Theory of the μ and the τ structures

Guy Michel M Stephan

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G. M. Stephan

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Résumé

A theory of the muon and the tau which is based on the properties of the electromagnetic potential is developed. The properties of the tensors formed by the partial derivatives of this potential in Minkowski spacetime are obtained from fundamental principles. In the proper frame of reference, the scalar potential obeys a Helmoltz equation whose solutions describe electromagnetic particles. These are characterized by a mass part and a field part. The mass part gives the potential energy. The muon and the tau particles result from an association of the first spherically symmetric solutions. We compute their energy and normalize it with respect to that of the electron. An agreement of 3 10^{-3} is obtained with experimental results.

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1 Introduction.

Like the electron, the muon[1] and the tau[2] are known to be point-like particles with the same elementary electric charge. These characteristics and the absence of quarks in their structure lead to classify them as elementary particles in the Standard Model. Masses of the muon and the tau are respectively 207 and 3477 times that of the electron and they are sometimes called heavy electrons. Up to now, there is no explanation for these numbers. This is only one example among the unexplained facts of the world of elementary particles. One aim of this article is to offer a new way to solve some of these mysteries.
The standard model, as well as modern theoretical physics, is known to be a vulnerable theory([3, 4, 5, 6, 7]) and new ways to decipher fundamental phenomena at the femtometer scale are highly desirable([8, 9]). We have recently begun to develop the "Potential Theory"[10] which is nothing but the study of the properties of the electromagnetic 4-potential A^i in the 4-dimensional Minkowski spacetime. The partial derivatives of A^i form tensors whose antisymmetric parts are related to the electromagnetic field. We have discovered that a symmetric part, when submitted to the constraints of symmetry invariance leads to a Helmholtz equation. Its solutions describe a concentration of potential around the origin of coordinates and represent electromagnetic (e.m.) particles. Each of them is characterized by 3 quantum numbers (n, ℓ and m) and a parity (even or odd). Far from the origin, the tensors reduce to waves in $1/r$ which extend to infinity. We have associated a Lagrangian to them which allows the calculation of the distribution of potential energy in space and the total energy of each particle. We were able to relate the electric charge to the tensor of derivatives. Now the question is to know if these e.m. particles have something to do with observed elementary particles. We have started this research with the idea that they are the elementary bricks forming them. Assembling these components together would ultimately give the structure, energy and properties of the atomic nuclei. One of the aims of this article is essentially to give arguments which sustain this idea. The first of these arguments is that the theory is very simple : the basic ingredient is the well-known electromagnetic potential to which two fundamental laws of nature are applied : these are the principle of least action (expressed by Euler-Lagrange equations) and the principle of symmetry (expressed by Noether's theorems). The second argument which is illustrated in this article is the comparison between the theoretical and experimental energies of the muon and the tau particles where we recover the numbers above.
The first section describes the potential theory and how it leads to the Helmholtz equation for the scalar potential. The potential vector and the tensor of derivatives at each event M in spacetime are obtained for each solution. These vectors and tensors are extensive quantities and a compound particle can tentatively be

1. email : gmstephan@wanadoo.fr
Retired from University de Rennes, (Unité 6082 associée au Centre National de la Recherche Scientifique), 6, rue de Kerampont, 22305 Lannion (France)

described by adding the potential of each component. In the second section we add several of these solutions to build the μ and the τ. In each case the puzzle was to find their composition in terms of e.m. particles by comparing experimental and calculated energies. These energies are referred to that of the electron. An excellent agreement is obtained when the frequencies of the fundamental solutions are properly chosen. These results bring the hope to understand the chemistry of matter at the scale of the atomic nucleus.

2 Potential theory.

2.1 Overview.

The main lines of potential theory are described in this section.
Potential theory is classical, very simple, without any new concept : it rests on the existence at each point (event) M in spacetime of an electromagnetic 4-potential A^i. M is defined by its temporal and spatial coordinates. The potential A^i is defined by its temporal component ϕ/c and the vector $\overrightarrow{A}$. These components are functions of coordinates.
The gradient $D(A^i)$ of A^i is the basic object of the theory. The partial derivatives which appear in $D(A^i)$ should be split into two independant parts, one of which is related to the external and the other to the internal degrees of freedom by analogy to the classical theory of a moving body. The first part gives birth to Maxwell equations (the particle moves with respect to the observer), the second part gives the internal description of the particles we are interested in (the center of gravity of the particle is at rest with respect to the observer).
There are 16 partial derivatives $A^{\ i}_{,k} = \partial A^i/\partial x^k$ which are the local slopes of components A^i in directions x^k. These are the components of the second rank gradient tensor $D(A^i) = [A^{\ i}_{,k}]$ in the direct space. The properties of $D(A^i)$ are developed, using its invariants and symmetries in operations of the Poincaré group. One finds that a proper time exists, in which the observer is at rest with respect to the tensor and where a time slope compensates for a space slope. When this property is associated to the invariance of the trace of the tensor in a time translation, one finds a Helmholtz equation for the scalar potential in the permanent oscillatory regime. Solutions of this equation describe the "Electromagnetic Particles"([10]). These are characterized by 3 quantum numbers : n, ℓ and m . Each triplet n, ℓ, m correspond to an odd and an even solution. A vector field A^i and a tensor field $D(A^i)$ describe each particle n, ℓ, m in spacetime. Around the origin, there is a concentration of potential but, far from it, the particle reduces to incoming and outgoing spherical waves in $1/x$ extending to infinity. It is the balance between the energies of these waves which ensures the stability of the particle. This result exactly fits the Wheeler-Feynman's absorber theory([13])
The closely related tensor $D(A_i) = [\partial A_i/\partial x^k]$ needs the existence of the reciprocal space and is related to the external degrees of freedom. This tensor has properties which are different from those of $D(A^i)$ and is the basic object of classical electromagnetism([12]). It is necessary to describe the properties of a moving particle as seen by an external observer. It is not used here.
The local Lagrangian density of the particle is found to be proportional to the determinant of the tensor $D(A^i)$. This Lagrangian leads to the expressions for the field and mass energies which are respectively associated to the antisymmetric and symmetric parts of $D(A^i)$. The duality field-matter naturally appears to be a consequence of this splitting. The electric charge can also be obtained from $D(A^i)$: one finds that all solutions are globally neutral but solutions (1, 0, 0) which represent the electron and the positron following the parity. A complete description of their structure, including the spin, has been given([11]).
The next step that we are developing in this article concerns the association of electromagnetic particles around the same center in order to see if the potential theory is still able to describe the elementary particles which have been discovered essentially in the second half of the XXth century. We have concentrated the present study on the case of the muon and the tau particles which have a spherical symmetry.

2.2 The field and the mass tensors.

In Minkowski spacetime, a contravariant 4-potential is written $A^i = (\phi/c, \overrightarrow{A})$ as indicated above. An event M has coordinates cT, X, Y, Z with respect to an origin 0_o. Now we introduce another system of coordinates whose origin O is at the center of gravity of the particle. In this system, the sets t, x, y, z are the coordinates of M with respect to origin 0 (internal degrees of freedom) and the sets $\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}$ are the coordinates of O with respect to O_o (external degrees of freedom). The simplest relations between these sets are :

$$T = t + \tilde{t} \quad , \; X = x + \tilde{x} \quad , \; Y = y + \tilde{y} \quad , \; Z = z + \tilde{z} \tag{1}$$

Derivatives are split into two parts, each of them is related to a coordinate with or without tilde. For example $\partial A^i/\partial X = \partial A^i/\partial \tilde{x} + \partial A^i/\partial x$. The gradient tensor of the potential A^i can thus be split into two independent parts which describe its properties with respect to the observers at O_o and O. In the following we focus our attention on the description of the properties of the second part .
The tensor of derivatives $[a_k{}^i]$, can be written in the cartesian frame as :

$$[a_k{}^i] = \left[\partial A^i/\partial x^k\right] \tag{2}$$

where $x^k = ct, x, y, z$. $[a_k{}^i]$ is defined in the direct (or real) space. A derivative $\partial A^i/\partial x^k$ will be noted as $A^i_{,k}$ and we can write $[a_k{}^i]$ in the extended form as :

$$[a_k{}^i] = \frac{1}{2}\begin{bmatrix} (\phi/c)_{,t} & A^x{}_{,t} & A^y{}_{,t} & A^z{}_{,t} \\ (\phi/c)_{,x} & A^x{}_{,x} & A^y{}_{,x} & A^z{}_{,x} \\ (\phi/c)_{,y} & A^x{}_{,y} & A^y{}_{,y} & A^z{}_{,y} \\ (\phi/c)_{,z} & A^x{}_{,z} & A^y{}_{,z} & A^z{}_{,z} \end{bmatrix} \tag{3}$$

A fundamental operation in tensor theory is the splitting into a symmetric and an antisymmetric part. There are two ways to obtain such tensors from $[a_k{}^i]$:
1- The first way uses the reciprocal space[14] together with the direct space to define the covariant tensor : $[a_{ki}] = [a_k{}^j][g_{ji}]$ where $[g_{ji}] = (1, -1, -1, -1)$ is the metric tensor. One obtains :

$$[a_{ki}] = \frac{1}{2}\begin{bmatrix} (\phi/c)_{,t} & -A^x{}_{,t} & -A^y{}_{,t} & -A^z{}_{,t} \\ (\phi/c)_{,x} & -A^x{}_{,x} & -A^y{}_{,x} & -A^z{}_{,x} \\ (\phi/c)_{,y} & -A^x{}_{,y} & -A^y{}_{,y} & -A^z{}_{,y} \\ (\phi/c)_{,z} & -A^x{}_{,z} & -A^y{}_{,z} & -A^z{}_{,z} \end{bmatrix} \tag{4}$$

The transpose $[a_{ki}]^T = [a_{ik}]$ is used to split $[a_{ki}]$ into its symmetric and antisymmetric parts :

$$[a_{ki}] = [f_{ki}] + [s_{ki}] \tag{5}$$

with $[s_{ki}] = 1/2\,(a_{ki} + a_{ik})$ and $[f_{ki} = 1/2\,(a_{ki} - a_{ik})$. The electromagnetic field tensor is the antisymmetric part :

$$\begin{aligned} [F_{ki}] &= \frac{1}{2}\begin{bmatrix} 0 & -A^x{}_{,t} - (\phi/c)_{,x} & -A^y{}_{,t} - (\phi/c)_{,y} & -A^z{}_{,t} - (\phi/c)_{,z} \\ (\phi/c)_{,x} + A^x{}_{,t} & 0 & -A^y{}_{,x} + A^x{}_{,y} & -A^z{}_{,x} + A^x{}_{,z} \\ (\phi/c)_{,y} + A^y{}_{,t} & -A^x{}_{,y} + A^y{}_{,x} & 0 & -A^z{}_{,y} + A^y{}_{,z} \\ (\phi/c)_{,z} + A^z{}_{,t} & -A^x{}_{,z} + A^z{}_{,x} & -A^y{}_{,z} + A^z{}_{,y} & 0 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 0 & E^x/c & E^y/c & E^z/c \\ -E^x/c & 0 & -B^z & B^y \\ -E^y/c & B^z & 0 & -B^x \\ -E^z/c & -B^y & B^x & 0 \end{bmatrix} \end{aligned} \tag{6}$$

We have used[15] the standard notations and definitions of the components of the electric and magnetic fields $\overrightarrow{E}$ and $\overrightarrow{B}$:

$$\overrightarrow{E} = -\overrightarrow{grad}\phi - \frac{\partial \overrightarrow{A}}{\partial t} \quad \text{and} \quad \overrightarrow{B} = \overrightarrow{curl}\,\overrightarrow{A} \tag{7}$$

This tensor, is the cornerstone of classical electromagnetism. It remains antisymmetric whatever the coordinate system. Maxwell equations and the Lorentz force can be deduced([12]) from $[a_{ki}]$ using the principle of least action (Euler-Lagrange equations) and the principle of symmetry (Noether's theorems). The field propagates **transversally** and its source originates from the motion of electric charges. This way of obtaining the symmetric and antisymmetric parts of the tensor of derivatives $D(A^i)$ is well adapted to the first part of $D(A^i)$, the part which contains the derivatives with respect to the coordinates of O in the reference frame attached to the external observer O_o.

2- The second way is the splitting of $[a_k{}^i]$ into its symmetric and antisymmetric parts in the direct space (without any appeal to the reciprocal space). The symmetric part writes :

$$[\overline{s}_k{}^i] = \frac{1}{2}\begin{bmatrix} 2(\phi/c)_{,t} & A^x{}_{,t} + (\phi/c)_{,x} & A^y{}_{,t} + (\phi/c)_{,y} & A^z{}_{,t} + (\phi/c)_{,z} \\ (\phi/c)_{,x} + A^x{}_{,t} & 2A^x{}_{,x} & A^y{}_{,x} + A^x{}_{,y} & A^z{}_{,x} + A^x{}_{,z} \\ (\phi/c)_{,y} + A^y{}_{,t} & A^x{}_{,y} + A^y{}_{,x} & 2A^y{}_{,y} & A^z{}_{,y} + A^y{}_{,z} \\ (\phi/c)_{,z} + A^z{}_{,t} & A^x{}_{,z} + A^z{}_{,x} & A^y{}_{,z} + A^z{}_{,y} & 2A^z{}_{,z} \end{bmatrix} \tag{8}$$

The antisymmetric part $[\overline{f}_k{}^i]$ has zeros on the diagonal and differences instead of sums in each non-diagonal element.
$[\overline{s}_k{}^i]$ is named the mass tensor and $[\overline{f}_k{}^i]$ the field tensor in this system of coordinates. Their properties lead to a completely new aspect of electromagnetism as shown below.

2.3 The Helmholtz equation.

Being symmetric, $[\overline{s}_k{}^i]$ can be diagonalized provided its determinant does not vanish. This means that a time coordinate $\bar{t}$ exists such that terms $\overline{s}_1{}^{(2,3,4)} = \overline{s}_{(2,3,4)}{}^1 = 0$. We name $\bar{t}$ the proper time of the tensor. In this coordinate frame, we use the bared symbols $c\bar{t}, \bar{x}, \bar{y}, \bar{z}$ and : $\overline{A}^i = (\overline{\phi}/c, \overline{A^x}, \overline{A^y}, \overline{A^z})$. The ordinary derivatives are replaced by absolute derivatives. In the proper time system, one has the relations :

$$\overline{A^x}_{,t} + \overline{(\phi/c)}_{,x} = \overline{A^y}_{,t} + \overline{(\phi/c)}_{,y} = \overline{A^z}_{,t} + \overline{(\phi/c)}_{,z} = 0 \qquad \text{or} \qquad \text{grad}\overline{(\phi/c)} + \frac{\partial \overrightarrow{\overline{A}}}{c\partial\bar{t}} = 0. \tag{9}$$

Here a symbol like $\overline{(\phi/c)}_{,x}$ generally stands for the absolute derivative (including Christoffel's coefficient) of $\overline{(\phi/c)}$ with respect to $\bar{x}$. The second formulation is tensorial. Eq.(9) means that in the proper time, the temporal derivative $\overline{A^i}_{,t}$ is compensated by the spatial derivative of $\overline{(\phi/c)}$ in the direction $\bar{i}$. Note that the (transverse) field defined by eq.(7) vanishes in the proper frame where there is no motion of the particle.
Now we will use the invariants of $[\overline{s}_k{}^i]$ in a coordinate transformation. There are four scalar invariants which are the coefficients of the characteristic polynomial. The most well-known are the trace and the determinant. We use the property of the trace of $[\overline{s}_k{}^i]$ to be invariant in a time translation to obtain :

$$\frac{\partial}{c\partial\bar{t}}\left(\frac{\partial\overline{(\phi/c)}}{c\partial\bar{t}} + \frac{\partial\overline{A^x}}{\partial\bar{x}} + \frac{\partial\overline{A^y}}{\partial\bar{y}} + \frac{\partial\overline{A^z}}{\partial\bar{z}}\right) = 0 \qquad \text{or} \qquad \frac{\partial^2\overline{(\phi/c)}}{c^2\partial\bar{t}^2} + \text{div}\frac{\partial\overrightarrow{\overline{A}}}{c\partial\bar{t}} = 0. \tag{10}$$

Equations (9) and (10) are combined to give :

$$\frac{\partial^2\overline{(\phi/c)}}{c^2\partial\bar{t}^2} - \Delta\overline{(\phi/c)} = 0 \quad , \tag{11}$$

where the symbol Δ stands for the Laplacian.
In the following, we will be interested in permanent oscillatory potentials which are proportional to $\cos\omega t$ or $\sin\omega t$. These potentials obey a Helmholtz-type equation :

$$\frac{\omega^2}{c^2}\ \overline{(\phi/c)}_{spatial} + \Delta\overline{(\phi/c)}_{spatial} = 0, \tag{12}$$

where $\overline{(\phi/c)}_{spatial}$ represents the spatial part of $\overline{(\phi/c)}$. This equation is a tensor equation which remains the same in any geometrical system of coordinates.

2.4 Solutions of the Helmholtz equation : Electromagnetic particles.

Helmholtz equation can be written in the spherical reference frame attached to M with the proper time $\bar{t}$ and the geometrical coordinates (r, θ, φ) such that :

$$\bar{x} = r\sin\theta\cos\varphi \quad , \quad \bar{y} = r\sin\theta\sin\varphi \quad , \quad \bar{z} = r\cos\theta \quad , \tag{13}$$

The advantage of the (r, θ, φ) system is that it makes use of the spherical or cylindrical symmetry of the solutions that we are going to use.
We introduce the *normalized* distance to the origin O of coordinates : x= $\omega r/c$. (Note the typography which is different from that of the coordinate x). This distance will thus be measured in units of the reference length c/ω.
Eq.(12) has extensively been studied in the context of the hydrogen atom where some of its solutions describe the electronic orbitals[16].
Solutions of eq.(12) can be split into normal and coupled modes :

1- Coupled angular-radial modes describe simultaneous vibrations on the three coordinates. They are obtained from the *ansatz* : $(\overline{\phi/c})_{spatial} = R(r)\ \Theta(\theta)\ \Phi(\varphi)$ where $R(r)$, $\Theta(\theta)$, and $\Phi(\varphi)$ are functions of r, θ, and φ respectively. One thus obtains the coupled angular-radial modes in terms of spherical Bessel functions of order ℓ $J_\ell(\mathrm{x})$ and spherical harmonics $Y_\ell^m(\theta,\varphi)$:

$$\phi_{\ell,m}(\mathrm{x},\theta,\phi) = \mathcal{A}_{\ell,m}\ J_\ell(\mathrm{x})\ Y_\ell^m(\theta,\varphi)\ \left\{ \begin{array}{l} \cos\omega_{\ell,m}\ t \\ \sin\omega_{\ell,m}\ t \end{array} \right. \quad . \tag{14}$$

Here $J_\ell(\mathrm{x})$ is a solution of the radial equation :

$$\mathrm{x}^2 + \frac{1}{R}\ \frac{\partial}{\partial \mathrm{x}}\ \left(\mathrm{x}^2\ \frac{\partial R}{\partial \mathrm{x}}\right) = \ell(\ell+1), \tag{15}$$

$Y_\ell^m(\theta,\varphi)$ describe the solutions of the angular part of eq.(12). The term $\ell(\ell+1)$ originates from the radial angular coupling and can be considered as a source term which fixes the properties of the radial solution. Quantities $\mathcal{A}_{\ell,m}$ and $\omega_{\ell,m}$ are unknown for the moment. They will be noted as $\mathcal{A}_n$ and ω_n for brevity. Amplitude $\mathcal{A}_n$ has the dimension $[\mathcal{A}_n] = ML^2T^{-2}Q^{-1}$ in the standard nomenclature.

2- Normal modes describe vibrations on one of the three coordinates. They are obtained from the *ansatz* : $(\overline{\phi/c})_{spatial} = R(r)\ + \Theta(\theta)\ + \Phi(\varphi)$. As the particle is embedded in noise, it is necessary to take it into account. At this stage, we consider it to be isotropic (independent of θ and ϕ), in such a way that it can excite the radial part $R(r)$ only. The potential N which represents the noise can be expanded on the basis of spherical Bessel functions and each component $N_n(\mathrm{x})$ obeys a spherical Bessel equation :

$$\mathrm{x}^2\ \frac{\partial^2 N_n}{\partial \mathrm{x}^2}\ + 2\mathrm{x}\ \frac{\partial N_n}{\partial \mathrm{x}}\ + \mathrm{x}^2\ N_n\ = \mathcal{C}_n\ \ , \tag{16}$$

This equation becomes "well-behaved" with the solution J_n if $\mathcal{C}_N = n(n+1)$. Functions N_n are the local building elements of the noise. They correspond to any integer value of n and are the source for the excitation of the spherical normal modes n.
Grouping coupled and normal modes together shows that the potential which describes a particle finally depends on three quantum number n, ℓ, m : $\phi/c = \phi/c(n,\ell,m)$. The general expression for a solution of Helmholtz equation is :

$$\phi_{n,\ell,m}(\mathrm{x},\theta,\phi) = \mathcal{A}_{n,\ell,m}\ J_n(\mathrm{x})\ Y_\ell^m(\theta,\varphi)\ \left\{ \begin{array}{ll} \cos\omega_{n,\ell,m}\ t & \text{(even solutions)} \\ \sin\omega_{n,\ell,m}\ t & \text{(odd solutions)} \end{array} \right. \quad . \tag{17}$$

We will need the explicit values of the first spherical harmonics :

$$Y_0^0 = \sqrt{\frac{1}{4\pi}} \qquad Y_1^0 = \sqrt{\frac{3}{4\pi}}\ \cos\theta$$
$$Y_1^1 = -\sqrt{\frac{3}{2\pi}}\ \sin\theta\ \cos\varphi \qquad Y_1^{-1} = \sqrt{\frac{3}{2\pi}}\ \sin\theta\ \sin\varphi$$

and the first spherical Bessel functions :

$$\begin{aligned} J_0 &= J_0(\mathrm{x}_0) = \frac{\sin \mathrm{x}_0}{\mathrm{x}_0} \\ J_1 &= J_1(\mathrm{x}_1) = \frac{\sin \mathrm{x}_1}{\mathrm{x}_1^2} - \frac{\cos \mathrm{x}_1}{\mathrm{x}_1} \end{aligned} \tag{18a}$$

Expression (9) allows the computation of the components of the vector potential in the proper time frame from the scalar potential.
These solutions obey the physical boundary conditions : they become asymptotically null far from the origin and they are either finite or null at the origin.
Let us now write explicitly the two tensors that we will need to describe the μ and the τ particles. The general expression of the gradient tensor in the spherical system of coordinates associated to point M is :

$$[a_k{}^i] = \left[\begin{array}{cccc} \phi_{,t} & A^r_{,t} & A^\theta_{,t} & A^\varphi_{,t} \\ \phi_{,r} & A^r_{,r} & A^\theta_{,r} & A^\varphi_{,r} \\ \phi_{,\theta}/r & \frac{1}{r}\left(A^r_{,\theta} - A^\theta\right) & \frac{1}{r}\left(A^\theta_{,\theta} + A^r\right) & \frac{1}{r}\ A^\varphi_{,\theta} \\ \frac{\phi_{,\varphi}}{r\sin\theta} & \frac{A^r_{,\varphi}}{r\sin\theta} - \frac{A^\varphi}{r} & \frac{1}{r\sin\theta}\left(A^\theta_{,\varphi} - \cos\theta A^\varphi\right) & \frac{1}{r\sin\theta}\ A^\varphi_{,\varphi} + \frac{1}{r}\ A^r + \frac{1}{r}\frac{\cos\theta}{\sin\theta}\ A^\theta \end{array} \right] \tag{19}$$

Its symmetric part (the mass part) is :

$$[s_k{}^i] = \frac{1}{2}\begin{bmatrix} 2\phi_{,t} & A^r_{,t}+\phi_{,r} & A^\theta_{,t}+\phi_{,\theta}/r & A^\varphi_{,t}+\frac{\phi_{,\varphi}}{r\sin\theta} \\ \phi_{,r}+A^r_{,t} & 2A^r_{,r} & A^\theta_{,r}+\frac{1}{r}\left(A^r_{,\theta}-A^\theta\right) & A^\varphi_{,r}+\frac{A^r_{,\varphi}}{r\sin\theta}-\frac{A^\varphi}{r} \\ \phi_{,\theta}/r+A^\theta_{,t} & \frac{1}{r}\left(A^r_{,\theta}-A^\theta\right)+A^\theta_{,r} & 2\frac{1}{r}\left(A^\theta_{,\theta}+A^r\right) & \frac{1}{r}\ A^\varphi_{,\theta}+\frac{1}{r\sin\theta}\left(A^\theta_{,\varphi}-\cos\theta A^\varphi\right) \\ \frac{\phi_{,\varphi}}{r\sin\theta}+A^\varphi_{,t} & \frac{A^r_{,\varphi}}{r\sin\theta}-\frac{A^\varphi}{r}+A^\varphi_{,r} & \frac{1}{r\sin\theta}\left(A^\theta_{,\varphi}-\cos\theta A^\varphi\right)+\frac{1}{r}\ A^\varphi_{,\theta} & 2\left(\frac{1}{r\sin\theta}\ A^\varphi_{,\varphi}+\frac{1}{r}\ A^r+\frac{1}{r}\frac{\cos\theta}{\sin\theta}\ A^\theta\right) \end{bmatrix} \tag{20}$$

The antisymmetric part (the field part) is :

$$[s_k{}^i] = \frac{1}{2}\begin{bmatrix} 0 & A^r_{,t}-\phi_{,r} & A^\theta_{,t}-\phi_{,\theta}/r & A^\varphi_{,t}-\frac{\phi_{,\varphi}}{r\sin\theta} \\ \phi_{,r}-A^r_{,t} & 0 & A^\theta_{,r}-\frac{1}{r}\left(A^r_{,\theta}-A^\theta\right) & A^\varphi_{,r}-\frac{A^r_{,\varphi}}{r\sin\theta}+\frac{A^\varphi}{r} \\ \phi_{,\theta}/r-A^\theta_{,t} & \frac{1}{r}\left(A^r_{,\theta}-A^\theta\right)-A^\theta_{,r} & 0 & \frac{1}{r}\ A^\varphi_{,\theta}-\frac{1}{r\sin\theta}\left(A^\theta_{,\varphi}-\cos\theta A^\varphi\right) \\ \frac{\phi_{,\varphi}}{r\sin\theta}-A^\varphi_{,t} & \frac{A^r_{,\varphi}}{r\sin\theta}-\frac{A^\varphi}{r}-A^\varphi_{,r} & \frac{1}{r\sin\theta}\left(A^\theta_{,\varphi}-\cos\theta A^\varphi\right)-\frac{1}{r}\ A^\varphi_{,\theta} & 0 \end{bmatrix} \tag{21}$$

Expressions of these tensors are obtained with the proper time :

$$[s_k{}^i] = \frac{1}{2}\begin{bmatrix} 2\phi_{,t} & 0 & 0 & 0 \\ 0 & 2A^r_{,r} & A^\theta_{,r}+\frac{1}{r}\left(A^r_{,\theta}-A^\theta\right) & A^\varphi_{,r}+\frac{A^r_{,\varphi}}{r\sin\theta}-\frac{A^\varphi}{r} \\ 0 & \frac{1}{r}\left(A^r_{,\theta}-A^\theta\right)+A^\theta_{,r} & 2\frac{1}{r}\left(A^\theta_{,\theta}+A^r\right) & \frac{1}{r}\ A^\varphi_{,\theta}+\frac{1}{r\sin\theta}\left(A^\theta_{,\varphi}-\cos\theta A^\varphi\right) \\ 0 & \frac{A^r_{,\varphi}}{r\sin\theta}-\frac{A^\varphi}{r}+A^\varphi_{,r} & \frac{1}{r\sin\theta}\left(A^\theta_{,\varphi}-\cos\theta A^\varphi\right)+\frac{1}{r}\ A^\varphi_{,\theta} & 2\left(\frac{1}{r\sin\theta}\ A^\varphi_{,\varphi}+\frac{1}{r}\ A^r+\frac{1}{r}\frac{\cos\theta}{\sin\theta}\ A^\theta\right) \end{bmatrix} \tag{22}$$

The antisymmetric part (the field part) is :

$$[s_k{}^i] = \frac{1}{2}\begin{bmatrix} 0 & 2E^r & 2E^\theta & 2E^\varphi \\ -2E^r & 0 & A^\theta_{,r}-\frac{1}{r}\left(A^r_{,\theta}-A^\theta\right) & A^\varphi_{,r}-\frac{A^r_{,\varphi}}{r\sin\theta}+\frac{A^\varphi}{r} \\ -2E^\theta & \frac{1}{r}\left(A^r_{,\theta}-A^\theta\right)-A^\theta_{,r} & 0 & \frac{1}{r}\ A^\varphi_{,\theta}-\frac{1}{r\sin\theta}\left(A^\theta_{,\varphi}-\cos\theta A^\varphi\right) \\ -2E^\varphi & \frac{A^r_{,\varphi}}{r\sin\theta}-\frac{A^\varphi}{r}-A^\varphi_{,r} & \frac{1}{r\sin\theta}\left(A^\theta_{,\varphi}-\cos\theta A^\varphi\right)-\frac{1}{r}\ A^\varphi_{,\theta} & 0 \end{bmatrix} \tag{23}$$

where the components of the electric field are :

$$E^r = \frac{1}{2}\ (A^r_{,t}-\phi_{,r}) = -\phi_{,r} = A^r_{,t} \quad , \quad E^\theta = -\phi_{,\theta}/r \quad , \quad E^\varphi = -\frac{\phi_{,\varphi}}{r\sin\theta} \tag{24}$$

It is remarkable that in the long range, when r becomes large as compared to the size of the particle, the leading term in E^r varies like $1/r$ while it varies like $1/r^2$ in E^θ and E^φ. Far from the particle, the field is longitudinal and spherically symmetric ; it is proportional to the inverse of the distance to the particle. These are the characteristics of the usual electrostatic field where the observer is at rest with respect to the particle.

Explicit formulas for the potential components and their derivatives are given in Appendix A. The tensors needed to describe the μ and the τ particles correspond to $n = \ell = m = 0$ (solutions g and g^*) and $n = 1, \ell = m = 0$ (solutions e or e^*). They can be written as :

- Solution g :

$$\left[a_k{}^i\right]_g \;=\; \frac{1}{\sqrt{4\pi}}\ \frac{\mathcal{A}_g\omega_g}{c^2}\begin{bmatrix} -\sin\omega_g t\ J_0 & -\cos\omega_g t\ J'_0 & 0 & 0 \\ \cos\omega_g t\ J'_0 & -\sin\omega_g t\ J"_0 & 0 & 0 \\ 0 & 0 & -\sin\omega_g t\ \frac{J'_0}{\mathrm{x}} & 0 \\ 0 & 0 & 0 & -\sin\omega_g t\ \frac{J'_0}{\mathrm{x}} \end{bmatrix} \tag{25}$$

- Solution g^* :

$$\left[a_k{}^i\right]_{g^*} \;=\; \frac{1}{\sqrt{4\pi}}\ \frac{\mathcal{A}_g\omega_g}{c^2}\begin{bmatrix} \cos\omega_g t\ J_0 & -\sin\omega_g t\ J'_0 & 0 & 0 \\ \sin\omega_g t\ J'_0 & \cos\omega_g t\ J"_0 & 0 & 0 \\ 0 & 0 & \cos\omega_g t\ \frac{J'_0}{\mathrm{x}} & 0 \\ 0 & 0 & 0 & \cos\omega_g t\ \frac{J'_0}{\mathrm{x}} \end{bmatrix} \tag{26}$$

Here x $= \omega_g r/c$, $J_0 = J_0(\mathrm{x})$ and the derivatives J'_0 and $J"_0$ are taken with respect to x. The normalized length x will be taken as the reference for the other solutions.
- Solution e :

$$\left[a_k{}^i\right]_e = \frac{1}{\sqrt{4\pi}} \frac{\mathcal{A}_e \omega_e}{c^2} \begin{bmatrix} -\sin\omega_e t\ J_1 & -\cos\omega_e t\ J'_1 & 0 & 0 \\ \cos\omega_e t\ J'_1 & -\sin\omega_e t\ J"_1 & 0 & 0 \\ 0 & 0 & -\sin\omega_e t\ \frac{J'_1}{\mathrm{x}_1} & 0 \\ 0 & 0 & 0 & -\sin\omega_e t\ \frac{J'_1}{\mathrm{x}_1} \end{bmatrix} \tag{27}$$

Here $J_1 = J_1(\mathrm{x}_e)$ with $\mathrm{x}_e = \omega_e r/c$ and the derivatives J'_1 and $J"_1$ are taken with respect to x_e.
Tensors for even solutions $q_0(n=1,\ell=1,m=0)$, $q_1(n=1,\ell=1,m=1)$, $q_{-1}(n=1,\ell=1,m=-1)$ are given in Annex B. They are not used in this article.

2.5 Energy.

Our objective is to compute the potential energy associated with a superposition such as $g+g^*+e$ and to compare it to that of particle e (the electron) which is taken as the reference, both for the computed and the experimentally measured figures. The energy is described by the Hamiltonian and is related to the Lagrangian through a Legendre transform.
We have two hints to find the Lagrangian associated with an electromagnetic particle :
- The first is that a Lagrangian **must** be invariant in a coordinate change. Among the four invariants of $[\overline{a}_k{}^i]$, one is proportional to the Lagrangian density $\mathcal{L}$.
- The second is that the global Lagrangian of a particle should be finite. In other words, the Hamiltonian density $\mathcal{H}$, when integrated over the whole volume, should converge, giving the total energy of the particle. This integral is : $\int_0^\infty\ r^2\ dr\ \int_0^\pi\ \sin\theta\ d\theta\ \int_0^{2\pi}\ d\varphi\ \mathcal{H}$ in the spherical system of coordinates.
These conditions are met by the determinant of $[\overline{a}_k{}^i]$ and we are led to the equation :

$$\mathcal{L} = \mathcal{C}\ \|a_k{}^i\| \tag{28}$$

where the double bar is the symbol for the determinant. The proportionality constant $\mathcal{C}$ is a physical quantity which has the dimensions $[\mathcal{C}] = M^{-3}L^{-2}T^2Q^4$.
The Legendre transform writes :

$$\mathcal{H} = \sum_{ik}\ a_k{}^i\ \frac{\partial\mathcal{L}}{\partial\overline{a}_k{}^i} - \mathcal{L} \tag{29}$$

This equation introduces the canonical momentum $\mathcal{L}'^k{}_i = \partial\mathcal{L}/\partial a_k{}^i$ associated with $a_k{}^i$.
$\mathcal{L}$ being the determinant of $[a_k{}^i]$, one sees that $\mathcal{L}'^k{}_i$ is the minor relative to the element $a_k{}^i$ and that $\sum_k\ \partial\mathcal{L}/\partial a_k{}^i\ a_k{}^i = \mathcal{L}$ (development of the determinant with respect to the elements of the line k). The simple relation follows :

$$\mathcal{H} = 3\mathcal{L}\ \ . \tag{30}$$

The tensor $a_k{}^i$ is divided into its symmetric (mass) and its antisymmetric (field) parts : The total energy density dW_t is found from the determinant of $a_k{}^i$. The mass energy is the potential energy dW_m, it is found from the determinant of $s_k{}^i$. The field energy dW_f is found from the difference $dW_t - dW_m$ or, equivalently, from the part of (29) which contains the field. A fundamental result which is illustrated below in the case of solution g is $W_m + W_f = 0$. the total mass energy and the total field energy associated with an e.m. particle are equal and opposite in sign. This result holds for each e.m. particle n, ℓ, m.

2.6 Electric charge.

This section briefly describes the way to find the electric charge Q associated with an e.m. particle. The main steps are the following :
1- Maxwell's equation of classical electrostatics relates the electric displacement $\overrightarrow{D}$ and the electric charge density ρ :

$$div\overrightarrow{D} = \rho \tag{31}$$

$\overrightarrow{D}$ is proportional to the electric induction which is the derivative of the Lagrangian with respect to the field[12]. Eq.(31) can thus be used as the relation of definition of ρ.

2- Lagrange's equations are applied to the potential components A^j which are considered to be the generalized coordinates. For each component A^i :

$$\sum_k \left[\frac{\partial \mathcal{L}}{\partial A^i} - \frac{\partial}{\partial x^k} \left(\frac{\partial \mathcal{L}}{\partial \left(\frac{\partial A^i}{\partial x^k} \right)} \right) \right] = 0 \quad . \tag{32}$$

The derivative $\partial \mathcal{L} / \partial \left(\frac{\partial A^i}{\partial x^k} \right)$ is the canonical momentum corresponding to $\partial A^i / \partial x^k$. Let us apply this equation to the component ϕ/c. It can be written in the extended form as :

$$\frac{\partial \mathcal{L}}{\partial(\phi/c)} - \frac{\partial}{c\partial t} \left(\frac{\partial \mathcal{L}}{\partial \left(\frac{\partial(\phi/c)}{c\partial t} \right)} \right) - \frac{\partial}{\partial r} \left(\frac{\partial \mathcal{L}}{\partial \left(\frac{\partial(\phi/c)}{\partial r} \right)} \right) - \frac{\partial}{r\partial\theta} \left(\frac{\partial \mathcal{L}}{\partial \left(\frac{\partial(\phi/c)}{r\partial\theta} \right)} \right) - \frac{\partial}{r \sin\theta \ \partial\theta} \left(\frac{\partial \mathcal{L}}{\partial \left(\frac{\partial(\phi/c)}{r \sin\theta \ \partial\theta} \right)} \right) = 0 \quad . \tag{33}$$

The first term nullifies because $\mathcal{L}$ does not depend on ϕ/c explicitly. The three last terms represent the divergence of the electric induction. This induction is the derivative of the Lagrangian with respect to the field or the canonical momentum of the field. If we use expression(24) we see that the induction reduces to a radial component :

$$D_r = \frac{\partial \mathcal{L}}{\partial E^r} = - \frac{\partial \mathcal{L}}{\partial \left(\frac{\partial(\phi/c)}{\partial r} \right)} \quad . \tag{34}$$

In the general case where E_θ and E_φ do not vanish, one obtains the relation :

$$\frac{\partial}{c\partial t} \left(\frac{\partial \mathcal{L}}{\partial \left(\frac{\partial(\phi/c)}{c\partial t} \right)} \right) = -div \overrightarrow{D} \tag{35}$$

0ne sees that the term $\partial \mathcal{L} / \partial \left(\frac{\partial(\phi/c)}{c\partial t} \right)$ is simply the minor relative to $\frac{\partial(\phi/c)}{c\partial t}$ in $a_k{}^i$. This minor is the determinant of the block 3X3 which groups the spatial derivatives in $a_k{}^i$. One gets :

$$div \overrightarrow{D} = - \frac{\partial}{c\partial t} \left\| \begin{array}{ccc} A^r_{,r} & A^\theta_{,r} & A^\varphi_{,r} \\ \frac{1}{r} \left(A^r_{,\theta} - A^\theta \right) & \frac{1}{r} \left(A^\theta_{,\theta} + A^r \right) & \frac{1}{r} \ A^\varphi_{,\theta} \\ \frac{A^r_{,\varphi}}{r \sin\theta} - \frac{A^\varphi}{r} & \frac{1}{r \sin\theta} \left(A^\theta_{,\varphi} - \cos\theta A^\varphi \right) & \frac{1}{r \sin\theta} \ A^\varphi_{,\varphi} + \frac{1}{r} \ A^r + \frac{1}{r} \frac{\cos\theta}{\sin\theta} \ A^\theta \end{array} \right\| \tag{36}$$

3- An integration over spacetime gives the total charge of the particle.
We have found that this integration gives a null result except for the solution $e(1,0,0)$ which leads to name it "the electron" and that charges have an opposite sign for even and odd solutions.
This result led us to consider that a compound particle is electrically charged if and only if it contains either one of the solutions $e(1,0,0)$ or $e^*(1,0,0)$. Another consequence is the null charge of the $e + e^*$ system.

2.7 Energies of g and e particles.

We will see below that the muon and the tau structures are superpositions of g and e particles. We give now their potential energies $\mathcal{H}_g$ and $\mathcal{H}_e$ with the scaling of $\mathcal{H}_e$ with respect to $\mathcal{H}_g$.

2.7.1 Energy of solution g.

The Lagrangian density corresponding to solution g is proportional to the determinant of (25) :

$$\mathcal{L}_g = \mathcal{C} \ \left(\frac{1}{\sqrt{4\pi}} \ \frac{\mathcal{A}_g \ \omega_g}{c^2} \right)^4 \ \left(\sin^4 \omega_g t \ J_0 \ J"_0 \ \frac{J_0'^2}{\mathrm{x}^2} + \sin^2 \omega_g t \ \cos^2 \omega_g t \ \frac{J_0'^4}{\mathrm{x}^2} \right) \tag{37}$$

The energy density in 4-space is given by the Hamiltonian :$d\mathcal{H}_g = 3\mathcal{L}_g$. It includes two parts, the first of which being the determinant of the symmetric part $[\bar{s}_k{}^i]$:

$$d\mathcal{H}_{mg} = 3 \ \mathcal{C} \ \left(\frac{1}{\sqrt{4\pi}} \ \frac{\mathcal{A}_g \ \omega_g}{c^2} \right)^4 \ \sin^4 \omega_g t \ J_0 \ J"_0 \ \frac{J_0'^2}{\mathrm{x}^2} \tag{38}$$

This is the mass part of solution g and is the potential energy density. The second part is the field part :

$$d\mathcal{H}_{fg} = 3\ \mathcal{C}\ \left(\frac{1}{\sqrt{4\pi}}\ \frac{\mathcal{A}_g\ \omega_g}{c^2}\right)^4\ \sin^2\omega_g t\ \cos^2\omega_g t\ \frac{J_0^{'4}}{\mathrm{x}^2} \tag{39}$$

The total mass energy $_{mg}$ is obtained by taking the mean value over a period $T = 2\pi/\omega_g$, or a "time length" $c\bar{T} = 2\pi c/\omega_g$ followed by an integration over the whole geometrical volume :

$$\mathcal{H}_{mg} \ = \ 3\ \mathcal{C}\ \left(\frac{1}{\sqrt{4\pi}}\ \frac{\mathcal{A}_g\ \omega_g}{c^2}\right)^4\ \frac{1}{2\pi}\int_0^{2\pi}\ \sin^4\omega_g t\ d(\omega_g t)\ \int_0^{2\pi}\ d\varphi\ \int_0^{\pi}\ \sin\theta\ d\theta\ \int_0^{\infty}\ \left(J_0\ J"_0\ \frac{J_0^{'2}}{\mathrm{x}^2}\right)\ \mathrm{x}^2\ d\mathrm{x} \tag{40}$$

Integration over time gives 3/8, that on angles gives 4π. Integration over x gives :

$$\int_0^{\infty}\ J_0\ J"_0\ J_0^{'2}\ d\mathrm{x} = \int_0^{\infty}\ \left(\frac{2\sin^4\mathrm{x}}{\mathrm{x}^8} - \frac{6\sin^3\mathrm{x}\cos\mathrm{x}}{\mathrm{x}^7} - \frac{\sin^4\mathrm{x}}{\mathrm{x}^6} + \frac{6\sin^2\mathrm{x}\cos^2\mathrm{x}}{\mathrm{x}^6}\right.$$
$$\left. - \frac{2\sin\mathrm{x}\cos^3\mathrm{x}}{\mathrm{x}^5} + \frac{2\sin^3\mathrm{x}\cos\mathrm{x}}{\mathrm{x}^5} - \frac{\sin^2\mathrm{x}\cos^2\mathrm{x}}{\mathrm{x}^4}\right) d\mathrm{x} = -\frac{2\pi}{315} = -0.0199466 \tag{41}$$

Finally :

$$\mathcal{H}_{mg} = -\frac{1}{560}\ \mathcal{C}\ \left(\frac{\mathcal{A}_g\ \omega_g}{c^2}\right)^4 \tag{42}$$

Let us compute the total energy corresponding to the total Lagrangian(37). This total energy is proportional to the radial integral :

$$\mathcal{H}_{total} \ \sim \ \int_0^{\infty}\ \left(3\ J_0\ J"_0\ \frac{J_0^{'2}}{\mathrm{x}^2} + \frac{J_0^{'4}}{\mathrm{x}^2}\right)\ \mathrm{x}^2\ d\mathrm{x} = \int_0^{\infty}\ \left(3\ J_0\ J"_0\ J_0^{'2} + J_0^{'4}\right)\ d\mathrm{x} =$$
$$= \int_0^{\infty}\ \frac{d(J_0\ J_0^{'3})}{d\mathrm{x}}\ d\mathrm{x} = \left(J_0\ J_0^{'3}\right)_0^{\infty} = 0 \tag{43}$$

The integral nullifies because J_0' vanishes at the origin and both J_0 nd J_0' nullify at infinity. The conclusion is that the mass energy and the field energy are equal in absolute value and opposite in sign. This property holds equally well for the e solution where J_0 is replaced by J_1. More generally, it holds for all other e.m. particles and, as seen below, for the μ and the τ.

2.7.2 Energy of solution e.

The procedure to obtain the density of energy of solution e is the same as before with J_1 instead of J_0. x_e is the normalized length : $\mathrm{x}_e = \omega_e\ r/c$. $J_1'(\mathrm{x}_e)$ and $J"_1(\mathrm{x}_e)$ are the derivatives with respect to x_e. We take x as the reference and write : $\mathrm{x}_e = n_e\mathrm{x}$. The radial integral which describes the radial potential energy density is :

$$\int_0^{\infty}\ J_1\ J"_1\ \frac{J_1^{'2}}{\mathrm{x}_e^2}\ \mathrm{x}^2\ d\mathrm{x} = \frac{1}{n_e^2}\ \int_0^{\infty}\ \left(\frac{24\sin^4(n_e\mathrm{x})}{(n_e\mathrm{x})^{12}} - \frac{96\sin^3(n_e\mathrm{x})\cos(n_e\mathrm{x})}{(n_e\mathrm{x})^{11}} + \frac{144\sin^2(n_e\mathrm{x})\cos^2(n_e\mathrm{x})}{(n_e\mathrm{x})^{10}}\right.$$
$$- \frac{96\sin(n_e\mathrm{x})\cos^3(n_e\mathrm{x})}{(n_e\mathrm{x})^9} + \frac{112\sin^3(n_e\mathrm{x})\cos(n_e\mathrm{x})}{(n_e\mathrm{x})^9} + \frac{18\sin^4(n_e\mathrm{x})}{(n_e\mathrm{x})^8} + \frac{24\cos^4(n_e\mathrm{x})}{(n_e\mathrm{x})^8} - \frac{120\sin^2(n_e\mathrm{x})\cos^2(n_e\mathrm{x})}{(n_e\mathrm{x})^8}$$
$$+ \frac{48\sin(n_e\mathrm{x})\cos^3(n_e\mathrm{x})}{(n_e\mathrm{x})^7} - \frac{40\sin^3(n_e\mathrm{x})\ \cos\mathrm{x}_e}{(n_e\mathrm{x})^7} - \frac{3\sin^4(n_e\mathrm{x})}{(n_e\mathrm{x})^6} - \frac{4\cos^4(n_e\mathrm{x})}{(n_e\mathrm{x})^6} + \frac{26\sin^2((n_e\mathrm{x})\cos^2(n_e\mathrm{x})}{(n_e\mathrm{x})^6}$$
$$\left. - \frac{4\sin\mathrm{x}_e\ \cos^3(n_e\mathrm{x})}{(n_e\mathrm{x})^5} + \frac{4\sin^3(n_e\mathrm{x})\cos(n_e\mathrm{x})}{(n_e\mathrm{x})^5} - \frac{\sin^2(n_e\mathrm{x})\cos^2(n_e\mathrm{x})}{(n_e\mathrm{x})^4} - \frac{36\sin^4(n_e\mathrm{x})}{(n_e\mathrm{x})^{10}}\right)\ d\mathrm{x} \tag{44}$$

This integral is also exactly computed and gives :

$$\frac{1}{n_e^2}\ \int_0^{\infty}\ J_1\ J"_1\ J_1^{'2}\ d\mathrm{x} = -\frac{17\pi}{10395}\ \frac{1}{n_e^3} \tag{45}$$

and the total potential energy is :

$$\mathcal{H}_e = -\frac{17}{36960}\ \frac{1}{n_e^3}\ \mathcal{C}\ \left(\frac{\mathcal{A}_e\ \omega_e}{c^2}\right)^4 \tag{46}$$

2.7.3 scaling solutions e and g.

1- The normalized length which appears in the spherical Bessel functions is x=$\omega_g r/c$ for solution g and $x_e = \omega_e r/c$ for solution e. One has to use the same unit of length when these particles are added together. ω_g is taken to be the reference of frequency and we have written $\omega_e = n_e\ \omega_g$ to compute the radial integral of solution e.
2- The amplitudes $\mathcal{A}_e$ and $\mathcal{A}_g$ are also different for both solutions and one has to find how they can be related. For this purpose, we remember that a particle reduces in the far field to spherical incoming and outgoing waves. The former originates from the noise and the latter from the particle. Both waves should have the same energy to realize the stability condition *"gain=loss"* in the stationary regime. We make the **hypothesis** that the noise which feeds the particles is an $1/f$ ("flicker") noise. It is such that its intensity is inversely proportional to its frequency which gives the relation :

$$\mathcal{A}_e^2\ \omega_e = \mathcal{A}_g^2\ \omega_g \quad . \tag{47}$$

or :

$$\mathcal{A}_e\ \omega_e = \sqrt{n_e}\ \mathcal{A}_g\ \omega_g \quad . \tag{48}$$

The energy in eq.(46) becomes :

$$\mathcal{H}_e = -\frac{17}{36960}\ \frac{1}{n_e}\ \mathcal{C}\ \left(\frac{\mathcal{A}_g\ \omega_g}{c^2}\right)^4 \tag{49}$$

In the following, all energies will be normalized by the factor $\mathcal{C}\ \left(\frac{\mathcal{A}_g\ \omega_g}{c^2}\right)^4$ and we will use the formula :

$$W_e = -\frac{17}{36960}\ \frac{1}{n_e} \tag{50}$$

for the scaled energy of the free electron. This formula will be used to compare W_e with the muon and the tau energies.
Fig.(1) represents the normalized energy of the electron as a function of $n_e = \omega_e/\omega_g$.

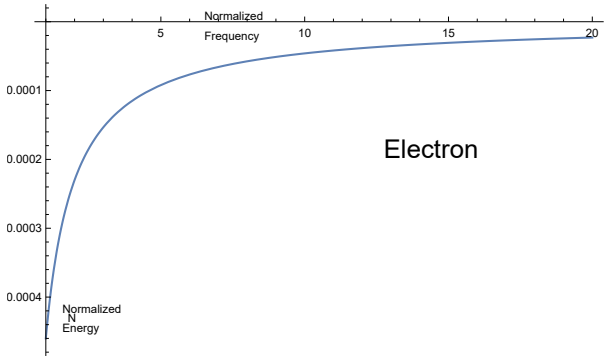

FIGURE 1 – Normalized energy $-\frac{17}{36960}\ \frac{1}{n_e}$ of the electron *vs* its normalized frequency $n_e = \omega_e/\omega_g$.

In passing, we note that the ratio of the potential energies $\mathcal{H}_{mg}$ (eq.42) and W_e for the same frequency ω_g is :

$$\frac{\mathcal{H}_{mg}}{\mathcal{H}_e} = \frac{1}{560}\ \frac{36960}{17} = 3.88... \tag{51}$$

The g is about 4 times heavier than the electron.

3 Potential energy of compound particles.

3.0.1 Generalities.

The aim of this section is to compare experimental and theoretical potential energies observed for the muon and the tau particles.

The formers are edited by the "Particle data group" ([17]). Results are 105.658 Mev/c^2 for the μ, 1776.86 Mev/c^2 for the τ and 0.511 Mev/c^2 for the electron which will be used as the reference, or the normalization factor. The normalized energies of the μ and the τ are respectively 206.77 and 3477.
The latters are obtained by combining electromagnetic particles and especially solutions g, g^*, e and e^*. We have used the property of potentials to be extensive quantities : The vector field and the tensor field of a compound particle are both linear combinations of vectors and tensors representing each constitutive solution (n, ℓ, m). We have taken the simplest case of electromagnetic particles centered around the same origin O.
Solutions g, g^* and e, e^* have a spherical symmetry. Solutions $1, 1, (-1, 0, 1)$ have a cylindrical symmetry[10]. We have guessed that these last solutions are associated to quarks. As there is no quark in the muon nor in the tau, we have supposed that their structure is spherically symmetric and includes solutions g, g^* and e, e^* only. We computed the energy associated with different structures as a function of the frequency n_μ or n_τ of the trapped electron(s) in the μ or the τ. There is no reason to have the same values for n_μ, n_τ and n_e : the frequency of the electron can vary following its environment. The computed energy was divided by 207 for the μ and 3477 for the τ to find the energy of the free electron which will be used as the reference for all other studied particles. As described below, the composition $g+g^*+e$ (or $g+g^*+e^*$) for the muon$^\pm$ gives very satisfactory results. Then we studied the combination $2(g+g^*)+e$ for the tau. While their order of magnitude was correct, the results were not satisfactory. We have considered the particles emitted when a tau disintegrates. There are many disintegration processes[17] but all of them show that many electrons and positrons are emitted. Again we computed the energy corresponding to several structures and finally arrived at the composition : $2(g+g^*)+2(e+e^*)+e$ (or $2(g+g^*)+2(e+e^*)+e^*$). These combinations are justified *a posteriori*, when the corresponding energies and charges are compared to the experimental results. The disintegration process changes the frequencies n_τ and n_μ of the trapped electrons into n_e, that of the free electron. This process is probably related to the emission of neutrinos.
The computation of the energy associated with a linear combination of solutions g, g^*, e, e^* is done with a computer and a symbolic and numeric computation software (*Mathematica*^{TM}). The entries are the number of each solution and the matrices representing them. The total matrix of a compound particle is the linear combination of these matrices. The determinant of this matrix allows the calculation of the total energy (field + mass) of the system while the determinant of the symmetric part leads to the potential energy. A frequency $n_e\omega_g$ is associated with the trapped electron(s) and the frequency ω_g of solution g is taken as the reference as described above. Then the determinant (the Lagrangian) is computed. The Hamiltonian density is 3 times this Lagrangian. The mean value over a period $T = 2\pi/\omega_g$ is first obtained. The second integration is over the angles and gives 4 π for spherically symmetric systems. The third integration is over the radial variable and gives the total energy, the potential energy and the ratio we are looking for when it is divided by the energy of the electron taken as the reference. We have found that the radial integral converges exactly only for integer values of n_τ or n_μ and gives exact values as in the case of the g and the e particles. Formulas for the total energy and the potential energy can be divided into three parts :
- The first contains the terms relative to solutions g and g^*,
- The second contains the terms relative to solutions e and e^*,
- The third is a mix of terms relative to all solutions and is the interaction term.

One can integrate each term separately : the magnitude of the interaction term gives an hint on the solidity or the fragility of the structure but we have not investigated this property further.
The same frequency and amplitude are given to even and odd solutions. The remaining white noise or other sources of noise are neglected, and, for the moment, we have abandoned the idea of introducing a phenomenological adjustable parameter ε_n to introduce corrections to the $1/f$ noise law.

3.1 Results.

A first characteristic of the experimentally observed elementary particles is the wide variety of masses as illustrated here by the muon and the tau. The second characteristic is the huge difference between lifetimes : the muon lives 2.2 10^{-6}s and the tau 2.9 10^{-13} s. The first characteristic is explained from the non-linearities appearing in the expressions for the energies. The second characteristic would need the calculation of the Lyapunov coefficient (the coefficient of stability) under perturbations brought by noise or collisions.
We give below the ratios of the computed potential energies of the μ and the τ particles referred to that of the lone electron. We have computed the energies corresponding to the structures $g+g^*+e$ and $g+g^*+2(e+e^*)+e$ as functions of the frequency n_μ or n_τ of the electron(s) trapped in these structures. We first compute the determinant of the corresponding sum of matrices then take the mean value over the period $2\pi/\omega_g$. Expression for the radial distribution of energy is obtained after integration over angles (which is simply a

multiplication by 4 π). It contains two parts, the first of which is a slowly-varying function of n_μ or n_τ and the other contains trigonometrical functions like $\cos(2n_\mu\pi)$ or $\sin(2n_\mu\pi)$. We found that the first part is about 1000 times greater than the second. The last integration is over x, the distance to the center. Formulas simplify enormously when n_μ or n_τ are integers and in this case the radial integrals are exactly computed. Results presented below are limited to this case.

3.1.1 Muon.

The muon has a mass m_μ= 105.6584 MeV/c^2 and the electron :m_e = 0.511 MeV/c^2. The experimental mass ratio is $m_\mu^\pm/m_e = 206.768$. The muon has a lifetime of 2.97 10^{-6} s and can be considered as a quasi-stable particle. It decays by emitting an electron ($\pm$) and neutrinos. Our basic hypothesis was that particles g and g^* are invisible particles (they are not electrically charged) which can escape in opposite directions in a collision or in a disintegration process. We have computed the energies corresponding to the structure $g + g^* + e$. The total (mass+field) energy density dW_μ of the muon is given by the formula written for integer values of n_μ :

$$dW_\mu = \frac{3\mathrm{x}^2}{128\pi^2}\ \Big(8(J_1'/\mathrm{x}_\mu)\ (J_0'/\mathrm{x})\ J"_1\ J_0 + J_1^{'2}\ ((J_1'/\mathrm{x}_\mu)^2 + 4(J_0'/\mathrm{x})^2) + 4(J_1'/\mathrm{x}_\mu)^2\ (J_0^{'2} + J_0\ J"_0) \\ +4(J_0'/\mathrm{x})^2\ (J_0^{'2} + 3J_0\ J"_0) + J_1(3(J_1'/\mathrm{x}_\mu)^2\ J"_1 + 4J"_1\ (J_0'/\mathrm{x})^2 + 8(J_1'/\mathrm{x}_\mu)\ (J_0'/\mathrm{x})\ J"_0)\Big) \quad (52)$$

This formula can be divided into three parts which describe (1) the electron (dW_{el}), (2) the $g + g^*$ components (dW_{g+g^*}) and (3), the interaction (dW_{int}) :

$$dW_{el} = \frac{3\mathrm{x}^2}{128\pi^2}\ \Big(J_1\ J"_1\ (J_1'/\mathrm{x}_\mu)^2 + J_1^{'4}/\mathrm{x}_\mu^2\Big) \quad (53)$$

$$dW_{g+g^*} = \frac{3\mathrm{x}^2}{128\pi^2}\ \Big(3J_0\ J"_0 + 4J_0^{'4}/\mathrm{x}^2\Big) \quad (54)$$

$$dW_{int} = \frac{3\mathrm{x}^2}{128\pi^2}\ \Big(8(J_1'/\mathrm{x}_\mu)\ (J_0'/\mathrm{x})\ J"_1\ J_0 + 4J_1^{'2}\ (J_0'/\mathrm{x})^2) + J_1\ (4J"_1\ (J_0'/\mathrm{x})^2\ + 8J"_0\ J_1'/\mathrm{x}_\mu)\ J_0'/\mathrm{x}))\Big) \quad (55)$$

In these formulas, J_1 and its derivatives are functions of n_μ :

$$\begin{aligned} J_1 &= \sqrt{n_\mu}\ \left(\frac{\sin \mathrm{x}_\mu}{\mathrm{x}_1^2} - \frac{\cos \mathrm{x}_\mu}{\mathrm{x}_\mu}\right) \\ J_1 &= \frac{\partial J_1}{\partial \mathrm{x}_\mu} \quad , \quad J"1 = \frac{\partial J_1'}{\partial \mathrm{x}_\mu} \\ \mathrm{x}_\mu &= n_\mu \mathrm{x} \end{aligned}$$

Integer values between 4 and 20 of the normalized electron frequency $n_\mu = \omega_\mu/\omega_g$ were used. Then, these energies were divided by that of an electron, taken from the formula used to draw Fig.(2). The goal was also to find the frequency n_e of the free electron, or the reference of energy which will be used as the normalization factor not only for the muon, but also for the τ and later, for all other particles. Fig.2 shows the ratios of the potential energy of the muon (which vary with n_μ) and that of a reference electron obtained with $n_e = 11, 12, 13, 14$. The ratio should be 207. One sees that the fit happens for $n_\mu = 15$ and $n_e = 12$. This finding needs a further confirmation.
In this way, we obtained a ratio 207.3 close to the experimental value 206.768 when the electron inside the muon (the μ electron) had the normalized frequency $n_\mu = 15$.

Equations (53), (54), and (55) are exactly integrated when n_μ is an integer. For $n_\mu = 15$, one finds :

$$\begin{aligned} \int_0^\infty dW_{el}\ d\mathrm{x} &= -17/554400 \sim -0.0000306638 \quad , \quad \int_0^\infty dW_g\ d\mathrm{x} = -1/140 \sim -0.00714286 \quad , \\ \int_0^\infty dW_{int}\ d\mathrm{x} &= -200222779/258339375000 \sim -0.00794856 \end{aligned} \quad (56)$$

which gives the total potential energy : $W(\mu 15) = -0.00794856$. This energy, when divided by the experimental ratio 206.768 should give the energy, or the normalized frequency n_e of the reference electron which

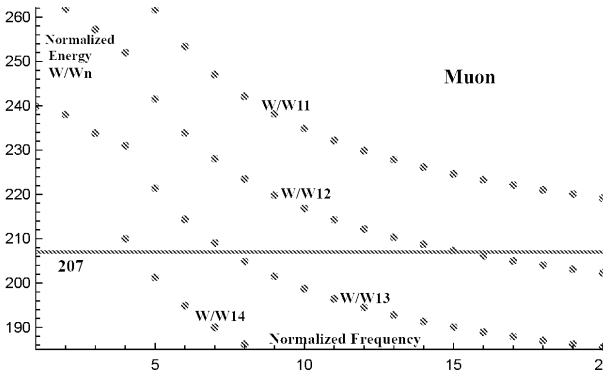

FIGURE 2 – W/Wn is the ratio of the potential energy of the muon and that of a free electron Wn *vs* the frequency n_μ. n_μ is the ratio of the frequency of the trapped electron and the frequency reference ω_g and varies from 1 to 20. 4 values have been tried for the normalized frequency $n_e = \omega_e/\omega_g$ of the free electron. Values $n_\mu = 15$ and $n_e = 12$ give $W(n_\mu)/W(n_e = 12) = 207.3$ close to the experimental figure 206.768.

is given by formula (50) . One finds $n_e = 11.965$, a value very close to the fit obtained from fig.(2).
The set of four figures (3,4,5,6) describe the evolution of the functions (52), (53), (54), (55) along the radial distance. The spatial extension of the electron is smaller than that of the $g + g^*$ component as can be seen in Figs.(3) and (4). One can consider that the electron is trapped inside these components.
The oscillating behavior of the interaction term in Fig.(10) originates from the leading term in eq.(55) when it is developped as a function of x :

$$\frac{1}{3645000000\ \pi\ \mathrm{x}^{10}}\left((-8172 - 698834\ \mathrm{x}^2 + 405000\ \mathrm{x}^4 + 202500\ \mathrm{x}^6)\cos 2\mathrm{x} + (-16280\mathrm{x} - 1415700\ \mathrm{x}^3 - 405000\ \mathrm{x}^5)\sin 2\mathrm{x}\right)$$

Radial oscillations are described by these terms in $\sin 2\mathrm{x}$ and $\cos 2\mathrm{x}$. There are many other oscillating terms containing $\sin 28\mathrm{x}$, $\sin 30\mathrm{x}$, $\sin 22\mathrm{x}$ and $\cos 28\mathrm{x}$, $\cos 30\mathrm{x}$, $\cos 22\mathrm{x}$ which are too small to be visible on the graph (10). The integral of functions (52, 53, 54, 55) nullifies which shows that the property *"potential energy = - field energy"* which was described for solution g holds also for a compound particle.
The next set of figures (7,8,9,10) describe the evolution of the potential energy (or the mass distribution) along the radial distance to the center. It is interesting to note that this potential energy is essentially hollow and negative around the center but it is surrounded by a spherical positive part. The curves for the $(g+g^*)$ components and the electron display also the same behavior.

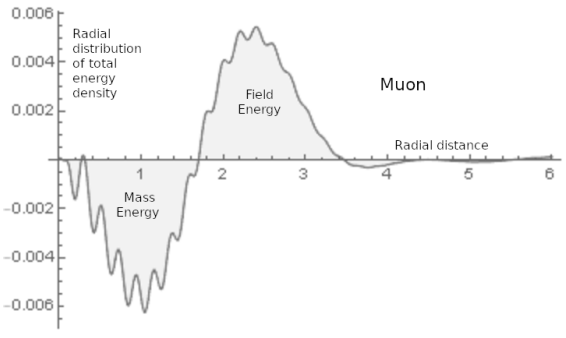

FIGURE 3 – Total energy density of the muon *vs* the radial distance to the center. The general shape is given by the $(g+g^*)$ components and the modulation by the interaction illustrated in Fig.(6). The integral of this curve vanishes. The negative (positive) part is essentially the mass (field) part.

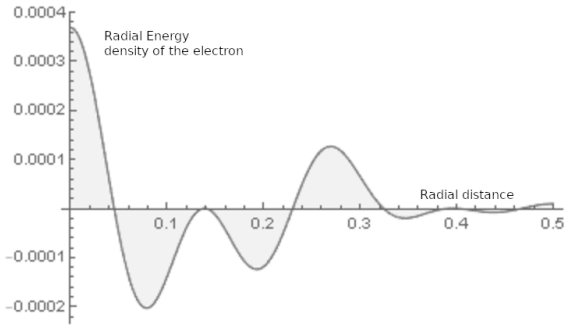

FIGURE 4 – Total energy density of the electron trapped in the muon *vs* the radial distance to the center. Note the differences in the energy scale and in the spatial extension of the muon in the preceding figure.The integral of this curve vanishes.

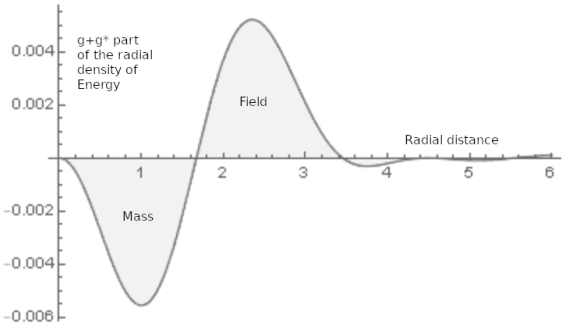

FIGURE 5 – Total energy density of the $g+g^*$ particles of the muon *vs* the radial distance to the center. The integral of this curve vanishes.

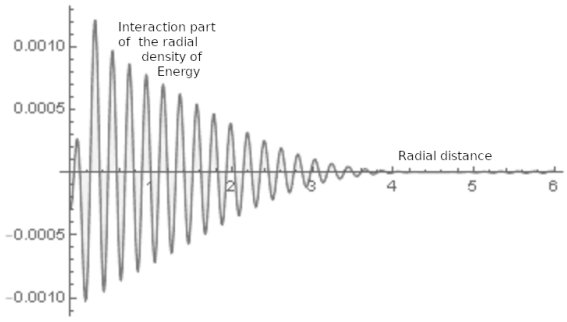

FIGURE 6 – Interaction part of the energy density between the $g + g^*$ particles and the electron *vs* the radial distance to the center. The integral of this curve vanishes.

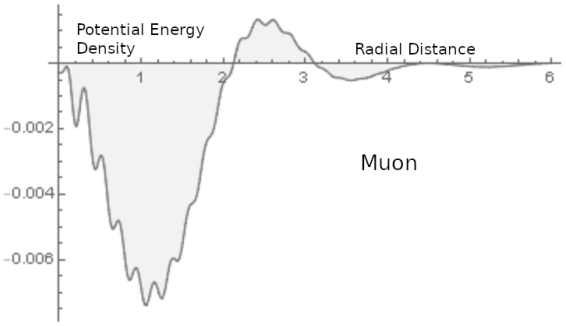

FIGURE 7 – Potential energy density of the muon *vs* the radial distance to the center. The general shape is given by the $(g+g^*)$ components and the modulation by the interaction illustrated in Fig.(7)

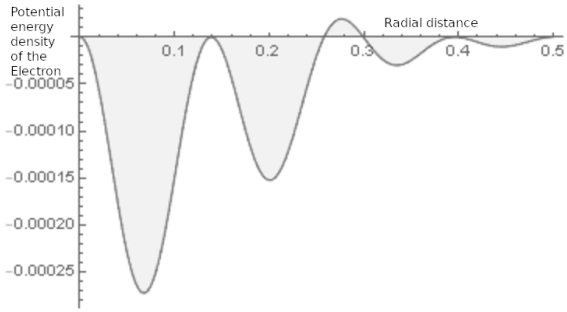

FIGURE 8 – Potential energy density of the electron trapped in the muon *vs* the radial distance to the center. Note the differences in the energy scale and in the spatial extension of the muon in the preceding figure.

3.2 tau.

After the muon, the τ particle is the next "heavy electron" (normalized energy 3477). Its products of desintegration[17] essentially include modes with one charged particle like kaons or pions and also modes with 3 charged particles like a system $2\pi^- + \pi^+$ accompanied by the emission of a neutrino τ. We were led to study the structure $2(g + g^*) + 2e + e^*$ (or, equivalently $2(g + g^*) + 2e^* + e$). Particles $2(g + g^*)$ give the essential part of the mass and particles $2e + e^*$ (or $2e^* + e$) give the charge + or -. The disintegration process should give either 3 charges or one charge with a γ ray originating from the e, e^* annihilation. The captured charged particles have a frequency $\omega_\tau = n_\tau \omega_g$. We computed the energy of this structure for integer values of n_τ varying between 4 and 20. Then we divided these results by the energy of reference electrons with $n_e = 11, 12, 13, 14$. This is illustrated in Fig.11 which shows that a ratio 3466 is obtained with $n_\tau = 13,\ n_e = 12$. The reference remains the electron 12 as in the case of the muon.
This result confirms the use of electron 12 as the reference. It brings also the conclusion that the g particles are responsible for the huge difference in energy between the electron, the muon and the tau particles.

The following sets of figures are similar to those describing the muon. Calculations have been done with $n_\tau = 13$. The first four figures describe the total energy density (Fig.(12)), the part coming from the charged particles $2e + e^*$ (Fig.(13)), the part coming from the particles $2(g + g^*)$ (Fig.(14)), and the interaction part (Fig.(15)).
The second set of four figures describe the potential energy density (Fig.(**??**)), with the part coming from the charged particles $2e + e^*$ (Fig.(**??**)), the part coming from the particles $2(g + g^*)$ (Fig.(18)), and the interaction part (Fig.(19)).
Formulas for obtaining these figures are similar to those written for the muon (52),(53), (54),(55), with

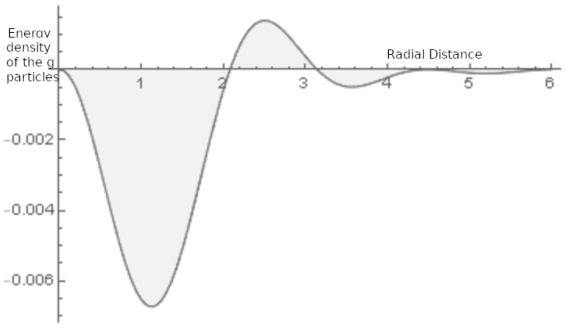

FIGURE 9 – Potential energy density of the $g + g^*$ particles of the nmuon *vs* the radial distance to the center.

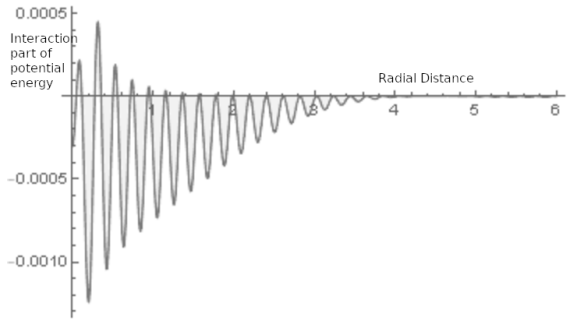

FIGURE 10 – Interaction part of the potential energy density between the $g + g^*$ particles and the electron *vs* the radial distance to the center.

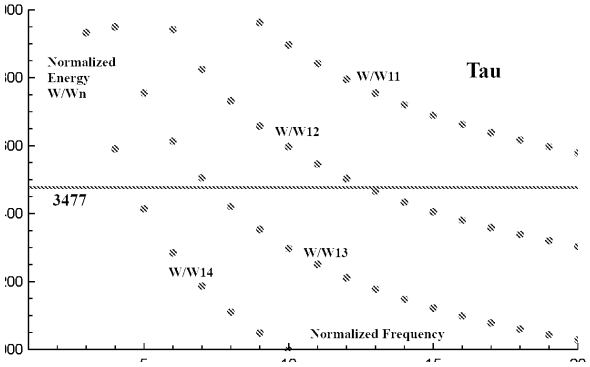

FIGURE 11 – Potential energy W of the tau normalized with respect to that of a free electron Wn *vs* the frequency ω of the captured electron normalized with respect to that of solution g, ω_g, which is taken as the reference. ω is an integer multiple of ω_g and varies from 1 to 20. 4 values have been tried for the frequency ω_e of the free electron. Values $\omega = 13$ and $\omega_e = 12$ give $W/W12 = 3466$ close to the experimental figure $W/Wn = 3477$.

different coefficients. They are also exactly integrated and the obtained energies are :

$$\begin{aligned}
\int_0^\infty dW_{\tau,el}\ d\mathrm{x} &= -(85/96096) \sim -0.000884532 \\
\int_0^\infty dW_{\tau,g}\ d\mathrm{x} &= -(4/35) \sim -0.114286 \\
\int_0^\infty dW_{\tau,int}\ d\mathrm{x} &= -(419619415/23718939426) \sim -0.0176913
\end{aligned}$$

Adding these energies together gives the total potential energy of the τ : $W_\tau = -0.132862$. Dividing this result by 3477.228, gives the energy of the reference electron W_{ref} =-0.000038209. This energy should obey the equation $W_{ref} = -17/(36960 n_e)$. Equating both terms gives $n_e = 12.04$. This result is close to $n_e = 11.965$ obtained with the muon. The relative difference ($3\ 10^{-3}$) with the energy of the electron 12 gives an idea of the precision of the results.

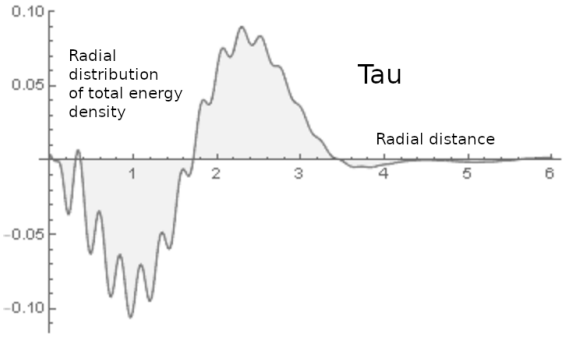

FIGURE 12 – Total energy density of the τ *vs* the radial distance to the center. The general shape is given by the $2(g + g^*)$ components Fig.(14) and the modulation by the interaction illustrated in Fig.(15). The integral of this curve vanishes.

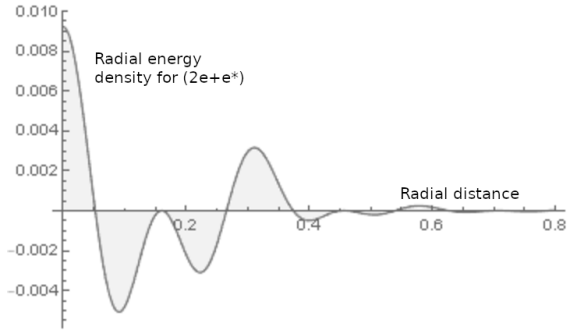

FIGURE 13 – Toial energy density of the charged particles $2e + e^*)$ trapped in the muon *vs* the radial distance to the center. Note the differences in the energy scale and in the spatial extension with respect to the total density in the preceding figure.The integral of this curve vanishes.

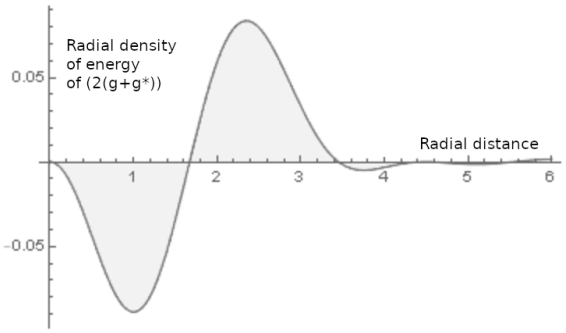

FIGURE 14 – Total energy density of the $2(g + g^*)$ particles in the tau *vs* the radial distance to the center. The integral of this curve vanishes.

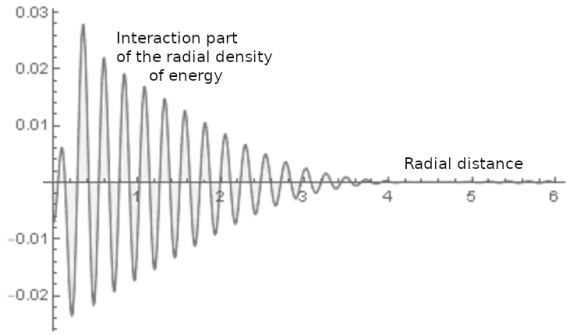

FIGURE 15 – Interaction part of the energy density between the $2(g + g^*)$ and the $2e + e^*)$ particles *vs* the radial distance to the center. The integral of this curve vanishes.

3.3 Hypothetical particles.

The following table gives the energies computed for several structures and normalized with respect to that of the electron 12. The frequency of the trapped electron can vary following the structure but is chosen to be $n = 13\omega_g$ for all results (this is why the energy $\mathrm{W}((g + g^*) + e)$ does not exactly coincide with that of the muon). Energies are non-linear functions of the particles number : for instance $\mathrm{W}(2(g + g^*)) = 16$ $\mathrm{W}(g + g^*)$ and more generally, $\mathrm{W}(k(g + g^*)) = \mathrm{k}^4$ $\mathrm{W}(g + g^*)$ which explains their wide variations.

TABLE 1 – Energy calculated for several structures based on combinations of electromagnetic particles g, g^*, e, e^*.

$\mathrm{W}(g + g^*) = 186.353$		$\mathrm{W}(2(g + g^*)) = 2982$		$\mathrm{W}(3(g + g^*)) = 15095$
$\mathrm{W}((g + g^*) + e) = 209.7$		$\mathrm{W}(2(g + g^*) + e) = 3075$		$\mathrm{W}(3(g + g^*) + e) = 15303$
$\mathrm{W}((g + g^*) + (e + e*) + e) = 325$		$\mathrm{W}(2(g + g^*) + (e + e*) + e) = 3466$		$\mathrm{W}(3(g + g^*) + (e + e*) + e) = 16156$
$\mathrm{W}((g + g^*) + 2(e + e*) + e) = 642$		$\mathrm{W}(2(g + g^*) + 2(e + e*) + e) = 4336$		$\mathrm{W}(3(g + g^*) + 2(e + e*) + e) = 17950$

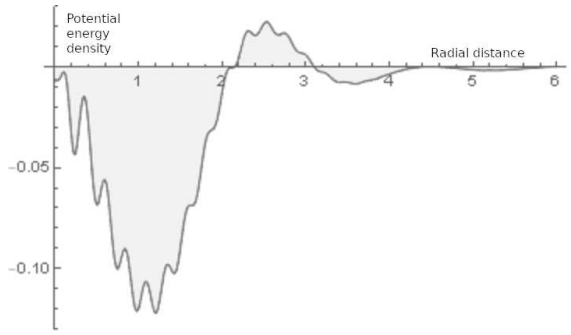

FIGURE 16 – Potential energy density of the tau *vs* the radial distance to the center. The general shape is given by the $(g+g^*)$ components and the modulation by the interaction illustrated in Fig.(**??**)

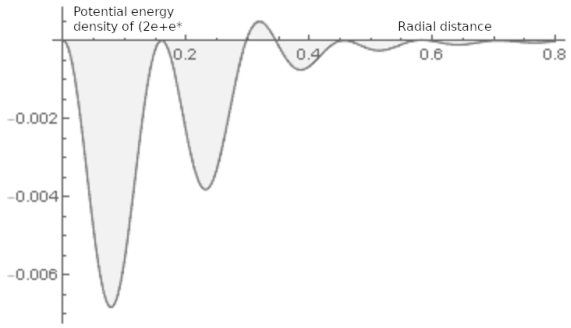

FIGURE 17 – Potential energy density of the $2e+e^*$ particles trapped in the tau *vs* the radial distance to the center. Note the differences in the energy scale and in the spatial extension of the tau in the preceding figure.

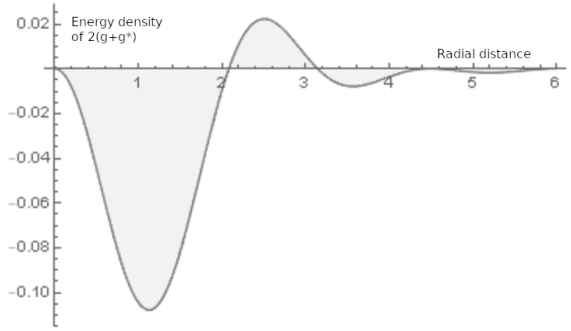

FIGURE 18 – Potential energy density of the $g+g^*$ particles of the tau *vs* the radial distance to the center.

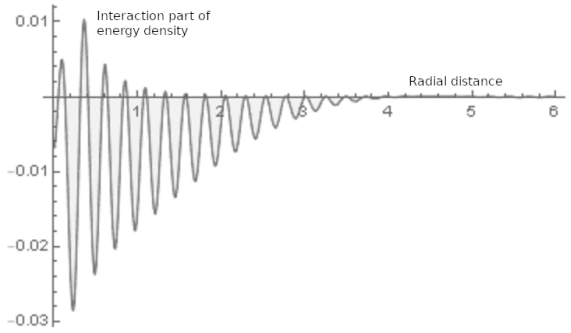

FIGURE 19 – Interaction part of the potential energy density between the $2(g+g^*)$ and the $2e+e^*$ particles *vs* the radial distance to the center.

4 Conclusion.

This article has presented first the general potential theory which is the study of the tensors formed by the derivatives of the 4-potential in the 4-dimensional spacetime. The first result is that the scalar potential obeys a Helmholtz equation in his proper frame of coordinates. Solutions of this equation describe electromagnetic particles. The vector potential and the tensor of derivatives $a_k^{\ i}$ are obtained at each event M for the five first solutions with their even (g, e, q_0, q_1, q_{-1}) and odd ($g^*, e^*, q_0^*, q_1^*, q_{-1}^*$) aspects. One then obtains the energy and the electric charge associated with each solution from these tensors. The energy density is obtained from the Lagrangian density which is proportional to the determinant of $a_k^{\ i}$ while the electric charge is related to the determinant of its geometrical part. In a second part, we have used these results to see if these e.m. particles are the fundamental elements of the μ and the τ which constitute the lepton family with the electron. A numerical application compared the theoretical and experimental potential energies when they are normalized by the electron energy. The μ and the τ structures are $g+g^*+e$ and $2(g+g^*)+2e+e^*$, respectively. They are compatible with the disintegration processes.The $g+g^*$ or the $2(g+g^*)$ components are responsible for the huge mass difference between the members of the lepton family. The general theory does not give the amplitude and frequency of each solution. We used the stability condition for the existence of a particle : this condition expresses the balance between the radiated and absorbed energy. The latter originates from the noise which we supposed to be an $1/f$ (flicker) noise. However, the frequency was still unknown and should be determined through a fit with experimental values. The fit with the μ led to a value 11.96 for the normalized frequency of the reference electron and the fit with the τ led to the value 12.03. We consider this result to be satisfactory enough to encourage the effort to improve and develop the general potential theory and its applications to the study of other elementary particles. An improvement would be a study of the contribution of the noise : the particle is coupled to the noise which is its surrounding medium. This picture has been developed by Wheeler and Feynman([13]) in their absorber theory : using causality, they showed that an electron should radiate an outgoing and absorb an incoming wave from the

noise. We have used the same concept to fix the stability of an e.m. particle : however adding the noise to the potential would give modified ("dressed") tensors and slightly different results than those described here. There are also many developments of the theory. For instance the study of the electron trapped inside the potential well of the $g + g^*$ particles is particularly interesting. Applying this classical theory based on firm fundamental concepts opens new ways to understand yoctoscopic behaviour of fields and matter.

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5 Appendix A.

This appendix gives explicit formulas for the potentials and their derivatives which are used to describe elementary particles.
Contravariant components of the potential vector are noted as : $A^i = (\phi/c, A^r, A^\theta, A^\varphi)$ in the spherical

frame of coordinates.
Even solutions of the Helmholtz equation are :

$$\begin{aligned} \phi/c &= \frac{\mathcal{A}_n}{c}\ J_n\ Y_\ell^m\ \cos\omega t \qquad A^r = -\frac{\mathcal{A}_n}{c}\ J'_n\ Y_\ell^m\ \sin\omega t\ , \\ A^\theta &= -\frac{\mathcal{A}_n}{c}\ \frac{J_n}{\mathrm{x}}\ Y_{\ell,\theta}^m\ \sin\omega t \qquad A^\varphi = -\frac{\mathcal{A}_n}{c}\ \frac{J_n}{\mathrm{x}\sin\theta}\ Y_{\ell,\varphi}^m\ \sin\omega t \quad . \end{aligned}$$

Derivatives are :

$$\begin{aligned} \frac{\partial(\phi/c)}{c\partial t} &= -\frac{\mathcal{A}\ \omega}{c^2}\ J_1\ Y_\ell^m\ \sin\omega t \quad , \quad \frac{\partial(\phi/c)}{\partial r} = \frac{\mathcal{A}\ \omega}{c^2}\ J'_1\ Y_\ell^m\ \cos\omega t \\ \frac{\partial(\phi/c)}{r\partial\theta} &= \frac{\mathcal{A}\ \omega}{c^2}\ \frac{J_1}{\mathrm{x}}\ Y_{\ell,\theta}^m\ \cos\omega t \quad , \quad \frac{\partial(\phi/c)}{r\sin\theta\partial\varphi} = \frac{\mathcal{A}\ \omega}{c^2}\ \frac{J_1}{\mathrm{x}}\ \frac{Y_{\ell,\varphi}^m}{\sin\theta}\ \cos\omega t \\ \frac{\partial A^r}{c\partial t} &= -\frac{\mathcal{A}\ \omega}{c^2}\ J'_1\ Y_\ell^m\ \cos\omega t \quad , \quad \frac{\partial A^r}{\partial r} = -\frac{\mathcal{A}\ \omega}{c^2}\ J"_1\ Y_\ell^m\ \sin\omega t \\ \frac{\partial A^r}{r\partial\theta} &= -\frac{\mathcal{A}\ \omega}{c^2}\ \frac{J'_1}{\mathrm{x}}\ Y_{\ell,\theta}^m\ \sin\omega t \quad , \quad \frac{\partial A^r}{r\sin\theta\partial\varphi} = -\frac{\mathcal{A}\ \omega}{c^2}\ \frac{J'_1}{\mathrm{x}}\ \frac{Y_{\ell,\varphi}^m}{\sin\theta}\ \sin\omega t \\ \frac{\partial A^\theta}{c\partial t} &= -\frac{\mathcal{A}\ \omega}{c^2}\ \frac{J_1}{\mathrm{x}}\ Y_{\ell,\theta}^m\ \cos\omega t \quad , \quad \frac{\partial A^\theta}{\partial r} = -\frac{\mathcal{A}\ \omega}{c^2}\ \left(\frac{J'_1}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right)\ Y_{\ell,\theta}^m\ \sin\omega t \\ \frac{\partial A^\theta}{r\partial\theta} &= -\frac{\mathcal{A}\ \omega}{c^2}\ \frac{J_1}{\mathrm{x}^2}\ Y_{\ell,\theta,\theta}^m\ \sin\omega t \quad , \quad \frac{\partial A^\theta}{r\sin\theta\partial\varphi} = -\frac{\mathcal{A}\ \omega}{c^2}\ \frac{J_1}{\mathrm{x}^2}\ \frac{Y_{\ell,\theta,\varphi}^m}{\sin\theta}\ \sin\omega t \\ \frac{\partial A^\varphi}{c\partial t} &= -\frac{\mathcal{A}\ \omega}{c^2}\ \frac{J_1}{\mathrm{x}}\ \frac{Y_{\ell,\varphi}^m}{\sin\theta}\ \cos\omega t \quad , \quad \frac{\partial A^\varphi}{\partial r} = -\frac{\mathcal{A}\ \omega}{c^2}\ \left(\frac{J'_1}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right)\ \frac{Y_{\ell,\varphi}^m}{\sin\theta}\ \sin\omega t \\ \frac{\partial A^\varphi}{r\partial\theta} &= \text{(will be 0)} \qquad , \quad \frac{\partial A^\varphi}{r\sin\theta\partial\varphi} = -\frac{\mathcal{A}\ \omega}{c^2}\ \frac{J_1}{\mathrm{x}^2}\ \frac{Y_{\ell,\varphi,\varphi}^m}{\sin\theta}\ \sin\omega t \quad . \end{aligned}$$

Formulas for corresponding odd solutions , $g^*(1,0,0)$, $e^*(1,0,0)$, $q_0^*(1,1,0)$, $q_1^*(1,1,1)$ and $q_{-1}^*(1,1,\text{-}1)$ are obtained by changing ωt into $\omega t - \pi/2$.

6 Annex B : Tensors of the first Electromagnetic particles.

The tensor of derivatives is given below for the five first even solutions.
- Even solution "g" $(n = \ell = m = 0)$:

$$\left[a_k^{\ i}\right]_g = \frac{\mathcal{A}_g\ \omega_g}{c^2} \begin{bmatrix} -\sin\omega_g t\ J_0 & -\cos\omega_g t\ J'_0 & 0 & 0 \\ \cos\omega_g t\ J'_0 & -\sin\omega_g t\ J"_0 & 0 & 0 \\ 0 & 0 & -\sin\omega_g t\ \frac{J'_0}{\mathrm{x}} & 0 \\ 0 & 0 & 0 & -\sin\omega_g t\ \frac{J'_0}{\mathrm{x}} \end{bmatrix} \tag{57}$$

- Odd solution "g^*" $(n = \ell = m = 0)$:

$$\left[a_k^{\ i}\right]_g = \frac{\mathcal{A}_g\ \omega_g}{c^2} \begin{bmatrix} \cos\omega_g t\ J_0 & -\sin\omega_g t\ J'_0 & 0 & 0 \\ \sin\omega_g t\ J'_0 & \cos\omega_g t\ J"_0 & 0 & 0 \\ 0 & 0 & \cos\omega_g t\ \frac{J'_0}{\mathrm{x}} & 0 \\ 0 & 0 & 0 & \cos\omega_g t\ \frac{J'_0}{\mathrm{x}} \end{bmatrix} \tag{58}$$

- Solution "e" $(n = 1, \ell = m = 0)$: Formulas for solution e are the same when $\mathcal{A}_g$ and ω_g are replaced by $\mathcal{A}_e$ and ω_e and $J_0(\mathrm{x})$ by $J_1(\mathrm{x}_e)$.

- Solution "q_0" $(n=1, \ell=1, m=0)$:

$$
\left[a_k^{\ i}\right] = \sqrt{\frac{3}{4\pi}}\ \frac{\mathcal{A}_1\omega_1}{c^2}
$$

$$
\left[\begin{array}{cccc}
-\sin\omega_1 t\ J_1\ \cos\theta & -\cos\omega_1 t\ J_1'\ \cos\theta & \cos\omega_1 t\ \frac{J_1}{\mathrm{x}}\ \sin\theta & 0 \\
\cos\omega_1 t\ J_1'\ \cos\theta & -\sin\omega_1 t\ J"_1\ \cos\theta & \sin\omega_1 t\ \sin\theta\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right) & 0 \\
-\cos\omega_1 t\ \frac{J_1}{\mathrm{x}}\ \sin\theta & \sin\omega_1 t\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right)\ \sin\theta & -\sin\omega_1 t\ \cos\theta\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right) & 0 \\
0 & 0 & 0 & -\sin\omega_1 t\ \cos\theta\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right)
\end{array}\right]
\tag{59}
$$

- Solution "q_1" $(n=1, \ell=1, m=1)$:

$$
\begin{aligned}
\left[a_k^{\ i}\right] \quad = \quad & \frac{\mathcal{A}_1\omega_1}{c^2}\ \sqrt{\frac{3}{2\pi}} \\
& \left[\begin{array}{cc}
-\sin\omega_1 t\ J_1\ \sin\theta\ \cos\varphi & -\cos\omega_1 t\ J_1'\ \sin\theta\ \cos\varphi \\
\cos\omega_1 t\ J_1'\ \sin\theta\ \cos\varphi & -\sin\omega_1 t\ J"_1\ \sin\theta\ \cos\varphi \\
\cos\omega_1 t\ \frac{J_1}{\mathrm{x}}\ \cos\theta\ \cos\varphi & -\sin\omega_1 t\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right)\ \cos\theta\ \cos\varphi \\
-\cos\omega_1 t\ \frac{J_1}{\mathrm{x}}\ \sin\varphi & \sin\omega_1 t\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right)\ \sin\varphi
\end{array}\right. \\
& \left.\begin{array}{cc}
-\cos\omega_1 t\ \frac{J_1}{\mathrm{x}}\ \cos\theta\ \cos\varphi & \cos\omega_1 t\ \frac{J_1}{\mathrm{x}}\ \sin\varphi \\
-\sin\omega_1 t\ \cos\theta\ \cos\varphi\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right) & \sin\omega_1 t\ \sin\varphi\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right) \\
-\sin\omega_1 t\ \sin\theta\ \cos\varphi\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right) & 0 \\
0 & -\sin\omega_1 t\ \sin\theta\ \cos\varphi\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right)
\end{array}\right]
\end{aligned}
\tag{60}
$$

- Solution "q_{-1}" $(n=1, \ell=1, m=-1)$:

$$
\begin{aligned}
\left[a_k^{\ i}\right] \quad = \quad & \frac{\mathcal{A}_1\omega_1}{c^2}\ \sqrt{\frac{3}{2\pi}} \\
& \left[\begin{array}{cc}
\sin\omega_1 t\ J_1\ \sin\theta\ \sin\varphi & \cos\omega_1 t\ J_1'\ \sin\theta\ \sin\varphi \\
-\cos\omega_1 t\ J_1'\ \sin\theta\ \sin\varphi & \sin\omega_1 t\ J"_1\ \sin\theta\ \sin\varphi \\
-\cos\omega_1 t\ \frac{J_1}{\mathrm{x}}\ \cos\theta\ \sin\varphi & \sin\omega_1 t\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right)\ \cos\theta\ \sin\varphi \\
-\cos\omega_1 t\ \frac{J_1}{\mathrm{x}}\ \cos\varphi & \sin\omega_1 t\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right)\ \cos\varphi
\end{array}\right. \\
& \left.\begin{array}{cc}
\cos\omega_1 t\ \frac{J_1}{\mathrm{x}}\ \cos\theta\ \sin\varphi & \cos\omega_1 t\ \frac{J_1}{\mathrm{x}}\ \cos\varphi \\
\sin\omega_1 t\ \cos\theta\ \sin\varphi\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right) & \sin\omega_1 t\ \cos\varphi\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right) \\
\sin\omega_1 t\ \sin\theta\ \sin\varphi\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right) & 0 \\
0 & \sin\omega_1 t\ \sin\theta\ \sin\varphi\ \left(\frac{J_1'}{\mathrm{x}} - \frac{J_1}{\mathrm{x}^2}\right)
\end{array}\right]
\end{aligned}
\tag{61}
$$