

# A VARIATIONAL DATA ASSIMILATION METHOD BASED ON COMPLEXITY REDUCTION AND MODEL ERROR INCLUSION FOR REAL-TIME MONITORING APPLICATIONS

W. Haik<sup>1,2,\*</sup>, Y. Maday<sup>1</sup> and L. Chamoin<sup>2</sup>

<sup>1</sup> Sorbonne Université, CNRS, Université de Paris, Laboratoire Jacques-Louis Lions (LJLL)  
4, Place Jussieu, 75005 Paris, France

*e-mail:* {willy.haik,yvon.maday}@sorbonne-universite.fr

<sup>2</sup> Université Paris-Saclay, ENS Paris-Saclay, CNRS, Laboratoire de Mécanique et Technologie  
61, av. du Président Wilson, 94235 Cachan, France

*e-mail:* {willy.haik,ludovic.chamoin}@ens-paris-saclay.fr

**Key Words:** variational data assimilation, model order reduction, parametrized partial differential equations, real-time monitoring

For several years, the explosion of the access to experimental data in industrial systems and the considerable change in computing tools have led to the development of data assimilation methods that aim to be used for system monitoring. From data measured on the physical system and a mathematical model, these methods provide an approximation of the physical state. Modeling affects the effectiveness of the method; complex models based on physics can indeed provide accurate solutions, however such models involve a very expensive computational cost. The use of reduced order modeling (ROM) methods and the reduction of the complexity of the model can overcome the difficulties at the cost of controlling and integrating the model error. The Parameterized Background Data-Weak (PBDW) formulation introduced in [1] is a non-intrusive, reduced basis, real-time and in-situ data assimilation framework for physical systems modeled by parametrized Partial Differential Equations (PDE) for steady-state problems. The formulation integrates a set of solutions from reduced modeling of the model and an update state informed by the experimental observations (correction of model bias). Further research [2] developed a nonlinear framework to deal with noisy data. The present study aims at extending the PBDW for time-dependent problems with a sequential formulation which combines a PBDW solution at the current time step and a prediction from an extrapolation of previous estimated states. Moreover, the set of solutions of the model is enhanced with prior updates in order to learn part of the physics which is not initially included in the model. The numerical experiments show that the method significantly reduces the online computational time while providing relevant evaluations and predictions.

## REFERENCES

- [1] Maday, Y., Patera, A. T., Penn, J. D., and Yano, M. (2015). A parameterized background data-weak approach to variational data assimilation: formulation, analysis, and application to acoustics. *International Journal for Numerical Methods in Engineering*, 102(5), 933-965.
- [2] Gong, H., Maday, Y., Mula, O., and Taddei, T. (2019). PBDW method for state estimation: error analysis for noisy data and nonlinear formulation. *arXiv preprint arXiv:1906.00810*.