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# Resonant whistler-electron interactions: MMS observations vs. test-particle simulation

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## Key Points:

- Characteristic double-branch signatures in the electron Velocity Distribution Function (VDF) are observed simultaneously with a whistler wave.
- The wave, applied to test-particles, produces signatures in the VDF through Landau and cyclotron resonances.
- This resonant wave-particle interaction cannot be diagnosed in the Magnetospheric MultiScale (MMS) observations through the dissipative  $\mathbf{E} \cdot \mathbf{J}$  term.

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**Abstract**

Simultaneous observation of characteristic 3-dimensional (3D) signatures in the electron velocity distribution function (VDF) and intense quasi-monochromatic waves by the Magnetospheric Multiscale (MMS) spacecraft in the terrestrial magnetosheath are investigated. The intense wave packets are characterised and modeled analytically as quasi-parallel circularly-polarized whistler waves and applied to a test-particle simulation in view of gaining insight into the signature of the wave-particle resonances in velocity space. Both the Landau and the cyclotron resonances were evidenced in the test-particle simulations. The location and general shape of the test-particle signatures do account for the observations, but the finer details, such as the symmetry of the observed signatures are not matched, indicating either the limits of the test-particle approach, or a more fundamental physical mechanism not yet grasped. Finally, it is shown that the energisation of the electrons in this precise resonance case cannot be diagnosed using the moments of the distribution function, as done with the classical E.J “dissipation” estimate.

**1 Introduction**

Resonant wave-particle interactions are one of the few mechanisms in collisionless plasmas that enable a net transfer of energy from oscillating electromagnetic fields to moving charged particles. They play a fundamental role in various regions of the near-Earth plasma environment, such as the bowshock, the radiation belts, the polar cusp or the magneto-tail (Mazelle et al., 2000; Grison et al., 2005; Thorne, 2010; Fujimoto et al., 2011; Krasnoselskikh et al., 2013). In (fully developed) plasma turbulence, wave-particle interactions are also thought to play a leading role in dissipating energy as the turbulent cascade proceeds from large (fluid) to small (kinetic) scales (Bruno & Carbone, 2013; Sahraoui et al., 2020). In the solar wind (and to some extent the magnetosheath), the most debated dissipation processes are the Landau damping (Landau, 1946; Howes et al., 2008; Schekochihin et al., 2009; Gary & Smith, 2009; Sahraoui et al., 2010; Podesta et al., 2010; Sulem & Passot, 2015; Kobayashi et al., 2017), cyclotron damping (Leamon et al., 1998; Kasper et al., 2008; Cranmer, 2014; He et al., 2015) and stochastic heating (Chandran et al., 2010), which all would involve different spatial or temporal scales. Often magnetic reconnection is also evoked as a potential dissipation process in localized current sheets that self-consistently form in turbulence plasmas (Matthaeus et al., 1984; Retinò et al., 2007; Sundkvist et al., 2007; Chasapis et al., 2015). However, even within such localized coherent structures, Landau damping is shown to be very effective in numerical simulations of collisionless magnetic reconnection (TenBarge & Howes, 2013; F. et al., 2013; Numata & Loureiro, 2015).

Despite their role in energy dissipation, a *direct* diagnosis of wave-particle resonances in numerical simulations and in-situ data remains elusive. The difficulty to approach these processes stems from the need to measure *simultaneously* the 7D VDF (3 spatial dimensions, 3 velocity dimensions, and time) with high temporal and velocity space resolutions to access the kinetic scales, and the 4D structure of the electric and magnetic fields. While the latter could have been achieved at the magnetohydrodynamic (MHD) and ion scales using the Cluster data and appropriate data analysis techniques (Sahraoui et al., 2006; Narita et al., 2010; Sahraoui et al., 2010), the former became possible only in recent years thanks to the MMS mission (Burch et al., 2016). MMS indeed provides us with the highest ever achieved resolution of the particle VDFs, both in time and velocity space (Pollock et al., 2016). Furthermore, thanks to its small inter-spacecraft separations ( $\sim 10$  km) MMS allows us to probe in 3D kinetic spatial scales of the fluctuations fields. On the other hand, the increasing computer capabilities allows achieving Vlasov simulations with high-enough phase space resolution to unravel the complex nature of the kinetic dissipation in turbulent collisionless plasmas (Cerri et al., 2018). The present article is part of the ongoing efforts in this direction.

69 A few observational approaches on signatures of such mechanisms were previously  
 70 proposed, and here we focus on studies manipulating 3-dimensional VDFs. Kitamura  
 71 et al. (2018) reported the unambiguous observation of wave-ion resonances leading  
 72 to particle acceleration, associated with an ion cyclotron wave, in the Earth mag-  
 73 netosphere, displaying clear agyrotropic signatures (phase bunching) and their time  
 74 evolution. In the work of Gershman et al. (2017), at the magnetopause, the energy  
 75 exchange between electrons and kinetic Alfvén waves was studied in terms of the dis-  
 76 sipative  $\mathbf{E}\cdot\mathbf{J}$  term, and trapped electrons were found in the wave minima. In the  
 77 radiation belts, Min et al. (2014) proposed empirical indications for Landau resonance  
 78 signatures in the electron VDFs, as local minima in their velocity derivatives close to  
 79 the parallel resonant velocity, in the presence of Chorus waves. He et al. (2015) and  
 80 Marsch and Bourouaine (2011) show indications in the solar wind of resonances in the  
 81 proton VDFs, composed of a diffused, anisotropic core and secondary beam, which  
 82 were linked to kinetic waves. In the Earth magnetosheath and using a field-particle  
 83 correlation technique, Chen et al. (2019) presented structures in the fluctuating elec-  
 84 tron VDF close to the electron thermal speed, which the authors linked to electron  
 85 Landau damping.

86 The method followed in the present study is similar to the approach of Kitamura  
 87 et al. (2018) and Gershman et al. (2017), in that we first identify a neat electromagnetic  
 88 wave, as intense and monochromatic as possible, study its potential effect on particle  
 89 Velocity Distribution Functions (VDF), and then compare the expected signatures  
 90 with the VDF observed outside and inside the region where the wave is observed. This  
 91 approach enables an unequivocal, 3-dimensional comparison of resonant signatures in  
 92 the VDF between observations and the simulation.

93 The wave studied in this work is a high-frequency quasi-parallel whistler mode,  
 94 ubiquitous in both magnetospheric and the solar wind plasmas (Tao et al., 2012;  
 95 Lacombe et al., 2014; Stansby et al., 2016). Electrons are therefore the species of  
 96 interest, and the frequencies of both the wave and the electron motion (gyration)  
 97 are much higher than the fastest particle instruments operating in space, though in  
 98 the reach of wave sensors. Anisotropies in the electron distribution functions are  
 99 fundamental for the generation of whistler waves, as shown in various contexts such as  
 100 the solar wind (Tong et al., 2019), magnetic reconnection regions (Huang et al., 2016;  
 101 Yoo et al., 2018, 2019), or mirror mode magnetic holes or other coherent structures in  
 102 the magnetosheath (Huang et al., 2017; Ahmadi et al., 2018). Verscharen et al. (2019),  
 103 Vocks et al. (2005) and Seough et al. (2015) have explored the theoretical link between  
 104 whistler waves and solar wind electrons, in either the formation or the scattering  
 105 of strahl and halo electrons. In their numerical approach, Hsieh and Omura (2017)  
 106 show how oblique whistler mode chorus in the magnetosphere can lead to electron  
 107 acceleration up to a few MeV, via Landau, cyclotron, and higher order resonances.

108 To go beyond the 1-dimensional description of wave-particle resonances, in which  
 109 a resonance is reduced to its associated (scalar) parallel speed, we explore the possi-  
 110 bilities offered by a test-particle approach for a more comprehensive description of  
 111 the mechanism and a direct comparison with the observations. This approach also  
 112 presents the great advantage of isolating the effect of the wave on the particles, with  
 113 no feedback allowed. A succinct view on particle energisation is also proposed, in order  
 114 to appreciate whether the energy gained by the resonant particles can be quantified in  
 115 observations.

## 116 2 Particle data analysis

117 The particle data used in this study were recorded by the Fast Plasma Investi-  
 118 gation (FPI) of the Magnetospheric Multiscale (MMS) mission (Burch et al., 2016;  
 119 Pollock et al., 2016). We work in a reference frame in which the average plasma flow  
 120 velocity  $\mathbf{u}(t) = \frac{m_e \mathbf{u}_e(t) + m_i \mathbf{u}_i(t)}{m_e + m_i}$  is zero, and the *local* magnetic field  $\mathbf{B}_0$  – averaged over

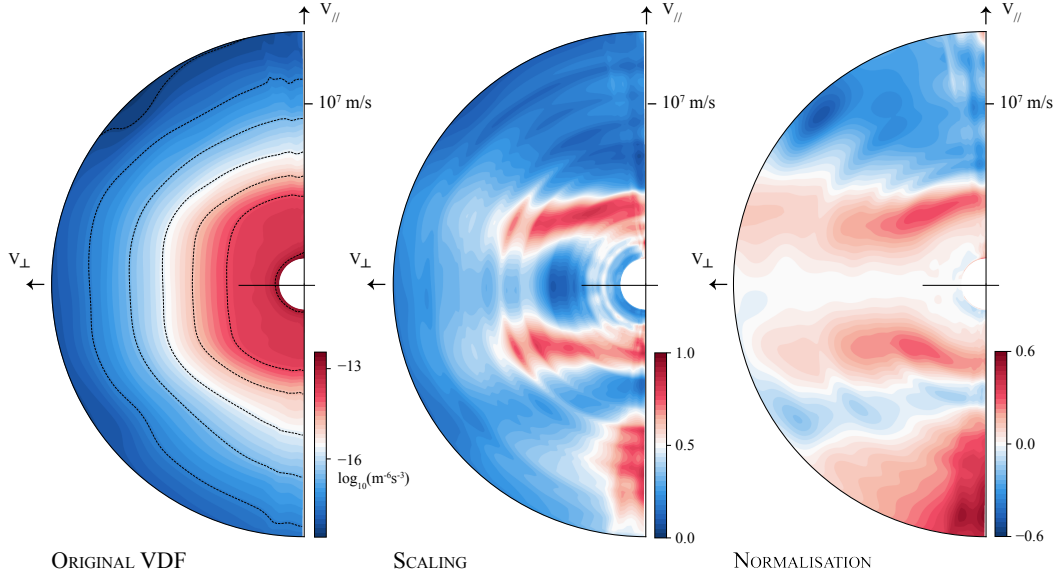
each 30 ms FPI measurement – is aligned with the  $z$ -direction. We define a spherical grid-of-interest in this reference frame, of arbitrary extent and resolution. For each FPI measurement of the VDF, this grid-of-interest is rotated and shifted to the instrument reference frame (cf. Appendix A) using the measured background magnetic field  $\mathbf{B}_0$  and flow velocity, as well as the probe motion. The VDF is then interpolated at each node of the transformed grid, using a tricubic interpolation scheme, documented and tested in Appendix B. The use of a spherical grid allows us to represent an averaged VDF in the  $(v_{\perp}, v_{\parallel})$ -plane without the need of a binning process, which is a source of systematic artifacts when working with multidimensional data. These aspects are illustrated in Appendix A. In Figure 1, the left-hand polar plot shows the result for 10 time-averaged VDFs, with the parallel velocity given along the vertical axis. A regular spherical grid-of-interest of  $200 \times 200 \times 200$  nodes was used with a maximum extent of  $1.5 \cdot 10^7$  m/s. To further ease the reading of the plots, a filled-contour representation is used.

Our goal is to study how the shape of the VDF may be affected by the presence of a wave. We wish to go further than the reduced description of the VDF given by its first order moments, namely its number density, its number flux density ( $\sim$  bulk velocity) and its momentum flux density tensor ( $\sim$  temperature in the thermal equilibrium case). For this purpose, we need a process that enhances potential patterns which might be “hidden” by the order zero distribution, or *background* distribution (not necessarily Gaussian/Maxwellian). In the example used in Figure 1, the VDF stretches over more than 6 orders of magnitude in the covered velocity space. The most obvious, order-0 shape found in the raw, original VDF is a centered, somewhat isotropic peak. A closer look may reveal obvious departure from the isotropy, with noticeably straight isocontours for parallel velocities around 0 m/s. We want a process which highlights these characteristics.

In a numerical context, such a background, or equilibrium distribution  $f_0$  is usually subtracted from  $f(t)$ , with the similar goal of enhancing higher order features (in such a case, the departure from  $f_0$ ). It is often defined as the initial VDF  $f(t = 0)$ , and sometimes as the time averaged distribution  $\langle f(t) \rangle$ . But the resulting *fluctuating* distribution  $\delta f(t) = f(t) - f_0$  might not be a valid concept when dealing with observations, as the *background*, order-0 distribution itself is generally varying – slightly or significantly – during the time interval of interest. In other words, as the plasma flows and the spacecraft moves, we never probe plasmas with the same parameters, which may be the case in a controlled simulation box. Using such a  $f_0$  with observations results in significant patterns in velocity space, which should be avoided for studying instantaneous, fine details of the VDF.

With this motivation, we propose two different treatments of the VDF which do not rely on any other information than the instantaneous distribution itself. These two treatments provide two complementary views of the VDF, with different advantages and drawbacks discussed in the following sections. The first treatment is a *scaling*, during which we consider each energy level of the spherical grid-of-interest separately. For each of these spherical shells, the minimum value of the interpolated VDF is set to 0 and the maximum value set to 1. Values are then averaged over the gyro-angle (angle around the background magnetic field). This scaling is closely related to classical pitch-angle distributions, in which the VDF is shown for a few selected energy ranges, with the color-plot dynamics ranging from the lowest to the highest distribution value of each energy range (see also Appendix B). In the proposed scaling, we virtually display 200 concentric, time-averaged pitch-angle distributions. In the next section, we will use a *scaled* pitch-angle distribution, by selecting only one limited energy range of the scaled VDF and plotting it over time (Figure 5).

In the second treatment, each VDF values is *normalized* to a reference value, which we choose to be the VDF value at the same energy for  $v_{\parallel} = 0$  (i.e., a pitch-



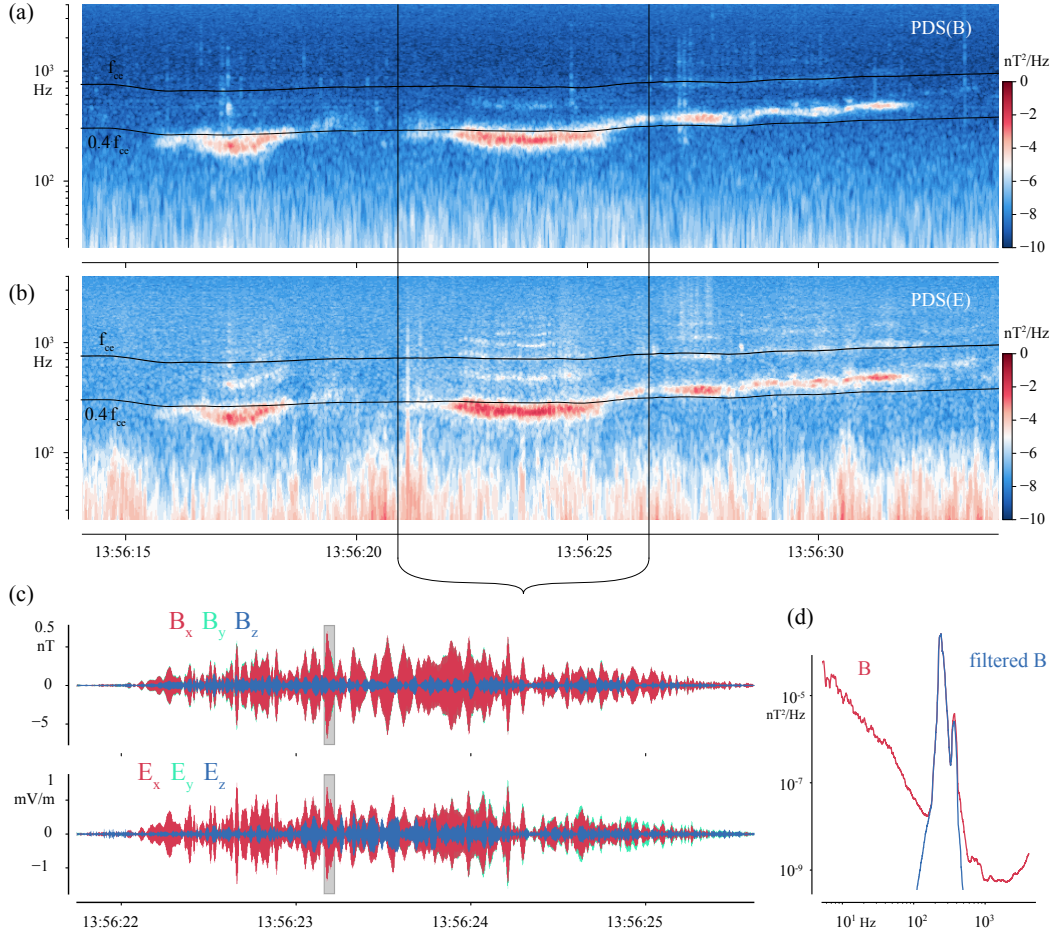
**Figure 1.** An example of an original VDF to the left, integrated over 300 ms, its scaled view in the middle, and its normalized view to the right.

174 angle of  $90^\circ$ ). All these central (equatorial) values are therefore set to one, appearing  
 175 in white tones, while higher values appear as red tones and lower values as blue. A  
 176 decimal logarithm is applied to the normalized values when plotted.

177 These two treatments do not have a physical motivation, they are only arbitrarily  
 178 chosen to highlight VDF features.

179 In Figure 1, the effect of the scaling on the original distribution is remarkable,  
 180 with two parallel branches of values larger than 0.5 (i.e. red tones) stretching at  
 181 constant parallel velocities. Properly speaking, these two structures are thick disks in  
 182 the 3-dimensional velocity space. Another structure is found along the anti-parallel  
 183 direction, a strahl-like structure greatly highlighted in comparison with the original  
 184 VDF. The circular features of constant speed are unavoidable artifacts of this scaling,  
 185 in which all energy levels are treated regardless of the others: the continuity of the  
 186 VDF across energy is lost, and some energy shells are scaled differently compared  
 187 to the neighbouring shells, resulting in circular visual artifacts. The normalization  
 188 we introduced here-above was applied on the original VDF, in order to conserve this  
 189 continuity across energy, for a complementary representation given in the right-most  
 190 plot of Figure 1. The circular artifacts vanished from the two branches – or disks – of  
 191 higher density, which are now reaching to even higher perpendicular velocities. In this  
 192 view, the strahl-like component is perceived as broader than in the scaled VDF. The  
 193 normalization puts different weights on details compared to the scaling, and though  
 194 being much less sharp, it appears in this case to be more sensitive to details at high  
 195 velocities.

196 We now have a comprehensive and constraining way of examining an instantane-  
 197 ous VDF, with two different views of it. The 0-to-1 scaling and the normalization  
 198 have different properties, already illustrated above and further appreciated in the fol-  
 199 lowing test-particle approach, which makes them a great tool for comparing numerical  
 200 and observational results. These two methods are easy to implement and come with  
 201 barely any computational cost. Together with the interpolation approach, they may



**Figure 2.** (a)-(b) B-field and E-field power density spectrograms. The electron gyrofrequency is indicated by the solid lines ( $f_{ce}$  and  $0.4 f_{ce}$ ). (c) Filtered E- and B-field wave forms. (d) Original B-field Power Density Spectra and its Butterworth filtered copy.

202 offer interesting applications for characterising the multi-dimensional VDF in other  
 203 heliophysics and planetary physics contexts.

### 204 3 Wave analysis and theoretical linear model

205 In order to study wave-particle resonances in observational data, we have isolated  
 206 one case displaying a clear wave activity within an otherwise “quiet” magnetosheath  
 207 environment. Captured on the 8th of March 2019, the 3 minute-long interval starting  
 208 at 13:56:10 displays contrasted, strong wave packets seen in the power density spec-  
 209 trograms of Figure 2 (a-b). The central frequency of these packets is about 250 Hz,  
 210 getting slightly higher on the second half of the observation. It is comprised between  
 211 0.3 and 0.5 electron gyrofrequency  $f_{ce}$ .

212 We focus on the central event of constant frequency, selected in Figure 2 (a-b).  
 213 The fields components are band-pass filtered using a Butterworth filter. The result is  
 214 displayed in Figure 2 (d). The reference frame is aligned with the background mag-  
 215 netic field  $\mathbf{B}_0$  averaged over the duration of the selected interval. We find that the parallel  
 216 component of both fields is significantly smaller than the perpendicular components,

217 however not null. In this frequency range, magnetic fluctuations up to 0.5 nT are  
 218 observed around an average background magnetic field 25 nT strong (Figure 2 (c)).  
 219 The electric field fluctuations are seen sometimes surpassing 1 mV/m.

220 We zoom-in once more to study a single wave packet, highlighted with a grey  
 221 background in the time series of Figure 2 (c) and expanded in Figure 3. The back-  
 222 ground magnetic field  $\mathbf{B}_0$  is now calculated over this shorter period. The fields compo-  
 223 nents exhibit remarkably clean sinusoids within the envelope defining the packet. The  
 224 two perpendicular ( $x$  and  $y$ ) components of both fields are almost equal in magnitude  
 225 and separated in time by a phase of  $\pi/2$ , corresponding to an almost perfect right-  
 226 handed circular polarisation. The polarisation is also nicely seen in the hodogram of  
 227 Figure 3, given in the local minimum variance reference frame (the frame in which the  
 228 variance along an axis – here the  $z$ -axis – of the vector time series is the smallest, 65  
 229 times smaller than the two other variances, almost equal to each other).

230 Whereas the four perpendicular components are defined by a similar envelope  
 231 in the fields  $\mathbf{E}$  and  $\mathbf{B}$ , the parallel (blue) component, also sinusoidal, follows a dif-  
 232 ferent time variation in  $\mathbf{E}$  and  $\mathbf{B}$ , and has a slightly higher frequency compared to  
 233 perpendicular components, in both fields (not shown). Given its right-handed circular  
 234 polarisation and its non-zero parallel component, we identify the wave as a quasi-  
 235 parallel whistler mode wave. Studying the same mode, Lacombe et al. (2014) and  
 236 Stansby et al. (2016) report its observations in a range from 0.1 to 0.6 gyro-frequency,  
 237 a range in which the present case falls perfectly. Because of the wave right-handed  
 238 circular polarity and its frequency, electrons are the species of interest for this study.

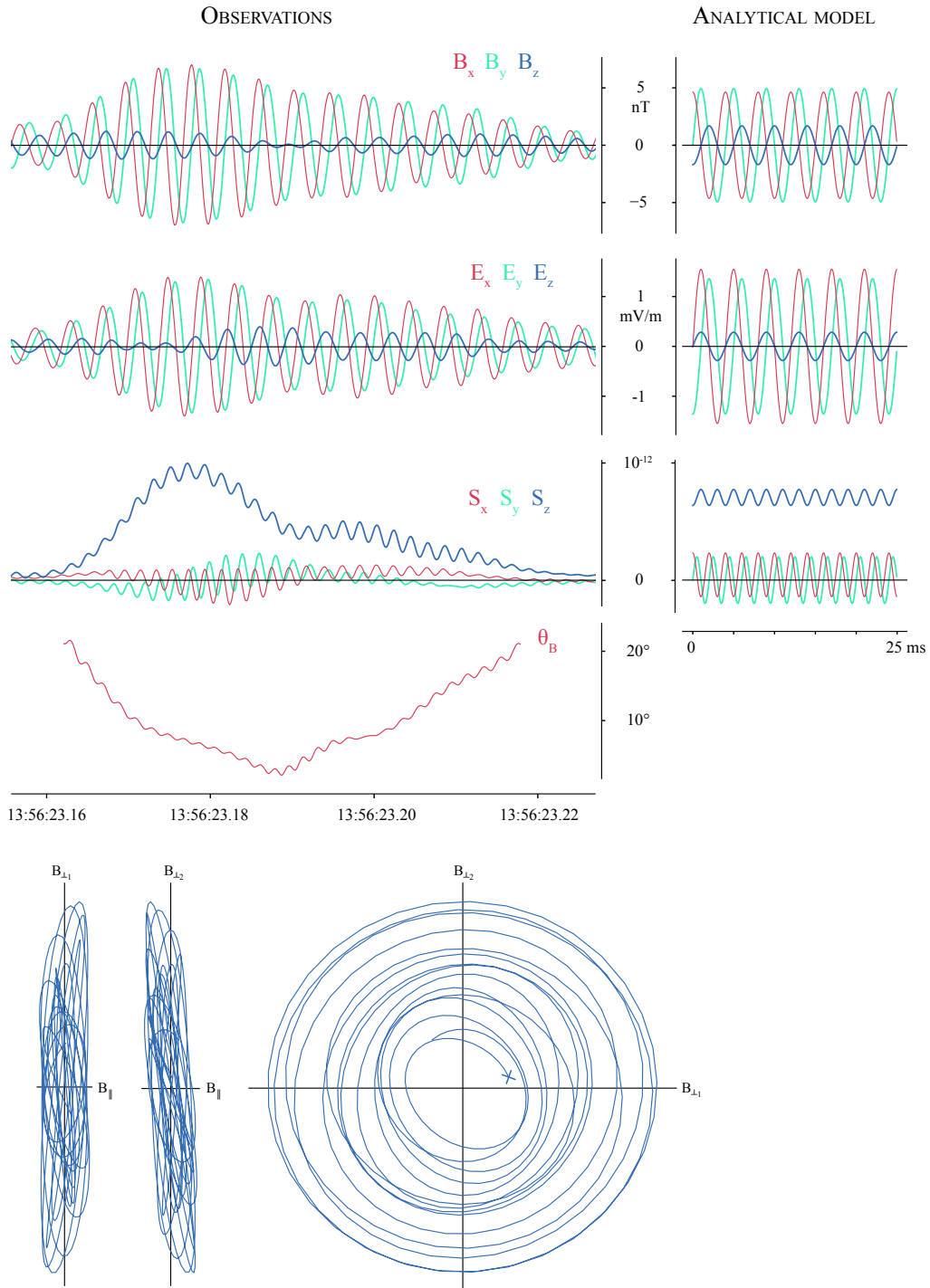
239 We use the following dispersion relation of the whistler mode, derived from the  
 240 Appleton-Hartree relation (Hsieh & Omura, 2017) in the limit of  $\omega \ll \omega_{pe}$ , valid in  
 241 our case with  $\omega < 10^{-2}\omega_{pe}$ :

$$Y = \frac{X^2}{X^2 + 1} \cos(\theta) \quad (1)$$

242 with the normalized spatial frequency  $X = k \cdot d_e$ , the normalized angular fre-  
 243 quency  $Y = \omega/\omega_{ce}$ ,  $k$  the amplitude of the wave vector,  $\omega$  the wave angular frequency,  
 244  $d_e$  the electron inertial length,  $\omega_{pe}$  the electron plasma frequency,  $\omega_{ce}$  the electron  
 245 (angular) gyrofrequency, and  $\theta$  the angle between the background magnetic field and  
 246 the wave vector.

247 Because of apparent monochromatic nature of the observed waveforms, we can  
 248 determine the wave vector angle  $\theta$  with regard to the magnetic field using a minimum  
 249 variance analysis over a sliding window as wide as the wave temporal period. In other  
 250 words, we consider one pseudo-circle at a time, described by the wave vector, and find  
 251 the orientation of the plane containing this circle. The result of this sliding minimum  
 252 variance analysis is shown in the fourth row of Figure 3 for the magnetic field. We find  
 253 that the wave is propagating with angles between 0 and 20 degrees from the background  
 254 magnetic field. Finally, to determine if this quasi-parallel wave is propagating along or  
 255 against the background magnetic field, we calculated the components of the Poynting  
 256 vector  $\mathbf{E} \times \mathbf{B}$ , which is parallel to the wave vector (see for instance the comprehensive  
 257 work of Stansby et al. (2016) on single-spacecraft estimation of whistler mode wave  
 258 properties). The Poynting vector  $\mathbf{S}$  (Figure 3 third row) is mostly parallel to the  
 259 background magnetic field, as evidenced by its largest and positive parallel component:  
 260 the wave propagates in a mostly parallel direction. We now have the frequency and the  
 261 wave vector direction of the wave, and an estimation of the wave vector amplitude.  
 262 It was also verified that the probe velocity in the plasma frame (in which  $\mathbf{u} = 0$ ,  
 263 see Section 2) is not significant with regard to the phase speed of the wave, and the  
 264 observed wave frequency found in the spectrograms is not Doppler affected.





**Figure 3.** One wave packet, indicated in Figure 2 (c) with a grey background, its analytical model, at position (0,0,0), and its hodogram in the minimum variance analysis frame. The first data point of the series is indicated by a cross.

265 Despite how clean the observed waves are, they are still limited to one point  
 266 in space. If we are to study the possible resonant interactions between the particles  
 267 and this precise type of wave, we need its temporal *and* spatial description, or model.  
 268 Hsieh and Omura (2017) developed an analytical expression for the components of a  
 269 quasi-parallel whistler mode wave, based on the electric field linear system proposed by  
 270 Stix (1962) and obeying the Faraday's law. With the  $k$ -vector lying in the  $(x, z)$ -plane,  
 271 the wave fields are given by the authors as

$$\begin{aligned} \mathbf{B}_w &= \mathbf{e}_x B_x^w \cos(\Psi) + \mathbf{e}_y B_y^w \sin(\Psi) - \mathbf{e}_z B_z^w \cos(\Psi) \\ \mathbf{E}_w &= \mathbf{e}_x E_x^w \sin(\Psi) - \mathbf{e}_y E_y^w \cos(\Psi) + \mathbf{e}_z E_z^w \sin(\Psi) \\ \Psi &= \omega t - k_x x - k_z z \end{aligned} \quad (2)$$

272 Following Hsieh and Omura (2017) in the limit  $\omega \ll \omega_{pe}$ , we get the following  
 273 polarizations

$$A_s = Y + \frac{Y^2 - 1}{\cos(\theta) - Y} \quad ; \quad A_p = \frac{\sin(\theta) \cos(\theta)}{\sin^2(\theta) - 1 + \cos(\theta)/Y} \quad (3)$$

274 Finally, the wave fields components are expressed as

$$\begin{aligned} B_y^w &= A_s(1 - A_p \tan(\theta)) B_x^w \quad ; \quad B_z^w = \tan(\theta) B_x^w, \\ E_x^w &= A_s v_{p\parallel} B_x^w \quad ; \quad E_y^w = v_{p\parallel} B_x^w \quad ; \quad E_z^w = A_s A_p v_{p\parallel} B_x^w, \\ B_x^w &= \frac{B_w}{\sqrt{\cos^2(\Psi) + A_s^2(1 - A_p \tan(\theta))^2 \sin^2(\Psi) + \tan^2(\theta) \cos^2(\Psi)}}, \end{aligned} \quad (4)$$

275 with the parallel phase speed  $v_{p\parallel} = \omega/k_{\parallel}$ .

276 The observed and modeled wave and plasma parameters for the packet shown in  
 277 Figure 3 are gathered in Table 1. Using these parameters, we have estimated the wave  
 278 vector amplitude using Equation 1, also given in the same table. We can now obtain  
 279 the fully analytical, temporal and spatial model of the wave, using Equation 4. We  
 280 note that no information about the electric field is fed to the analytical model, this  
 281 field is thus completely constrained by the model.

282 The result for a fixed point in space is given in the lower-right panels of Figure  
 283 2 over a few wave periods. The analytical model results in a slightly higher  $E_w/B_w$   
 284 ratio, but provides satisfactory wave forms and Poynting vector form and amplitude,  
 285 additionally obeying Maxwell's equations. We can now use this model to investigate  
 286 resonant wave-particle interactions.

#### 287 **4 Resonant test-particles vs. observed VDF**

288 We will now examine the potential effect of the modeled wave on the electron  
 289 dynamics, and see whether or not signatures in the electron VDF can be found. Before  
 290 considering simulating the situation with a self-consistent model (fields and particles  
 291 feedback on each other according to Maxwell's equations), we explore the possibilities  
 292 offered by a test-particle approach. The fields and waves are analytically described  
 293 with constant frequency and wave-vector through time: there is no feedback from  
 294 the particles on the fields. Thus, the presence of other particles, or the shape of the  
 295 distribution, does not affect a single particle dynamics, which can be solved on its  
 296 own. This allows for a very cheap, flexible first study of the resonant dynamics in the  
 297 case of our simplified wave. A considerable advantage is that by doing so, we isolate

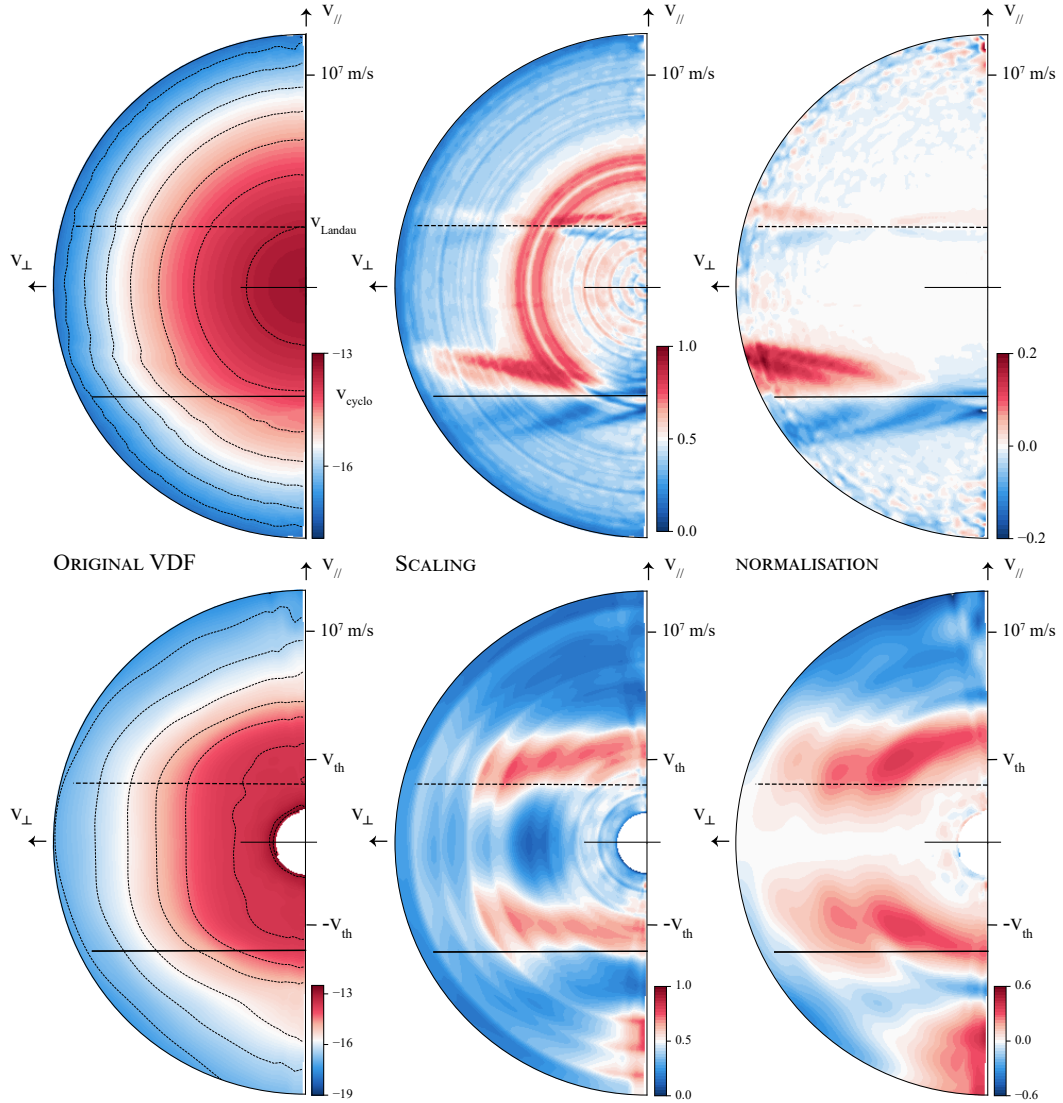
	Observed	modeled
$B_0$	25.4 nT	25.4 nT
$n_e$	$16.3 \text{ cm}^{-3}$	$16.3 \text{ cm}^{-3}$
$B_w$	[0.2, 0.7] nT	0.5 nT
$E_w$	[0.5, 1.4] mV/m	1.5 mV/m
$\omega$ ( $\omega/\omega_{ce}$ )	1571 rad/s (0.35)	1571 rad/s (0.35)
$k$ ( $k \cdot d_e$ )	$\emptyset$ ( $\emptyset$ )	$5.9 \cdot 10^{-4}$ rad/m (0.77)
$\theta$	[0°, 20°]	20°
$T_{\parallel}$	45 eV	45 eV
$T_{\perp}$	41 eV	45 eV
$\beta_e$	0.46	$\emptyset$

**Table 1.** Wave parameters for the observation and the model, and additional plasma parameters.

298 the effects *of* the wave *on* the particles, and not the other way around, which is not  
299 directly possible in a self-consistent description.

300 In the absence of collisions and gravity, and using the analytical wave discussed in  
301 the previous section, we know the force experienced by a particle at any time and any  
302 position. The particles dynamics are solved according to this force using the classical  
303 Boris scheme (Boris, 1970), widely used by the Particle-In-Cell community because of  
304 its straightforward implementation and its great accuracy for this precise problem –  
305 charged particles moving in electromagnetic fields. We initialise 200 million particles  
306 following a 3-dimensional isotropic non-drifting Maxwellian distribution, characterised  
307 by the observed electron (parallel) temperature of 45 eV. These particles are homoge-  
308 neously distributed in physical space, in a 2-dimensional periodic box, extending over  
309 one projected wavelength (parallel for one axis, perpendicular for the other). Results  
310 are given for a simulation time of 25 ms, corresponding to a bit more than 6 wave  
311 periods and about 15 electron gyrations.

312 The Landau resonance is obtained for particles with a parallel velocity close  
313 to the wave parallel phase speed: seen from these particles' perspective, the parallel  
314 component of the wave is almost static, and continuously accelerates them. This gives  
315 us a first resonant speed,  $v_{Landau} = \omega/k_{\parallel}$ . Because of their angle from the background  
316 magnetic field, the observed and modeled waves have a projected parallel component,  
317 which therefore enables this resonance. Particles with a parallel speed slightly slower  
318 than the parallel phase speed are being caught up by the wave and gain kinetic energy,  
319 whereas faster particles experience the opposite phenomenon. These resonant particles  
320 migrate in phase space, with a motion dependent on their initial phase space position.  
321 The wave we consider here has a dependence along the perpendicular direction: as  
322 a particle gyrates, its perpendicular position evolves, and so does the magnetic and  
323 electric wave components it experiences. A deeper, 3-dimensional description of these  
324 resonant motions, depending on  $v_{\parallel}$  and  $v_{\perp}$  is beyond the scope of this article, despite  
325 its great interest. These dynamics quickly result in a mixing of the particles in phase  
326 space around the parallel Landau speed. It is noteworthy that if one initialises the  
327 simulation with a flat velocity distribution (i.e. no density gradient in velocity space),  
328 no signature of this mixing can be found in velocity space. A density signature is indeed  
329 only visible if the resonant speed corresponds to high VDF gradients, which happens  
330 to be the case here. Very quickly, in just 6 wave periods, the resonant particles form an  
331 under/over-density centered on the Landau resonant speed, as seen in the scaled and  
332 normalized VDF of Figure 4 (the Landau resonant speed is displayed by the dashed  
333 horizontal line).



**Figure 4.** Test-particle results (top) compared to the observation (bottom), for an integration time of 60 ms. The Landau parallel resonant velocity is given by the horizontal dashed line and the cyclotron one by the solid lines.

334 The cyclotron resonance is only met in the presence of a circularly polarised wave.  
 335 Particles with different parallel speeds experience a wave with a different frequency, a  
 336 simple Doppler effect dependent on the particles velocity. For one particular parallel  
 337 speed, given by  $v_{Cyclo} = (\omega - \omega_{ce})/k_{\parallel}$ , with  $\omega_{ce}$  the electron gyrofrequency, the wave  
 338 is seen with an angular frequency equal to the electron (angular) gyrofrequency: the  
 339 particles gyrate synchronously with the rotation of the circularly polarised  $\mathbf{B}_w$  and  
 340  $\mathbf{E}_w$ . Note that because  $\omega < \omega_{ce}$ , this resonant speed is negative (if  $k_{\parallel} > 0$ ): resonant  
 341 particles are moving against the wave, which in turn significantly decreases the time  
 342 during which they may interact with the wave. As discussed in the Landau case, such  
 343 particles are continuously accelerated by the wave, and migrate in phase space, in  
 344 an even more complex manner. The mixing of a high density gradient region again  
 345 results in an under/over-density organised around the cyclotron speed given by the  
 346 solid line in Figure 4, with a sign opposite to the one of the Landau resonant speed.  
 347 The over-density is found for lower parallel speeds but higher perpendicular speeds,  
 348 corresponding to higher total speed for these particles.

349 The lower row of Figure 4 gives a comparison to the VDF observed during the  
 350 same wave packet. We use only two FPI observations, corresponding to an integration  
 351 time of 60 ms. There as well, two over-densities are found just above the two resonant  
 352 speeds, at the same location as for the test-particles. The scaled view of the test-  
 353 particle VDF exhibits two strong, perfectly circular artefacts, which were discussed  
 354 in the first section. The normalized view, conserving the continuity along the radial  
 355 dimension, does not display such structures. An additional feature is also found along  
 356 the anti-parallel direction. We will see in the next section that this beam does not  
 357 correlate with the presence of the wave. The thermal parallel speed is indicated in  
 358 the observed distributions. The speed is the same as the central speed of the over-  
 359 densities: the resonant mixing of particles indeed happens where the VDF velocity  
 360 gradient  $\partial f/\partial v_{\parallel}$  is high, which could lead to an efficient damping of the wave. We  
 361 note however that the relevance of this thermal speed is limited, when considering the  
 362 large departures of the observed VDF from a Maxwellian distribution.

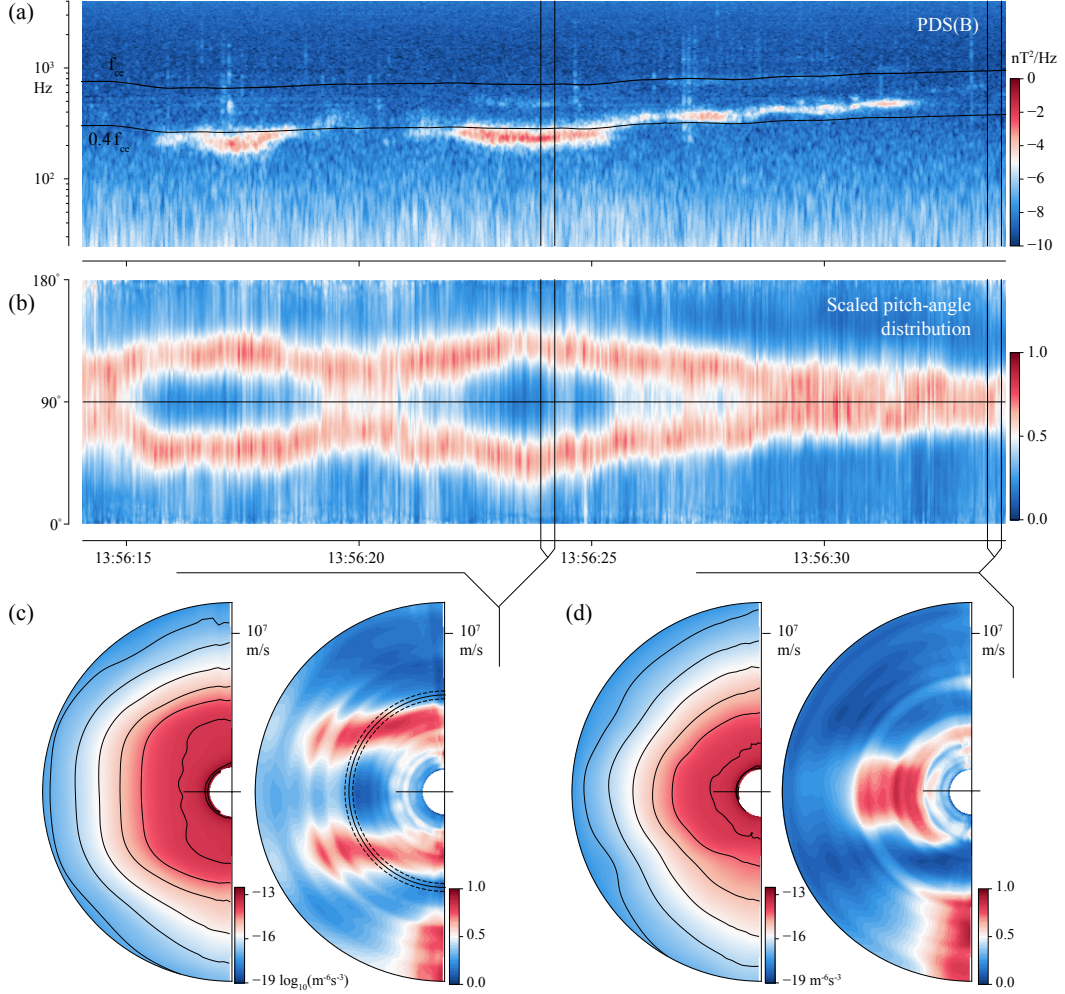
363 The over densities are observed around similar absolute parallel velocities, despite  
 364 the absolute value of the resonant parallel velocities being clearly different: the over-  
 365 densities tend to be fairly symmetric with regard to the ( $v_{\parallel} = 0$ )-plane. Note that this  
 366 fact holds for the plasma and wave parameters analysed here. If the position of the two  
 367 over-densities are matching nicely between the test-particles and the observed VDF,  
 368 strong discrepancies in the shape of these signatures exist and are discussed below.

## 369 5 VDF time evolution

370 To verify if this double branch signature indeed correlates with the presence of  
 371 the wave over longer time scales, we show In Figure 5 (c-d) two additional electron  
 372 VDFs integrated over a longer time of ten FPI measurements, corresponding to an  
 373 integration time of 300 ms. In both cases, a strahl-like (beam) component is found  
 374 along the anti-parallel direction, with a constant perpendicular width. In the absence  
 375 of wave activity (last VDFs), we find a clearly anisotropic distribution, with a strong  
 376 equatorial signature in the scaled view.

377 When analysed during the maximum power of the wave activity, the VDF dis-  
 378 plays the double-branch signature. It was checked that during the entire time interval,  
 379 no significant (i.e. higher than the measurement noise) agyrotropy are to be found.  
 380 Ions (not shown here) present a similarly anisotropic distribution, virtually static over  
 381 the time interval.

382 Figure 5 (b) gives a view on the temporal evolution of the scaled VDF and its  
 383 double-branch feature. For this representation, each 30 ms scaled VDF was averaged  
 384 over a limited speed range between  $5.8$  and  $6.3 \cdot 10^6$  m/s (indicated on the scaled view  
 385 in panel (c)), and the result was plotted in the time  $t$  and pitch-angle  $\Theta$  dimensions in

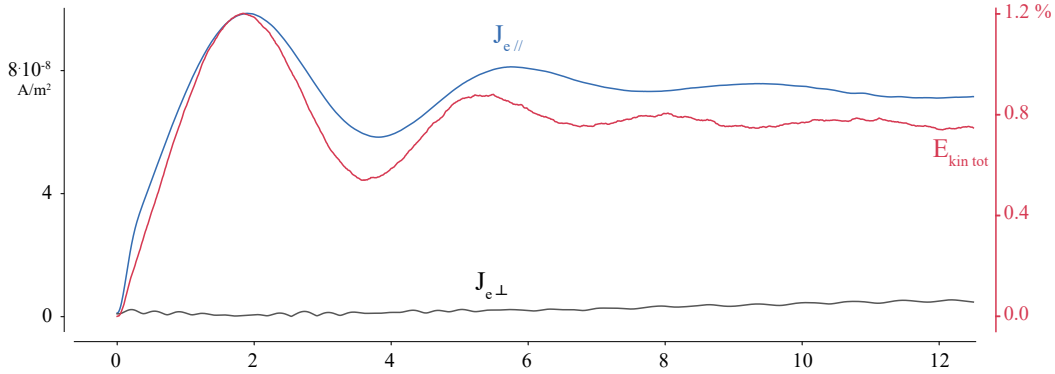


**Figure 5.** (a) B-field power density spectrogram. (b) scaled pitch-angle distribution for a speed range indicated in (c). (c)-(d) time-integrated VDFs and their scaled view, with the integration time indicated on the time series.

386 panel (b). Note that scaled pitch-angle distributions and classical pitch-angle distributions  
 387 qualitatively converge as the energy range decreases. In this energy range, scaled  
 388 values above 0.5 (red tones) form a striking feature centered on 90 degrees. When the  
 389 waves are observed in the magnetic field spectrogram (panel (a)), the feature splits into  
 390 two branches, corresponding to the double-branch pattern observed in velocity space.  
 391 We note that such a pitch-angle distribution looks very similar to electron distributions  
 392 in mirror modes, explored by Breuillard et al. (2018), where the envelope of the  
 393 pitch-angle distribution is shown to correlate with a critical pitch-angle, under which  
 394 electrons get trapped in one mirror structure. We note, however, that in our case, the  
 395 signature is ordered by (constant) parallel velocities, and not constant pitch-angles  
 396 when all the energy range is considered.

## 397 6 Particle energisation

398 We can follow the energisation of the test-particles either by summing up the  
 399 total kinetic energy of all particles, or simply by taking the center of mass of their



**Figure 6.** Test-particle total kinetic energy (relative to its value at  $t=0$ ) and current density, parallel and perpendicular. Time is given in unit of wave period  $T_{wave}$ .

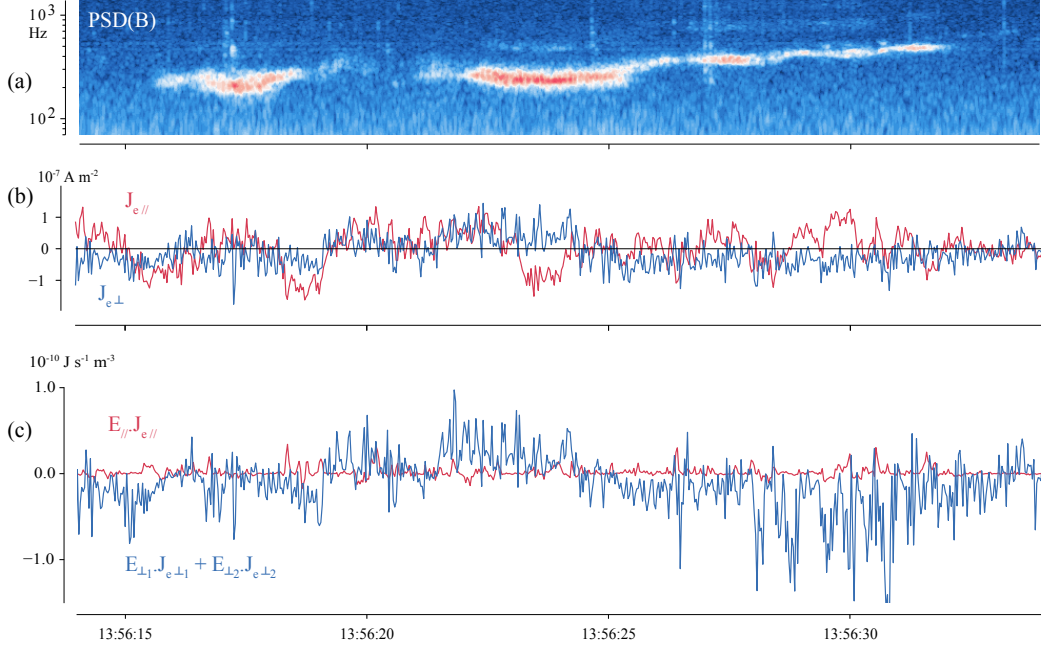
400 distribution (order 1 moment) and expressing it as a current density, with one parallel  
401 and one perpendicular component. The total kinetic energy and the two components  
402 of the current density are given in Figure 6. We find that with the physical parameters  
403 of our problem, the maximum energisation happens very fast, in about 2 wave periods,  
404 or 5 gyroperiods. Therefore, the wave packets are sufficiently long to exchange energy  
405 with the particles. After about 5 wave periods, the overall energy gained by the test-  
406 particles, as well as the parallel current density, stabilise to a non-zero value. The  
407 test-particle approximation does not hold anymore, as this current would necessarily  
408 alter the electric and magnetic fields, which would act to decrease it back to zero.  
409 For this reason, it is likely that this estimated current for the present resonant wave-  
410 particle interaction is an upper bound for the self-consistent interaction. Barely any  
411 perpendicular current is found, indicating that the VDF remains gyrotropic at all  
412 times. The fact that the total kinetic energy of all particles and their parallel current  
413 have almost the same time evolution illustrates that these resonant interactions mostly  
414 result in an increase of the bulk velocity and only some of the energy is transformed  
415 into “temperature”, or change of the pressure tensor.

416 A crucial point is indicated by this time evolution and verified in the time evolution  
417 of the simulated VDF (not shown) is that the resonant signatures are not periodic:  
418 they remain at the same position (in velocity space) with the same over/under densi-  
419 ties through time. This is a necessary condition for it to be observed by an instrument  
420 with an integration time longer than the wave period.

421 In Figure 7, the observed electron current density (parallel and perpendicular)  
422 are displayed, together with the cropped spectrogram of the magnetic field. The  
423 parallel current is varying around 0, and no correlation with the wave activity can be  
424 found. Despite the strength and duration of the wave packets, the *background* current  
425 density turns out to be significantly larger than the estimated (upper bound) current  
426 rising from the resonant particle signatures. Alternative moments were also calculated  
427 ignoring the core of the distribution (using different speed threshold), showing barely  
428 any changes in the electron current. In turn, the dissipative term  $\mathbf{E} \cdot \mathbf{J}_e$ , using an  
429 electric field averaged over the electron integration time, shows no correlation with the  
430 wave activity.

## 431 7 Discussion & Conclusions

432 We have first shown that VDF features originally hidden under high gradients  
433 of the background VDF can be highlighted using some scaling and normalization,



**Figure 7.** (a) Magnetic field spectrogram, (b) observed electron current and (c) dissipative term.

434 without the use of a reference VDF  $f_0$ . We then isolated a strong, fairly narrow-  
 435 band, quasi-parallel whistler mode wave, which could be analytically mimicked. We  
 436 applied this analytical wave to a collection of test-particles to get a first sense of its  
 437 potential effects on the electrons. We could map where and how particles resonate,  
 438 and have found that the initial Maxwellian distribution is reshaped by Landau and  
 439 cyclotron resonances. Test-particles and observed VDFs display two branches of higher  
 440 density, around constant parallel velocities, in the scaled and normalized views of the  
 441 VDF. The observed signatures were found to correlate nicely with the wave activity.  
 442 Finally, this wave-particle interaction could not be detected in the observed current  
 443 or the observed  $\mathbf{E} \cdot \mathbf{J}_e$  product, as both  $\mathbf{E}$  and  $\mathbf{J}_e$  (observed) present fluctuations of  
 444 larger amplitude than the current and electric field used by or resulting from the  
 445 model. Strong discrepancies between the simulation and the observations remain to  
 446 be discussed.

447 Firstly, the strongest discrepancy between the observed VDF and the simulated  
 448 one is the strong symmetry of the observed signatures around the ( $v_{\parallel} = 0$ )-plane,  
 449 further illustrated by the time evolution of the scaled pitch-angle distribution in Figure  
 450 5. As such, it is difficult to prove that the Landau resonance, providing an already faint  
 451 signature in the perfect test-particle set-up, may cause a resonant branch as strong as  
 452 the observation, and so symmetric to the cyclotron branch. It actually *appears* as if  
 453 a mirrored cyclotron branch shows up systematically with the wave activity, despite  
 454 our wave analysis only bringing out one single mode, propagating in one well defined  
 455 direction. Such symmetric signatures have been displayed in the works of Min et al.  
 456 (2014) (in the magnetosphere) and Chen et al. (2019) (in a nominal, or fully developed  
 457 turbulence case), in which the authors did not isolate one single mode, and interpret  
 458 the symmetric signatures as caused by waves propagating in both directions, parallel  
 459 and anti-parallel. This symmetry remains the foremost open question of our study.

460 Secondly, the test-particle model cannot account for the transformation from the  
 461 already strongly anisotropic VDF out of the wave activity (shown in panel (d) of Figure



462 5) to the 2-branch signature. The disturbance of the VDF caused by resonances in the  
 463 test-particle case is a local phenomenon in velocity space, it certainly cannot reshape  
 464 macroscopically the distribution, make a large amount of the electron population mi-  
 465 grate over large velocities. Were we able to easily initialise the particles according  
 466 to the observed VDF outside the wave activity, the strong equatorial structure would  
 467 remain there, with additional resonant signature expressed on the sides. While the  
 468 test-particle simulation enlighten us on the micro-physics of the interaction during a  
 469 very brief snapshot of the observations (when an intense wave activity is observed),  
 470 the macroscopic configuration of the fields (magnetic gradients, mirrors, etc) should  
 471 be considered to fully understand the time evolution of the electron VDF.

472 The limitations of the test-particle simulation are numerous, if one is to thor-  
 473 oughly compare its results and the observations. Instead, we suggest that this approach  
 474 only points at the very first step of the full self-consistent wave-particle interaction.  
 475 It shows us where and how in velocity space resonances should occur for a given a  
 476 monochromatic wave, and gives a first hint on their phase density signatures. Most  
 477 importantly, it also gives us a good sense of how fast these resonances can produce  
 478 signatures in the VDF, and their efficiency. But it obviously cannot go further in the  
 479 physics of the interaction, both an advantage and a drawback.

480 This numerical approach is limited to the case of a purely monochromatic wave  
 481 with a constant amplitude and a constant normal angle, whereas the observed wave  
 482 packets exhibit some spectral breadth, a parallel component at a slightly higher central  
 483 frequency, and additionally a propagation direction constantly evolving. Because of the  
 484 periodic boundaries necessary for the simple test-particle approach, such a wave cannot  
 485 be easily modeled. We refer to the work of Tao et al. (2012), in which the authors  
 486 specifically highlighted the effect of the amplitude modulation of chorus emissions  
 487 in the magnetosphere, on particle acceleration. They, however, do not reconstruct  
 488 distribution functions, and can therefore do without the periodicity constraint.

489 The feedback of the particles on the wave is a different problem entirely, in terms  
 490 of numerics at least. Thus the comparison should not be over-interpreted. We have  
 491 shown that the observed wave is expected to have clear signatures on the electron  
 492 distribution function, signatures matching well in position and somewhat in shape the  
 493 observed signatures.

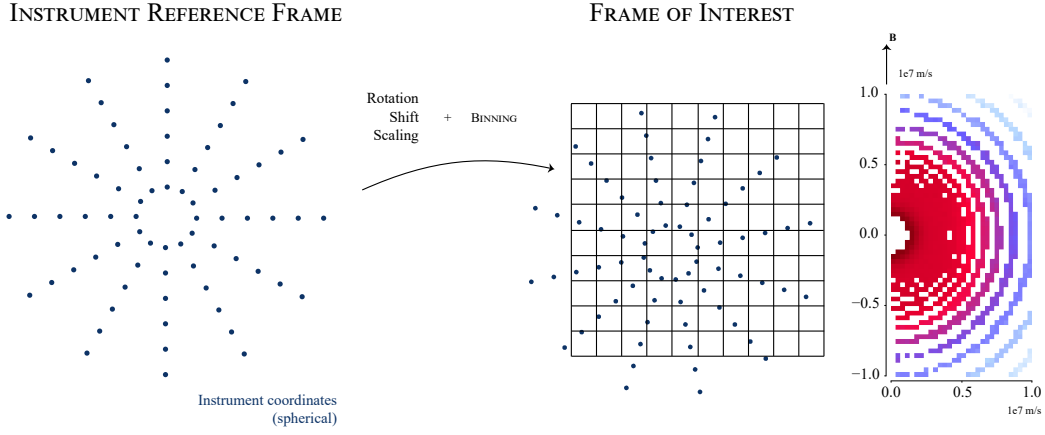
494 The visualisations (scaling and normalization) developed for this study are a key  
 495 element for a better, deeper characterisation of the particles distributions. It is in the  
 496 numerical aspect of our work that these views reveal best the effect on the distribution,  
 497 getting us rid of the need for an additional information (e.g. distributions at other  
 498 times). They show great flexibility and versatility, which may be useful for other  
 499 applications on observations and simulation data, and mostly their comparisons.

500 Such an exploration of wave-particle resonances in 3 dimensions, and its direct  
 501 comparison to observations, is – to the best of our knowledge – novel. It familiarises  
 502 us with resonant dynamics and signatures, at a low cost and without the intrinsic  
 503 complexity a self-consistent model adds to physical interpretations. This approach  
 504 may present real promises for a more systematic recognition of resonant signatures in  
 505 a more complex data, such as those of fully developed turbulence in the solar wind or  
 506 the magnetosheath  
 507

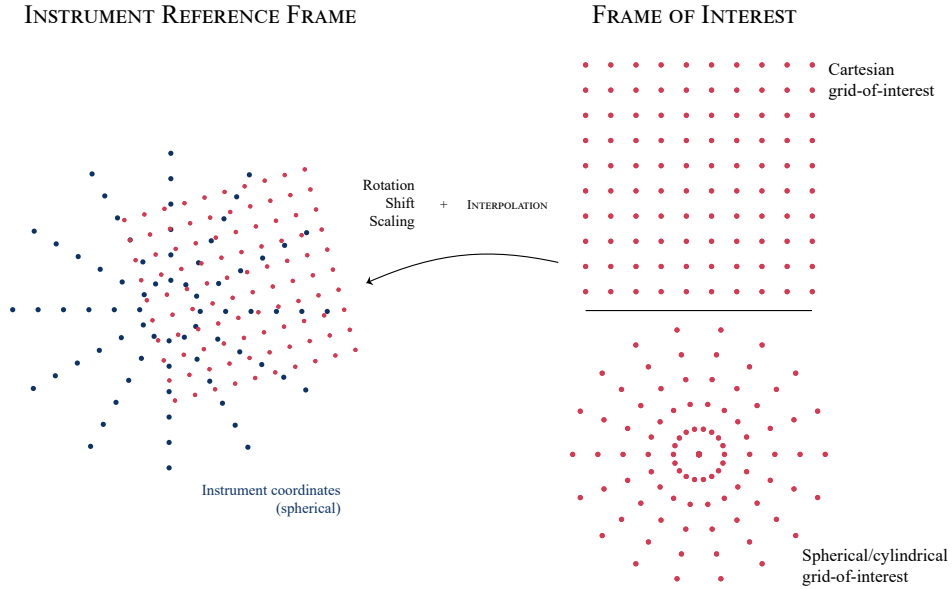
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 512 Contel and Gérard Belmont for valuable discussions that have broadened the context  
 513 of this study.

## BINNING



## INTERPOLATION



**Figure A1.** Illustration of a binning and an interpolation.

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## Appendix A Binning versus interpolating

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In various publication, when handling multidimensional Velocity Distribution Functions (VDF), authors choose the somewhat more intuitive of rotating the data from the instrument frame to the frame of interest, to then *bin* the data within a defined grid, as illustrated in Figure A1 in the first row. During this process, one measurement point ends up in one single cell of the grid-of-interest, and one cell of the grid-of-interest may receive zero one, or many measurements. Empty cells will appear on the representation, producing visual artifacts, and it is necessary to average the binned data over at least one dimension in order to *fill* as much as possible the grid-of-interest. We give such an example in a cylindrical representation of the VDF, right-most plot of the first row of Figure A1, which shows about thirty electron distributions

525 binned in a 2-dimensional grid defined, in the classical  $(v_{\parallel}, v_{\perp})$ -plane. The binned  
 526 data show strong artifact along the energy dimension, as the instrument energy levels  
 527 are log-distributed: the higher the energy, or speed, the greater the energy steps.  
 528 Therefore one regular Cartesian grid cannot be suitable for the entire energy range of  
 529 the instrument: its resolution will be too coarse at low energies – where weighting of  
 530 the binned data has to be taken into account properly – and too fine for high energies,  
 531 leaving empty most of the grid-of-interest. The binning approach can only be used over  
 532 a restricted range of energies, and is anyway not suitable for 3-dimensional analyses.

533 The interpolation approach is illustrated in the second row of Figure A1. First,  
 534 we define a 3-dimensional grid-of-interest – an array/set of coordinates – within the  
 535 reference frame of our choice, with arbitrary extent and resolution. We apply the  
 536 opposite rotation, shift, and scaling to a copy of the grid-of-interest, from the reference  
 537 frame to the instrument frame, as illustrated in Figure A1. The values of the data  
 538 are then interpolated at each node of the transformed grid (see next section): the  
 539 data are continuously evaluated over the entire grid, leaving no room for artifacts, and  
 540 avoiding additional weighing operation. All the results in the article use a spherical  
 541 grid-of-interest, allowing at no additional cost the scaling and normalization of the  
 542 VDF, and their straightforward representation in the  $(v_{\parallel}, v_{\perp})$ -plane.

543 But interpolation with an order higher than 2 presents complications, and this  
 544 approach is not universal and cannot be applied blindly.

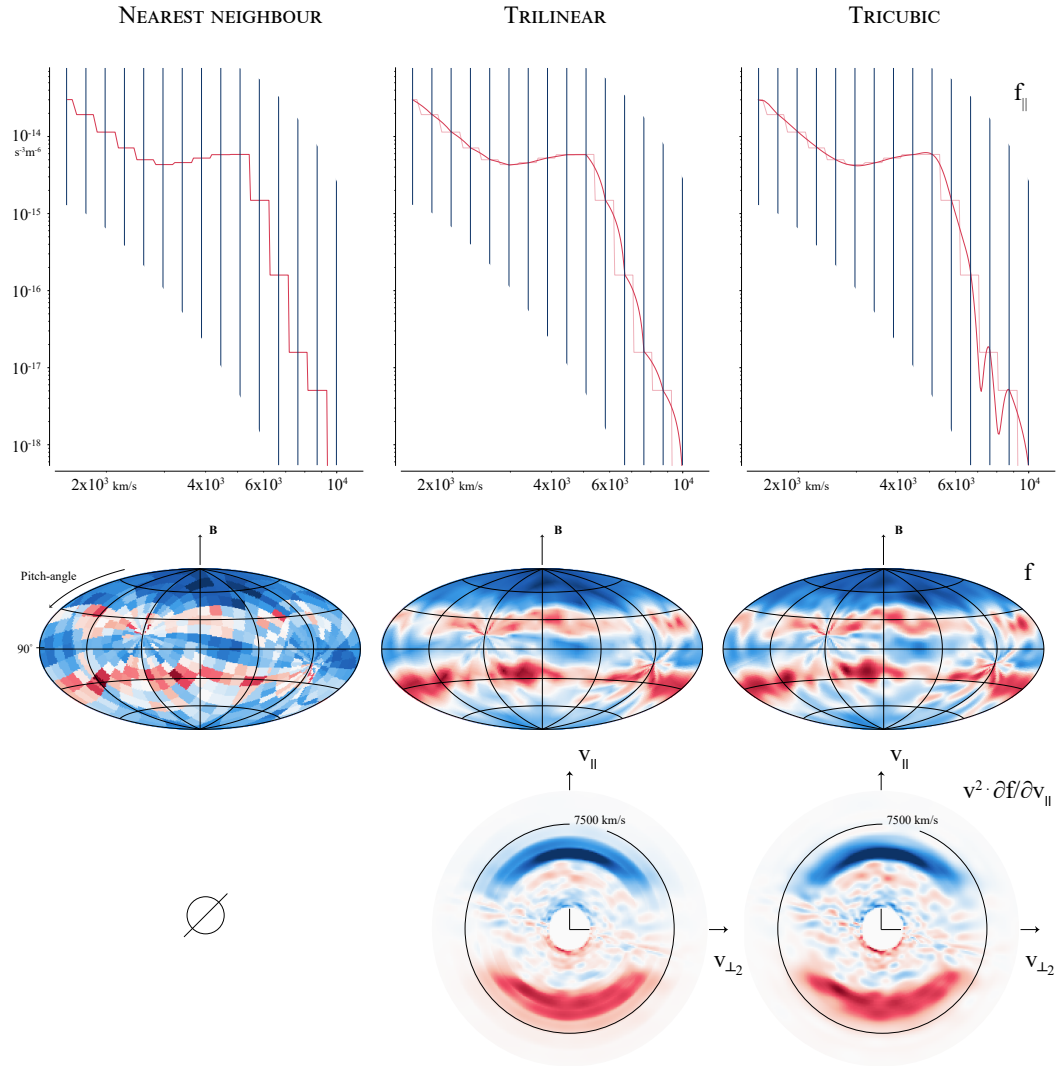
## 545 Appendix B Interpolating velocity space distributions

546 We have tested three different interpolation schemes, namely nearest-neighbour  
 547 (order 0, only one point of the measured VDF is used), trilinear (order 1, 8 data points  
 548 are used), and tricubic (order 2, 64 data points are used). The two first schemes are  
 549 taken from the main scientific python library, SciPy, while the third was implemented  
 550 by (Lekien & Marsden, 2005), with one wrapper made available by the authors for a  
 551 usage with Python.

552 We now have a 3-dimensional array of interpolated VDF values, with two different  
 553 sets of coordinates, one in the instrument frame (used for the interpolation), one in  
 554 the reference frame of the study (the originally defined Grid-of-Interest). This array  
 555 can be either analysed and visualised on its own, or added up to other arrays in order  
 556 to average the data over a longer duration, all within the same frame. For instance,  
 557 Figure B1 presents results using only one single electron VDF, which is the most  
 558 challenging test for the methods, because of low detection rates for higher energies.

559 This method has one invaluable interest: any resolution can be used without any  
 560 of the risks inherent to a binning process, illustrated in Figure A1. Another great  
 561 advantage when using a Cartesian grid-of-interest is that velocity derivatives of the  
 562 data can be easily and straightforwardly computed. The advantage of using a spherical  
 563 or cylindrical grid-of-interest was already illustrated. The method has one drawback,  
 564 namely the computational cost. The interpolation in itself is more computationally  
 565 demanding than a simple binning, and for most purposes we will have many more  
 566 elements in the grid-of-interest than in the instrument grid, resulting in as many more  
 567 calculations to compute.

568 In Figure B1, a single electron distribution is used to illustrate the differences  
 569 between the three interpolation schemes, using three different visualisations. In the  
 570 first row, we give the profile (1-dimensional) of the interpolated values along the paral-  
 571 lel direction. For this representation and in order to increase slightly the statistics, all  
 572 values within an angular distance from the parallel axis ( $+\mathbf{B}$ ) are selected and aver-  
 573 aged, using a conic selection  $\pi/6$ -wide. For this purpose, a spherical grid-of-interest is  
 574 a better choice, allowing one to simply average the interpolated data over one angular  
 575 dimension, avoiding a binning process.



**Figure B1.** Interpolation.

576 The energy/speed levels of the instrument are also indicated as vertical lines,  
 577 regularly spaced in this logarithmic representation. For this analysis, we only rotated  
 578 the frame, without a shift, so the energy levels of the instruments remain centered on  
 579 the origin of the Grid-of-Interest, making the results more readable.

580 Another representation is proposed on the second row of Figure B1, in which  
 581 we selected a specific speed range, averaged the data over this range, resulting in the  
 582 given angular maps (2-dimensional). Here as well, using a spherical Grid-of-Interest  
 583 makes this selection straightforward, selecting only one radial range in the array, again  
 584 avoiding a binning process. The parallel axis is vertical and intersects these maps at  
 585 their poles, and the equator correspond to a pitch angle of 90 degrees.

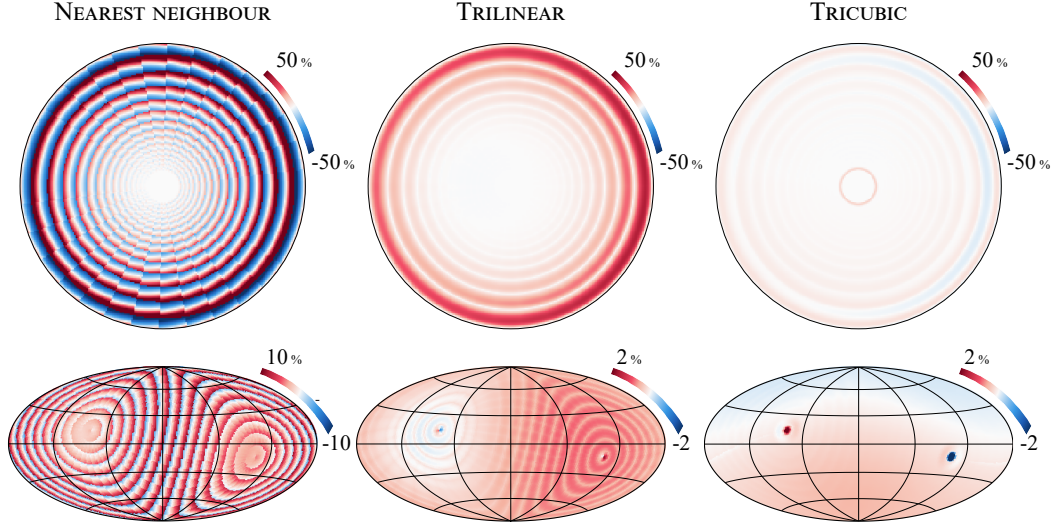
586 In the parallel 1-dimensional profiles previously described, the nearest-neighbour  
 587 interpolation results in steps centered around the instrument energy levels, as expected  
 588 from such a scheme. This curve is reported in the two other VDF profiles for the  
 589 trilinear and tricubic cases, in order to verify that all three interpolations indeed  
 590 meet at the instrument speed levels, an important convergence test. These steps in  
 591 the nearest-neighbour case directly correspond to the “pixels” found in the left-most  
 592 angular map. This is the way particle data are often displayed, implying that within an  
 593 instrument “pixel”, the observed flux is constant. This assumption is also widely used  
 594 when integrating the plasma moments, which simply corresponds to a Riemann sum  
 595 of the area under a function. The instrument poles are misaligned with the magnetic  
 596 field direction, and can be seen close to the equator, 180 degrees apart, with triangular  
 597 pixels meeting in one point.

598 The trilinear interpolation provides a smoother, more continuous profile. Think-  
 599 ing in one dimension, between two measurement points, interpolated values will follow  
 600 a linear relationship. In this logarithmic representation, these linear segments show  
 601 up as arc segments in-between each instrument energy/speed level. This is obviously  
 602 an unwanted visual artifact, generated by the choice of the representation. Just as the  
 603 1-dimensional profile, the angular map shows a smoother result than in the nearest-  
 604 neighbour case. In the same way that the nearest-neighbour can be linked to a Riemann  
 605 sum, this linear interpolation converge to a trapezoidal rule in terms of VDF integral.

606 Finally, the tricubic interpolation provides the smoothest and most continuous  
 607 curve for speeds up to 7000 km/s, for which the VDF is relatively high. At higher  
 608 energies, strong artifacts are found, with oscillations between the last instrument speed  
 609 levels. Two phenomena can account for this behaviour. The first could be the Runge’s  
 610 phenomenon, namely oscillations at the edges of the interpolation interval when using  
 611 polynomials of high degree (higher than two). We note however that we only use a  
 612 third degree polynomial interpolation, which should limit this phenomenon, and these  
 613 oscillations are only visually found for low fluxes, high speeds. The second phenomenon  
 614 occurs when the VDF contains zero values. But most of all, this interpolation scheme,  
 615 as derived and implemented by Lekien and Marsden (2005), can result in negative  
 616 VDF values. This nonphysical result should be monitored, so these negative values  
 617 remain small and insignificant for the analysis we want to perform.

618 *Remark:* these interpolations and their unavoidable artifacts and drawbacks are  
 619 not well suited for all purposes. For instance, the tricubic interpolation is valid only  
 620 for strong signals, high VDF values, and should anyway be used with great caution,  
 621 so these artifacts are not interpreted as physical features.

622 As a first test, we have already verified that the three schemes converge at the  
 623 instrument speed levels. A second important test is to make sure that the three  
 624 interpolations do not show artifacts at the instrument poles. Indeed, the trilinear and  
 625 tricubic schemes we use assume that the data are defined over a grid with cuboid  
 626 elements: evenly spaced along each dimension, with possibly different spacings along  
 627 each dimension. The poles of the instrument spherical grid should therefore present  
 628 errors, as the array elements there strongly depart from cuboids. To test the overall  
 629 error from each scheme, we have defined artificial, ideal instrument measurements,

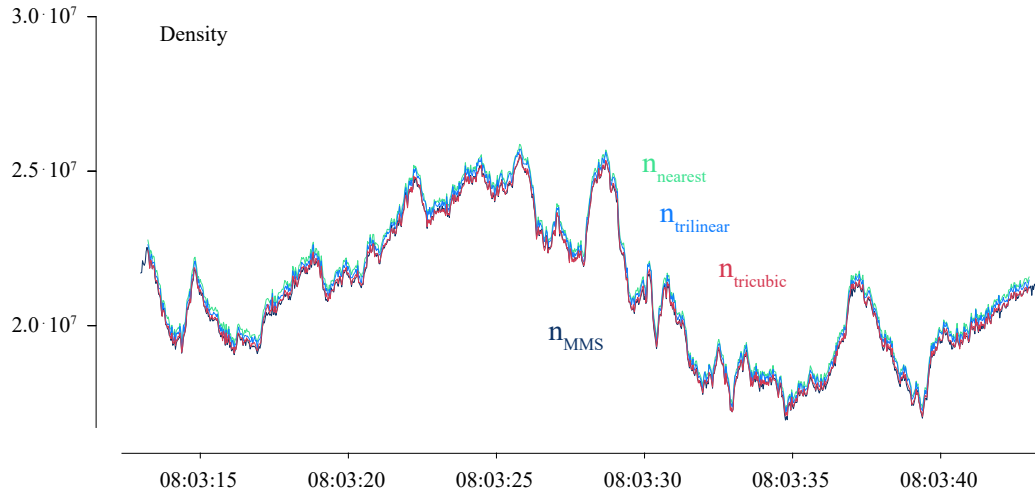


**Figure B2.** Error interpolation.

630 namely a drifting<sup>1</sup> Maxwellian distribution. This way, we also know the real value of  
 631 the distribution at the nodes of the rotated, shifted, scaled grid-of-interest, and can  
 632 compare these analytical values to the interpolated ones. It allows us to create error  
 633 maps, presented in Figure B2. The first row shows one chosen Cartesian cut through  
 634 the error distribution, while the second row gives one angular cut, or angular map. The  
 635 nearest-neighbour interpolation results in the largest errors, just as expected from the  
 636 steps seen in the 1-dimensional profiles of Figure B1: the interpolation is alternately  
 637 much smaller and much larger than the real analytical value of the distribution. These  
 638 errors can largely surpass the 50 percent level, because of the steepness of the VDF.  
 639 Since the Maxwellian distribution is convex for high energies/speed, the error given by  
 640 the linear interpolation is positive-only for high speed values, as seen in the first row,  
 641 middle panel. At these higher speeds, the error can reach up to 50 percent, whereas at  
 642 lower speed values – of greater interest for us – the error becomes arbitrarily low. The  
 643 tricubic interpolation shows the smallest error in this plane with mostly positive values.  
 644 We note a slightly stronger error-ring at the lowest speeds covered by the instrument,  
 645 which we link to the Runge’s phenomenon. At the high VDF values usually observed  
 646 at these energies, this error may become significant for some purposes, though it has  
 647 not been found to be the case for our analyses.

648 The instrument poles are not intersected by this plane, but we expect to find the  
 649 greater error there. They however show up in any angular map, as seen in the second  
 650 row of Figure B2. The poles still have the same position as previously, and can be  
 651 easily spotted in each interpolation error map. The worst qualitative result is obtained  
 652 for the tricubic interpolation, with relatively strong errors localised at the instrument  
 653 poles, though the absolute error remains low (a few percents) and indeed extremely  
 654 localised. We conclude that for our purpose, the three schemes are sound, resulting in  
 655 acceptable errors where the observed VDF values are high, i.e. for speeds lower than  
 656  $10^4$  m/s.

<sup>1</sup> The drift is important for the significance of the test, as it misaligns the center of the artificial distribution, and the center of the instrument grid, testing the errors over all three dimensions.



**Figure B3.** Moments tests.

657 As a third test, we have verified that the electron density, resulting from integrat-  
 658 ing the interpolated values using a simple Riemann integral, calculated for the three  
 659 interpolations, matches the density provided by the instrument team. Their densi-  
 660 ties are also the plasma moment of order 0, but directly integrated in the instrument  
 661 spherical coordinate system. The result is shown in the first row of Figure B3, and  
 662 one can find that the four curves are barely distinguishable.

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