of a new glare prediction model for daylight environments with the use of CCD cameras. *Energy Build.* 38(7):743–757.
- Wilkins AJ. 1986. What is visual discomfort? *Trends Neurosci.* 8:343–346.
- Wilkins AJ, Nimmo-Smith I, Tait A, McManus C, Della Salla S, Tilley A, Arnold K, Barrie M, Scott S. 1984. A neurological basis for visual discomfort. *Brain.* 107(4):989–1017.
- Winter J, Fotios S, Völker S. 2017. Gaze direction when driving after dark on main and residential roads: where is the dominant location? *Lighting Res Technol.* 49(5):574–585.

Appendix A: Index

- R : mean level of discomfort glare;
- a, b : parameters of discomfort glare models;
- L_s : the luminance of the source (cd/m^2 or fL);
- ω_s : the solid angle of the source (sr);
- p_s : the position of the source in the visual field (described by the eccentricity θ_s , or by the Guth position index p_{Guth} of the source);
- α : exponent parameter on the source luminance in discomfort glare models;
- β : exponent parameter on the solid angle in discomfort glare models;
- γ : exponent parameter on the background luminance in discomfort glare models;
- δ : exponent parameter on the source position in discomfort glare models;

- n : number of sources;
- G_{eq} : the contribution of the equivalent light source to the discomfort glare;
- L_{eq} : the luminance of the equivalent source (cd/m^2);
- ω_{eq} : the solid angle of the equivalent source (sr);
- p_{eq} : the position of the equivalent source (degree);
- G_i : the contribution of each light source i to the discomfort glare;
- θ_s : eccentricity of the source (degrees or arc minutes);
- d_{ij} : the distance between sources i and j (degree);
- D_i : unknown function;
- ϕ : unknown function;
- L_{Refj} : the luminance of the reference LED adjusted by participant j ;
- L_{Fij} : the luminance of the source of fixed luminance i for each participant j ;
- L_{Vij}^* : the luminance of the variable source i (cd/m^2) that allows the participant j to feel at the BCD when the variable source is simultaneously switched on with the light source(s) of fixed luminance;
- L_{Vij} : the luminance of the same variable source i (cd/m^2) at BCD for the participant j when it is switched on alone;
- r_{Lij} : luminance ratio L_{Vij}^*/L_{Vij} of participant j for variable source i ;
- r_L : Mean luminance Ratio;
- r_{L2Sij} : dependent variable of the two-source stimuli;
- r_{L3Sij} : dependent variable of the three-source stimuli;
- G_{BCD} : G_{eq} at BCD level;
- N : number of participants.

Appendix B: Mean BCD and mean ratios

Tables 7 and 8 provide the means and standard deviations of the BCD luminance of the variable sources, and the mean luminance ratios, respectively, of the two- and three-source stimuli.

When the position of the source varies, and if one tries to keep the discomfort at the BCD, the source luminance changes. In the two-stimuli and three-stimuli experiments, when varying the inter-distance, the variable source position changed, and consequently the luminance tuned at the BCD level by the participants. It is highlighted in Tables 7 and 8 that the mean BCD luminance increases with the eccentricity.

The BCD luminance is different for each participant due to the large inter-individual variability of discomfort sensitivity: it explains the large standard deviation. The coefficient of variation (mean SD/mean of mean values) is around 0.74.

Table 7. Mean values of two-source stimuli: Mean luminance of the variable sources and mean luminance ratios defined in Eq. 5. Standard deviations are under the bracket.

Eccentricity range (deg)	Inter-distance (deg)	Mean L_{BCD} (cd/m^2)	Mean Ratio r_L
< 10°	0.6°	7835.2 [6242.4]	0.85 [0.40]
< 10°	1.2°	7710.3 [5960.6]	0.82 [0.34]
< 10°	2.3°	7661.3 [5178.0]	0.86 [0.40]
< 10°	4.6°	9206.7 [6215.6]	0.86 [0.22]
> 10°	0.6°	10852.2 [8285.8]	0.80 [0.29]
> 10°	1.2°	11103.2 [8260.8]	0.82 [0.26]
> 10°	2.3°	11783.5 [9110.4]	0.81 [0.28]
> 10°	4.6°	10832.9 [7579.9]	0.77 [0.28]
Mean		9623.2 [7104.2]	0.82 [0.31]

Table 8. Mean values of three-source stimuli: Mean luminance of the variable sources and mean luminance ratios defined in Eq. 5. Standard deviations are under bracket.

DUO	Eccentricity range (deg)	Inter-distance (deg)	Mean L_{BCD} (cd/m^2)	Mean Ratio r_L
DUO 1	< 10°	0.6°	7523.9 [6651.3]	0.86 [0.39]
DUO 1	< 10°	1.2°	7383.7 [5381.6]	0.83 [0.35]
DUO 2	< 10°	1.2°	8129.7 [6487.6]	0.90 [0.35]
DUO 2	< 10°	2.3°	8737.7 [7983.5]	0.78 [0.34]
DUO 3	< 10°	2.3°	8608.8 [6254.7]	0.91 [0.32]
DUO 3	< 10°	4.6°	11022.5 [8123.3]	0.82 [0.31]
DUO 4	> 10°	0.6°	10291.1 [8587.6]	0.81 [0.43]
DUO 4	> 10°	1.2°	10718.1 [9011.7]	0.75 [0.36]
DUO 5	> 10°	1.2°	10,562.5 [8407.2]	0.79 [0.29]
DUO 5	> 10°	2.3°	11242.7 [9206.1]	0.78 [0.32]
DUO 6	> 10°	2.3°	10439.4 [9095.1]	0.77 [0.36]
DUO 6	> 10°	4.6°	9904.4 [8816.8]	0.75 [0.35]
Mean			9547.0 [7833.9]	0.81 [0.34]

In order to investigate the effect of inter-distance independently of the source position, we have considered the ratio of the luminance of the source when it is switched on with another source(s) at the BCD, and the luminance of this same source alone at the BCD. This ratio does not depend on the variable source position, only on the inter-distance. Computing the ratio for each participant also removes the variability due to the difference of discomfort sensitivity among the participants: the coefficient of variation is around 0.38, nearly half of what was found with the BCD luminance data (see also Section 7.4).