

External Didactic Transposition in Undergraduate Mathematics

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External Didactic Transposition in Undergraduate Mathematics

ABSTRACT. How is the content of university programmes selected and structured? How does it evolve? In the anthropological theory of the didactic, external didactic transposition designates the processes through which knowledge to be taught is delimited and developed for delivery within a school institution, including universities. This paper presents a first international comparative research project focused on the external didactic transposition in European and Canadian undergraduate programmes in mathematics. The project aims at analyzing the factors that motivate the evolutions of the programmes, the procedures to develop them, as well as the constraints that limit broader modifications and can explain their current state. The exploratory study presented in this paper considers 10 case studies from universities in Canada, Denmark, France, Germany, and Spain. Data is mainly collected from semi-structured interviews with faculty having participated in recent design of undergraduate mathematics programmes and syllabi. We identify both similarities and differences between programmes and syllabi, with the differences being relatively significant between Canadian and European universities when it comes to the overall structure of programmes, and less visible when considering the European programmes, as well as the processes in general.

Keywords: undergraduate mathematics, external didactic transposition, curricula, scale of levels of didactic codeterminacy.

Introduction, theoretical framework, methodology

Choosing a unit of analysis in university mathematics education research

The delimitation of the unit of analysis is one of the most important methodological gestures in any research. It defines the type of questions addressed and determines the empirical material used to support the analysis (Barquero, Bosch & Gascón, 2019; Ernest, 2017). Some investigations carried out in undergraduate mathematics education consider either no particular content, but rather overarching themes such as transitions, student identity or lectures. Many focus on a unit of analysis that tends to be limited to a specific curricular content, that is, a body of knowledge determined by university curricula and delivered within a single course unit. Few studies question the pertinence or selection of the mathematical content and the way it is structured in relation to other contents, topics or domains. Using a mathematical metaphor, we can say that mathematical content plays a central role in many studies on university mathematics education (UME), but almost always as a *parameter* of the problem, not as a *variable* susceptible to change. This phenomenon has already been identified by Winsløw, Gueudet, Hochmuth, & Nardi (2018, p. 71) when they comment on the limited impact

of research on innovation of curricula and policy. When the knowledge to be taught is taken as a given, researchers tend to locate themselves closer to the position of the teacher, without approaching the whole university teaching system from the outside.

The research presented in this paper adopts the latter, broader perspective. It aims at analyzing the development of undergraduate mathematics programmes, considering the kind of offer they provide to students in terms of knowledge and know-how – in a broad sense, including competencies, skills and attitudes. This paper describes an exploratory study on this direction, drawing on the comparison of mathematics programmes of 25 universities from Europe and Canada and on the interviews of 10 selected teachers who participated in the design of their university programmes.

Some pioneer studies about the evolution of university mathematics curricula can be found in (Huntington, 2015) and (Tucker, 2013), who investigated the evolution of undergraduate programmes in the USA in the 20th century. They distinguish three main periods: before 1950, 1950-1970, and after 1970. Until 1950, undergraduate mathematics programs remained relatively stable, with mainly calculus and geometry modules. In the 1950s, there was a huge effort to develop research and to attract students to the university. These correlated developments of research and education led to profound changes in UME, with the introduction of advanced modules such as abstract algebra and topology, and the increasing focus and volume of graduate programmes. However, there is then much less change after 1970, at least in what concerns the structure of the programmes.

In the European sphere, we were not able to find any research approaching the development of undergraduate mathematics programmes as a whole. A relatively large unit of analysis appears in Dorier's research on the teaching of linear algebra (Dorier 1990), that includes the evolution of the programmes in France for this mathematical domain and its relations to geometry and analysis. There are also more local accounts and studies of a historical nature, such as the Spanish mathematician Outerelo (2009) who presented an interesting historical overview of the evolution of the teaching of mathematics in the Complutense University of Madrid since the end of the 18th century. We refer to (Bosch & Winsløw, 2020) for a more extensive discussion of previous research which bears, in some way, on the evolution of mathematics programmes at the university level.

Theoretical framework and research questions

Research on the structure and genesis of *study programmes* requires adopting a theoretical perspective that includes the possibility to question, model, describe and analyze this type of empirical object. A central role of theories is to help conceptualizing – and questioning – the educational reality, the way teachers and students – but also scholars, politicians and plain citizens – intervene in educational processes. In this respect, the anthropological theory of the didactic (ATD), through the notion of didactic transposition (Chevallard, 1991), provides a pertinent framework for

research on the institutional production of the curricular organizations – the *knowledge to be taught*.

The notion of didactic transposition points to the fact that teaching and learning processes do not begin when teachers and students meet in a classroom. They start with the selection and elaboration of the *knowledge to be taught* from what is called the *scholarly knowledge*, that is, the organization of knowledge as it is produced and disseminated by research communities (in a broad sense). Three institutions intervene in didactic transposition processes: the *scholarly institution* of knowledge producers and users (the “experts” or scholars); the *school institution* where the scholarly knowledge has to be transposed (which in our case is a university); and the *noosphere* which is defined as the neighbourhood of the school institution and includes all those agents who make decisions about the teaching processes at play. In the case of UME, these three institutions have a large intersection. However, even if they share many of their agents, we can differentiate the *positions* occupied and the roles assumed by their subjects. Therefore, a mathematics researcher can act as a subject of the scholarly institution—a scholar—when she does research and acts as an expert of the discipline. She can also act as a subject of the noosphere when she makes decisions about the study programme of a given degree and the syllabus of its components. And she is the subject of the school institution—this or that university or faculty—when she teaches a given subject or module.

The process of selecting, adapting, organizing and declaring the *knowledge to be taught*—beginning from scholarly knowledge and ending with all the teaching materials that can be proposed in a given course—is called *external didactic transposition* (EDT), while the subsequent step towards the *actually taught knowledge* is called *internal didactic transposition* (Chevallard, 1991, p. 35). One can interpret almost all existing research on UME as primarily concerned with internal didactic transpositions when programmes and syllabi are considered a fixed context of teaching and learning processes, and the focus is on much more local conditions and constraints for teaching and learning.

The analysis and description of EDT processes lead to considering different types of questions, such as the way university study programmes are organized and modified over time, the institutional forces that influence this organization (from the scholarly institution, the *noosphere*, and the school) and those that influence its evolution or, on the contrary, hinder it. The ATD provides a second conceptual tool, the scale of levels of didactic codeterminacy (Chevallard 2002) to analyse the set of conditions and constraints that affect the evolution of educational processes within and outside the school institution. The *generic levels* of the scale include the general way of organizing teaching and learning processes – the *pedagogies* – that are not specific of a given discipline or field; the level of the *schools* that provide a variety of infrastructures (buildings, groupings, diplomas, time schedules, classrooms, libraries, etc.) where teaching and learning processes take place; the levels of *societies* and *civilizations* where schools and pedagogies take place (Figure 1). When teaching and learning

processes are determined within a given set of disciplines, the EDT processes produce new levels of codeterminacy, the *specific levels*, which provide a concrete structure of the disciplines into domains, sectors, themes and topics.

Civilizations ↔ Societies ↔ Schools ↔ Pedagogies ↔
↔ Disciplines ↔ Domains ↔ Sectors ↔ Themes ↔ Topics

Figure 1. The scale of levels of didactic codeterminacy

Our research aims at analysing the main evolutions and rationales in EDT processes leading to European and Canadian UME study programmes. More concretely, we want to describe the main tendencies and commonalities that appear in a sample of universities in the way they define their programmes and manage their evolution. We also want to study the institutional forces that influence the programmes and syllabi from different levels of the scale and different institutions within each level (the society, the scholarly institution, the *noosphere*, and the school). Finally, we want to see what are the main conceptual and organizational tools used to this endeavour. We will distinguish two main objects: on the one hand, the current situation of undergraduate mathematics programmes and the way they are conceived by faculty participating of the noosphere – the *statics* of the knowledge to be taught – and, on the other hand, how the evolution of programmes and its contents is organized – the *dynamics* of the knowledge to be taught.

To adopt a common and somehow aseptic language, we will refer to *study programmes* as the whole study processes that are proposed by universities to obtain a “degree” or “bachelor” in mathematics. Study programmes are built with what we will call a set of *modules* (usually called “subjects” or “course units”). Modules receive a name that can correspond to a sector or domain of a discipline (Linear Algebra, Analysis, Differential geometry, etc.) or to a specific selection of contents (Perspectives in mathematics, Mathematical language and reasoning, Mathematical modelling). At a second level of concretion, modules are described through a *syllabus* where the learning goals are more precisely defined in terms of subject-based contents and often also skills, learning outcomes, textbooks, prerequisites, etc. Taking the word “knowledge” in a broad sense—to include formal knowledge but also informal practices, skills and competencies, attitudes, etc.—we can say that study programmes, modules and syllabi together form the way universities define *knowledge to be taught*.

The research questions we wish to address are:

- (1) [Statics] What are the main similarities and variations in the (officially declared) structure and contents of undergraduate mathematical programmes and modules in the five countries considered? How do faculty refer to them, what terms do they find it necessary to define or discuss, and which other do they omit or take for granted?

- (2) [Dynamics] How do faculty describe and rationalize the procedures used to manage the revision and evolutions of study programmes and modules, the difficulties or limitations that motivate the changes and the results obtained? What conditions and constraints for EDT do the faculty members refer to as contributing to shape, hinder or promote major changes? At what levels of the scale can these conditions and constraints be located?

Methodology

This paper presents an exploratory study, based mainly on ten semi-structured interviews with experienced undergraduate mathematics teachers from Canada, Denmark, France, Germany and Spain – two from each. The research design is a development of the study presented in (Bosch, Hausberger, Hochmuth & Winslow, 2019)) with some methodological changes. First of all, the addition of more interviews for each country (doubling the initial number), and second of one more country (Canada). The interviewees are all university teachers and researchers in different areas of mathematics. The main selection criterion was their experience with recent curriculum design, their proximity with the authors of the paper, and their willingness to participate in the study. The bachelor programmes of mathematics of the ten universities were analysed before preparing the interviews and used as the empirical basis of our interaction with the interviewees. They are not necessarily representative of other programmes in the same country.

The interview guide used in the study contains four sections that correspond to the themes identified in the preliminary research: the first two correspond to the description of the current situation taken as a product of EDT, one for the programmes and another one for the modules; the last two refer to the functioning of the EDT as a dynamic process, distinguishing between the procedure followed to modify modules and programmes, and the general conditions and constraints promoting or limiting the evolution. The first section of the interview guide asks for a rough description of one of the latest undergraduate modules the interviewee had taught, focusing on how the syllabus influenced that teaching. The second section is about the history of this module, the genesis of its syllabus and possible conditions for changing it. The third section includes questions about the whole study program, compared to past programs and to programs of other universities. Finally, the fourth section refers to processes involved in the elaboration of the program, obstacles for changing it, and factors of influence within and outside the department.

To support the discussion, the informants were given a copy of the course structure diagram of their faculty, showing modules with titles, workload¹ credits and time organization. In particular, the informants drew arrows and the like on this diagram, to

1

A standard measure of student workload in Europe is the *European Credit Transfer and Accumulation System* (ECTS). Most bachelors in Europe are 180 ECTS (3 years). Spain is an exception with 4-year programmes of 240 ECTS. In Canada bachelor programmes require at least 120 credit hours, an equivalent of 40 courses completed in 4 years.

show the dependency between modules, and referred to it while commenting on the design of the study plan as a whole. Both the design of the interview guide and the choice of informants reflect our assumption that EDT in this context is closely related to the internal didactic transposition, including the daily teaching experience. The sequencing of the interviews in four parts aims to ground and relate the larger dimension (e.g. what interviewees say about the rationales for change in programmes) in the more concrete experiences with teaching and designing modules.

The interviews were audio-recorded and afterwards transcribed, and translated into English if not conducted in this language (three of the ten). The authors independently coded the interviews for utterances about the four themes: programme structure and contents; syllabi, modules and use; process of revising and maintaining the programme; external conditions and constraints for major changes and evolutions. After all transcriptions were coded, we developed a consensual structuring of the viewpoints on the EDT which appear in the interviews. In other words, the authors agreed on what sections of the interview correspond to each theme. We then structured the analysis by selecting the topics that appear in more than one interview and select the quotes that seem more representative of these topics. When quoting informants, we refer to their country by the first letter in its English name (C, D, S, F, G); for instance, C1 and C2 refer to the two interviewees from Canada.

For the first two themes, we complement the interviews with a set of documents from each concerned university, concerning the programme structures, content and goals, and the syllabi of modules. We also gathered documents describing three pure mathematics programmes and two applied mathematics programmes from other universities in the same countries for 25 universities (five in each country), to gauge the variation among institutions and between pure and applied programmes; this material is mainly used in the next section.

Programme structure and contents

This section presents general traits of undergraduate mathematics programmes in the 25 selected universities.

Programme characteristics

Study programmes can be characterized by their duration, flexibility and focus. Typical duration in Canada and Spain is four years, and three years in Denmark, France and Germany. In most European programmes, almost all students attend a fixed set of courses during the first year. In Canada and Denmark, programmes provide students with a list of required and ‘elective’ courses, a schedule of all offered courses as well as general rules and/or “advising templates that allow you to plot your way through the programme.” (C2) However, in Denmark, almost all students follow recommended schedules.

Some programmes do not allow much else than pure mathematics modules, which could be a subject of criticism:

very soon our students will do nothing else than math. And it is very difficult to understand math from the interior only... (F1)

Others require additional subjects and have a broader perspective (e.g., of science, culture, and society). Flexibility and breadth are more visible in Canada, reflecting the so-called ‘liberal education’ tradition in North America. Despite these general differences, when interviewees talked about mathematics programmes offered in their universities, they all referred to them as “classical”, “typical”, or “standard”. We will now look at some of the programmes’ commonalities.

Programmes generally consist of modules of the following types:

- 1) *Sequenced courses*, which fit into longer ‘strings’ of courses that require each other successively as prerequisites,
- 2) *Freestanding courses*, which “contrary to most other courses in mathematics, do not follow from other courses and do not lead up to other courses,” (D2)
- 3) *Project units* in which students choose their topic at least partly and where teachers act as supervisors.

Courses enjoy explicit teaching (through lectures, and possibly other formats), which may not be the case for project units.

Programme content

The following mathematical domains have compulsory *sequenced* courses in all programmes:

Analysis and calculus. Usually, a course in Real or Mathematical Analysis includes a rigorous study of real numbers and functions based on theorems and proofs. In many programmes, a first Analysis course is viewed as “the main foundation for all of the following courses in geometry and measure theory and complex analysis, and also to some extent differential equations” (D1). Further courses may include labels like Complex Analysis, Functional Analysis, Fourier Analysis, Ordinary and Partial Differential Equations. At least two courses in Analysis are required in all programs, and in all places, additional analysis courses are offered as a choice: “There is really a fairly strong interweaving at the level of the progression in Analysis” (F2). Topics in courses named Calculus are characterized by a stronger focus on calculations and standard methods, rather than proofs. In Canada, Analysis is preceded by a sequence of three or four Calculus courses. Fewer Calculus courses are also required in Spain and France, and they are absent in Germany and most of Denmark. However, in Germany calculus content could be provided in non-credit bridging courses or online materials as well as at the tutorials that supplement theoretical lectures in analysis.

Algebra. Programmes include one or more courses in linear algebra, typically beginning with systems of linear equations, matrices, and vector spaces. Abstract algebra as a study of groups, rings, and fields is explicitly required in all programs but to a different extent.

Probability and Statistics. Probability as a study of the likelihood of events and Statistics (Stochastic Methods) as a means for data analysis are required courses in all programs.

Courses with the following labels are offered in many of the programmes, but are required only in some: Geometry, Topology, Discrete Mathematics, Combinatorics, Optimization, Number theory, Differential geometry, Arithmetic, Logic, Finance/insurance mathematics, Databases, and various courses on proofs and on problem-solving. Some of these courses are freestanding rather than sequenced.

There are different reasons for making certain courses compulsory or not. Some are taken for granted: “We try to include here all the things that we think are taught everywhere. We are pretty standard” (S1) or not even mentioned. Others are very circumstantial, related to the department tradition and resources. For instance, we note that especially small size universities run into situations when they: “don’t have the capacity to teach all the courses [they] would like to teach every year. ... and if you say, Algebra 2 is required, then the number of students would be maybe too small.” (G1)

Common elements of programmes

Now we will look at the required elements, comparing the beginning, middle and final stages of the programmes.

Practice-focused courses. The aim of these courses is to ease students’ transition from secondary to tertiary level.

The first semester ... is spent completing the basics, consolidating the knowledge acquired in [high school] ... there is no theory, there is a lot of calculation and know-how. It’s a novelty. This gives students confidence. (F2)

This also frees some time in future courses for more theoretical studies because such courses introduce

tools that are needed for other subjects, such as polynomials, permutations, ... (S2)

and may give students a taste of mathematical reasoning from the university position:

then there is a kind of a general introduction to mathematical methods in this, ... it’s mainly about common methods of proof. (D2)

Programmes start from modules that teach new content, however possibly focusing on standard techniques of calculations with little or no formal theory (e.g. Calculus). Programmes differ in the amount of such practice-focused experience. If we compare

programmes specializing in Mathematics, we find a range of settings from the almost immediate focus on theory (in Germany, some Denmark and some France) to the gradual introduction of theory while laying a solid praxis basis that is essential for the degree at least from the institutional perspective (in Canada and some Spain).

At the same time in Canada the beginning of the study is more individualized. Depending on student's high school grades and/or placement test, they may be required to take a "remedial mathematics course" (normally not counted towards the degree). At another extreme, students may be placed in more theoretical courses if they have succeeded in the AP [Advanced placement] exam or completed required practice-focused courses while in the upper grade of high school. Starting with more advanced courses is also a possibility in Germany for students who have completed prerequisite university courses while in high school.

Courses in computer science and scientific programming are frequent requirements. First, there are introductory courses covering basic skills in programming and the use of mathematical software. Then there are courses on mathematical modelling, numerical methods and algorithms that could assist scientific research with the use of technology. Mathematics departments in both European and Canadian universities witness the emergence of this type of courses due to the rapid development of computer technology and its influence on the research in particular:

it is often easier to teach the algorithms once they have been discovered than the somewhat more difficult advanced theory.... In this respect, Computer Algebra and computer learning are also suitable for achieving something that is the focus of research more quickly. (G2)

Specifically, such a course could address the question: "Can you calculate anything at all and how efficiently can you calculate it?" (G2). This is particularly true for mathematical applications: "computer science is really essential for people who do applied maths." (F2)

Required first courses in Analysis and Algebra. In contrast with practice-focused courses that may include students specialized in other fields, these courses are more proof-oriented:

I tried to build the course, especially for an audience of mathematics students. (F1)

Still, they have to be broad enough to allow further students' specialization within mathematics:

this is, of course, a common problem in the course to have to cater to more than one programme. (D2)

Project units may be limited to a certain subject matter area, but their description is otherwise mostly done in terms of generic requirements and how these are assessed. In general, project units leave much more freedom to students and teachers:

It is individual, and you have a variety of options, from something very theoretical, an article, and they have to study a big piece or a proof, others do new things, others very applied things. (S1)

Several universities offer a first- or second-year project-based course. All considered programmes in Denmark, Germany and Spain conclude by a Bachelor project (10-15 ECTS), while in France only one university requires an oral presentation (3 ECTS) as a smaller scale exit project. In Canada, the exit project is typically done by students who take the honours degree (a usual route for students intending to proceed to graduate school in Mathematics), but it is not required by the majority of programs:

They do exist, these opportunities in a few programs and some students do take advantage of that. (C2)

In all except one of the programmes considered, project units take up less than 15% of the total credits, and they are more popular towards the end of the program.

Upper level courses, those taken in the last year(s) of study after students made their choice of specialization, present a greater variation across universities. They are generally challenging for students:

In the fifth semester, they discover topology and measure theory, a course on group theory, three major theories highly formalized and very difficult to learn. (F1)

The offering rather depends on the research interests of the faculty:

It is rather the special lectures that change when a colleague leaves or a colleague comes. (G2)

Research interests in a mathematics department may define the position of a course in the program:

The place where you put topology and the amount of material of topology says a lot about the faculty. Because applied mathematicians tend not to like it and pure mathematicians tend to overestimate it. (S1)

The research influence could be also seen in connection with graduate studies in math:

[We have] historical math modules; this is what is now called linear algebra and analysis, a bit of geometry and linear algebra, the rest is the offspring of Master degree programs. (F1)

As well,

We sometimes have to give the courses for last year bachelor and first year master students at the same time. (G1)

Since students are making choices between different routes, professors are interested in putting courses of their speciality earlier in the program. This creates a competition

among the subjects and may alter the natural flow in some strands. Thus, a course may appear earlier in a program with some omissions in rigor that will be recovered later:

they postpone some proofs and so on [in the probability course] ... so you can prove this when you have had the course in measure theory. (D2)

Some courses are required if there is a general feeling of their importance or if they are considered as part of the Faculty tradition:

We finally decided to include *Galois theory* because it has a historical component. (S2)

[About the course Philosophy of mathematics] we need as a faculty to say, you are a science graduate from this faculty, then you have to know something about these and these things. (D2)

In summary

All programs have sequenced modules in analysis and algebra, which often constitute a core of the program. Most programmes are structured in such a way that

you do obtain a solid background in analysis and algebra [and] it gives a solid foundation for most of the mathematical directions. (D1)

However, there are variations. In Canada, most programmes include practice-oriented courses before proof-based courses, as well as provide more flexibility and choices. In contrast, most of Germany, France and Denmark have a steeper start in the study of mathematical theory, in parallel with some practice-oriented experience. European countries also have their singular traditions in the course requirements, e.g. a Math Seminar in Germany or a course in Euclidean and Projective Geometry in France.

In any case, universities need to take into consideration the secondary school and the graduate school or job situations. The former is accommodated by remedial and practice-focused courses, while the latter is reflected in the emergence of various joint programmes and degrees in mathematics in combinations with either computer science, economics, engineering, physics or teaching of mathematics. The administrative models in which such combinations occur also vary from place to place.

Syllabi, modules and use

As observed above, modules are the building blocks of programmes - courses and project units. None of our informants talked about actually teaching project units. So in this section, we mainly focus on courses, the unit described by the syllabus and subsequently transposed to day-to-day teaching and subsequent assessment by the teacher.

Both sequenced and freestanding courses tend to be named after disciplinary domains (such as Combinatorics) or sectors (such as Integral calculus). We now zoom in on the form and functions of the courses as frameworks for teaching, especially the function of

their official descriptions (syllabi). Through the interviewees' accounts of how these descriptions are used in practice, we also get closer to the boundaries between external and internal transposition: indeed, we get a glimpse of how the latter is conditioned and constrained by the former, in this particular educational context.

The syllabi of courses contain different types of information, from the form of teaching and assessment to details of contents. They are actually very similar in all five countries. They invariably include a list of 3-20 "topics" indicated in terms of mathematical notions (metric spaces, compactness), theorems, rules and techniques; these lists can be considered the essence of the syllabus. They are sometimes - but far from always - complemented by a list of textbook references. Other details may or may not be included in the official module description, especially concerning the number of hours to be spent, e.g. on lectures.

Most of the informants refer to modules in which their main teaching took the form of lectures. Their accounts of "recent" teaching experiences confirm that in undergraduate mathematics programmes, lectures continue to be a dominant and mostly unquestioned form of teaching. In fact, several informants talk about the lectures (and, in close connection to these, exams) as the elements of the course that determine how much it covers:

...there's a list of topics to cover, and I felt compelled to cover them because I was the one who was involved in the development of the syllabus. No, I think that colleagues, indeed, feel bound by the list of subjects that are on the official list... (F2)

The actual mode of delivery. If you did not want to have it all in lectures, it's up to yourself.... As long as you cover the material. (...) As long as you put all topics on the assignments and final exam... Not a lot of freedom. (C1)

We notice here a general tendency in the way in which the informants talk about the syllabi when they take on the role as teachers (even if they previously acted as designers of syllabi or of entire programmes, i.e. as members of the noosphere): in this role, syllabi are taken as a given constraint, which is just as binding as the syllabus is for a school teacher. The freedom lies solely in minor details, such as the "mode of delivery".

While what gets covered in lectures and especially in exams seems to determine what the course as a whole covers, freestanding course units – such as the more recently added courses on "discrete mathematics", tend to offer much more freedom for the lecturer (and course leader) than does core courses:

You have margin but not a lot. Because other subjects depend on them. A subject that has no dependencies and is very pleasant is Discrete Mathematics (6 ECTS), in the second year. (S2)

Discrete mathematics, of course, has been introduced in many places quite late in comparison to linear algebra, analysis and so on, and it took some time before it was really

developed what kind of things would be a part of discrete mathematics and it's still not quite fixed (...) There is still some flexibility, particularly how far one would go with certain topics, for example, whether in the chapter on combinatorial fundamentals and combinatorial numbers, how much one would talk about partitions, partition theory and generating functions for partition identities and I could go a little bit further in that part, or one could also do some extra elementary methods maybe on counting or talk about summation tricks. (G1)

It is clear here and in several other descriptions of the “flexibility” that the “one” who could go more or less “far” is the lecturer - i.e. the course leader: “I could go a little bit further”. There is also an implicit assumption that this relative, but still limited lack of certain limits is a state of immaturity, before the contents - the “things” - will get “quite fixed” by a more established syllabus, like in sequence courses (“linear algebra, analysis, and so on”). The attraction towards stability is evident, as the mere possibility of “doing elementary methods maybe on counting or talk about summation tricks” does not after all sound as a very positive perspective. As the list of “fundamentals” - substantiated subjects to cover, such as “combinatorial numbers” and “generating functions” - grows longer, the mass of “things to part” of the module may, on the other hand, lead to a situation where only the delivery mode is left somewhat open. This seems, after all, the situation characterizing the maturity of a module.

In the European universities, almost all courses seem to be endowed with one course leader, who takes all decisions on what to cover in lectures and exams. Meanwhile, some of the Canadian teachers talk of courses with several sections where classes are given by different faculty members in parallel (but with different schedules); but these may still have a common programme of assignments and exams, which greatly reduces the individual lecturer's freedom:

I was teaching one of 25 sections, I think, and in that course, it was a fully coordinated course... the assignments were common across the 25 sections, and the exams were common across the 25 sections We had the freedom to teach what we wanted in individual lectures but ... we did know at the same time that by the end of the week they had to do this common assignment, so here is roughly what we need to cover (C2)

Teachers of sequenced courses may even feel obliged to teach certain topics which are not mandated by the syllabus and which they, themselves, find less attractive:

If you ask me, I will tell you which are the two theorems I usually give them in full detail. One is Heine-Borel. It is avoided by some of the lecturers because they say that this is a little beyond the goals of the course. And it's true. (...) Heine-Borel is purely analytic theory, and then you go on epsilon and delta. But I do it. And I know that people in analysis insist that we put Heine-Borel in detail. (S1)

They may still enjoy some marginal freedom, for instance, to pursue personal preferences or to adopt the range of topics taught to perceived student capacity and aspirations:

... I suppose I did have some influence on, you know of course the weight that you would put on various topics, and in particular, I think I spend more time on and focus on continuous functions, than was perhaps originally intended. (D1)

Yet, I didn't deal with the whole program because there were still differential equations and given the time format, I preferred not to rush the work (...) Indeed, I think that at the cost of losing 80% of the students, I could have treated this topic, but I don't think it's interesting. (F2)

In almost all courses, lecturers are followed by various forms of tutorials, often taught by teaching assistants. The importance and contents of these course elements depend on the form of assessment, which can be oral or written; but the main distinction seems to be the extent to which assessments concern mainly exercises (as in most Calculus courses) or more theoretical questions (in more advanced courses). Assessment is a concern to almost all informants, as is the failure of students to meet the official demands. As a result, they also find themselves constrained in terms of selecting tasks for students, particularly at exams:

... the tasks - let's say, roughly speaking - are actually fixed. Either they are old exercises that you explicitly take and modify, or they are old exercises that only occur to you when you think they are new, but in reality, they are old exercises which you remember darkly. So there are always somehow 'modified old exercises' in the environment. (G2)

If you put an exam that is surprising by the kind of questions you ask, it will be a disaster for sure. So the key for 60 or 70% of the students to pass is to do an exam that is not essentially different from the previous ones. That's basic. The course would collapse if you start asking completely new types of questions. (S1)

The course is highly constrained. (...) The topics on the exams have been there for a number of years. (C1)

...what I find difficult for the students is that, when you add another topic that you find important, and you put it on the exam (...) it's difficult for the students from the previous year that have to repeat the course they had to, because they failed (...) So, I feel that it's rather better to have, in some sense, standard topics as core topics. (G1)

To sum up, most teachers mention informal constraints – beyond the wording of the syllabus – in the way student assignments are chosen, particularly when it comes to assignments used in a formal assessment. One can discern a certain paradox in the fact that as teachers, both formal and informal constraints are pointed out as both inevitable and as obstacles to choices which might, in fact, make more sense (such as not using almost the same tasks in both teaching and exams, year after year), and at the same time that such constraints are characteristic of what is considered more safely established – and indeed, sequenced – courses. The reference to student failure as both a recurring issue and as something which results, in some sense, from the constraints inherent in the programme as a whole, as well as the syllabus, is striking, given that the same

informants (and peer colleagues) are in fact setting these constraints. The force of “tradition” – what has been there “for a number of years”, as “standard topics” – is ever-present, especially when talking about the sequenced courses. It is at the same time a characteristic of the “core” parts of the curriculum, which seem to have always been there.

External conditions and constraints for major changes and evolutions

In this section, we will focus on conditions and constraints on the EDT that contribute to shape, hinder or foster major changes in study programmes. We will restrict to conditions and constraints that are *external* in the sense that they are situated higher than the discipline on the scale of levels of didactic codeterminacy. We will thus consider the levels of Pedagogy, School, Society and Civilisation. Pedagogy includes all the constraints on the curriculum that arise from the interactions between mathematics and other disciplines, how students impact teaching principles, and the conditions for pedagogical innovation. At the school level, one may find conditions and constraints coming from the interrelationships between research and teaching, for instance, the composition of the academic staff. Constraints from the state are situated at the Society level, which also includes interactions with the professional world. Finally, the requirements of the Bologna process may be envisaged as constraints at the Civilization level, considering that the countries considered here belong to a culturally homogeneous group of societies that aligned with each other through centuries of interactions.

Those conditions and constraints may be detected through the interviewees’ answers in different sections of the interview guide: when they discuss possible conditions for changing the syllabus (section 2), the comparison between the present study programme and past programmes or programmes from other universities (section 3), or obstacles for changing a study programme, outside the department (section 4). A synthesis of the arguments will be presented below, organized according to the four higher levels of the scale of didactic codeterminacy.

Pedagogy

Many informants pointed out constraints coming from the interweaving of study programmes, particularly in the first undergraduate years, which condition the place of shared modules like a jigsaw puzzle. At a deeper epistemological level, the interrelations between mathematics and other disciplines create inter-dependencies between modules and require adequate institutional conditions for the creation of a bi-disciplinary programme:

In the applied math L3 [third year Bachelor], there are twice six credits in informatics, not because of openness or greatness of mind, but because they have a crucial need for informatics. [...] As long as we don’t have a common Master degree that is bi-disciplinary, we won’t have any real need or impetus to create a common study programme. (F2)

In fact, the main constraint at the pedagogy level of the scale, strongly underlined by all the informants, is related to students. It comprises various dimensions (institutional, cognitive, epistemological, economical) as illustrated by the following quotes:

The real constraint is the students. That is to say, we can offer a curriculum that would be tailored to a certain type of students, but if these students are not there, then it will not work. [...] The school has changed a lot, and we cannot catch up in two years the loss in high school math education. (F1).

The sort of classical mathematical rigorous way of doing things is under pressure, you could say, globally [...]. And sometimes I also think we have to give up because it's simply too abstract for the incoming students from high school. (D1)

Right now, in Denmark, we have a system where the money follows the students; only when the student passes a course, money is given to the university. [...] It has been instrumental in turning more attention to what students are actually saying and doing, and it has been institutionalized in a lot of ways now with them. (D2).

Other pedagogical aspects relate to assessment and innovation. They were mentioned by a few informants, for instance, in terms of the role that new technologies may play, but they do not give rise to specific constraints in view of the academic pedagogical freedom. An informant also highlighted the complexity of implementing new pedagogies such as project work in a context of academic inertia that hinders their development:

Finding the problems to propose to the students is very difficult, and the way it is organized, too. [...] Some teachers, if they don't have a textbook to follow, they are lazy about it. These open modules... We are all heirs of what we have learnt, and we live in a comfort zone that makes us very lazy. (S2)

School

A university is an institution dedicated to both teaching and research. Conditions and constraints related to the impact of research were explicitly investigated in the interview guideline. Several informants reported on the impact of the composition of the teaching staff, with institutional, epistemological and political concerns:

The obstacles would be economical and institutional because these courses need to be research-based. [...] So either I should change my research agenda to accommodate that, or we should hire someone or use someone or co-teach it with someone who had that as a research agenda (D2)

If people from the staff are more, say, predominantly from applied mathematics, they tend to shift pure mathematics from the syllabus. And the reverse is true too. (S1)

People have been appointed to work in this direction. It was a political process. [...] This is certainly a university policy decision by an institute that says "we now want to focus on Discrete Mathematics, Discrete Optimization". (G2)

Nevertheless, the proximity of scholarly knowledge and teaching practices remains limited at the undergraduate level. In such study programmes, the impact of research is indirectly visible through the epistemology of the institution:

When we put things from our own research, the consequences are a complete disaster. It's too advanced. [...] Even when we put an exam, we have to be very careful because our sense of what is easy and what is difficult is distorted by our own knowledge. It's very dangerous. (S1)

I would say it was maybe influenced indirectly by research interests because some of the people who were the most vocal were colleagues in pure math who felt that some of the presentations of the rigor of that language were inconsistent with what they believed actually happens in mathematics... and the latest round of revision was some of the reframings of that. (C2)

Several informants also mentioned financial constraints on the institution that affect pedagogical means; for instance, the cost of hiring teaching assistants to offer computer lab sessions based on new technologies (C1). These constraints may also hinder the possibility to accommodate the needs of specific students. For instance, it may not be viable financially to offer dedicated study programmes to future mathematics teachers (D2).

Society

Unlike secondary education, the influence of the state seems limited and seldom appears in the interviews. An informant pointed out that the asynchronicity of study programme accreditation cycles and high school reforms hindered the attempts to soften the secondary-tertiary transition in France. As a consequence, several years were needed for university programmes to adjust to upper secondary school programmes.

Conditions and constraints coming from the interactions with the professional world were mentioned by all the informants. Nevertheless, discourses may be contrasted with respect to the declared impact on study programmes. This reflects different financial contexts throughout countries and regions and probably different educational views as well:

The two biggest changes in the 19 years I've been here on faculty are... the evolution and expansion of our less traditional math programmes, things like math and business, financial analysis and management... and some more specific areas of... cryptography, artificial intelligence. (C2)

There is another point regarding the curriculum. We have a lot of students who think about a career in this region. And of course, here you have lots of insurance companies, you have banks. So it's quite natural to go in this direction. (G1)

We should explore how to get students more skills to make them employable and introduce more of it in the program... doing this without compromising the integrity of the standards,

ensuring that students have the basic mathematics that they need. And those things are always negotiations, you know. (C1)

But the industry, they say, anyway we don't care because they have the general competencies they need, so we can quickly give them a course in the specific programming tool they need to use. (D2)

A few informants underlined the difficulties to cater simultaneously for all types of professional outcomes: the academic career, the needs of the industry, banking and finance sectors, and also training high school teachers at the same time is a complex agenda. Finally, one informant stressed positive aspects of interactions with the professional world, in terms of the positive effects of internships that made students realize the value of the knowledge and skills gained at university.

Civilization

The main aspects at this level, mentioned by all the European informants, are related to the Bologna Process. Many universities had to change the structure of their study programmes to adapt to the uniform framework and its new features (ECTS, competences, etc.). It meant quite a cultural change within institutions, to understand and meet the new formal requirements:

With Bologna, the change is radical because there is nothing established. [...] We explored what was done in other countries, and we saw that there was no agreement in Europe, except partially for the first year. A mathematician should know one-variable calculus, several variables, and linear algebra. This would be the intersection in Europe. Besides, we were clearer about the competencies, what a mathematician should be able to do: solving problems, analysing complex situations, abstraction, modelling. This kind of things are in the White Book and appeared in all the bachelors. (S2)

In fact, the competencies framework didn't seem to induce major effects on the contents of study programmes. If the constraints are strong with respect to structural aspects, they are now looser with respect to the syllabi. As a negative consequence of the requirement to divide into semesters in a context of increased autonomy of universities, a few informants pointed out a tendency towards disciplinary purism and the compartmentalization of knowledge.

The contents are not the important things, should anything else have been written instead, this would have had the same effect [...] What is important is the shell, the structure, and the integration in the local environment [...] The disciplinary refocusing is very strong [...] I think BMD [Bachelor-Master-Degree] leads to disciplinary compartmentalization. Namely, in the division of modules, we were able to cut knowledge into many modules. (F1)

Finally, our informants from Canada didn't point out such similar general reforms, which naturally reflects a different historical development of curriculum policies in North America.

Process of revising and maintaining the programme inside “department”

The maintenance, adaptation and modification of study programs and modules are linked to ongoing negotiations, communication and administrative processes within mathematics departments. The interviewees describe reactions to and processes regarding internal and external requirements addressed in the preceding section. In particular, phenomena and circumstances were noted which were considered to be reasons or causes for activities in mathematics departments (and beyond). This section now focuses on related aspects of decision-making processes, types of activities, including, for example, the distribution of responsibilities, and what personal, administrative, and possibly legal steps are mentioned in relation to the different levels of codetermination.

To account for the dynamics of the EDT we focus in particular on the following: First, we look at the opportunities for action and the limits addressed and second for distinctive, e.g. affect-related, characteristics in statements about processes. Thirdly, we are interested in individual positions and how they relate to the informant’s context. We consider these three points with respect to the levels of codetermination.

Pedagogy

When the informants talk about processes of change regarding pedagogy, they mainly focus on particularly problematic teaching situations. The problems may concern both the content of courses and their pedagogy and are often visible in students failing to pass exams. Interview statements frequently contain affective judgements of such situations. In the following quotations, we write sentences that address such aspects in italics:

So, for instance, we had a *really really bad probability course* and each year, I think *the students hated it, and not a lot of care was given for the course*. (D2)

Now, I *resurrected a very old inactive* numerical optimization course. (C1)

These are things that *broadly break a little bit the routine*; these are techniques I would say, methodological techniques. But changes on the content, *those are dangerous*. (S1)

Processes at this level are often described as altogether rather informal and as direct implementations of individual activities. However, the specificity of the individual positions are also addressed as a limiting factor:

Actually, only own restrictions [to change something in the course]. So, the lecture has the advantage that it is relatively free for the lecturer. So, you could do something completely different when a good idea comes along. For me, of course, restrictions are that the lecture makes a certain sense to me. (G2)

And I made significant changes to that since I came. I was basically given the freedom to do that. There were structural changes. I added a lab component to the course; it was never there before. I reworked some topics of the course to make it a more cohesive part of the other courses on numerical analysis and scientific computing at the undergraduate level. So basically, reshaped the whole offering in this area. (C1)

Both aspects, the expression of affects and the experience of individual opportunities to act, indicate that on the pedagogical level, external and internal didactic transpositions are intimately related. In the interviews, this also becomes evident in comments on changes from more abstract to more concrete and less formal courses.

So, I thought that this is maybe not such a great idea to have the oriented matroids. So, it's one of the special topics chosen by a previous colleague. I decided to take this off in the module description (G1)

algebra was moved to the second year from 1st year because it was too, it was deemed to be too abstract in a way, and also it was taught at a place, it was taught next to another really heavy course, in analysis. So, we had two difficult and very abstract courses simultaneously (D2)

The “modern math” movement affected universities in many countries, but often persons and their positions were instrumental to its actual effects. The following quote describes a change process in a very personalized form, which could also be understood as an expression of or in connection to effects and direct activities.

Strasbourg was really the bastion of Bourbaki. Introducing applied math to Strasbourg was done by [name1]. President [name1], when he was president of the university, when he was director of the IRMA, he brought in [name2], a policy of openness towards applied maths was set up, it was encouraged at the national level. (F2)

The last quote already refers also to the school level, which we will discuss in the next section.

School

Changes and processes are described as natural and in a certain sense, relatively autonomous. In line with the level of the School, respondents essentially act as spokespersons or representatives of their institutional role and how they assume it, or, according to the changes and developments in mathematics as a science. Actions are described as limited by faculty positions, by links to other programmes, and by other university-level processes.

And it depends on where the department comes from. For example, if graph theory is big at that place, then it should definitely be a big part. If discrete optimization is big at that place, then optimization would be a bigger part. (G1)

it is rather the sociological evolution of the laboratory, that is to say, the number of colleagues who are in this or that team, what the thematic focus of a particular team is, rather in terms of the sociology of individuals than on their own research. (F1)

There will be different obstacles, so one obstacle is, the program here, even though formally the study board kind of owns a program, the program is still embedded in the mathematics department, and the head of study is part of the mathematics program. (D2)

The latter quote touches discussions and negotiation processes within the department. In the following quotes, the interaction with other parts of the university is described:

The computer scientists wanted to have only the first semester in common because they found that in the first year of the preceding study programme we did too much math and therefore that this discouraged some students with computer science profiles. It's not that there was too much math overall, but too much math all at once. (F2)

Actually, when you send one of those proposals, there is a consultation report with all affected units. So any change that is proposed gets sent to other units that may be affected somehow, and they get an opportunity to write back, and this feedback is included with all the documentation for the approval. This consultation would take place with every department in the faculty of Science and also faculty of Arts because a math degree could be done as a either BSc [Bachelor of Science] or BA [Bachelor of Arts]. Also, send it to Engineering. Wide consultations. (C1)

Society

Regarding the level of Society, the interviewees essentially describe adaptation processes in terms of reactions to changes which are perceived as external: students with changed entry requirements, ministerial decisions or the development of new tools relevant to mathematics.

The lecture is, or the idea of this lecture has arisen from the observation that beginner students have big problems in Mathematics. Mainly the students for grammar school teaching, but also the bachelor students, and we have made a change in the study plans there. (G2)

As I told you, the basic material has to be the one which is prescribed by the ministry. We don't have very much to play, but we can distribute as we want. (S1)

I mean a course like the Introduction to Mathematical Statistics, that has changed over the years, because they're using computers much more in courses like that than what people did before, and also, I guess when I took a course like that. (D1)

The prevalence of adaptation (as opposed to more radical change) probably reflects the fact that the interviewees see themselves primarily as teachers and actors in their respective teaching and research institutions, and that they were addressed by us precisely in these roles. With regard to specific societal aspects (for example, concerning decisions about entry requirements for students), they do not see themselves

as experts or decision-makers. It is interesting that the respondents tend to address themselves essentially in relation to (more or less) directly experienced processes of external didactic transpositions as experts, but not in relation to university level and societal issues. In general, this attitude seems to shape the processes and dynamics of EDT and has already been noticed above at school level.

Civilization

The latter remarks also apply to the level of civilization. However, in this respect, there is the additional aspect that processes are described as formal and bureaucratic:

You have three levels: the management, then the math commission. It is a little complicated; there is the legislative and the executive. For me, the math Commission is a bit like the legislative body; it is the control body. And the executive was us, and the people in charge of the study programmes. Then, once the political guidelines were validated, there was a lot of communication. Then we continued to work with the heads of the study programmes, who solicited the colleagues most directly concerned. (F2)

in the province of Ontario, there is a formal cyclical process for reviewing programs. That happens in Ontario every 7 years. We are obliged to go through this formal provincial process of having external reviewers coming to review programs. (C2)

Changes related to this level, while now affecting larger areas, are described as mostly superficial and formal:

I said it was part of the major change in our study program and basically the whole Bachelor programme was redesigned. And in particular, these first-year courses were redesigned, because as I said we removed Calculus for our own students, and so we had actually two semesters for doing analysis. (D1)

There are modules of 10 credits and then many 7 ½ credits modules, whereas before there were only five credit modules. [...] “Numbers and structures” disappeared and was integrated into the large analysis module, “analysis and algebra”. That is to say, it is no longer a separate module, it is one of the many changes introduced in the different study programmes through the years, and there is now a big ten credit module which was created that agglomerates these issues rather than have small elements here and there. (F1)

To sum up, it is clear that informants are more directly engaged - affectively and practically - in developing the programme at the course level, i.e. below the discipline level. Changes at higher levels are mainly adapted to. This reflects, to some extent, the roles of the interviewees within their institution, and the questions raised to them during the interview. We also note that a certain shared aversion to major and fundamental change seems to exist and that indeed adaptation to higher-level requirements are made to avoid such changes.

Discussion and conclusion

From this study, we can identify some striking similarities in the external didactic transposition as it is relayed and observed in the universities considered, both when viewed as a product and as a process. Viewed as a product, the document analysis shows that the domains and structure of programmes are surprisingly similar, particularly within Europe; in all countries, sequenced modules within Analysis and Algebra are central elements in the pure mathematics programme. There seems to be a general difference among European and Canadian universities in terms of how quickly formal proof becomes a main factor in pure mathematics programmes. The Canadian programmes usually offer several calculation-oriented courses labelled Calculus, while the European programmes (with Spain as a possible exception) contain fewer or even no courses of this type. In Europe as well as Canada, an important factor for including or not including such courses in the programme can be the extent to which the first courses of the mathematics programme are also taken by students of other programmes (in Engineering, Computer Science, Natural Science etc.). Moreover, there is a tendency that European programmes offer less freedom of choice to students (in some, options exist only in the final year).

Viewed as processes, the interviews describe the EDT processes as more or less bureaucratic changes which happen in response to constraints and requests that are external to the mathematics department. Only very local innovation, for instance of a single module, seems to originate in personal or departmental initiatives (which are then not viewed as bureaucratic). For instance, official regulations and requirements may be changed at the university level, at the national level or even (in Europe) at higher levels, and then the programmes need to adapt, with consequences also for the modules and their structure; there is a tendency, then, to minimize the changes. Substantial changes mainly occur as a result of a crisis, such as students' failure. In some universities, there is also an ongoing tension between the professional needs of students and the mathematical norms of department faculty, which in some cases lead to new ("applied") degrees. Less visible change may operate at the level of the courses, when students' needs or capacities lead faculty to focus more on elementary or practical subject matter.

Another important observation is that both when the interviewees talk as teachers and as occasional *noospherians*, they claim to have very limited influence on the syllabi. As teachers, they work with syllabi and programmes much as teachers in other school institutions: namely, as given conditions and constraints which they have to obey, with only some local freedom, for instance in the choice of exercises for students. As *noospherians* they have to consider both internal and external requirements for the programme; here, change is often minimized and remains, in fact, quite limited except in cases where external requirements change. In the interviews, it is sometimes hard to distinguish whether hindrances to desirable change come from external constraints or from faculty members' preference to avoid change.

There seems to be a general difference among European and Canadian universities in terms of how quickly formal proof becomes a main factor in pure mathematics programmes. The Canadian programmes usually offer several calculation-oriented courses labelled Calculus, while the European programmes (with Spain as a possible exception) contain fewer or even no courses of this type. In Europe as well as Canada, an important factor for including or not including such courses in the programme can be the extent to which the first courses of the mathematics programme are also taken by students of other programmes (in Engineering, computer science, natural science etc.). Moreover, there is a tendency that European programmes offer less freedom of choice to students (in some, options exist only in the final year).

In several of the interviews, we also observe that changing power balances between domains of research are sometimes related to the volume attributed to the corresponding domains of teaching within the programme. At the same time, some of our respondents find that their own teaching and research tasks are quite distant. The day-to-day impact of current research on the teaching is thus very limited, while more long-term tendencies (such as the growing importance of discrete mathematics and computers) seem to occur both in teaching and research.

It must be underlined that all the tendencies and observations presented in this paper are based on a limited material and on new and tentative methods of analysis; also, all observations are made through the filter of our respondents' perspectives and viewpoints. In particular, when it comes to analysing how programmes and their development are influenced from higher levels of didactical codeterminacy, our choice to interview mathematics faculty obviously gives a highly partial viewpoint, for instance, for assessing whether changes are desirable, needed, imposed etc.; and also, to identify what actors in the system ultimately make decisions, and on what grounds. It is clear, though, that many of the conditions accepted as parameters by most research on UME are variables which do change over time. The rationales and institutional mechanisms behind such changes must be investigated further, as an essential condition for increasing the impact of research-based knowledge on UME as a whole.

At the same time, such an impact requires more than an external sociological or anthropological perspective. We consider that a major strength of the ATD, illustrated in Figure 1, is to propose a coherent theoretical framework to analyse and question *both* the fine details of disciplinary practices and theories, *and* the institutional conditions and constraints that determine the role of mathematics in universities and, in particular, the shape and contents of UME. Substantial developments in the external didactical transposition could be motivated by developments in the mathematical sciences or in the wider mathematical practices in society; however it appears from the present study that changes imposed from outside can easily remain superficial and isolated. To achieve real and aligned innovations of the internal and external didactic transpositions is evidently a challenge in all teaching institutions, but presents specific challenges in the case of universities. Research on UME cannot support innovation efficiently by

assuming a purely internal or a purely external position relative to the teaching institution. In particular, systematic experiments in the context of specific modules is necessary but not sufficient.

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