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► **To cite this version:**

Maxime Christ, Luc Forget, Florent de Dinechin. Lossless Differential Table Compression for Hardware Function Evaluation. IEEE Transactions on Circuits and Systems II: Express Briefs, 2021. hal-03040364v1

HAL Id: hal-03040364

<https://hal.science/hal-03040364v1>

Submitted on 4 Dec 2020 (v1), last revised 6 Jan 2022 (v2)

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Lossless Differential Table Compression for Hardware Function Evaluation

Maxime Christ, Luc Forget, Florent de Dinechin

Abstract—Hsiao *et al.* recently introduced, in the context of multipartite table methods, a lossless compression technique that replaces a table of numerical values with two smaller tables and one addition. The present work improves this technique and the resulting architecture by exposing a wider implementation space, and an exhaustive but fast algorithm exploring this space. It also shows that this technique has many more applications than originally published, and that in many of these applications the addition is for free in practice. These contributions are implemented in the open-source FloPoCo core generator and evaluated on FPGA and ASIC, reducing area up to a factor 2.

Index Terms—Table of numerical values, hardware function evaluation, compression, computer arithmetic, ASIC, FPGA.

I. INTRODUCTION

Tables of precomputed values are pervasive in the design of application-specific hardware, especially in the field of elementary function evaluation [1], [2]. For low precisions (typically up to 12 bits), a look-up table may store the value of a function for all the possible input values. For larger precisions, many evaluation methods may be used [1], [2]. These methods often rely on tables of precomputed values [3], [4], [5], [6], [7], [8]. These table-based methods expose a trade-off between storage and computation. This enables FPGA designers to finely tune their architecture to the target device, and ASIC designers to match the silicon budget or performance requirements of an application.

Hsiao *et al.* introduced [6] then improved [7] a technique for compressing one specific table appearing in multipartite table methods [3]. This *lossless differential table compression* (LDTC) replaces one table with two smaller tables and an addition (Fig. 1). The present article extends this work in several ways.

A first contribution is, in Section II, an improvement to the compression method itself: the space of compression opportunities is wider than previous works suggest, and can be explored exhaustively by a simple and fast algorithm.

A second contribution is to show in Section III that this technique is not limited to multipartite table methods: it is applicable as soon as the tabulated function presents small local variations, which is a very common case. Although LDTC was developed for low-precision function evaluation (up to 24 bits), it actually improves most function evaluation

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 This work was supported by the MIAI and the ANR Imprenum project.

TABLE I
 NOTATIONS USED IN THIS ARTICLE

original table	$T : A \mapsto R$
input and output sizes	w_A, w_R
subsampling table	$T_{ss} : B \mapsto H$
input and output sizes	$w_B = w_A - s, w_H$
difference table	$T_d : A \mapsto L$
input and output sizes	w_A, w_L
number of overlap bits	$v = w_H + w_L - w_R$

methods, including those that scale to 64-bit precision and beyond. It could even be used in some software contexts. Besides, since this compression is errorless, it is very easy to plug into existing table-based methods, as illustrated with examples in the open-source core generator FloPoCo.

A last contribution in Section IV is the observation that in many of these applications, LDTC is lossless in terms of functionality, but also in terms of performance. Indeed, the addition in LDTC adds two numbers with only a few bits of overlap (Fig. 1, Fig. 4). When the table value is itself added to a bit array [13] to be computed thanks to a compressor tree [14], [15], then the area overhead will be very little, and there will usually be no timing overhead.

Section V gathers experimental results that support all the previous claims.

II. LOSSLESS DIFFERENTIAL TABLE COMPRESSION

Fig. 1(a) shows an uncompressed table T with $w_A = 8$ address bits and $w_R = 19$ output bits. A table T has some potential for compression if its output for consecutive values of the address A present small variations with respect to the full output range of the table¹. For instance, the TIV (Table of Initial Values) of the original article [6] samples a continuous and differentiable function at regularly spaced points. What is important, however, is not the possible mathematical properties (here continuity) of the underlying function, but the “small local variations” property of the discrete table. For illustration, Fig. 3 plots the content of two tables that are the result of a numerical optimization process [16]. There is no closed-form real function of which such a table is a sampling, the content of the C_3 table is not even monotonic, and still these tables are perfectly suitable for the compression studied here.

¹The only really formal and accurate formulation of this condition is probably: “a table can be compressed if Algorithm 1 succeeds”, see Section II-C.

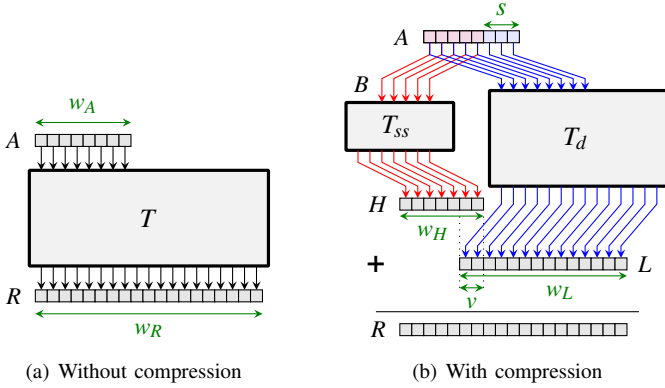


Fig. 1. Table compression example (here for the TIV of a multipartite architecture for $\sin(\frac{\pi}{4}x)$ on $[0, 1)$ with 16-bit inputs and outputs)

A. Previous work

The core idea [6] of LDTC is the following. The original table T is sub-sampled by a factor 2^s , which gives a sub-sampling table T_{ss} . Obviously, T_{ss} is smaller than T since it has fewer entries (2^{w_A-s} instead of 2^{w_A}). Each value of the original table is then reconstructed by adding, to one entry of T_{ss} , the difference of this entry to the original value of T . This difference is stored in a second table T_d of 2^{w_A} entries – as many as the original table. However, thanks to “small local variations” property of the original table, the output range w_L of T_d is smaller than that of T (as T_d only contains differences). Hence T_d has fewer output bits than the original table, and is therefore also smaller. There is a compression as soon as the sum of the sizes of the two smaller tables is smaller than the original size of T [6]. Reconstructing the value of T requires an addition, whose architectural cost will be discussed in Section IV.

In all the following, we call a *slice* of T the subset of 2^s consecutive values to be reconstructed from one value of T_{ss} . If built as exposed previously, T_d systematically has 2^s entries equal to 0, one for each slice. This suggests that a further optimization is possible (these systematic zeroes should themselves be somehow compressed). The solution proposed in [7] is to add, to each entry of T_d , the value of the $w_R - w_L$ least significant bits (LSB) of the corresponding T_{ss} entry. Thus, these bits can be removed from T_{ss} , reducing its output size by $w_R - w_L$ bits. However, the addition may overflow, enlarging the output size of T_d by one bit.

B. A wider implementation space

The possible overflow bit in T_d is expensive, since it is added to 2^{w_A} entries. The refinement introduced in the present paper attempts to avoid this T_d overflow bit. Instead of always removing the maximum number of output bits from T_{ss} as in [7], we consider leaving some of these bits, in the hope that it allows to avoid the overflow bit in T_d . If k extra output bits in T_{ss} allows for a T_d without overflow, the extra cost is $k \times 2^s$ bits and the benefit is 2^{w_A} bits, so there is a potential net gain in storage.

Conversely, once we acknowledge that T_d may overflow and that its output size w_L must be enlarged, it is worth attempting

to reduce w_H by one bit (its least significant bit) to benefit from the new freedom that a wider w_L provides.

More simply, for a given table T with its input and output sizes w_A and w_R , a compression parameter vector is defined as the triplet (s, w_H, w_L) . A vector is valid if it possible to achieve LDTC with these parameters. A vector also has an implementation cost, estimated thanks to a cost function $\text{cost}(w_A, s, w_H, w_L)$, further discussed in Section II-D.

The implementation space thus defined is a strict superset of the one explored in [7]. In particular, as Section V will show, the optimal solution often shows $v = 2$ bits of overlap between H and L (see Figures 1 and 4), and is therefore out of the space explored by previous approaches [6], [7].

C. Improved LDTC optimization algorithm

A generic LDTC optimization is then provided by Algorithm 1. It simply enumerates this parameter space, and selects among the valid vectors the one with the smallest cost. This space is fairly small since s , w_H , and w_L are numbers of bits.

Algorithm 1: Generic LDTC optimization

```

function optimizeLDTC( $T, w_A, w_R$ )
   $bestVector \leftarrow (0, w_R, 0)$  ; // no compression
   $bestCost \leftarrow \text{cost}(bestVector)$  ;
  forall ( $s, w_H, w_L$ ) do
     $c \leftarrow \text{cost}(w_A, s, w_H, w_L)$ ;
    if  $c < bestCost$  then
      if isValid( $T, w_A, w_R, s, w_H, w_L$ ) then
         $bestCost \leftarrow c$ ;
         $bestVector \leftarrow (s, w_H, w_L)$ ;
      end if
    end if
  end forall
  return  $bestVector$ 

```

Actually, Algorithm 1 first filters by cost, then by validity, because cost (see Section II-D) is faster to evaluate than validity. Algorithm 2 determines if a parameter vector is valid.

Algorithm 2: Is a parameter vector valid ?

```

function isValid( $T, w_A, w_R, s, w_H, w_L$ )
  for  $B \in (0, 1, \dots, 2^{w_A-s} - 1)$  ; // loop on slices
  do
     $S \leftarrow \{T[j]\}_{j \in \{B \cdot 2^s \dots (B+1) \cdot 2^s - 1\}}$  ; // slice
     $M \leftarrow \max(S)$  ; // max on slice
     $m \leftarrow \min(S)$  ; // min on slice
     $mask \leftarrow 2^{w_R-w_H} - 1$  ;
     $H \leftarrow m - (m \& mask)$  ; //  $w_H$  upper bits of  $m$ 
     $M_{low} \leftarrow M - H$  ; // max diff value on this slice
    if  $M_{low} \geq 2^{w_L}$  then
      return false ; // one slice won't fit: exit with false
    end if
  end for
  return true

```

Note that Algorithm 2 is faster than attempting to fill the tables: it only needs the max and min of T on each slice,

TABLE II
POSSIBLE TABLE COST FUNCTIONS

number of bits	$c_{\text{bit}}(m, n) = 2^m \times n$
number of FPGA LUT	$c_{\text{LUT}}(m, n) = 2^{\min(m-\ell, 0)} \times n$
ASIC standard cells [17]	$c_{\text{SC}}(m, n) = 2^{0.65 \min(m, n)} \times 2^{0.19 m-n }$

which can be computed only once for each value of s , and memoized. Therefore one invocation of Algorithm 2 require time proportional to $2^{w_A - s}$, not to 2^{w_A} .

Altogether, with a little obvious pruning of this parameter space, its exploration is almost instantaneous on current computers for any practical size.

D. Cost functions

The cost of a solution may be evaluated as:

$$\text{cost}(w_A, s, w_H, w_L) = c(T_{ss}) + c(T_d) + c(\text{add}).$$

The cost of the addition will be discussed in Section IV. The hardware cost of a table with m input bits and n output bits can be estimated in various ways, e.g. those given in Table II. Most previous works [3], [6], [7] use $c_{\text{bit}}(m, n)$, which counts the total number of stored bits. On FPGAs, $c_{\text{LUT}}(m, n)$ estimates the number of FPGA architectural LUTs with ℓ inputs. This model is both pessimistic (it ignores the optimizations performed by synthesis tools) and optimistic (it doesn't count LUTs used as address decoding multiplexers for large m), but it is accurate for small tables. The third function, $c_{\text{SC}}(m, n)$, defined empirically [17], estimates the cost of a table implemented in ASIC as standard cells.

III. A REVIEW OF APPLICATIONS

This section gives a non-exhaustive list of applications of the LDTC technique to function evaluation [1], [2] beyond the original multipartite approximation.

First, LDTC works, and even works extremely well, for plain function tables with $w_A = w_R$. Such tables are routinely used for very low precisions (up to 12 bits). Table III shows that the gain in such cases exceed 50%.

For larger precisions, approximation techniques must be used. Many generic function evaluation methods (including the multipartite methods) are variations or refinements of piecewise polynomial approximation. Fig. 2 shows a typical uniform piecewise approximation architecture [18], [19], [8]. The input domain of the function is decomposed in 2^{w_A} segments of identical size by a simple splitting of the input word X in its w_A leading bits (which become the segment address A) and its remaining least significant bits (which become the index Y within a segment). On each segment, a good polynomial approximation is precomputed and its coefficients C_i are stored in a table indexed by A . It is possible to use more complex architectures where the segments of the input domain may have different sizes [5], [20], there is nevertheless a coefficient table.

For LDTC, instead of the wide table of Fig. 2, we consider as many independent tables as there are coefficients. It turns

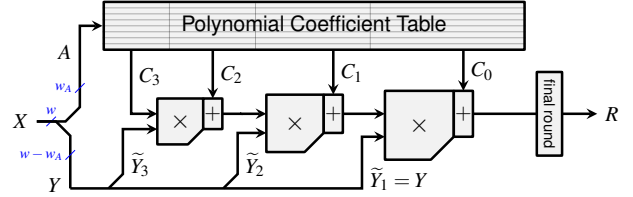


Fig. 2. A fixed-point polynomial evaluator, using uniform segmentation and a Horner scheme with truncated multipliers – see [18] for more details.

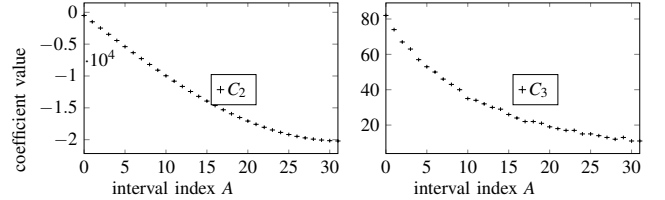


Fig. 3. Plot of all the degree-2 and degree-3 coefficients (the values of C_2 and C_3 in Fig. 2) of a 24-bit, degree 3 piecewise ($w_A = 5$) polynomial approximation to $\log(1+x)$ when using machine-efficient polynomials [16].

out that each coefficient table has the property of small local variations. This is an indirect consequence of a requirement of all these approximation methods, namely that the function must be differentiable up to a certain order. For instance, the degree-one coefficient C_1 is closely related to the derivative of the function, taken somewhere in each segment. As long as the derivative itself is continuous, $C_1(A)$ will present small local variations. For illustration, Fig. 3 shows $C_2(A)$ and $C_3(A)$ for a degree-3 approximation to $\log(1+x)$, and Table III shows the compression achieved in each of the coefficients of a degree-2 approximation to the sine function (all obtained using FloPoCo's `FixFunctionByPiecewisePoly` generator).

Another large class of applications consists in function-specific range reduction algorithms that rely on large tables of precomputed values. It was pioneered in software by Tang [21] then used in hardware to implement e.g. exponential [22, Fig. 2], [23, Fig. 10], logarithm [23, Fig. 6] or trigonometric functions [12, Fig. 2]. These tables all present small local variations and are suitable for LDTC, as illustrated by Table III on the e^A table of [22] (obtained using FloPoCo's `FPEXP`).

Such range reduction techniques may be used iteratively, as in most CORDIC variants [1], in which case the table is addressed by the iteration index. It is unclear if there is compression potential in this case, however there is in high-radix iterative algorithms [1] [24, Fig. 1].

Finally, table-based methods have been also used to implement multiplication by constants [9], [10], [11] which are themselves used in elementary function implementations [22, Fig. 2] [23, Fig. 6] [12, Fig. 2]. Again the tables there are perfectly suited to LDTC. However, it is less obvious here that this potential can be exploited, as the tables are already finely tailored to the LUT-based logic of the FPGAs [10], [11].

IV. WITH COMPRESSION TREES, LDTC IS FOR FREE

If an adder is used to compute the sum, it should be obvious from Fig. 1(b) that the adder size is only w_H bits (the $w_R - w_H$ lower bits of the sum are those of L).

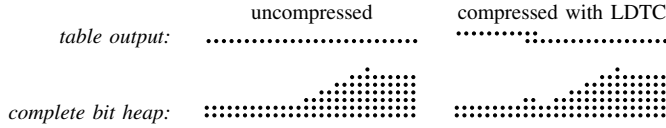


Fig. 4. The dot-diagram overhead of lossless table compression, here for the 24-bit multipartite implementation of $\sin(\frac{\pi}{4}x)$

However, in many of the applications reviewed in Section III, the table result is added to a value that it itself computed by a compression tree. This is the case in the original multipartite method (where it is added to multi-operand addition) [3, Fig. 7]. It is also the case for all the coefficients except C_3 in Fig. 2: in the Horner evaluation scheme [1] used there, each C_i is added to a product, and the best way to implement this product is also a compression tree [25], [15] (if a parallel evaluation scheme is used [8] only C_0 is added to a product). The table outputs are also summed in table-based constant multiplication techniques [10], [11], and in many other cases (e.g. [23, Fig. 6] where the tabulated $E \times \log(2)$ is added to another term).

In all these cases, thanks to merged arithmetic [13], the addition adds only $v = w_H + w_L - w_R$ bits to the bit array. This is illustrated in Fig. 4. The area cost of the addition therefore becomes proportional to the overlap v , in other words negligible. Furthermore, as long as the addition of these v bits does not entail one more compression stage [15] (this is definitely the case in Fig. 4), the delay overhead will be zero.

V. EVALUATION

LDTC is now available in FloPoCo (git master branch), where it will benefit to all the table-based operators. Table III reports the compression rate in terms of bit count for a representative range of application (see section III). Compression ratio up to 0.36 are possible, to be compared to the best ratio of 0.74 in the previous state of the art [7, Table VII].

A general observation on Table III is that the best compression ratios are observed for functions with the smaller difference between w_R and w_A . The intuition here is that the larger table is always T_d , and that T_d has very few output bits when $w_R \approx w_A$. Note that LDTC even works when $w_R < w_A$.

Looking at the pt results, the compression ratio seems to improve with the initial table size.

The optimal often has an overlap of $v = 2$ bits. As the ptl line shows, this is not a strict rule.

Table IV compares the c_{bit} of the previous state of the art in multipartite methods [7, Tables 2 and 3] to that achieved by the multipartite implementation of FloPoCo when enhanced with LDTC. The function used here is $\sin(\frac{\pi}{4}x)$, the only one on which we could reproduce the results of [7]. LDTC always improves upon MP, but not always over the hierarchical multipartite method [7] which brings other improvements. There is one more addition in MP+LDTC than in HMP, however remark that the total number of bits input to the compressor tree is smaller for MP+LDTC ($7+14+10+8+6+4=49$, versus $20+12+10+8+6=56$ for HMP) — both observations also hold in the 24-bit case, not detailed for space. Therefore the cost of the compressor tree is likely to be smaller for MP+LDTC.

TABLE III
COMPRESSION RESULTS IN TERMS OF BIT STORAGE

	plain table	LDTC (ratio)	T_{ss}	T_d	v
pt:8	2,048 $8 \cdot 2^8$	992 (0.48)	224 $7 \cdot 2^5$	768 $3 \cdot 2^8$	2
pt:9	4,608 $9 \cdot 2^9$	2,048 (0.44)	512 $8 \cdot 2^6$	1,536 $3 \cdot 2^9$	2
pt:10	10,240 $10 \cdot 2^{10}$	4,224 (0.41)	1,152 $9 \cdot 2^7$	3,072 $3 \cdot 2^{10}$	2
pt:12	49,152 $12 \cdot 2^{12}$	17,920 (0.36)	5,632 $11 \cdot 2^9$	12,288 $3 \cdot 2^{12}$	2
ptl:12	49,152 $12 \cdot 2^{12}$	18,432 (0.38)	6,144 $12 \cdot 2^9$	12,288 $3 \cdot 2^{12}$	3
mpt:12	960 $15 \cdot 2^6$	800 (0.83)	96 $6 \cdot 2^4$	704 $11 \cdot 2^6$	2
mpt:14	2,048 $16 \cdot 2^7$	1,632 (0.8)	224 $7 \cdot 2^5$	1,408 $11 \cdot 2^7$	2
mpt:16	4,864 $19 \cdot 2^8$	3,808 (0.78)	224 $7 \cdot 2^5$	3,584 $14 \cdot 2^8$	2
mpt:20	24,576 $24 \cdot 2^{10}$	18,560 (0.76)	1,152 $9 \cdot 2^7$	17,408 $17 \cdot 2^{10}$	2
mpt:24	114,688 $28 \cdot 2^{12}$	83,456 (0.73)	5,632 $11 \cdot 2^9$	77,824 $19 \cdot 2^{12}$	2
ea:sp:10	28,672 $28 \cdot 2^{10}$	22,656 (0.79)	1,152 $9 \cdot 2^7$	21,504 $21 \cdot 2^{10}$	2
ea:dp:10	58,368 $57 \cdot 2^{10}$	52,352 (0.90)	1,152 $9 \cdot 2^7$	51,200 $50 \cdot 2^{10}$	2
ea:dp:12	233,472 $57 \cdot 2^{12}$	202,240 (0.87)	5,632 $11 \cdot 2^9$	196,608 $48 \cdot 2^{12}$	2
ea:dp:14	933,888 $57 \cdot 2^{14}$	780,288 (0.84)	26,624 $13 \cdot 2^{11}$	753,664 $46 \cdot 2^{14}$	2
C_0	17,920 $35 \cdot 2^9$	15,360 (0.86)	512 $8 \cdot 2^6$	14,848 $29 \cdot 2^9$	2
C_1	12,800 $25 \cdot 2^9$	9,792 (0.76)	576 $9 \cdot 2^6$	9,216 $18 \cdot 2^9$	2
C_2	6,656 $13 \cdot 2^9$	4,160 (0.62)	576 $9 \cdot 2^6$	3,584 $7 \cdot 2^9$	2
all C_i	73 · 2 ⁹	29,312 (0.78)	26 · 2 ⁶	54 · 2 ⁹	

pt, ptl plain tabulation of $\sin(\frac{\pi}{4}x)$ (pt) or $\log(1+x)$ (ptl) on $[0, 1)$
mpt first table of a multipartite approximation [6] to $\sin(\frac{\pi}{4}x)$ on $[0, 1)$
ea e^A table of [22], for single (sp) or double (dp) precision
 C_i coefficients of a degree-2 uniform piecewise approximation [18] to $\sin(\frac{\pi}{4}x)$ on $[0, 1)$ for 32-bit accuracy ($w_A = 9$)

TABLE IV
COMPARISON WITH THE STATE OF THE ART IN MULTIPARTITE METHODS

	24 bits	16 bits
HMP [7]	166,528	$6,272 = 20 \cdot 2^7 + 12 \cdot 2^7 + 10 \cdot 2^7 + 8 \cdot 2^6 + 6 \cdot 2^6$
MP+LDTC	158,208	$6,752 = 7 \cdot 2^5 + 14 \cdot 2^8 + 10 \cdot 2^7 + 8 \cdot 2^7 + 6 \cdot 2^6 + 4 \cdot 2^6$

TABLE V
FPGA IMPLEMENTATION RESULTS FOR $\sin(\frac{\pi}{4}x)$ (ADDITION INCLUDED)

case	without LDTC		with LDTC (ratio)	
pt:8	27 LUT	5.5 ns	26 LUT (0.96)	6.4 ns (1.17)
pt:9	61 LUT	6.3 ns	46 LUT (0.75)	6.7 ns (1.08)
pt:10	134 LUT	6.8 ns	83 LUT (0.62)	7.5 ns (1.1)
pt:12	536 LUT	9.8 ns	287 LUT (0.54)	8.7 ns (0.89)
mpt:12	76 LUT	7.6 ns	73 LUT (0.96)	7.7 ns (1.02)
mpt:14	102 LUT	8.1 ns	98 LUT (0.96)	8.1 ns (1.0)
mpt:16	184 LUT	8.9 ns	190 LUT (1.03)	9.3 ns (1.04)
mpt:20	676 LUT	12.8 ns	637 LUT (0.94)	12.5 ns (0.98)
mpt:24	2489 LUT	18.1 ns	2322 LUT (0.93)	16.1 ns (0.89)

Results after implementation on Kintex7 using Vivado 2020.2. The compressor tree used for mpt is FloPoCo's default.

TABLE VI
ASIC IMPLEMENTATION RESULTS FOR $\sin(\frac{\pi}{4}x)$ (ADDITION INCLUDED)

case	LDTC results (ratio WRT uncompressed table)		
pt:8	130 μm^2 (0.82)	0.3 ns (1.44)	0.12 mW (0.91)
pt:9	184 μm^2 (0.56)	0.3 ns (1.12)	0.16 mW (0.62)
pt:10	298 μm^2 (0.54)	0.4 ns (0.82)	0.26 mW (0.58)
pt:12	1,190 μm^2 (0.63)	0.6 ns (0.94)	1.06 mW (0.66)
mpt:12	269 μm^2 (0.98)	0.6 ns (1.00)	0.34 mW (0.96)
mpt:14	424 μm^2 (0.98)	0.7 ns (1.00)	0.50 mW (0.96)
mpt:16	793 μm^2 (0.92)	0.8 ns (1.00)	0.88 mW (0.88)
mpt:20	2,888 μm^2 (0.84)	1.2 ns (1.12)	2.76 mW (0.67)
mpt:24	11,801 μm^2 (0.87)	1.8 ns (1.01)	11.13 mW (0.72)

Results obtained with Synopsys design compiler using STMicroelectronics 28nm FDSOI standard cell library. The compressor tree used for mpt is FloPoCo's default, which is currently optimized for FPGAs.

Results for FPGA and ASIC synthesis are reported in Tables V and VI respectively. All the LDTC were optimized using the c_{bit} cost function, the other cost functions of Table II failing to provide any significant improvement so far.

The area compression ratios in the synthesis results are not as good as those of Table III, and the difference cannot be explained by the addition cost only: the logic optimization of synthesis tools somehow discover some of the compression opportunity exploited by LDTC [17].

The compressed plain tables (pt lines in Table V) include an adder. Its area overhead is smaller than the area gained on the tables for precisions larger than 8 bits, however it adds to the critical path delay. Still, for large tables, LDTC also reduces the delay, both on ASIC and on FPGA.

In the multipartite cases (mpt lines in Table V), this addition is merged in the compressor tree (see Figure 4) and the delay is essentially unchanged in most cases, as expected². However the area reduction is less dramatic as only one table out of several is compressed, and for some precisions LDTC may even be counter-productive area-wise.

We do not report results targeting FPGAs with block RAM. Actual savings will depend on the block RAM capabilities of the target, which are very discrete (e.g. M20k blocks on Altera/Intel devices can be configured as $2^9 \times 40$, $2^{10} \times 20$, or $2^{11} \times 10$ bits, while the Xilinx/AMD 36kbit blocks can do from $2^9 \times 72$ to $2^{15} \times 1$). The larger the table, the higher the chances that LDTC is useful.

VI. CONCLUSION

This work adds one optimization technique to the bag of tricks of arithmetic designers: lossless differential table compression can reduce up to a factor two the storage requirement of most tables used in function evaluation, at the cost of one small integer addition that can be hidden for free in an existing compressor tree. The optimal solution can be found by an exhaustive enumeration. This technique is available in the open-source FloPoCo core generator.

Acknowledgement: Many thanks to Frédéric Pétrot for his comments and his help with ASIC synthesis.

²The mpt timing variation in Table V are routing artifacts – even the improvement for mpt:24 is actually due to variations in input buffer net delay.

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