SYNTHESIS OF THE VIBROACOUSTIC RESPONSE OF A PANEL UNDER DIFFUSE ACOUSTIC FIELD USING THE SYNTHETIC ARRAY PRINCIPLE

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ABSTRACT

The reproduction of the vibration and acoustic responses of structures under random excitation such as the diffuse acoustic field or the turbulent boundary layer is of particular interest to researchers and the transportation industry (automobile, aeronautics, etc.) as well. Indeed, the determination of these vibroacoustic responses requires making *in-situ* measurements or using test facilities such as the anechoic wind tunnel, which are complex and costly methods. Another drawback of these test means is the variability of the results when for instance; the same structure is tested in different facilities of the same kind. Based on the previous considerations, the necessity of finding a simple, cost-efficient and reproducible alternative method becomes obvious. In the present paper, a method of achieving this goal using a single acoustic source and the synthetic array principle is proposed. To assess the validity of this method, we propose an academic case study consisting of a baffled and simply supported aluminum panel under diffuse acoustic field and turbulent boundary layer excitations. The vibration response of the plate as well as the transmission loss are determined with the proposed process and compared to results from random vibration theory. These comparisons show good agreement between both the results obtained with the proposed approach and the theoretical ones.

1. INTRODUCTION

The experimental characterization of structures under random excitations such as the diffuse acoustic field (DAF) and the turbulent boundary layer (TBL) is of great interest to the transportation industry and the building sector. However, the test facilities used (reverberant chamber for the DAF and anechoic wind tunnel or *in-situ* tests for the TBL) can sometimes be complex and costly. Moreover, the results obtained for a given structure can be very different from one facility to another even though the same setup is implemented.

The experimental reproduction of the vibroacoustic response of structures under stochastic excitations using an array of acoustic sources has been theoretically shown some decades ago. But due to technical limitations, this method could not be experimentally validated back then. Since 2000, many researchers have addressed this problem using various approaches. Maury, Bravo, Elliott and Gardonio [1–4] have widely discussed the reproduction of a TBL excitation using an array of loudspeakers. This method works well when it comes to the reproduction of a DAF excitation but due to the limited number of sources in the array, it fails to simulate the wall-pressure fluctuations of a subsonic TBL excitation because of the high wavenumbers involved meaning that a denser source array would be needed. In this paper, an improvement of the previous approach using the synthetic array principle is proposed. This technique aims at simulating the vibroacoustic response of structures under these random excitations independently of the environment. This process has been used by Aucejo et al. [5] under the name of *Source Scanning Technique* (SST) in order to reproduce the vibration response of a steel panel to a TBL excitation in the low frequency domain (up to 300 Hz).

This paper is organized as follows: first the theoretical background on the vibroacoustic response of a simple structure under random excitation is given. Secondly, the source scanning technique is briefly described. Finally, after presenting the experimental setup, some results are presented.

2. WAVENUMBER FORMULATION

This analysis considers the response of two dimensional rectangular structures to a random pressure field excitation. This pressure field is assumed to be stationary in time and homogeneous in space. We will be interested in two types of random excitations: the diffuse acoustic field and the turbulent boundary layer excitation.

The geometric configurations of the studied structure is shown in Fig. 1. In the following, we will assume that the wall-pressure fluctuations are not affected by the vibrations of the structure which means that the excitation is not modified by the structural re-

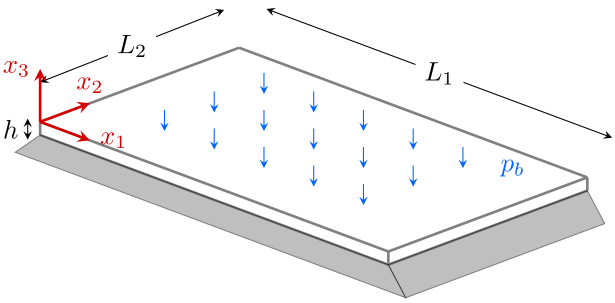

Figure 1. Simply supported plate on all edges.

sponse. Thus the random excitations considered in this paper are modeled by the wall pressure fluctuations that would be observed on a smooth rigid wall, also known as the *blocked pressure* p_b [6].

2.1 Response of Panels to Random Pressure Fields

The response of the panel when excited by the blocked-pressure $p_b(\mathbf{y},t)$ is denoted $\alpha(\mathbf{x},t)$ and designates the panel velocity response $v(\mathbf{x},t)$ if $\mathbf{x}$ is located on its surface Σ_p. Otherwise, it corresponds to the radiated pressure by the panel $p_r(\mathbf{x},t)$ or the particle velocity response $v_0(\mathbf{x},t)$. It is given by the following convolution product

$$\alpha(\mathbf{x},t) = \int_0^t \iint_{\Sigma_p} \gamma_\alpha(\mathbf{x},\mathbf{y},t-\tau)\, p_b(\mathbf{y},\tau)\, \mathrm{d}\mathbf{x}\mathrm{d}\tau \tag{1}$$

where $\gamma_\alpha(\mathbf{x},\mathbf{y},t)$ is the space-time impulse response of the panel at point $\mathbf{x}$ when excited by a normal unit force at point $\mathbf{y}$. With the previous assumptions made on the random processes involved, the cross-correlation function $R_{\alpha\alpha'}(\mathbf{x},t)$ can be written

$$R_{\alpha\alpha'}(\mathbf{x},t) = \int_{-\infty}^{+\infty} \alpha(\mathbf{x},t)\, \alpha'(\mathbf{x},t+\tau)\, \mathrm{d}\tau \tag{2}$$

where $\alpha'(\mathbf{x},t)$ also designates v, p_r or v_0. Performing a time-Fourier transform of the cross-correlation function after introducing Eq. (1) in Eq. (2) yields the following space-frequency spectrum of the panel response (see Ref. [7] for details)

$$S_{\alpha\alpha'}(\mathbf{x},\omega) = \iint_{\Sigma_p} \iint_{\Sigma_p} \Gamma_\alpha(\mathbf{x},\mathbf{y},\omega)\, S_{p_bp_b}(\mathbf{y},\mathbf{z},\omega) \times \Gamma^*_\alpha(\mathbf{x},\mathbf{z},\omega)\, \mathrm{d}\mathbf{y}\mathrm{d}\mathbf{z} \tag{3}$$

where $\Gamma_\alpha(\mathbf{x},\mathbf{y},\omega)$ is the time-Fourier transform of $\gamma_\alpha(\mathbf{x},\mathbf{y},t)$ and corresponds to the panel frequency response function at point $\mathbf{x}$ when excited by a normal force at point $\mathbf{y}$; $S_{p_bp_b}(\mathbf{y},\mathbf{z},\omega)$ is the time-Fourier transform of the blocked-pressure cross-correlation function and the superscript "*" represents the complex conjugate.

Defining the wavenumber-frequency spectrum of the blocked-pressure $S_{p_bp_b}(\mathbf{k},\omega)$ as the wavenumber transform of its space-frequency spectrum $S_{p_bp_b}(\mathbf{x},\mathbf{y},\omega)$ yields

$$S_{p_bp_b}(\mathbf{x},\mathbf{y},\omega) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} S_{p_bp_b}(\mathbf{k},\omega)\, e^{j\mathbf{k}(\mathbf{x}-\mathbf{y})}\mathrm{d}\mathbf{k} \tag{4}$$

Introducing Eq. (4) in Eq. (3) and re-arranging, one obtains the following expression of the panel response

$$S_{\alpha\alpha'}(\mathbf{x},\omega) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} H_\alpha(\mathbf{x},\mathbf{k},\omega)\, S_{p_bp_b}(\mathbf{k},\omega) \times H^*_{\alpha'}(\mathbf{x},\mathbf{k},\omega)\, \mathrm{d}\mathbf{k} \tag{5}$$

where

$$H_\alpha(\mathbf{x},\mathbf{k},\omega) = \iint_{\Sigma_p} \Gamma_\alpha(\mathbf{x},\mathbf{y},\omega)\, e^{-j\mathbf{k}\mathbf{y}}\mathrm{d}\mathbf{y} \tag{6}$$

is called the sensitivity function and it characterizes the vibroacoustic behavior of the panel. In practice, this response is approximated by

$$S_{\alpha\alpha'}(\mathbf{x},\omega) \approx \frac{1}{4\pi^2} \sum_{\mathbf{k}\in\Omega_\mathbf{k}} H_\alpha(\mathbf{x},\mathbf{k},\omega)\, S_{p_bp_b}(\mathbf{k},\omega) \times H^*_{\alpha'}(\mathbf{x},\mathbf{k},\omega)\, \delta\mathbf{k} \tag{7}$$

where $\Omega_\mathbf{k}$ is a set of properly chosen wave-vectors.

The DAF is very well known random excitation. In fact, there is a closed-form solution that exactly describes it. The space-frequency spectrum of a DAF is defined by the following equation [8]

$$S_{p_bp_b}(r,\omega) = \Phi_{p_bp_b}(\omega)\, \frac{\sin(k_0 r)}{k_0 r} \tag{8}$$

where $r = |\mathbf{x}-\mathbf{x}'|$; ω is the frequency; $k_0 = \omega/c_0$ is the acoustic wavenumber and c_0 the speed of sound in the medium. $\Phi_{p_bp_b}(\omega)$ designates the wall-pressure auto-spectral density function. The wavenumber transform of the space-frequency spectrum yields the frequency-wavenumber spectrum of the DAF blocked-pressure

$$S_{p_bp_b}(\mathbf{k},\omega) = \begin{cases} \Phi_{p_bp_b}(\omega)\, \dfrac{2\pi}{k_0} \dfrac{1}{\sqrt{k_0^2 - |\mathbf{k}|^2}} & \text{if } |\mathbf{k}| < k_0 \\ 0 & \text{if } |\mathbf{k}| \geq k_0 \end{cases} \tag{9}$$

where $|\mathbf{k}| = \sqrt{k_1^2 + k_2^2}$, k_1 and k_2 are the wavenumbers in the x_1 and x_2 directions, respectively.

2.2 Radiated Power

The radiated power is defined by the following equation

$$\Pi_r(\omega) = \iint_{\Sigma_p} I_{act}(\mathbf{x},\omega)\, \mathrm{d}\mathbf{x} \tag{10}$$

where $\mathrm{d}\mathbf{x}$ is the surface element and $I_{act}(\mathbf{x},\omega)$ is the normal component of the active sound intensity at point $\mathbf{x}$. The active sound intensity is directly related to the CSD function $S_{pv_0}(\mathbf{x},\omega)$ between the sound pressure and the particle velocity at point $\mathbf{x}$ [9]

$$I_{act}(\mathbf{x},\omega) = \Re\left[S_{pv_0}(\mathbf{x},\omega)\right] \tag{11}$$

where $\Re$ designates the real part and from Eq. (7), one has

$$S_{pv_0}(\mathbf{x},\omega) \approx \frac{1}{4\pi^2}\sum_{\mathbf{k}\in\Omega_\mathbf{k}} H_p(\mathbf{x},\mathbf{k},\omega)\, S_{pp}(\mathbf{k},\omega)\, H_{v_0}^*(\mathbf{x},\mathbf{k},\omega)\,\delta\mathbf{k} \tag{12}$$

In practice, the radiated power will be estimated by an approximation of the integral of Eq. (10) with the rectangular rule

$$\Pi_r(\omega) \approx \sum_{\mathbf{x}\in\Sigma_r} I_{act}(\mathbf{x},\omega)\,\delta\mathbf{x} \tag{13}$$

where Σ_r is an elemental surface at a distance x_3 on the radiating side of the panel.

3. SOURCE SCANNING TECHNIQUE

The Source Scanning Technique (SST) relies on the linearity of the involved phenomena and on the principle of wave superposition. SST also relies on the assumption that the random pressure fields of interest can be modeled as a set of uncorrelated wall-pressure plane waves. This technique is used for the measurement of the sensitivity functions which allow to compute the vibroacoustic response of the panel when the excitation is known, namely $S_{p_bp_b}(\mathbf{k},\omega)$. The synthetic array principle consists in using a unique monopole source which is spatially displaced to different positions thereby creating virtually the array of monopole sources. It is closely related to the concept of Synthetic Aperture Radar (SAR), which consists in post-processing the signals received by a moving radar to produce fine resolution images from an intrinsically resolution-limited radar system in the along-track direction [10]. The proposed approach is based on the mathematical formulation of the problem in the wavenumber domain. This formulation is appropriate because it allows an explicit separation of the contributions of the excitation via the wall-pressure cross-spectrum density function from those of the vibroacoustic behavior of the structure via the sensitivity functions. This formulation is also computationally more efficient than the space-frequency formulation [7].

Given a target pressure field $p(\mathbf{x},\mathbf{k},\omega) = e^{-i\mathbf{kx}}$ consisting of wall-pressure plane waves of wavevector $\mathbf{k} = (k_1,k_2)$, there are three main steps for the reproduction of this target pressure field using SST

(a) **Characterization of the acoustic source:** measurement of the transfer functions (G_{sp}) between source positions s and observation points p on the plate (microphones), see Fig. 2

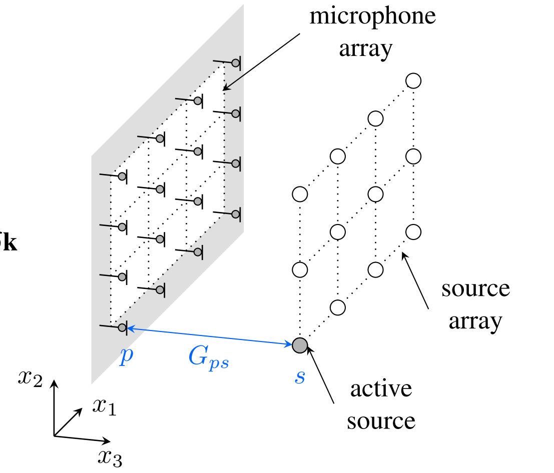

Figure 2. Measurement of $G_{ps}(\omega)$

(b) **Computation of the source amplitudes** Q_s at each position by inversing the equation below

$$\sum_s G_{ps}(\omega)\, q_s(\mathbf{k},\omega) = p_p(\mathbf{k},\omega) \tag{14}$$

(c) **Synthesis of the target pressure field:** in order to determine the vibroacoustic response of the structure, one needs to determine the following sensitivity functions

$$H_\alpha(\mathbf{x},\mathbf{k},\omega) = \sum_s Q_s(\mathbf{k},\omega)\,\Gamma_\alpha^s(\mathbf{x},\omega) \tag{15}$$

where $\alpha = (v,p,v_0)$ and $\Gamma_\alpha^s(\mathbf{x},\omega)$ represents the frequency response function (FRF) of the structure at point $\mathbf{x}$ which is to be measured.

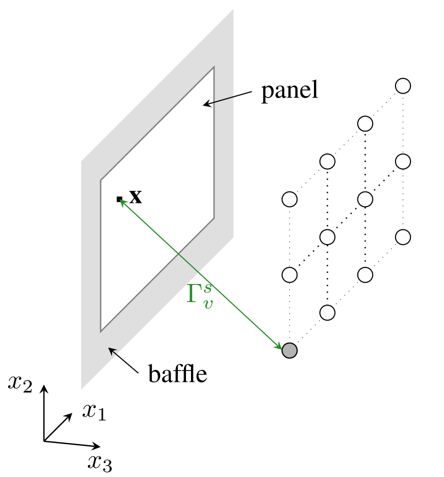

Figure 3. Measurement of the velocity FRF $\Gamma_v^s(\mathbf{x},\omega)$

4. EXPERIMENTAL SETUP

The plate was manufactured using the protocol presented by Robin et al. [11] and was placed in a baffle consisting of a $2\,\mathrm{cm}$ thick square plywood with a $1\,\mathrm{m}$ side and in which there is an aperture the size of the plate, see Fig. 5. A *mid-high frequency monopole* source manufactured by *Microflown* is used to generate the sweep signals necessary for the determination of the transfer functions between the source and an array $1/4''$ ROGA RG-50 microphones. This source was placed on the arm of a 3 axis Cartesian robot in order to automatize the displacement of the source. The vibration response of the structure was measured using a Brüel & Kjær type 4508 accelerometer. It is important to note that the monopole source used for the experiment was only efficient from approximately 300 Hz to 7000 Hz.

Figure 4. Measuring the transfer functions G_{sp}

The measurements were done in a room where the three walls are covered with absorbing wedges and $10\,\mathrm{cm}$ absorbing foam panels were placed on the floor and around the structure inside the baffle in order to prevent the potential reflections and noises coming from the robot and acquisition system from polluting the measurements.

Figure 5. Baffled simply supported panel

5. RESULTS AND DISCUSSION

In all the results shown below, there is a horizontal shift (along the frequency axis) which is due to the fact that in the theoretical model, the boundary conditions are considered perfect which they are not in practice.

Fig. 6 shows the auto-spectrum density function of the structural velocity response at point $\mathbf{x} = (0.06, 0.3, 0)\ \mathrm{m}$ (in dB units) when excited by a DAF.

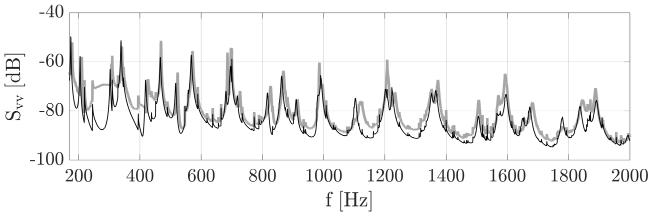

Figure 6. DAF vibration response: theory (thin black line), SST (thick gray line).

It can be observed that the vibration response determined using SST do not match the theoretical ones under 300 Hz: this is due to the fact that the source is not efficient in that frequency range as stated before. The vertical offsets that can be observed at some frequencies are due to the fact that for the theoretical case, the modal damping of the plate is taken constant in the entire frequency range whereas it is not the case in real conditions.

Fig. 7 shows the inverse of the radiated power (in dB units) by the panel when excited by a DAF.

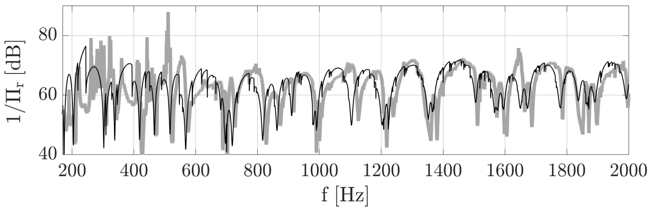

Figure 7. Inverse of the radiated power under DAF: theory (thin black line), SST (thick gray line).

The radiated power was determined using the *two microphone* method. The experimental results do not match the theoretical ones under approximately $600\,\mathrm{Hz}$: this is probably due to the fact that the monopole source was not very efficient to induce a sufficient radiation amplitude for the measurement of the pressure and particle velocity sensitivity functions. Above that frequency, there is a good agreement between the theoretical results and those obtained using SST.

6. CONCLUSION

The results obtained using SST for the simulation of the DAF induced vibrations of a rectangular panel were compared to numerical results. This comparison showed a fairly good agreement between both results. The process was automatized with two Cartesian robots controlled by a Matlab code: this will allow us to characterize more complex panels in the future.

Further details on this study as well as results for a turbulent boundary layer excitation will be available in the submitted journal paper [12].

7. ACKNOWLEDGMENT

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