# Generation of orthogonal codes with chaotic optical systems

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We propose to use an electro-optic oscillator based on two Mach-Zehnder modulators in two different delayed feedback loops to generate two orthogonal chaotic spreading sequences (codes). We numerically demonstrate, for such codes, spectrally-efficient multiplexing and demultiplexing of two digital data streams at multi-Gb/s rates using chaos synchronization and covariance-based detection. 

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Optical chaos-based communications are spread-spectrum techniques that typically exploit noise-like waveforms to convey and encrypt at the physical layer a single data stream and chaos synchronization to decrypt it [1]. Interest has grown recently in multiplexing chaotic signals [2–4] and multiple data streams [5,6] using nonlinear and delayed systems. In optics, only an application of the principle of wavelength division multiplexing (WDM) was considered with chaotic lasers [7–9]. A different approach to multiplexing relies on code-division multiple access (CDMA); this techniques makes use of multiple fixed pseudo-random binary

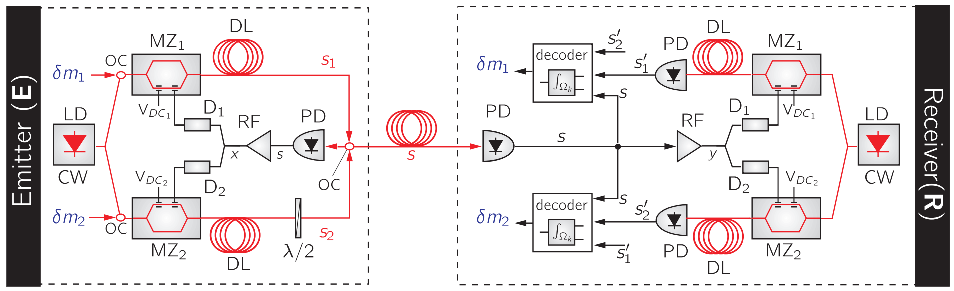


Fig. 1. Chaotic transmission chain using for the emitter E and receiver R a single optoelectronic oscillator with two cosine-square nonlinearities with different internal gain. LD: laser diode, OC: optical coupler, $\text{MZ}_{1,2}$: Mach-Zehnder modulator, DL: optical delay line, $\lambda/2$: half-wave plate, PD: photodetector, RF: band-pass amplifier, $\text{D}_{1,2}$: voltage divider, $m_{1,2}$: messages to be encrypted.

signals (known as *codes*) to spread out the spectrum of various binary data streams, which then overlap spectrally. At the receiver, duplicates of the codes are used to recover data using correlation-based detection [10]. Near-perfect orthogonality, defined as having a dot product between the binary codes close to zero, is crucial in CDMA to properly recover each user's binary message. We propose to substitute these fixed codes with optical chaotic signals (chaotic codes). In order to do so, the following issues must be solved: (i) the duplication of the chaotic codes at the receiver, (ii) their joint modulating and spectrum-spreading effect, and (iii) their non-stationarity resulting in difficulty of ensuring orthogonality [5].

In this letter, we propose to generate two orthogonal chaotic optical codes using an opto-electronic system. For this, the chaotic electro-optic oscillator (EOO) of ref. [11] is modified; two delayed feedback loops, each comprising a Mach-Zehnder modulator, are added. We later exploit the statistics of their outputs to generate the desired codes. We discuss the conditions that ensure their orthogonality and then numerically demonstrate how they can be used in a multiplexed two-user encryption/decryption at 2.5 Gb/s per user.

Figure 1 depicts a two-user optical chaos-based transmission chain based on our modified EOO with two feedback loops. Emitter (E) is composed of a monochromatic CW laser diode with optical power $P$ divided in two separate arms. In each arm the light is modulated by a Mach-Zehnder modulator ($\text{MZ}_i$) biased by voltage $V_{dc_i}$ and with constant-valued rf and dc half-wave voltages $V_{\pi_{rf_i}}$ and $V_{\pi_{dc_i}}$. The linearly polarized optical signals travel through different optical fibers $\mathrm{DL}_i$ imposing time delays $T_i$. Before being recombined and detected by a single photodetector PD (of efficiency $S$), the polarization direction of one of the signals is rotated by $\pi/2$ relative to the other with a half-wave plate ($\lambda/2$) to prevent optical interferences. The electrical signal generated by the PD is then amplified with gain $G$ and filtered by a band-pass filter with low and high cut-off frequencies $f_L$ and $f_H$. The total attenuation of each loop is denoted $g_i < 1$ and is obtained for instance by using a voltage divider $\mathrm{D}_i$, inducing different internal gains $\omega_i$ in the cosine-square nonlinearities because they reduce the electrical voltage $V(t)$ before driving the respective Mach-Zehnder modulator $\mathrm{MZ}_i$. The receiver (R) has a similar structure. This results in the following two dynamical models using similar notations to [11]:

$$\tau \frac{dx}{dt} + x + \frac{1}{\theta}\int_{t_0}^{t} x\,(u)\,du \quad = \quad s(t), \tag{1}$$

$$\tau \frac{dy}{dt} + y + \frac{1}{\theta}\int_{t_0}^{t} y\,(u)\,du \quad = \quad s(t - T_c), \tag{2}$$

with $x(t)$ and $y(t)$ the respective dimensionless RF variable of E and R, $x(t) = \pi g_1 G V(t)/2V_{\pi r f_1}$, $\theta = (2\pi f_L)^{-1}$, $\tau = (2\pi f_H)^{-1}$. $s(t) = \sum_{i=1}^{n=2} \beta \cos^2 (\omega_i x_{T_i} + \phi_{0i})$ is the multiplexed signal with $x_{T_i} = x(t - T_i)$, $\beta = g_1 G S P \pi/(2nV_{\pi_{rf_1}})$, $\phi_{0i} = \pi V_{dc_i}/(2V_{\pi_{dc_i}})$, and $\omega_i = g_i/g_1 V_{\pi_{rf_1}}/V_{\pi_{rf_i}}$. The dynamics of the synchronization error $e(t) = y(t) - x(t - \tau_c)$ between E and R is that of a damped oscillator, converging exponentially fast to zero. This ensures E and R to be chaotically synchronized. In the simulations, the transmission time $T_c$ is taken equal to zero without loss of generality.

The codes being generated from the state variable $x(t)$, we must first analyze its statistical properties. It is already known that an EOO with a single cosine-square nonlinearity can generate high-dimensional chaos with Gaussian statistics similar to Ikeda-like systems [12]. Figure 2 confirms that such statistics can be observed in our architecture with two feedback loops (for sufficiently large values of $\beta_i$). This can also be explained by the combination of the band-passing action of our EOO architecture with the “random-like” driving action of the delayed nonlinear feedbacks (a key feature of the feedback to observe Gaussian statistics with a single delayed EOO).

We propose to use the delayed output of each Mach-Zehnder modulator $s_i(t) =$

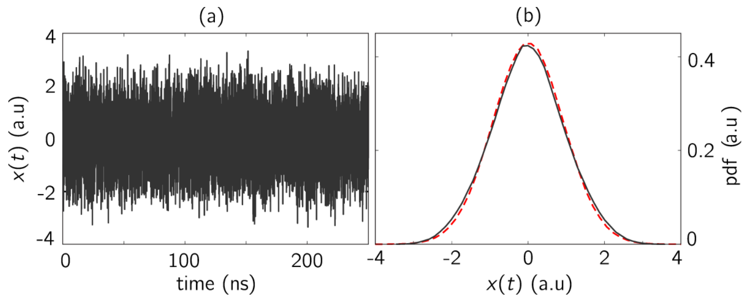


Fig. 2. (a) Time series of $x(t)$, (b) probability density function (gray line) of $x(t)$ and a theoretical Gaussian distribution (red dashed line). The parameters are $\tau = 25$ ps, $\theta = 10$ $\mu$s, $T_{i=1,2} = 30$ ns, $\beta_{i|i=1,2} = 5$, $\phi_{0i|i=1,2} = -\pi/4$, $\omega_2 = 2\omega_1 = 2$, and time step $\Delta t = 5$ ps.

$\beta_i \cos^2(\omega_i x_{T_i} + \phi_{0i})$ $(i = 1, 2)$ for the optical chaotic codes, as they process $x(t)$ with two different and strong nonlinearities. The properties of each code are controlled by the nonlinear gain $\beta_i$, the internal gain $\omega_i$, the offset phase $\phi_{0i}$, and bit duration $T_b$. To understand the influence of these four parameters on the orthogonality between the two chaotic codes $s_i(t)$, which corresponds to zero cross-covariance, we carry out a systematic numerical investigation. Orthogonality is studied as a function of internal gain difference $\Delta\omega_{ij} = \omega_i - \omega_j$, nonlinear gain $\beta_i$ (taken identical for the two nonlinearities for the codes to have approximately identical variance), and relative phase difference $\Delta\phi_{0ij} = \phi_{0i} - \phi_{0j}$. The bit duration $T_b$ may also impact orthogonality significantly. In particular, if $T_b$ is comparable or smaller than the time scale of the chaotic fluctuations of $x(t)$, a sound estimate of the

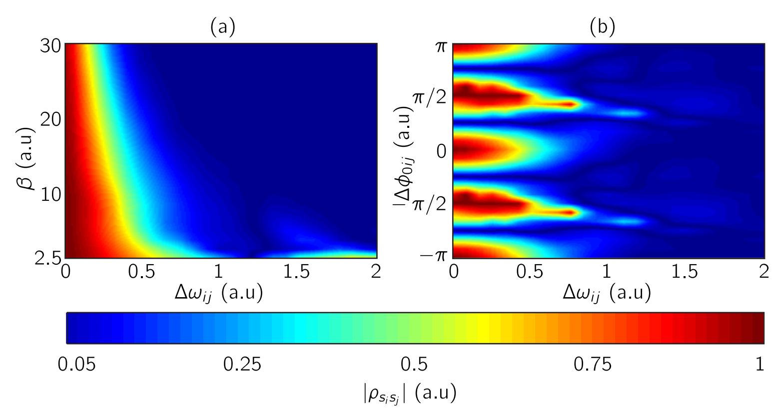


Fig. 3. Evolution of the normalized cross-covariance coefficient $\rho_{s_is_j}$ in the two-parameter space $(\Delta\omega_{ij}, \beta)$ in (a) and $(\Delta\omega_{ij}, \Delta\phi_{0ij})$ in (b).

cross-covariance cannot be obtained. To study orthogonality, we consider the normalized cross-covariance coefficient $\rho_{s_is_j} = \Gamma_{s_is_j}/[\Gamma_{s_is_i}\Gamma_{s_js_j}]^{1/2}$ calculated over a finite period $T_b$ with $\Gamma_{s_is_j} = \langle(s_i(t) - \langle s_i\rangle)(s_j(t) - \langle s_j\rangle)\rangle$ and $\langle\cdot\rangle$ the time-average operator.

Figure 3 plots $|\rho_{s_is_j}|$ as a function of $(\Delta\omega_{ij}, \beta)$ in Fig. 3(a), and $(\Delta\omega_{ij}, \Delta\phi_{0ij})$ in Fig. 3(b). The results are averaged over 5000 realizations of the chaotic codes with duration $T_b = 0.4$ ns. Figure 3(a), in which the phases are identical $\phi_{0i} = -\pi/4$ (i=1,2), confirms the existence of quasi-perfect orthogonality between the codes with short duration $T_b$ over a large region and highlights to this regard the key role of $\Delta\omega_{ij}$ and $\beta_i$. In Fig. 3(b), similar parameters to those of Fig. 3(a) are used except for the gains $\beta_i = 5$ (to ensure a hyperchaotic regime) and for the relative phase-shift $\Delta\phi_{0ij}$ that varies. It shows the existence, for small $\Delta\omega_{ij}$, of only four narrow zones of orthogonality, which merge as the internal gain difference $\Delta\omega_{ij}$ increases until quasi-perfect orthogonality is reached for any value of $\Delta\phi_{0ij}$.

Additional insight into the properties of $\Gamma_{s_is_j}$ may be gained if its analytical form is found. Toward this end, we assume $x(t)$ to be perfectly Gaussian with zero mean and variance $\sigma_x^2$ [based on our observation of Fig. 2(b)] and obtain in the $T_b \to \infty$ limit

$$\begin{aligned}\Gamma_{s_is_j} = \frac{\beta^2}{8}\left(1 - e^{-4\omega_i\omega_j\sigma_x^2}\right) e^{-2\Delta\omega_{ij}^2\sigma_x^2}\times \\ \left(\cos 2\Delta\phi_{0ij} + \cos(2\phi_{0i} + 2\phi_{0j})e^{-4\omega_i\omega_j\sigma_x^2}\right).\end{aligned} \tag{3}$$

Equation 3 explains how cross-covariance behaves with $\Delta\omega_{ij}$, $\beta$, and $\Delta\phi_{0ij}$. Since $\sigma_x^2$ varies like $\beta^2$ for large $\beta$ [12], Eq. 3 demonstrates that $\Gamma_{s_is_j}$ decreases exponentially fast with $\beta^2$ and $\Delta\omega_{ij}$, hence explaining the decorrelation obtained for large values of these two parameters.

Our orthogonal codes can be used to multiplex and demultiplex two messages in the spirit of CDMA. This can be achieved by digitally modulating each nonlinear gain $\beta_i$ at the rate $1/T_b$ with the binary message of the $i$th user $m_i$. The signal $s(t)$ now reads $s(t) = \sum_{i=1}^{n=2}\beta_i(1 + \delta m_i)\cos^2(\omega_i x_{T_i} + \phi_{0i})$, where $\delta$ is modulation depth that satisfies $|\delta| \ll 1$ and $m_i(t) = \pm 1$. For the decryption, R produces the duplicates $s_i'(t)$ of each code $s_i(t)$ with chaos synchronization and then performs covariance-based detection. The extraction of each message is based on

$$\hat{m}_{i=\{1,2\}} \simeq \frac{1}{\delta\Gamma_{s_i's_i'}}\left(\Gamma_{ss_i'} - \Gamma_{s_i's_i'} - \Gamma_{s_i's_j'}\right), \tag{4}$$

with $s_i'(t) = \beta_i\cos^2(\omega_i y_{T_i} + \phi_{0i})$. $\Gamma_{ss_i'}$ and $\Gamma_{s_is_j'}$ are the cross-covariance of the $(s, s_i')$ and $(s_j', s_i')$

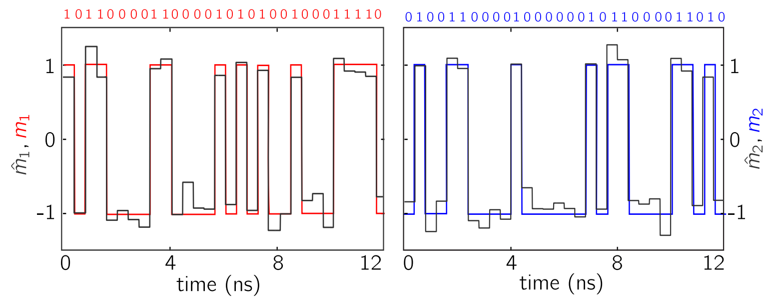


Fig. 4. Decryption of two binary messages transmitted simultaneously at 2.5 Gb/s per user. The colored and gray lines represent the encrypted and decrypted messages, respectively. The simulation parameters are identical to those of Fig. 1.

pairs calculated during $T_b$, respectively. The decoding equation for the $i$th user originates from the calculation of $\Gamma_{ss'_i} = (1 + \delta m_i)\Gamma_{s_i s'_i} + (1 + \delta m_j)\Gamma_{s_j s'_i}$. Neglecting $\delta m_j \Gamma_{s_j s'_i}$ thanks to the quasi-perfect orthogonality between the two codes leads to $\Gamma_{ss'_i} \simeq (1 + \delta m_i)\Gamma_{s_i s'_i} + \Gamma_{s_j s'_i}$. E and R being chaotically synchronized, signals $s_j$ and $s'_j$ are equal, which finally leads to Eq. 4. It is similar to the decoding equation used in [5], except that cross-covariance are used instead of cross-correlation. Figure 4 illustrates our approach with the demultiplexing of two data streams encoded at 2.5 Gb/s (bit rate of the OC-48 standard), thus resulting in a cumulative bit rate of 5 Gb/s. We have also checked that the simulations are robust to realistic levels of noise and parameter mismatch. Furthermore, a crucial property of our encoding technique is that significant increase of spectral efficiency is achieved by comparison to a similar encryption technique using an EOO-based architecture with a single loop. Using identical parameters to those of Fig. 4 (including bit rate) for both the single and double-loop EOO, it is found that the spectral efficiency approximately doubles in the latter case.

We have restricted ourselves to the case of two optical lines to avoid optical interferences at the photodetector resulting form the optical summation, but our structure can be generalized to more than two optical lines to achieve even better spectral efficiency. However, the unavoidable optical interferences would impact the dynamics, the statistics of $x(t)$ and $s(t)$, and the decoding equation. These effects and their consequences on the modeling and performance of the architecture will be further discussed elsewhere.

In this letter, we have demonstrated how to advantageously use a single electro-optic oscillator (EOO) with two delayed feedback loops comprising Mach-Zehnder modulators to generate two orthogonal chaotic codes. Orthogonality is easily achieved when the gain $\beta_i$ of each cosine-square nonlinearity and the difference in internal gain $\Delta\omega_{ij}$ are sufficiently large. This allows for the orthogonal codes to be used as carriers and ensures a cross-talk free decryption of messages. Encryption and decryption of two messages at 2.5 Gb/s per user were numerically demonstrated. Consequently, our approach may constitute a first significant step towards multiplexed and spectrally-efficient optical chaos-based communications.

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