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1 **Topography curvature effects in thin-layer models for**
2 **gravity-driven flows without bed erosion**

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9 **Key Points:**

- 10 • Interpretation of topography curvature terms in thin-layer models.
11 • Comparison of methods to include curvature effects in thin-layer models.
12 • Quantification of curvature effects in thin-layer model simulations with synthetic
13 and real topographies.

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Abstract

Depth-averaged thin-layer models are commonly used to model rapid gravity-driven flows such as debris flows or debris avalanches. However, the formal derivation of thin-layer equations for general topographies is not straightforward. The curvature of the topography results in a force that keeps the velocity tangent to the topography. Another curvature term appears in the bottom friction force when frictional rheologies are used. In this work, we present the main lines of the mathematical derivation for these curvature terms that are proportional to the square velocity. Then, with the SHALTOP numerical model, we quantify their influence on flow dynamics and deposits over synthetic and real topographies. This is done by comparing simulations in which curvature terms are exact, disregarded or approximated. With the Coulomb rheology, for slopes $\theta = 10^\circ$ and for friction coefficients below $\mu = \tan(5^\circ)$, neglecting the curvature force increases travel times by up to 10% and 30%, for synthetic and real topographies respectively. When the curvature in the friction force is neglected, the travel distance may be increased by several hundred meters on real topographies, whatever the topography slopes and friction coefficients. We observe similar effects on a synthetic channel with slope $\theta = 25^\circ$ and $\mu = \tan(15^\circ)$, with a 50% increase of the kinetic energy. Finally, approximations of curvature in the friction force can break the non-invariance of the equations and decelerate the flow. With the Voellmy rheology, these discrepancies are less significant. Curvature effects can thus have significant impact for model calibration and for overflows prediction, both being critical for hazard assessment.

1 Introduction

The propagation of rapid gravity-driven flows [Iverson and Denlinger, 2001] occurring in mountainous or volcanic areas is a complex and hazardous phenomenon. A wide variety of events are associated with these flows, such as rock avalanches, debris avalanches and debris, mud or hyper-concentrated flows [Hungr et al., 2014]. The understanding and estimation of their propagation processes is important for sediment fluxes quantification, for the study of landscapes dynamics. Besides, gravity-driven flows can have a significant economic impact and endanger local populations [Hungr et al., 2005; Petley, 2012; Froude and Petley, 2018]. In order to mitigate these risks, it is of prior importance to estimate the runout, dynamic impact and travel time of potential gravitational flows.

This can be done empirically, but physically-based modeling is needed to investigate more precisely the dynamics of the flow, in particular due to the first-order role of local topography. Over the past decades, thin-layer models (also called shallow-water models) have been increasingly used by practitioners. Their main assumption is that the flow extent is much larger than its thickness, so that the kinematic unknowns are reduced to two variables: the flow thickness and its depth-averaged velocity. The dimension of the problem is thus lower, allowing for relatively fast numerical computations. The first and simplest form of thin-layer equations was given by Barré de Saint-Venant [1871] for almost flat topographies. The 1D formulation (i.e. for topographies given by a 1D graph $Z = Z(X)$) for any bed inclination and small curvatures was derived by Savage and Hutter [1991]. This model has since been extended to real 2D topographies (i.e. given by a 2D graph $Z = Z(X, Y)$). Some of the software products based on thin-layer equations are currently used for hazard assessment to derive, for instance, maps of maximum flow height and velocity. Examples include RAMMS [Christen et al., 2010, 2012], 3d-DMM [GEO, 2011; Law et al., 2017], DAN3D [McDougall and Hungr, 2004; Moase et al., 2018] and FLO-2D [O'Brien et al., 1993]. A non exhaustive overview of some existing models used for field scale modeling is given in Table 1. Yavari-Ramshe and Ataie-Ashtiani [2016] and Delannay et al. [2017] give a more comprehensive review of thin-layer models. Current research focuses include modeling of multi-layer flows [Fernández-Nieto et al., 2018; Garres-Díaz et al., 2020], bed erosion along the flow path [Hungr, 1995; Bouchut et al., 2008; Iverson, 2012; Pirulli and Pastor, 2012] and the description of two-phase flows [e.g.

66 *Pudasaini, 2012; Rosatti and Begnudelli, 2013; Iverson and George, 2014; Bouchut et al.,*
 67 *2015, 2016; Pastor et al., 2018a).*

68 In addition to the complexity of choosing realistic constitutive equations to model
 69 the flow physical properties, there is also a purely methodological difficulty in deriving the
 70 thin-layer equations for a complex topography, with acceleration forces arising from the
 71 curvature of the topography. Their influence in 1D thin-layer models was investigated by
 72 *Hutter and Koch [1991], Greve and Hutter [1993] and Bouchut et al. [2003]. Koch et al.*
 73 *[1994] investigated curvature effects for unconfined flows on simple 2D topographies.*
 74 *Their work was completed by Gray et al. [1999] and Wieland et al. [1999] for channel-*
 75 *ized flows in straight channels. Later on Pudasaini and Hutter [2003] and Pudasaini et al.*
 76 *[2003] considered flows in curved and twisted channels. The generalization of curvature*
 77 *forces to general topographies was done by Bouchut and Westdickenberg [2004], Luca*
 78 *et al. [2009a] and Rauter and Tukovic [2018]. To our knowledge, only one study focused*
 79 *on quantifying curvature effects in simulations on general topographies: Fischer et al.*
 80 *[2012] showed curvature terms have a substantial effect for model calibration. However,*
 81 *it focuses on curvature terms in the bottom friction and does not consider other curvature*
 82 *terms that are independent from the chosen rheology.*

83 In this work, we aim at quantifying more generally and precisely the influence of
 84 curvature terms in depth-averaged thin-layers simulations. This is important for practition-
 85 ers using thin-layers models: we will identify situations (in terms of topographic settings
 86 and rheological parametrization) where curvature effects may significantly impact their
 87 simulation results, and thus are worth taking into account for hazard assessment. We fo-
 88 cus on the modeling of single-phase incompressible flows, with an Eulerian description.
 89 We also disregard bed erosion and internal friction. The resulting equations may be over-
 90 simplified in comparison to the physical processes at stake in real geophysical flows. How-
 91 ever, such equations are now widely used to simulate debris flows, debris avalanches and
 92 rock avalanches [*Hungr et al., 2007; Pastor et al., 2018b*]. Thus, we deem important to
 93 assess quantitatively the importance of curvature terms for field applications. However, we
 94 acknowledge that it goes along with major uncertainties on the rheology and rheological
 95 parameters needed to reproduce correctly real gravity flows.

96 In the following section, we present the depth-averaged thin-layer equations for flows
 97 on complex topographies. We detail the derivation of two curvature terms: one that does
 98 not depend on the rheology and the other appearing in the bottom friction when a fric-
 99 tional rheology is used. We also introduce the SHALTOP numerical model [*Mangeney*
 100 *et al., 2007*] and its modified version without curvature forces, that will be used to carry
 101 out simulations. The curvature terms will be formally analyzed and compared to previous
 102 studies in Section 3. Then, in Section 4, we illustrate for synthetic topographies the im-
 103 portance of taking into account curvature forces. Finally, in Section 5, we consider two
 104 real Digital Elevation Models, with a non-viscous debris flow in the Prêcheur river (Mar-
 105 tinique, French Caribbean) and a potential massive debris avalanche from the Soufrière de
 106 Guadeloupe volcano (Guadeloupe, French Caribbean).

107 **2 Modeling approach using thin-layer equations**

108 Thin-layer equations model the propagation of a thin layer of fluid following the top-
 109 ography. As opposed to full 3D models, thin-layer models no longer simulate the move-
 110 ment of each solid or fluid element. Instead, they integrate their dynamics over a column
 111 of fluid in the direction normal to the topography and consider the mean flow velocity
 112 over this column. Although the resulting equations are relatively simple, their rigorous
 113 derivation is not straight-forward. As a matter of fact, the momentum and mass equations
 114 must first be written in a reference frame that allows a convenient integration. Its mere
 115 definition is difficult, not to mention the expression of the constitutive equations in the
 116 resulting coordinate system. In Supplementary Note 1, we describe into details how cur-

Table 1. Non exhaustive list of 2D thin-layer models used at the field scale.

Model name	Numerical scheme	Bed erosion	Two-phase flows	References
MassMov2D		-	-	[<i>Beguertia et al., 2009</i>]
Flo-2D	Finite Differences	-	-	[<i>O'Brien et al., 1993</i>]
Volcflow		x	-	[<i>Kelfoun and Druitt, 2005</i>]
SHALTOP		-	-	[<i>Bouchut and Westdickenberg, 2004; Mangeney et al., 2007</i>]
RAMMS		x	-	[<i>Christen et al., 2010</i>]
RASH3D		x	-	[<i>Pirulli et al., 2007; Pirulli and Pastor, 2012</i>]
r.avaflow	Finite Volumes	x	x	[<i>Pudasaini, 2012; Mergili et al., 2017</i>]
GeoClaw		-	-	[<i>Berger et al., 2011; LeVeque et al., 2011</i>]
D-Claw		-	x	[<i>Iverson and George, 2014; George and Iverson, 2014</i>]
Titan2D		-	x	[<i>Patra et al., 2005; Pitman and Le, 2005</i>]
TRENT-2D		-	x	[<i>Armanini et al., 2009; Rosati and Begnudelli, 2013</i>]
DAN3D	SPH	x	-	[<i>McDougall and Hungr, 2004</i>]
GeoFlow_SPH	(Smoothed Particle Hydrodynamics)	x	x	[<i>Pastor et al., 2009a, 2018a</i>]
3d-DMM		x	-	[<i>Kwan and Sun, 2007; Law et al., 2017</i>]

117 vature terms appear in the thin-layer equations derivation in *Bouchut and Westdickenberg*
 118 [2004]. In the following, we will only present the chosen parametrization and the final
 119 equations.

120 2.1 Mass and momentum equations and boundary conditions

121 Most thin-layer models are based on the incompressible mass and momentum equa-
 122 tions

$$123 \partial_t \vec{U} + (\vec{U} \cdot \nabla_{\vec{X}}) \vec{U} = -\vec{g} + \nabla_{\vec{X}} \cdot \sigma, \quad (1)$$

$$124 \nabla_{\vec{X}} \cdot \vec{U} = 0, \quad (2)$$

126 where $-\vec{g}$ is gravity and σ the Cauchy stress tensor normalized by the flow density. $\vec{X} =$
 127 (X, Y, Z) is the cartesian coordinate system associated with the orthonormal base $(\vec{e}_X, \vec{e}_Y, \vec{e}_Z)$.
 128 In the following we will write $\mathbf{X} = (X, Y) \in \mathbb{R}^2$ for the horizontal coordinates. In the fol-
 129 lowing, 3D vectors will be identified by an arrow and 2D vectors will be in bold. For in-
 130 stance, $\vec{U}(\vec{X}) = (U_X, U_Y, U_Z) = (\mathbf{U}, U_Z)$ gives the components of the 3D velocity field in
 131 the cartesian reference frame. $\nabla_{\vec{X}}$ is the gradient operator.

132 The base of the flow matches the topography and is given by a 2D surface $Z =$
 133 $b(\mathbf{X})$, with upward unit normal vector \vec{n} (Figure 1a for 1D topographies, Figure 1b for 2D
 134 topographies),

$$135 \vec{n} = c \left(-\frac{\partial b}{\partial X}, -\frac{\partial b}{\partial Y}, 1 \right) = (-\mathbf{s}, c), \quad (3)$$

136 with

$$137 c = \cos(\theta) = \left(1 + \|\nabla_{\mathbf{X}} b\|^2 \right)^{-\frac{1}{2}}, \quad (4)$$

$$138 \mathbf{s} = c \nabla_{\mathbf{X}} b, \quad (5)$$

139 where θ is the topography steepest slope angle. Along with boundary conditions detailed
 140 in Supplementary Note 1, a constitutive equation for the stress tensor σ is needed to close
 141 the problem. The latter can be divided into pressure and deviatoric parts, namely

$$142 \sigma = \sigma' - pI_3, \quad (6)$$

143 with σ' the deviatoric stress tensor, p the pressure field (divided by the flow density) and
 144 I_3 the identity matrix. For scale analysis and to allow for a rigorous mathematical deriva-
 145 tion, *Bouchut and Westdickenberg* [2004] chose a Newtonian approach with a linear stress
 146 constitutive equation

$$147 \sigma' = \nu \left(\nabla_{\vec{X}} \vec{U} + (\nabla_{\vec{X}} \vec{U})^t \right), \quad (7)$$

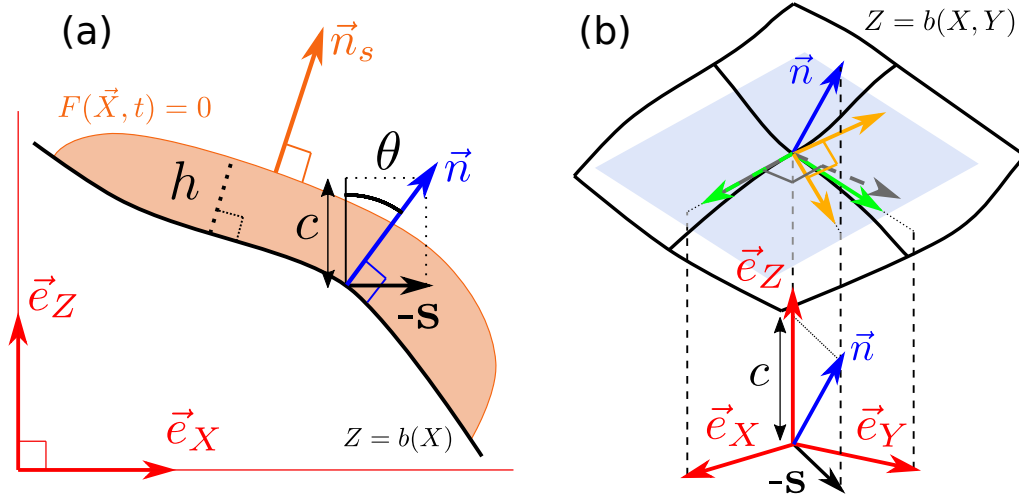
148 with ν the kinematic viscosity, that is assumed to be small (see Supplementary Note 1).
 149 They furthermore imposed a friction boundary condition at the bed

$$150 \sigma \vec{n} - (\vec{n} \cdot \sigma \vec{n}) \vec{n} = \mu \frac{\vec{U}}{\|\vec{U}\|} (-\vec{n} \cdot \sigma \vec{n})_+, \quad (8)$$

151 where $\mu = \tan(\delta)$ is the friction coefficient and δ is the friction angle. The key point here
 152 is the transformation of the equations in a convenient reference frame, in which they can
 153 be integrated.

154 2.2 Coordinate system and reference frame

155 The simplest way to derive the thin-layer equations is to use cartesian coordinates
 156 and integrate the Navier-Stoke equations along the vertical direction [*Barré de Saint-Venant,*
 157 1871; *Pitman et al., 2003; Berger et al., 2011*]. This is done in particular to model the
 158 propagation of tsunamis because the wavelength of waves is small in comparison to the



177 **Figure 1.** (a) Topography and flow description, for a 1D topography $Z = b(X)$. The orange area is the
 178 flow region, with thickness h in the direction normal to the topography. (b) 2D topography $Z = b(X, Y)$
 179 description, with reference frames commonly used in the literature to derive thin-layer equations. Red arrows:
 180 Cartesian reference frame. Blue arrows: topography normal unit vector. $-s$: main slope horizontal direc-
 181 tion. All other arrows are in the topography tangent plane (blue plane). Green arrows: *Christen et al.* [2010].
 182 Dashed gray arrows: *Mangeney-Castelnau et al.* [2003]. Orange arrows: *Iverson and George* [2014].

159 water vertical depth and the main driving forces are horizontal pressure gradients [e.g.
 160 *Berger et al.*, 2011; *LeVeque et al.*, 2011]. On the contrary, the shallowness of landslides
 161 propagating on potentially steep slopes must be regarded in the direction normal to the
 162 topography. Moreover, the flow velocity is (at least for a first approximation) tangent to
 163 the topography. Thus the velocity in the normal direction is small. In order to translate
 164 this property it is appropriate to write these equations in a reference frame linked to the
 165 topography with one vector in the direction normal to the topography. In Figure 1b, we
 166 give some reference frames used in previous studies. A proper definition is of prior impor-
 167 tance, as the reference frame varies spatially. Spatial differential operators in the flow
 168 equations, with respect to this reference frame, will thus describe the spatial variations of
 169 the fluid thickness and velocity as well as the variations of the reference frame itself.

170 In order to characterize these variations, a functional relation must be found to re-
 171 late the new coordinates to the cartesian coordinates, from which the spatial derivative
 172 operators in the new reference frame can be deduced. It is therefore somehow more nat-
 173 ural and mathematically simple to first define the new coordinate system and to derive the
 174 associated reference frame, instead of the contrary. With this method, the reference frame
 175 may not be orthonormal but this does not entail any loss of generality or accuracy com-
 176 pared to models using an orthonormal reference frame.

183 The most straightforward way to localize a point M above the topography is to con-
 184 sider its projection M' on the topography, along the direction normal to the topography
 185 (Figure 2a). The point M , which has coordinates $\vec{X} = (X, Y, Z)$ in the cartesian refer-
 186 ence frame, can then be localized with a new set of coordinates (x_1, x_2, x_3) : $(x_1, x_2) = \mathbf{x}$ are the
 187 horizontal coordinates of M' in the cartesian reference frame and $x_3 = MM'$ is the dis-
 188 tance to the topography (Figure 2a). Provided we remain in a sufficiently small neigh-
 189 borhood above the topography, this new coordinate system is non-ambiguous: one (and only
 190 one) triplet (x_1, x_2, x_3) can be associated with any point in this neighborhood and vice-
 191 versa. More formally, the link between the cartesian coordinates $\vec{X} = (\mathbf{X}, Z)$ and the new

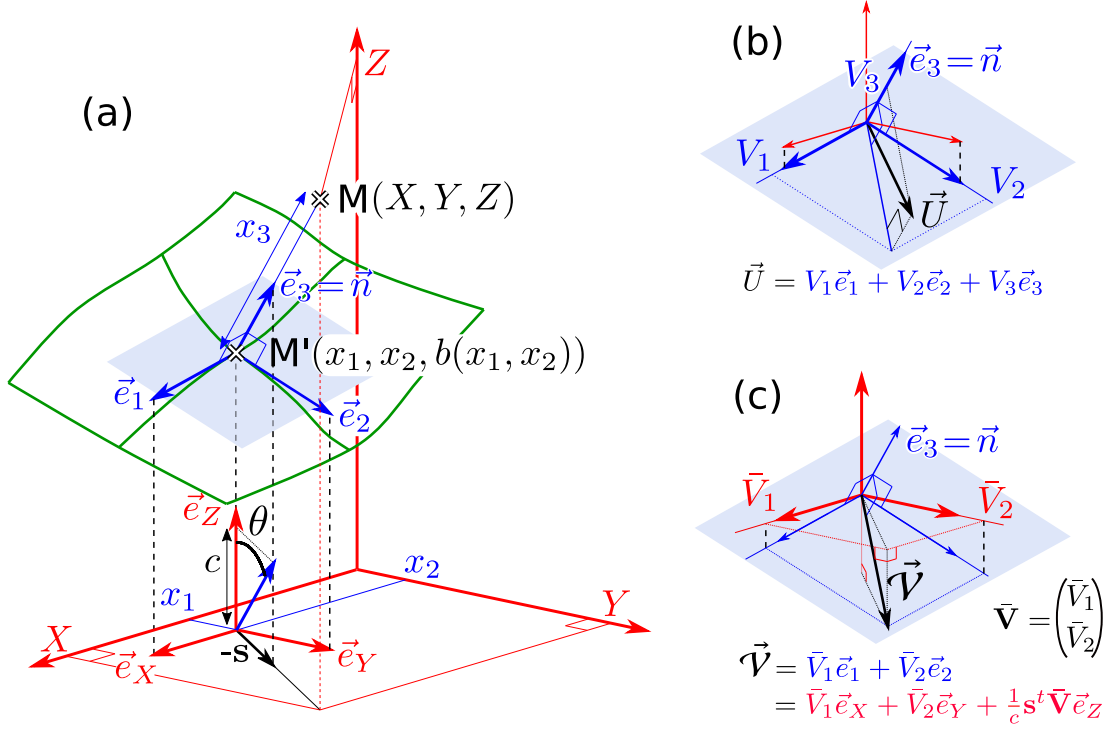


Figure 2. Notations and reference frames for the thin-layer equation derivations. (a) Coordinates of a material point M in the cartesian reference frame $(\vec{e}_X, \vec{e}_Y, \vec{e}_Z)$ (red arrows) are given by (X, Y, Z) and by (x_1, x_2, x_3) in the topography reference frame $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ (blue arrows). M' is the projection of M on the topography: it has cartesian coordinates $(x_1, x_2, b(x_1, x_2))$. \vec{e}_3 is the unit normal vector to the topography and \vec{e}_1, \vec{e}_2 are the projections parallel to \vec{e}_Z of \vec{e}_X and \vec{e}_Y on the plane tangent to the topography (blue feature). (b) Parametrization of the physical velocity \vec{U} of a material point in the topography reference frame. (c) Parametrization of the physical average velocity \vec{V} of the flow. \vec{V} is tangent to the topography and is parametrized in the cartesian reference frame (red) and in the topography reference frame (blue).

coordinates $\vec{x} = (x_1, x_2, x_3) = (\mathbf{x}, x_3)$ of a same physical point is given by

$$(\mathbf{X}, Z) = \vec{X}(\mathbf{x}, x_3) = M' + x_3 \vec{n} = \begin{pmatrix} \mathbf{x} \\ b(\mathbf{x}) \end{pmatrix} + x_3 \vec{n}(\mathbf{x}). \quad (9)$$

As previously, $\vec{n} = \vec{n}(\mathbf{x})$ is the unit upward vector normal to the topography. The same coordinate system was used by *Bouchut et al.* [2003] for 1D topographies and *Bouchut and Westdickenberg* [2004] and *Luca et al.* [2009a] for 2D topographies. A more general formulation with a curvilinear coordinate system $\mathbf{x} = \mathbf{x}(\xi)$ is presented in *Bouchut and Westdickenberg* [2004]. For instance, for 1D topographies, we can choose to locate M' by its curvilinear coordinates along the topography, instead of its cartesian X -coordinate [*Savage and Hutter*, 1991]. For simplicity, we shall keep the Cartesian coordinate system to locate M' . However, this does not limit in any way the type of topographies that can be described in the model.

The reference frame $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ associated with the new coordinates $\vec{x} = (x_1, x_2, x_3)$ follows coordinate lines, so we obtain, with the Einstein notation

$$d\vec{X} = \vec{e}_i dx_i = \vec{e}_1 dx_1 + \vec{e}_2 dx_2 + \vec{e}_3 dx_3. \quad (10)$$

214 We therefore have, for instance, $\vec{e}_1 = \partial_{x_1} \vec{X}$. In this base, the velocity field has coordi-
 215 nates $\vec{V} = (V_1, V_2, V_3) = (\mathbf{V}, V_3)$, such that (Figure 2b)

$$216 \quad \vec{U} = U_X \vec{e}_X + U_Y \vec{e}_Y + U_Z \vec{e}_Z = V_1 \vec{e}_1 + V_2 \vec{e}_2 + V_3 \vec{e}_3. \quad (11)$$

217 We can show (see Supplementary Note 1) that $\vec{e}_3 = \vec{n}$ and thus that V_3 is the topography
 218 normal component of the velocity (Figure 2b).

219 Note that in the previous equation, \vec{U} must be seen as the physical 3D velocity of
 220 the fluid, in the sense that $\|\vec{U}\| = (U_X^2 + U_Y^2 + U_Z^2)^{\frac{1}{2}}$ is the real velocity. In comparison,
 221 \vec{V} is only a parametrization of the velocity field. In particular, as the topography reference
 222 frame is in general not orthonormal, we have

$$223 \quad \|\vec{V}\| = \left(V_1^2 + V_2^2 + V_3^2 \right)^{\frac{1}{2}} \neq \|\vec{U}\|. \quad (12)$$

224 It is not straightforward to replace \vec{U} by \vec{V} in the Navier-Stokes equations. This deriva-
 225 tion can be found in [Bouchut and Westdickenberg, 2004], or in [Luca et al., 2009a] with a
 226 different formalism. However, the resulting equations can be significantly simplified with
 227 the thin-layer approximations. In the following, we simply give the final thin-layer equa-
 228 tions and analyze the resulting curvature terms. More details on the formal derivation and
 229 hypotheses are given in Supplementary Note 1.

230 2.3 Thin-layer equations

231 In the thin-layer approximation, we describe the dynamics of a fluid layer with thick-
 232 ness $h(\mathbf{x})$. We assume this thickness to be small in comparison to the flow extent. Its
 233 physical depth-averaged velocity $\vec{\mathcal{V}}$ is tangent to the topography and thus can be written
 234 in the topography frame

$$235 \quad \vec{\mathcal{V}} = \bar{V}_1 \vec{e}_1 + \bar{V}_2 \vec{e}_2 \quad (13)$$

236 and has coordinates $(\mathcal{V}, \mathcal{V}_3)$ in the cartesian coordinate system. We can show (see Supple-
 237 mentary Note 1) that it is written in the cartesian reference frame:

$$238 \quad \vec{\mathcal{V}} = \bar{V}_1 \vec{e}_X + \bar{V}_2 \vec{e}_Y + \frac{1}{c} \mathcal{V} \vec{e}_Z \quad (14)$$

239 We show that $\bar{V}_1 \vec{e}_1$ and $\bar{V}_2 \vec{e}_2$ are respectively the projections of $\bar{V}_1 \vec{e}_X$ and $\bar{V}_2 \vec{e}_Y$, on the
 240 topography-tangent plane, parallel to \vec{e}_Z (Figure 2c).

241 The resulting equation for $\vec{\mathcal{V}} = (\bar{V}_1, \bar{V}_2)$ is given by

$$242 \quad \partial_t \vec{\mathcal{V}} + (\vec{\mathcal{V}} \cdot \nabla_{\mathbf{x}}) \vec{\mathcal{V}} + (I_2 - \mathbf{ss}^t) \nabla_{\mathbf{x}} (g(hc + b)) =$$

$$243 \quad -c \left(\vec{\mathcal{V}}^t (\partial_{\mathbf{xx}}^2 b) \vec{\mathcal{V}} \right) \mathbf{s} - \frac{\mu g c \vec{\mathcal{V}}}{\sqrt{\|\vec{\mathcal{V}}\|^2 + \left(\frac{1}{c} \mathcal{V} \right)^2}} \left(1 + \frac{\vec{\mathcal{V}}^t (\partial_{\mathbf{xx}}^2 b) \vec{\mathcal{V}}}{g} \right)_+. \quad (15)$$

244 Two curvature terms, involving $\partial_{\mathbf{xx}}^2 b$, appear in (15). One does not depend on the rheol-
 245 ogy (red term in (15)) and the other is included in the friction force (blue term in (15)).
 246 These terms arise from the expression of the pressure at the bottom of the flow (see Sup-
 247 plementary Note 1). They will be interpreted in Section 3. Note that (15) is equivalent to
 248 equation (9.32) in Luca et al. [2009a]. The viscosity ν does not appear in (15), because
 249 we chose it to be negligible, which allows for a rigorous mathematical derivation. To our
 250 knowledge, there exist no formal derivation of thin-layer equations with non negligible vis-
 251 cosity, an no assumption on the velocity profile, on general topographies.

252 The mass equation does not entail any curvature term. With the same formalism as
 253 in our development, Bouchut and Westdickenberg [2004] show that it reads :

$$254 \quad \partial_t \left(\frac{h}{c} \right) + \nabla_{\mathbf{x}} \cdot \left(\frac{h}{c} \vec{\mathcal{V}} \right) = 0. \quad (16)$$

2.4 The SHALTOP numerical model

In order to investigate the influence of curvature forces in numerical simulations, we use the SHALTOP numerical model [Mangeney *et al.*, 2007]. It has been used to reproduce both experimental dry granular flows [Mangeney *et al.*, 2007] and real landslides [Lucas and Mangeney, 2007; Favreau *et al.*, 2010; Lucas *et al.*, 2011, 2014; Moretti *et al.*, 2015, 2020; Brunet *et al.*, 2017; Peruzzetto *et al.*, 2019]. We choose not to compare SHALTOP to another code that would not describe precisely the topography effects. We would not be able to tell whether discrepancies in results originate from curvature effects or, for instance, from the different numerical scheme. A proper benchmarking exercise would be needed, but is beyond the scope of this work. Instead we shall use the same code, but modify it in order to reflect several approximations or remove the curvature.

In SHALTOP, the flow equations are written in terms of the variable $\mathbf{u} = \bar{\mathbf{V}}/c$. This parametrization will be discussed later on. The corresponding momentum equation is:

$$\begin{aligned} \partial_t \mathbf{u} + c(\mathbf{u} \cdot \nabla_{\mathbf{x}}) \mathbf{u} + \frac{1}{c}(Id - \mathbf{s}\mathbf{s}^t) \nabla_{\mathbf{x}}(g(hc + b)) \\ = -\frac{1}{c}(\mathbf{u}^t \mathcal{H}(\mathbf{u}) \mathbf{s} + \frac{1}{c}(\mathbf{s}^t \mathcal{H}(\mathbf{u}) \mathbf{u}) - \frac{\mu g c \mathbf{u}}{\sqrt{c^2 \|\mathbf{u}\|^2 + (\mathbf{s}^t \mathbf{u})^2}} \left(1 + \frac{c^2 \mathbf{u}^t (\partial_{\mathbf{x}\mathbf{x}}^2 b) \mathbf{u}}{g}\right). \end{aligned} \quad (17)$$

with curvature terms colored as in (15). SHALTOP solves the conservative form of (17) with a finite-volume numerical scheme (see Mangeney *et al.* [2007])

We will show in Section 3.1 that the curvature force (first two terms on the right-hand side of (17)) ensures the velocity remains tangent to the topography at all time. Thus, to model this effect, a tangent transport is applied [e.g. Knebelman, 1951]. Considering the physical velocity $\bar{\mathbf{V}} = (c\mathbf{u}, \mathbf{s}^t \mathbf{u})$ in one cell with topography normal vectors \vec{n} , the transported velocity $\bar{\mathbf{V}}'$ in a neighboring cell with normal vector \vec{n}' is computed with:

$$\bar{\mathbf{V}}' = \bar{\mathbf{V}} - \frac{\bar{\mathbf{V}} \cdot \vec{n}'}{1 + \vec{n} \cdot \vec{n}'} (\vec{n} + \vec{n}') \quad (18)$$

Since the curvature force involves the slope variations, for real data with small scale variations it is often necessary to slightly smooth the topography to avoid numerical instabilities. Indeed, when the topography radius of curvature is smaller than the flow thickness, lines normal to the topographies can cross within the flow. In turn the coordinate system defined in (9) is ambiguous: several coordinates (x_1, x_2, x_3) can be associated to a single physical point.

In SHALTOP, the friction coefficient μ can be a function of the flow thickness and velocity. We can thus change the expression of the bottom stress T as in other classical rheologies. For instance, in the semi-empirical Voellmy rheology [Voellmy, 1955; Salm, 1993], the bottom stress reads:

$$T = \rho h \mu (gc + \gamma \|\bar{\mathbf{V}}\|^2) + \rho g \frac{\|\bar{\mathbf{V}}\|^2}{\xi}, \quad (19)$$

with ρ the material density, γ the curvature along flow path (see next section for its computation) and ξ the turbulence coefficient. In numerical experiments (Sections 4 and (5)), we will consider both the Coulomb and the Voellmy rheology that are classically used for field application due to the small number of parameters involved while being able to reproduce first order observations [e.g. Hungr *et al.*, 2007; Lucas *et al.*, 2014; McDougall, 2017].

3 Formal analysis of curvature terms

3.1 Interpretation of curvature terms

The curvature terms appearing in the derived thin-layer equations can be interpreted as acceleration forces in the non-Galilean reference frame linked to the topography. This appears more clearly when we write the depth-averaged equations for the 3D velocity $\vec{\mathcal{V}}$, in the cartesian reference frame :

$$\partial_t \vec{\mathcal{V}} + (\mathcal{V} \cdot \nabla_{\mathbf{X}} \vec{\mathcal{V}}) = \vec{F}_g + \vec{F}_{\mathcal{H}} + \vec{F}_\mu, \quad (20)$$

with

$$\vec{F}_g = - \begin{pmatrix} Id - \mathbf{ss}^t \\ c\mathbf{s}^t \end{pmatrix} \nabla_{\mathbf{X}}(g(hc + b)), \quad (21)$$

$$\vec{F}_{\mathcal{H}} = c(\mathcal{V}^t \partial_{\mathbf{XX}}^2 b \mathcal{V}) \vec{n} = \frac{1}{c^2} (\mathcal{V}^t \mathcal{H} \mathcal{V}) \vec{n}, \quad (22)$$

$$\vec{F}_\mu = - \frac{\mu g c \vec{\mathcal{V}}}{\|\vec{\mathcal{V}}\|} \left(1 + \frac{\mathcal{V}^t (\partial_{\mathbf{XX}}^2 b) \mathcal{V}}{g} \right)_+, \quad (23)$$

where $\mathcal{H} = c^3 \partial_{\mathbf{XX}}^2 b$ is the curvature tensor. \vec{F}_g represents the gravity and lateral pressure forces, \vec{F}_μ is the friction force and $\vec{F}_{\mathcal{H}}$ is the curvature force.

For a material point advected by the velocity field $\vec{\mathcal{V}}$, we can compute

$$\frac{d}{dt} (\vec{\mathcal{V}} \cdot \vec{n}) = \left(\partial_t \vec{\mathcal{V}} + (\mathcal{V} \cdot \nabla_{\mathbf{X}}) \vec{\mathcal{V}} \right) \cdot \vec{n} + \vec{\mathcal{V}} \cdot ((\mathcal{V} \cdot \nabla_{\mathbf{X}}) \vec{n}) \quad (24)$$

$$= \left(\partial_t \vec{\mathcal{V}} + (\mathcal{V} \cdot \nabla_{\mathbf{X}}) \vec{\mathcal{V}} \right) \cdot \vec{n} - c \mathcal{V}^t \partial_{\mathbf{XX}}^2 b \mathcal{V}. \quad (25)$$

In the right-hand side of (24), we can use (20). As $\vec{F}_g \cdot \vec{n} = \vec{F}_\mu \cdot \vec{n} = 0$ and $\vec{F}_{\mathcal{H}} \cdot \vec{n} = c \mathcal{V}^t \partial_{\mathbf{XX}}^2 b \mathcal{V}$, (24) becomes

$$\frac{d}{dt} (\vec{\mathcal{V}} \cdot \vec{n}) = 0. \quad (26)$$

In other words, the curvature force $\vec{F}_{\mathcal{H}}$ ensures that the flow velocity remains parallel to the topography, i.e. $\vec{\mathcal{V}} \cdot \vec{n} = 0$. This force is normal to the topography and thus to the velocity and does no work: in the absence of gravity and friction, the material point would be advected on the topography at constant kinetic energy. Note that this acceleration force is still present, though the equations are written in the fixed cartesian coordinate system: that's because they arise in the intermediate step where the momentum equations are integrated in the direction normal to the topography. The Lagrangian form of equation (20) provides a direct expression of the curvature along a flow path. As a matter of fact, if $M(t)$ is the position of a material point, we have

$$\dot{M}(t) = \vec{\mathcal{V}}, \quad (27)$$

$$\ddot{M}(t) = \partial_t \vec{\mathcal{V}} + \mathcal{V} \cdot \nabla_{\mathbf{X}} \vec{\mathcal{V}}. \quad (28)$$

From classical analytical geometry results and using (20), the curvature of the topography along a flow path, γ , is thus given by

$$\gamma = \pm \frac{\|\dot{M} \wedge \ddot{M}\|}{\|\dot{M}\|^3} = c \frac{\mathcal{V}^t \partial_{\mathbf{XX}}^2 b \mathcal{V}}{\|\vec{\mathcal{V}}\|^2}. \quad (29)$$

In the previous equation, we used (28) and (20), and the fact that $\vec{F}_{\mathcal{H}}$ is the only force which is not colinear to the velocity $\dot{M} = \vec{\mathcal{V}}$: other terms are canceled by the cross product operation. γ is positive for a convex topography and negative otherwise. We thus obtain the classical expression of a centripetal force

$$\vec{F}_{\mathcal{H}} = c(\mathcal{V}^t \partial_{\mathbf{XX}}^2 b \mathcal{V}) \vec{n} = \gamma \|\vec{\mathcal{V}}\|^2 \vec{n}. \quad (30)$$

Note however that as for any acceleration force, the expression of the curvature force depends on the velocity parametrization. In the topography reference frame ($\vec{e}_1, \vec{e}_2, \vec{e}_3$), the velocity components are given by \vec{V} . Provided we impose $V_3 = 0$, (15) describes the evolution of the 2D velocity field \vec{V} along the topography. The curvature force in this reference frame is exactly

$$\mathbf{F}_{\mathcal{H}}^{\vec{V}} = -c \left(\vec{V}' (\partial_{\mathbf{X}\mathbf{X}}^2 b) \vec{V} \right) \mathbf{s}. \quad (31)$$

$\mathbf{F}_{\mathcal{H}}^{\vec{V}}$ has the direction of the main slope and can have a non-zero power ($\mathbf{F}_{\mathcal{H}}^{\vec{V}} \cdot \vec{V} \neq 0$). As a matter of fact, in the absence of gravity and friction, the kinetic energy must remain constant, however it is given by $\|\vec{V}\|^2 = \|\vec{V}\|^2 + \left(\frac{1}{c} \mathbf{s}' \vec{V}\right)$ and not by $\|\vec{V}\|^2$ which is not constant, explaining why the curvature force has a non-zero power in the topography reference frame.

3.2 Comparison with previous studies

3.2.1 Friction force

If we use (30) in (23) to introduce the curvature along flow path γ in the friction force, we get

$$\vec{F}_{\mu} = -\frac{\mu \vec{V}}{\|\vec{V}\|} \left(gc + \gamma \|\vec{V}\|^2 \right)_+, \quad (32)$$

This is the classical expression of the friction force. In 1D (for $b = b(X)$), the derivation of γ is simple. Thus, most 1D thin-layer models [e.g. *Savage and Hutter, 1991*] include the curvature in the friction force. As shown in the previous section, the computation is less self-evident for real 2D topographies ($b = b(X, Y)$), in particular because the flow path must take into account velocity variations (see Supplementary Note 2). The curvature term in the friction force is thus either neglected [*O'Brien et al., 1993*] or approximated. For instance, *Pitman et al. [2003]* use the curvature in the X and Y directions in the momentum equations for V_X and V_Y respectively. We could find only one reference (found in *Fischer et al. [2012]*) to the exact curvature expression mentioned above with a different numerical model than SHALTOP (that is based on the thin-layer equations derived previously). However, it is also possible to implicitly take into account this curvature by solving the equations for the pressure at the bottom of the flow, in addition to the flow thickness and velocity [*Rauter and Tukovic, 2018, see next section*].

If the curvature γ is positive, we see from (32) that neglecting the curvature decreases the bottom friction and accelerates the flow. The opposite effect is expected if γ is negative. On non-flat topographies, we can expect the flow to propagate on gradually decreasing slopes, at least in a first approximation. For instance, the longitudinal cross-sections of volcanoes are often modeled with an exponential fit [e.g. *Mangeny-Castelnau et al., 2003; Kelfoun, 2011; Levy et al., 2015*]. The topography is thus "globally" convex and the curvature is positive at most points. Without the curvature term in the friction, we can thus expect landslides to go further than in the model including curvature.

The effect of approximating the curvature depends of course on the chosen approximation. In Supplementary Note 2, we analyze these effects in some examples. In particular we can compute the curvature along topography in a straight direction given by the local velocity, that is, without taking into account changes in direction. If the flow is not moving in the main slope direction, then the curvature term will be over-estimated.

The numerical code Volcflow uses the following approximation (Karim Kelfoun, personal communication),

$$\gamma = \gamma_x |\cos(\alpha)| + \gamma_y |\sin(\alpha)|, \quad (33)$$

where α is the angle between the horizontal component of the velocity and the X -axis. In our study, we shall test the approximation

$$\gamma = \gamma_x \cos^2(\alpha) + \gamma_y \sin^2(\alpha), \quad (34)$$

387 which is a more classical weighting as $\cos^2(\alpha) + \sin^2(\alpha) = 1$. In both cases, the model
 388 is no longer invariant by rotation. For instance, in the case of a flow confined to a chan-
 389 nel, we show in Supplementary Note 2 that both approximations entail a deceleration of
 390 the flow in most realistic cases. When the channel is aligned with in the X or Y axes, the
 391 deviation from the exact equations is null, but significant differences can be expected oth-
 392 erwise. As the two previous approximations have similar effects, we will test only the sec-
 393 ond in the following. The effects of neglecting or approximating the curvature with (34)
 394 will be assessed in simulations in Sections 4 and 5.

395 3.2.2 Curvature force

396 The first detailed derivation of thin-layer equations for complex topographies was
 397 carried out by *Savage and Hutter* [1991] on 1D topographies. The curvature tensor \mathcal{H}
 398 was reduced to a scalar κ , the curvature of the topography graph $Z = b(X)$. The curva-
 399 ture term is present in their final expression of the friction force, but no curvature force
 400 appears. That is however expected, given their parametrization. They use a curvilinear co-
 401 ordinate system (ξ, η) , with η the distance from the topography (our coordinate x_3) and ξ
 402 the curvilinear coordinate along the topography graph. The associated orthonormal base
 403 is composed of the topography tangential vector \vec{T} and of the topography normal vector \vec{n} .
 404 To be consistent with *Savage and Hutter* [1991], let us choose the new parametrization

$$405 \quad \mathbf{u} = \frac{\vec{V}}{c}. \quad (35)$$

406 This is equivalent to changing our topography reference frame to $(\vec{i}_1, \vec{i}_2, \vec{i}_3) = (c\vec{e}_1, c\vec{e}_2, \vec{e}_3)$,
 407 such that in 1D, \vec{i}_1 is the downslope unit vector and (\vec{i}_1, \vec{i}_3) is an orthonormal base. With
 408 this parametrization, the physical velocity is $(c\mathbf{u}, \mathbf{s}^t \mathbf{u})$ and its norm is

$$409 \quad \|\vec{V}\|^2 = \|c\mathbf{u}\|^2 + (\mathbf{s}^t \mathbf{u})^2. \quad (36)$$

410 Substituting (35) in (15), we can show that the momentum equation for \mathbf{u} is (17), where
 411 the curvature force becomes:

$$412 \quad \mathbf{F}_{\mathcal{H}}^{\mathbf{u}} = -\frac{1}{c} (\mathbf{u}^t \mathcal{H} \mathbf{u}) \mathbf{s} + \frac{1}{c} (\mathbf{s}^t \mathcal{H} \mathbf{u}) \mathbf{u} \quad (37)$$

413 In comparison to (15), the new term $\frac{1}{c} (\mathbf{s}^t \mathcal{H} \mathbf{u}) \mathbf{u}$ comes from the computation of

$$414 \quad (\vec{V} \cdot \nabla_{\mathbf{X}}) \vec{V} = c(\nabla_{\mathbf{X}}(c\mathbf{u}))\mathbf{u}, \quad (38)$$

415 where $\partial_{\mathbf{X}} c$ appears. The curvature force is null when \mathbf{s} and \mathbf{u} are colinear (i.e. when the
 416 velocity is in the downslope direction). This is because in this case $\|\vec{V}\| = \|\mathbf{u}\|$, so no
 417 correction needs to be applied to ensure energy is preserved. In particular in 1D, with this
 418 parametrization, no curvature forces appear in the equations.

419 *Gray et al.* [1999] derived thin-layer equations in a similar fashion. But instead of
 420 choosing a reference frame linked to the topography, they used a simpler reference surface
 421 with the constraint that the deviation from the topography is of the order $\mathcal{O}(\epsilon)$, where ϵ is
 422 the ratio of a characteristic height of the flow over its characteristic length. In thin-layer
 423 models, ϵ is assumed to be very small (see Supplementary Note 1 for a discussion on the
 424 ordering of the equations, and for the mathematical meaning of $\mathcal{O}()$). The same approach
 425 was used, for instance, by *George and Iverson* [2014]. This in turn makes it possible to
 426 assume that the velocity component normal to the reference surface (and not normal to
 427 the topography) has magnitude $\mathcal{O}(\epsilon)$ and only the curvature of the reference surface needs
 428 to be accounted for. In *Gray et al.* [1999], this boils down to the curvature along the x -
 429 axis κ (see their equations (5.9) and (5.10)). With their ordering, it however disappears
 430 in the depth-averaged equations. The derivative of the curvature κ' also appears before
 431 the ordering of the equations in their work. It can also be found in the development of

432 *Bouchut and Westdickenberg* [2004] when curvilinear instead of cartesian coordinates are
 433 considered (without changing the accuracy of the resulting equations).

434 Another fine description of the topography was made by *Pudasaini and Hutter* [2003]
 435 for flows confined in channels. The thalweg is described by a 3D parametric curve $\vec{R}(s)$
 436 to which an orthonormal reference frame is associated with the Serret-Frénet formulas.
 437 *Pudasaini and Hutter* [2003] write the Navier-Stokes equations in this reference frame.
 438 The topography curvature is then rendered by the curvature κ and torsion τ of the thalweg
 439 $\vec{R}(s)$. However, they thus describe only a limited set of topographies, making a proper
 440 comparison with our model difficult.

441 *Fischer et al.* [2012] derive a curvature force by solving the Euler-Lagrange equa-
 442 tions for a free point mass m with coordinates $\vec{X}(t) = (X_1(t), X_2(t), X_3(t))$ subjected to
 443 gravity and evolving on the topography in a fixed cartesian reference frame. With our no-
 444 tation, the Lagrangian reads

$$445 \quad L = \frac{1}{2}m\|\vec{X}(t)\|^2 - mgX_3(t), \quad (39)$$

446 with the constraint

$$447 \quad f(\vec{X}) = X_3(t) - b(X_1, X_2) = 0. \quad (40)$$

448 Solving this system yields

$$449 \quad \frac{d^2\vec{X}}{dt^2} = -gc^2 \left(\frac{\nabla_{\mathbf{X}}b}{\|\nabla_{\mathbf{X}}b\|^2} \right) + c \left(\mathcal{V}' \partial_{\mathbf{X}\mathbf{X}}^2 b \mathcal{V} \right) \vec{n} = -gc^2 \left(\frac{\nabla_{\mathbf{X}}b}{\|\nabla_{\mathbf{X}}b\|^2} \right) + \vec{F}_{\mathcal{H}}. \quad (41)$$

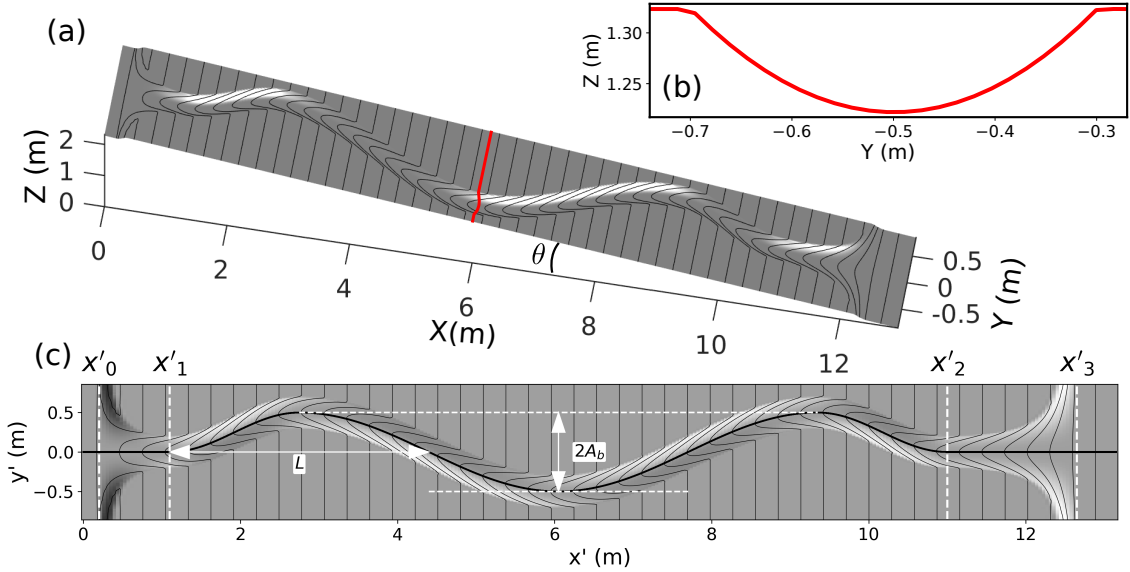
450 This is the Lagrangian form of the momentum equation (20), without the friction force
 451 \vec{F}_{μ} and lateral pressure forces in \vec{F}_g . *Fischer et al.* [2012] use (41) to justify the curvature
 452 term appearing in the friction force, but the curvature force $\vec{F}_{\mathcal{H}}$ is actually independent of
 453 the friction.

454 *Rauter and Tukovic* [2018] and *Rauter et al.* [2018] use an approach similar to that
 455 of *Bouchut and Westdickenberg* [2004]. However, while we use the momentum equation
 456 for the topography-normal component of the velocity to get an explicit expression of the
 457 pressure, they keep this equation and consider the basal pressure as another unknown to
 458 be numerically estimated. This is equivalent to considering the basal pressure as a La-
 459 grangian multiplier, respecting the constraint that the velocity is in the topography-tangent
 460 plane. With this method, *Rauter and Tukovic* [2018] do not need to explicitly describe the
 461 curvature. However, a rigorous derivation of their equations also requires complex differ-
 462 ential calculations, in particular related to the definition of a gradient operator along the
 463 topography.

464 Now that we have detailed the origin of the curvature effects in thin-layer models,
 465 we will investigate their influence, in practice, in simulations. We will first consider simu-
 466 lations on synthetic topographies to identify situations where curvature effects significantly
 467 influences the results. We will then carry out simulations on real topographies.

468 **4 Curvature effects in simulations with synthetic topographies**

469 As shown in Section 3.1, the curvature force $\vec{F}_{\mathcal{H}}$ is needed to ensure that the flow
 470 velocity remains tangent to the topography. It is thus particularly important when the flow
 471 changes direction in twisted channels. As proposed by *Gray and Hutter* [1998], we create
 472 a synthetic topography with a channel composed of n_b successive bends, superimposed
 473 on a plane (Figure 3) with inclination $\theta = 10^\circ$. The channel cross-section is a parabola
 474 (Figure 3b). At both extremities, there is a smooth transition between the end of the chan-
 475 nel and the bottom plane (Figure 3a and 3c). The thalweg is a sinusoidal of amplitude A_b
 476 and period $L = 2.1$ m (black curve in Figure 3c). We define the ratio $\bar{\gamma} = A_b/(L/2)$,
 477 that can be seen as a non-dimensionalized bend curvature. This is detailed in Supplemen-
 478 tary Note 3, along with the exact mathematical definition of this synthetic topography and
 479 some precisions on the simulation set-up.



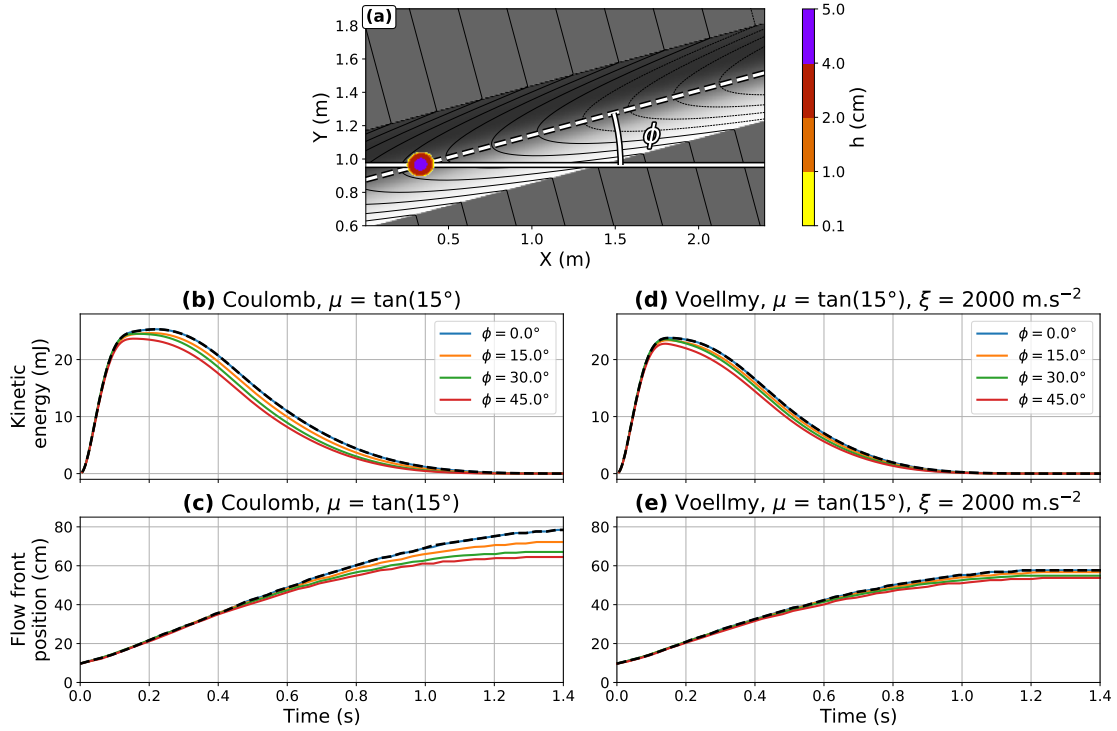
488 **Figure 3.** Synthetic topography with a twisted channel superimposed on a flat plane. (a) 3D view of the
 489 generated topography, in the fixed cartesian reference frame (b) Cross-section of the channel for $X = 6$ m (red
 490 curve in (a)). (c) Top view of the channel, with illustration of the parameters used to construct the topography
 491 are (see Supplementary Note 3). Here $L = 2.1$ m and $A_b = 0.5$ m. x' and y' are the curvilinear coordinates
 492 along the basal plane on which the channel is superimposed. The contour interval is 5 cm in both (a) and (b).

480 In the following, we will first investigate the effects of approximating curvature for a
 481 flow propagating in a straight channel (Section 4.1). We will then model flows in a chan-
 482 nel with only one bend, with the Coulomb and the Voellmy rheologies and analyze how
 483 curvature affects the flow direction, velocity and kinetic energy (Section 4.2). For hazard
 484 assessment, however, it is convenient to synthesize the overall flow dynamics with a few
 485 simple characteristics. In Section 4.3, we will thus investigate curvature effects on the flow
 486 travel duration within the channel and on the maximal dynamic force, for various channel
 487 geometries and rheological parameters.

493 4.1 Curvature approximation and non-invariance by rotation

494 To demonstrate the importance of solving equations that are invariant by rotation, we
 495 first consider the propagation of a flow with the Coulomb rheology ($\mu = \tan(15^\circ)$) and the
 496 Voellmy rheology ($\mu = \tan(15^\circ)$ and $\xi = 2000$ m s⁻²), in a channel without bends (that
 497 is, $A_b = 0$ m) and a slope inclination of $\theta = 10^\circ$. As the flow propagates at the bottom
 498 of the channel, the curvature in the flow direction is, as a first approximation, zero. As a
 499 consequence, no curvature effects are expected. Changing the angle ϕ between the X-axis
 500 and the thalweg (see Figure 4a) should not change the flow dynamics. However, when we
 501 implement the approximation of the curvature (34) in the friction force, we lose the rota-
 502 tional invariance of the model and the flow is slowed down when $\phi > 0$ (Figure 4b to 4e).
 503 For instance with $\phi = 45^\circ$, after 0.5 s, the total kinetic energy is decreased by 20% and
 504 15%, with the Coulomb (Figure 4a) and Voellmy rheology (Figure 4c) respectively. This
 505 directly impacts the travel distance, with 20% (Figure 4b) and 5% (Figure 4d) reductions
 506 respectively.

507 In the following, we will no longer consider the approximation of curvature in the
 508 friction force, and compare only simulations when it is properly taken into account (which



511 **Figure 4.** Modeling of a flow within a straight channel with inclination $\theta = 10^\circ$. (a) Top view of the chan-
 512 nel, with the initial mass (thickness is given by the color scale). ϕ is the angle between the channel direc-
 513 tion (white dashed line) and the X-axis (white solid line). (b) and (c): Kinetic energy and flow front position, with
 514 the Coulomb rheology ($\mu = \tan(15^\circ)$). Colored solid curves: results when the curvature term in the friction is
 515 approximated by weighting the curvature in the X and Y directions (see equation (34)), for different values of
 516 ϕ . Black dashed curves: result with the exact model, that does not depend on the channel orientation ϕ (up to
 517 small numerical errors, not shown here) as it should be. (d) and (e): Same as (b) and (c) but with the Voellmy
 518 rheology ($\mu = \tan(15^\circ)$ and $\xi = 2000 \text{ m}\cdot\text{s}^{-2}$)

509 is not numerically costly) or omitted. Comparisons with approximated curvature are how-
 510 ever provided in the Supplementary Figures and will be referred to briefly.

519 4.2 Thicknesses, velocity and kinetic energy

520 Let us now construct a channel with one bend of amplitude $A_b = 0.5 \text{ m}$ (and thus
 521 $\bar{\gamma} = 0.48$). We will first consider the case where $\mu = 0$ in the Coulomb rheology. We thus
 522 model a pure fluid and can highlight the influence of the curvature force, independently
 523 of the curvature term appearing in the friction force. This is however unrealistic when
 524 considering real geophysical flows, as there is no energy dissipation. We will thus also
 525 consider $\mu = \tan(6^\circ)$, which is a sensible friction coefficient for debris flow modeling
 526 [e.g. *Moretti et al.*, 2015], and the Voellmy rheology that is commonly used to model such
 527 flows. To obtain insight on curvature effects for debris and rock avalanche modeling, we
 528 will finally model flows propagating on a steeper slope ($\theta = 25^\circ$) with a higher friction
 529 coefficient $\mu = \tan(15^\circ)$ [e.g. *Moretti et al.*, 2020].

4.2.1 Channel with slope $\theta = 10^\circ$

For $\mu = 0$, the only acting forces are the curvature and gravity forces. The simulation results are displayed in Figure 5. As shown in Section 3.1, the curvature force horizontal component is in the steepest slope direction and thus tends to keep the flow at the bottom of the channel. This has a major impact on the flow direction at the exit of the channel (Figures 5a and 5b). It also results in a smoother increase of the flow velocity (Figure 5e, between 1 and 2 s), because without the curvature force, the flow bounces back and forth on the channels walls (Figures 5a and 5b). Thus, the effect of the curvature force cannot be neglected: its norm is indeed comparable to the norm of gravity and pressure forces when there are steep changes in the topography, as in the main bend and at the outlet of the channel (Supplementary Figures 1b and 1c). The maximum flow velocities are, however, of the same order: about 3 m s^{-1} at the outlet of the channel (Figures 5c and 5d).

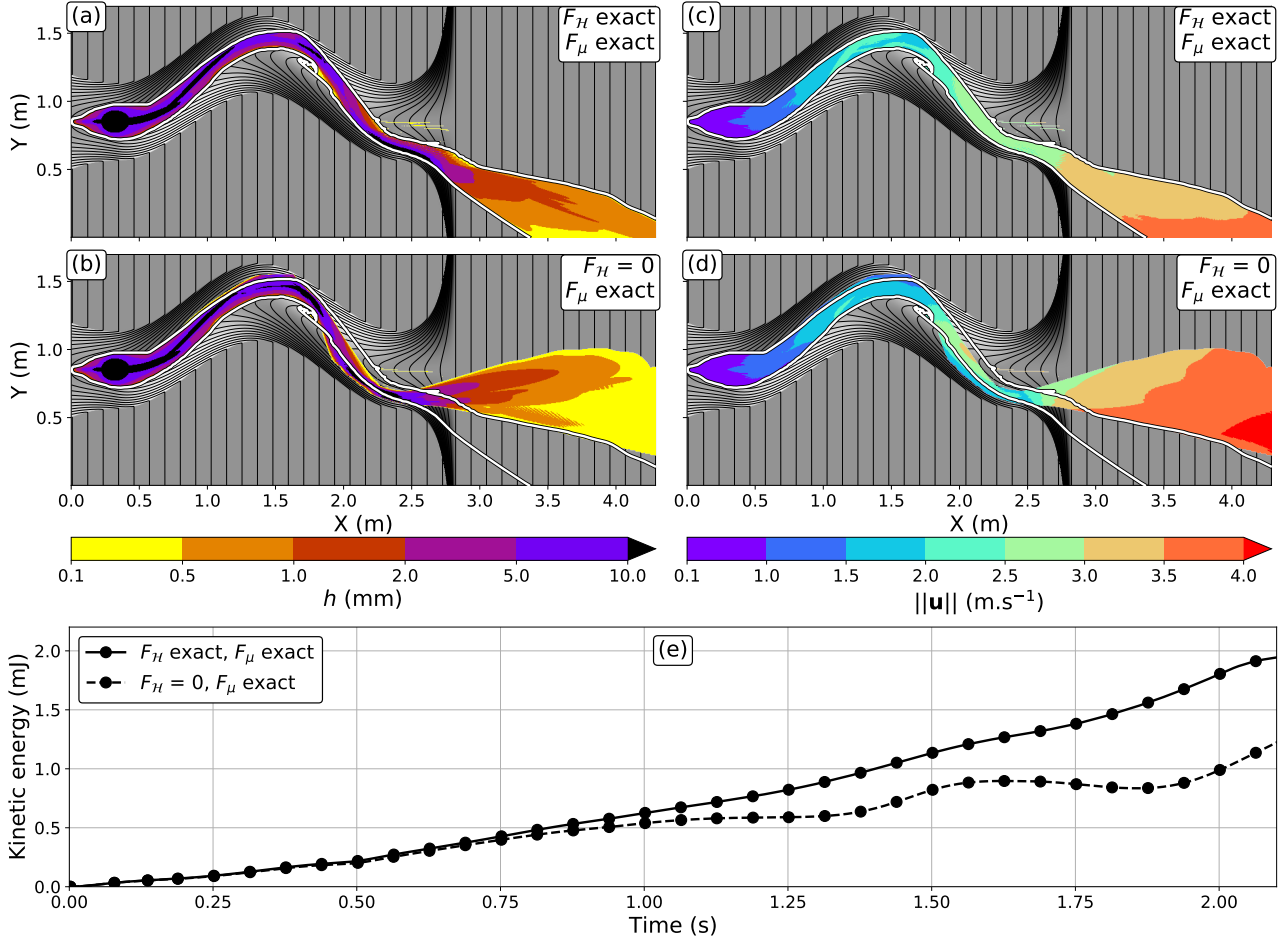
In order to model debris flows more realistically, we now use a friction coefficient $\mu = \tan(6^\circ)$. We can then analyze the influence of neglecting the curvature term in the friction force (Figures 6, $\vec{F}_\mu=0$). Because of friction, the flow is decelerated compared to the case without friction (only 2 m s^{-1} at the channel outlet). The curvature terms (both in friction and curvature forces), which are proportional to the square of velocity, are then only half as high as gravity and pressure forces (see Supplementary Figures 2b and 2d). However, neglecting the curvature force does still slow down the flow, with a 5% kinetic energy decrease at the channel outage (Figure 6 i, $F_H=0$). On the contrary, neglecting the curvature term in the friction force results in a slightly smaller friction force and thus increases the flow velocity (kinetic energy increased by 5% at the channel outage, Figure 6i, F_μ no curvature) and runout (e.g. Figures 6a and 6e). Approximating the curvature in the friction decelerates the flow, as expected (see Supplementary Figures 2 and 3).

In the literature, the empirical Voellmy rheology is also often used to model debris flows. We show in Supplementary Figures 4 and 5 that curvature effects have only limited influence with this rheology, which will be confirmed by further results.

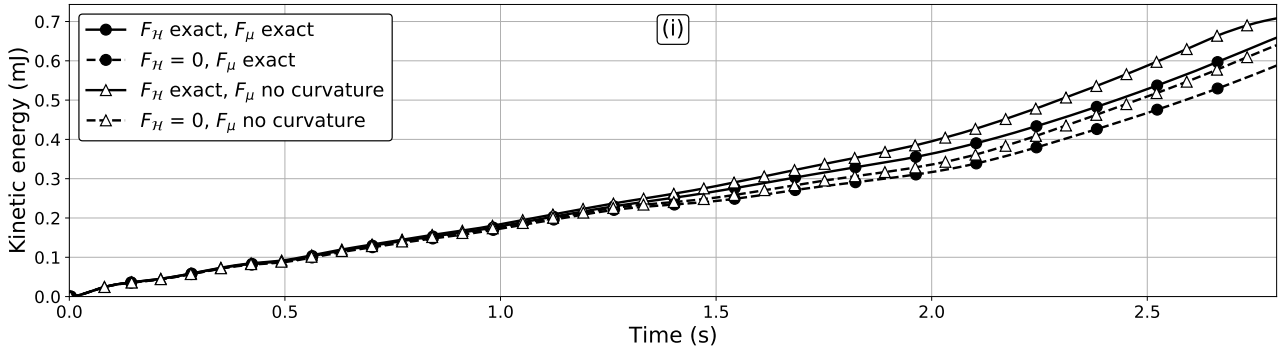
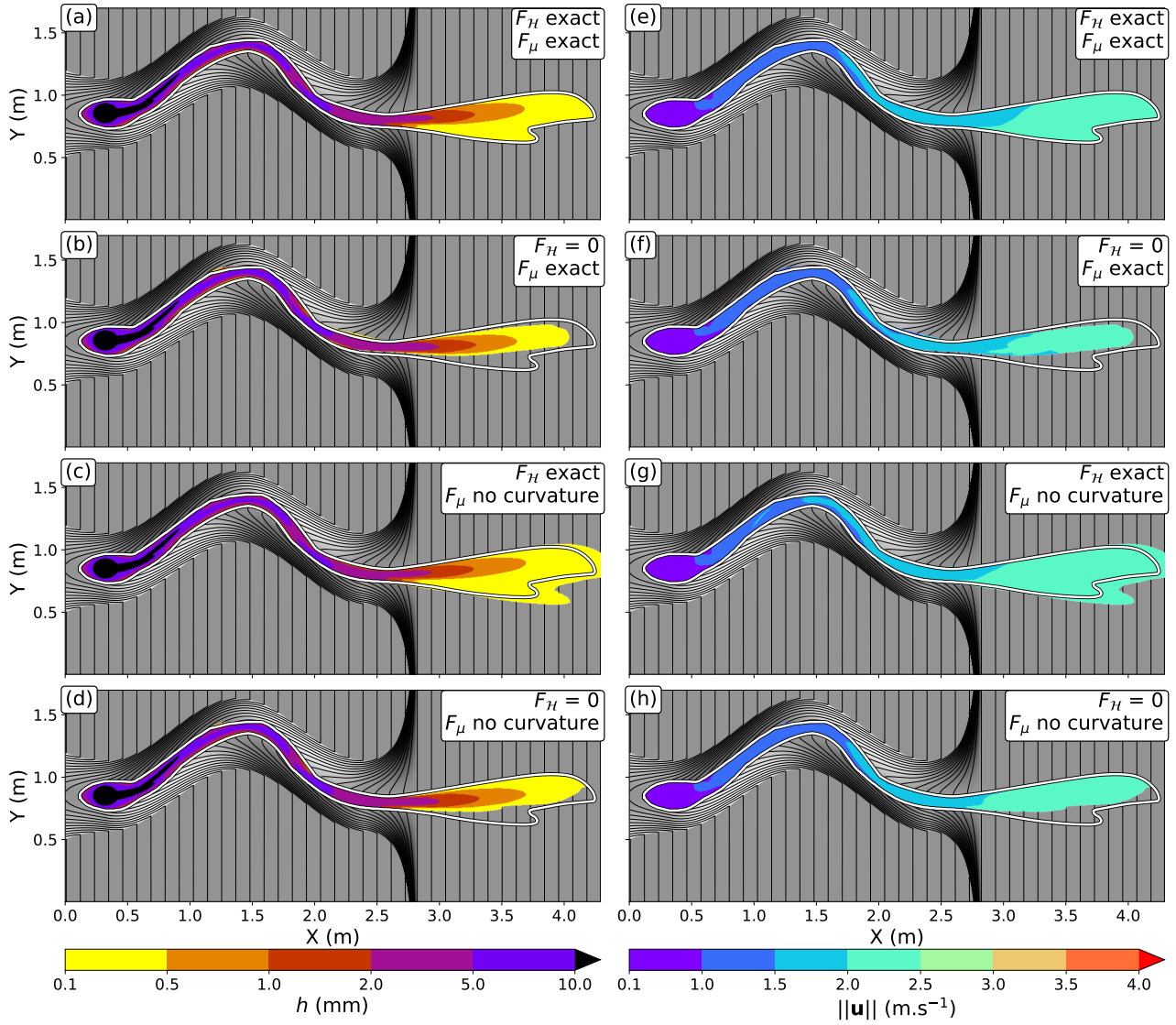
4.2.2 Channel with slope $\theta = 25^\circ$

The main slope of the channel and the parameters we have considered so far are reasonable estimates for modeling debris flows [e.g. *Moretti et al.*, 2015]. For debris and rock avalanches, it is more relevant to use steeper slopes and higher friction coefficients. In Figure 7, we investigate the curvature effects on a steeper slope ($\theta = 25^\circ$) and for a higher friction coefficient $\mu = \tan(15^\circ)$, which is still characteristic of mobile landslides [*Pirulli and Mangeney*, 2008]. The impact of neglecting curvature terms is qualitatively similar to the previous case with $\mu = \tan(6^\circ)$, but errors are amplified (see Supplementary Figures 6 and 7 for the simulations with approximated curvature in friction). In particular, neglecting the curvature term in the friction leads to a significant acceleration of the flow: at the channel outlet, the total kinetic energy is increased by 70% (Figure 8a, F_H exact and F_μ no curvature). It can be directly correlated to the 30% error induced on the friction in the channel bends, when curvature is not taken into account (Figures 8b and 8d).

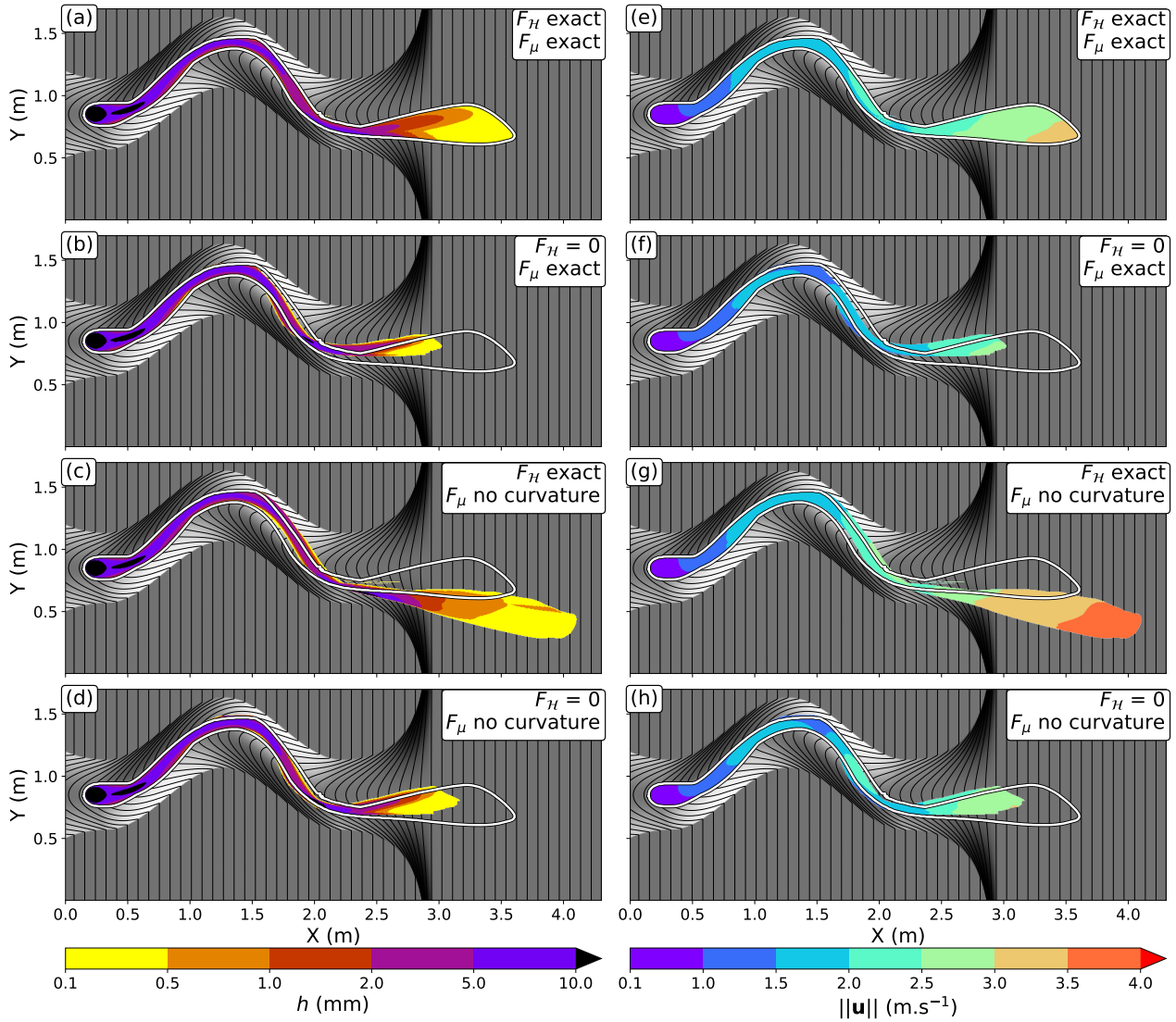
In the above simulations, we have shown that the direction of the flow at the channel outlet can change significantly when curvature effects are not accounted for. That is of course of prior importance in hazard mapping. In order to characterize the flow dynamics, two other indicators can refine the hazard assessment analysis.



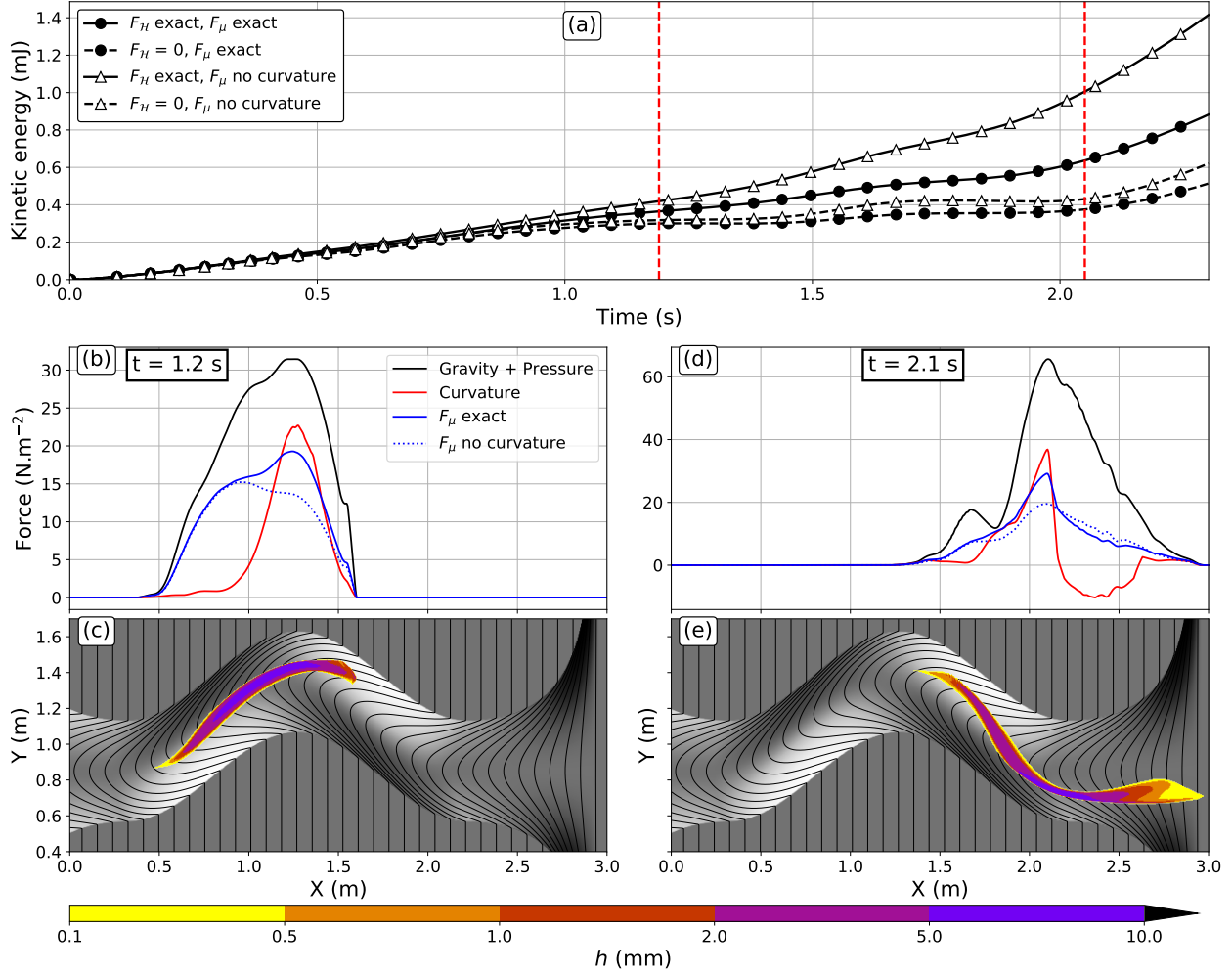
558 **Figure 5.** Flow simulation with the Coulomb rheology, $\mu = 0$ and a slope $\theta = 10^\circ$. (a) and (c): with the
 559 curvature force (F_H exact). (b) and (d): without the curvature force ($F_H = 0$). (a) and (b) give the maximum
 560 flow thickness during the simulation, (c) and (d) the maximum flow velocity. The white curve is the flow
 561 extent when the curvature force is taken into account. Simulation durations is 2.5 s. We give more details in
 562 Supplementary Note 4 on the derivation of maximum thickness and velocity maps.



563 **Figure 6.** Flow simulation with the Coulomb rheology, $\mu = \tan(6^\circ)$ and a slope $\theta = 10^\circ$. The first column
 564 is the maximum flow thickness (a-d) and the second column is the flow maximum velocity (e-h), both after
 565 2.8 s. Each subfigure displays the results of the simulation when the curvature force is taken into account (F_H
 566 exact) or neglected ($F_H = 0$) and when the curvature in the friction is exact (F_μ exact) or neglected (F_μ no
 567 curvature). (a) and (e) are the simulation results in the reference case, with exact curvature terms: the corre-
 568 sponding flow extent (white line) is reported in all figures. The contour interval is 2 cm. (i) Kinetic energy in
 569 the different simulations.



583 **Figure 7.** Same as 6, but with the Coulomb rheology, $\mu = \tan(15^\circ)$ and a slope $\theta = 25^\circ$. The contour inter-
 584 val is 4 cm. Simulation duration is 2.3 s. The kinetic energies are given in Figure 8.



585 **Figure 8.** (a) Total kinetic energy of the flow with the Coulomb rheology, $\mu = \tan(15^\circ)$ and a slope $\theta = 25^\circ$.
 586 (b) For the simulation with exact curvature terms, maximum norm of gravity and pressure force (\vec{F}_g^V , black
 587 curve), of the curvature force (\vec{F}_H^V , red curve, negative when $\vec{n} \cdot \vec{F}_H^V < 0$) and of the friction force (\vec{F}_H^μ ,
 588 blue curves). The friction force is computed with the exact curvature term (F_μ exact) or when it is neglected
 589 (F_μ no curvature). The maximum is computed for a constant X coordinate, at $t=1.2$ s. (c) Flow thickness at
 590 $t=1.2$ s. (d) and (e): Same as (b) and (c), respectively, but for $t=2.1$ s. These two times are indicated by the red
 591 dashed vertical line in (a).

4.3 Travel time and maximum dynamic force

The flow travel duration within the channel is often a key indicator for hazard assessment. The second indicator is the maximum dynamic force F_d ,

$$F_d = \max\left(\frac{1}{2}\rho h\|\vec{V}\|^2\right) = \max(hP_d), \quad (42)$$

where P_d is the dynamic pressure. In the following we choose $\rho = 1500 \text{ kg m}^{-3}$ for the density: it acts only as a scaling factor. To obtain a more systematic analysis of the influence of curvature terms on these indicators, we keep only one bend, but try three different bend amplitudes: $A_b = 0 \text{ m}$, $A_b = 0.25 \text{ m}$ and $A_b = 0.5 \text{ m}$ ($\bar{\gamma} = 0$, $\bar{\gamma} = 0.24$ and $\bar{\gamma} = 0.48$). Simulations are run in each configuration with the Coulomb and Voellmy rheologies, while varying the friction and turbulence coefficients.

Results are displayed in Figure 9 and summarized in Table 2. Unsurprisingly, for a straight channel, travel durations in the channel are very similar whatever the curvature description. There are however variations in the dynamic force (e.g. blue curves in Figure 9d), likely due to the initial spreading of the mass in all directions. When a bend is added ($A_b = 0.25 \text{ m}$ and $A_b = 0.5 \text{ m}$), in the case of small friction coefficients and thus small friction forces, neglecting the curvature in the friction force has less effect than neglecting the curvature force (e.g. Figure 9a, $A_b = 0.5 \text{ m}$ for $\mu < \tan(6^\circ)$). However the opposite occurs when the friction coefficient increases, as the friction force also increases. The error on maximum dynamic force is particularly high for fast flows, that is for small friction coefficients (e.g. for $A_b = 0.5 \text{ m}$, up to 40% for $\mu = \tan(2^\circ)$ and only 5% for $\mu = \tan(8^\circ)$, Figure 9b). Note, however, that when we increase the length of the channel by adding successive bends, the effect of using incorrect curvature terms is amplified due to successive errors. We have for instance at most 5% discrepancies in travel durations with one bend and $\mu = \tan(6^\circ)$, but up to 15% differences with 5 successive bends (see Table 2 and Supplementary Figure 8). With higher slope angles and friction coefficients corresponding to rock avalanches, the differences would be even more significant.

When we use the Voellmy rheology, as expected, differences in travel times are less striking: only 5% deviations for the flow travel time (Figure 9c), and 10% differences for the maximum dynamic force (Figure 9d).

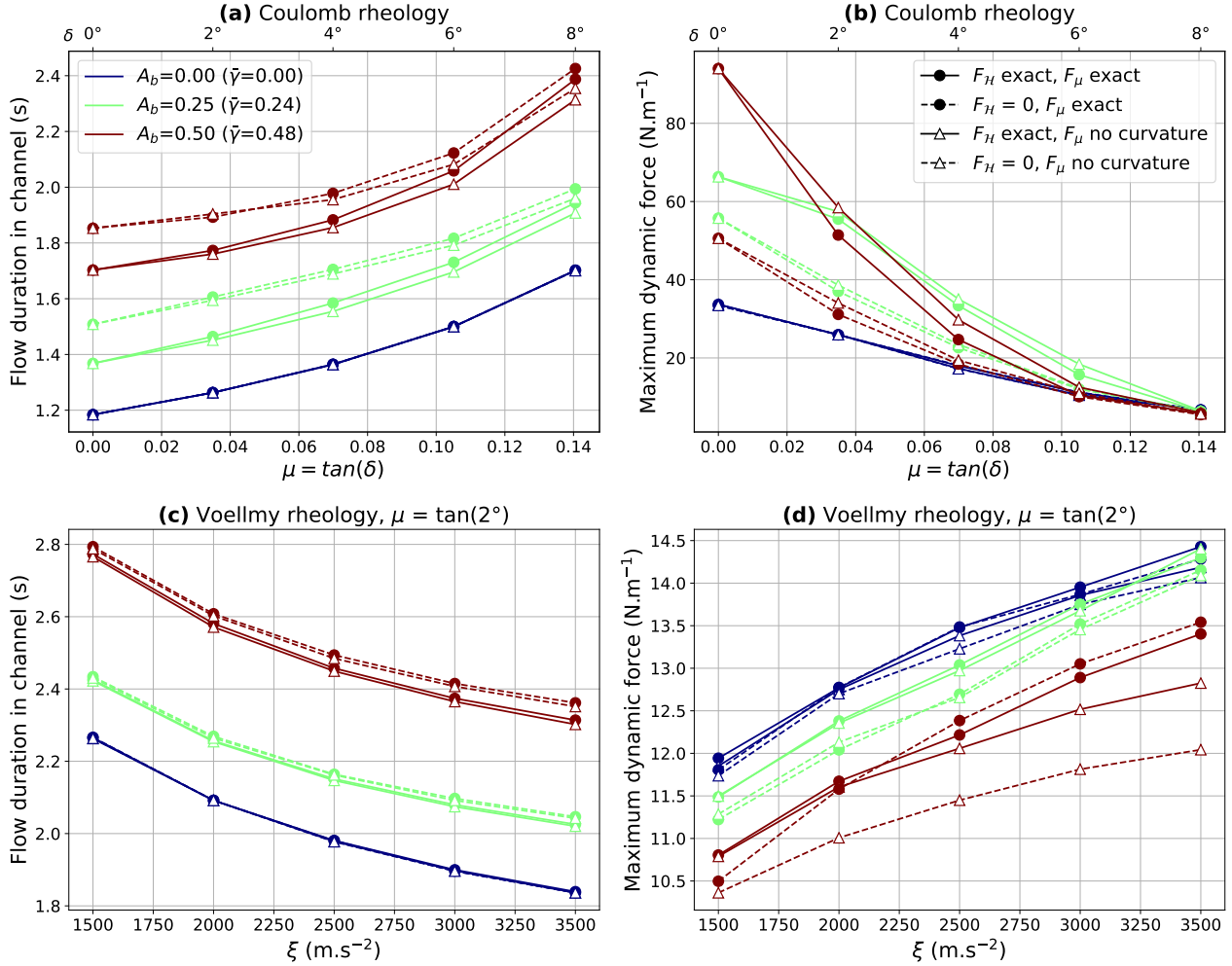
We may wonder whether our observations on synthetic and simple topographies can be extrapolated to more realistic scenarios. In the next section, we thus carry out simulations on real topographies.

5 Curvature effects in simulations over real topographies

We chose two case studies for our simulations on real topographies: the simulation of debris flows in the Prêcheur river, in Martinique (French Caribbean) and the simulation of a debris avalanche on the Soufrière de Guadeloupe volcano, in Guadeloupe (French Caribbean)

5.1 Debris flow in the Prêcheur river

The Prêcheur river is located on the western flank of Montagne Pelée, an active volcano for which the last eruption dates back to 1932. Debris flows and hyper-concentrated flows occur regularly in this 6 km long river [Clouard *et al.*, 2013; Nachbaur *et al.*, 2019], with the risk of overflow into the Prêcheur village, at the mouth of the river [Aubaud *et al.*, 2013; Queffélec, 2018]. In this context, numerical modeling can help constrain the prominent parameters controlling the flow dynamics and in turn be used for quantified risk assessment. However, a detailed analysis is beyond the scope of this paper. We only aim here to illustrate whether or not curvature effects have a significant impact on the flow dynamics. To that purpose, we release a hypothetical mass of $90,000 \text{ m}^3$ at the



625 **Figure 9.** Simulation of a flow in a channel with slope $\theta = 10^\circ$ and one bend with the Coulomb rheology
 626 (a and b) and the Voellmy rheology (c and d, with $\mu = \tan(2^\circ)$). The bend amplitude A_b is either 0 m, 0.25 m
 627 or 0.5 m (respectively, blue, green and red curves). The corresponding non-dimensionalized curvature is $\bar{\gamma}$.
 628 The flow duration in the channel (a and c) and the maximum impact pressure (b and d) are plotted as functions of
 629 the friction coefficients for the Coulomb rheology (the top x-axis gives the corresponding friction angle) and
 630 as functions of the turbulence coefficient for the Voellmy rheology. Different situations are considered: when
 631 the curvature force is taken into account (F_H exact) or neglected ($F_H = 0$) and when the curvature in the
 632 friction is exact (F_μ exact) or neglected (F_μ no curvature)

633 **Table 2.** Influence of curvature terms for synthetic topographies, with the Coulomb rheology, for different
 634 channel geometries with slope $\theta = 10^\circ$ (lines, with A_b the channel bend amplitude) and friction coefficients
 635 (columns). The relative maximum deviation from the reference simulation with exact curvature is given
 636 for the flow duration in the channel (Δt , bold) and for the maximum dynamic force (F_d , italic). We specify
 637 which curvature term has the more prominent influence on the flow dynamics: the curvature force ($F_{\mathcal{H}}$) or the
 638 curvature in the friction (F_μ). n/a indicates indicates no simulation was done.

		$\mu = \tan(0^\circ)$		$\mu = \tan(6^\circ)$		$\mu = \tan(8^\circ)$	
		Δt	F_d	Δt	F_d	Δt	F_d
$A_b = 0$ m		0%	0%	0%	-5%	0%	0%
$A_b = 0.25$ m	1 bend	+10%	-15%	+5%	-25%	+2.5%	-5%
		$F_{\mathcal{H}}$		$F_{\mathcal{H}}$		$F_{\mathcal{H}}, F_\mu$	
$A_b = 0.5$ m	1 bend	+10%	-45%	+5%	+20%	-2.5%	+5%
		$F_{\mathcal{H}}$		$F_{\mathcal{H}}, F_\mu$		F_μ	
	5 bends	not modeled		+15% ^a	-60% ^a		
				-10% ^b	+135% ^b		
				F_μ			

^a Differences for F_μ exact and $F_{\mathcal{H}} = 0$ neglected

^b Differences for F_μ without curvature and $F_{\mathcal{H}}$ exact.

657 bottom of the cliff and model its propagation for 10 minutes, on a 5-meter Digital Eleva-
 658 tion Model. We will first explore the possibility of overflows (Figures 10) in simulations
 659 with the Coulomb rheology. We will then conduct a more systematic analysis of curva-
 660 ture effects on the debris flow front position with the Coulomb and Voellmy rheologies
 661 and various rheological parameters, tracking the front position during the simulation with
 662 a thickness threshold of 1 cm (Figure 11). The results of approximating the curvature are
 663 displayed in Supplementary Figures 9 and 10.

664 5.1.1 Channel overflows with the Coulomb rheology

665 A critical point for hazard assessment is the possibility of overflows. In Figure 10,
 666 we show the maximum thickness of the flow in the Prêcheur river, simulated with the
 667 Coulomb rheology and $\mu = \tan(3^\circ)$, which is representative of a highly mobile mate-
 668 rial. Keeping the curvature force but neglecting the curvature in the friction not only in-
 669 creases the runout, but also leads to multiple overflows (Figure 10c). Neglecting the cur-
 670 vature force partly compensates artificially this effect (Figure 10d). However, in this case,
 671 overflows do not correspond to the ones modeled in the reference case (see Figure 10e
 672 and 10f, the white line is the extent of the flow in the simulation with exact curvature).
 673 Streaks outside the topography are artifacts explained in Supplementary Note 4.

674 5.1.2 Flow front position and travel distance

675 We now track the flow front position as the flow propagates in the river. With the
 676 Coulomb rheology, the travel distance is increased by several hundred meters when the
 677 curvature in the friction force is neglected (Figure 11a, solid and dashed curves with trian-
 678 gles). This difference may be reduced by choosing a thickness threshold higher than 1 cm.
 679 Nevertheless, it highlights the bias introduced by an improper curvature description. Cur-
 680 vature effects have a particularly strong influence in the upper part of the river which is
 681 narrow, twisted and with slopes above $\theta = 7^\circ$ (see Figure 10): there are significant vari-
 682 ations in the time needed by the flow to travel the first 1.5 km (Figure 11b). Depending
 683 on the friction coefficient, neglecting the curvature in the friction increases the flow veloc-

741 **Table 3.** Qualitative summary of the simulations results, with the different topographies (columns) and
 742 rheologies (lines). For the Coulomb rheology, we give the curvature description that gives the biggest error in
 743 comparison to simulations with the exact curvature. $F_{\mathcal{H}}$ refers to the curvature force and F_{μ} to the curvature
 744 in the friction force.

		Error max for channelized flow		Error max for non-channelized flow
		Synthetic topography	Prêcheur river	Soufrière de Guadeloupe
Coulomb	$\mu < \tan(5^\circ)$	$F_{\mathcal{H}} = 0$	F_{μ} without curvature and $F_{\mathcal{H}}$ exact	<i>n/a</i>
	$\mu > \tan(5^\circ)$	F_{μ} approximated	F_{μ} without curvature	F_{μ} without curvature
Voellmy		Limited influence of curvature effects		<i>n/a</i>

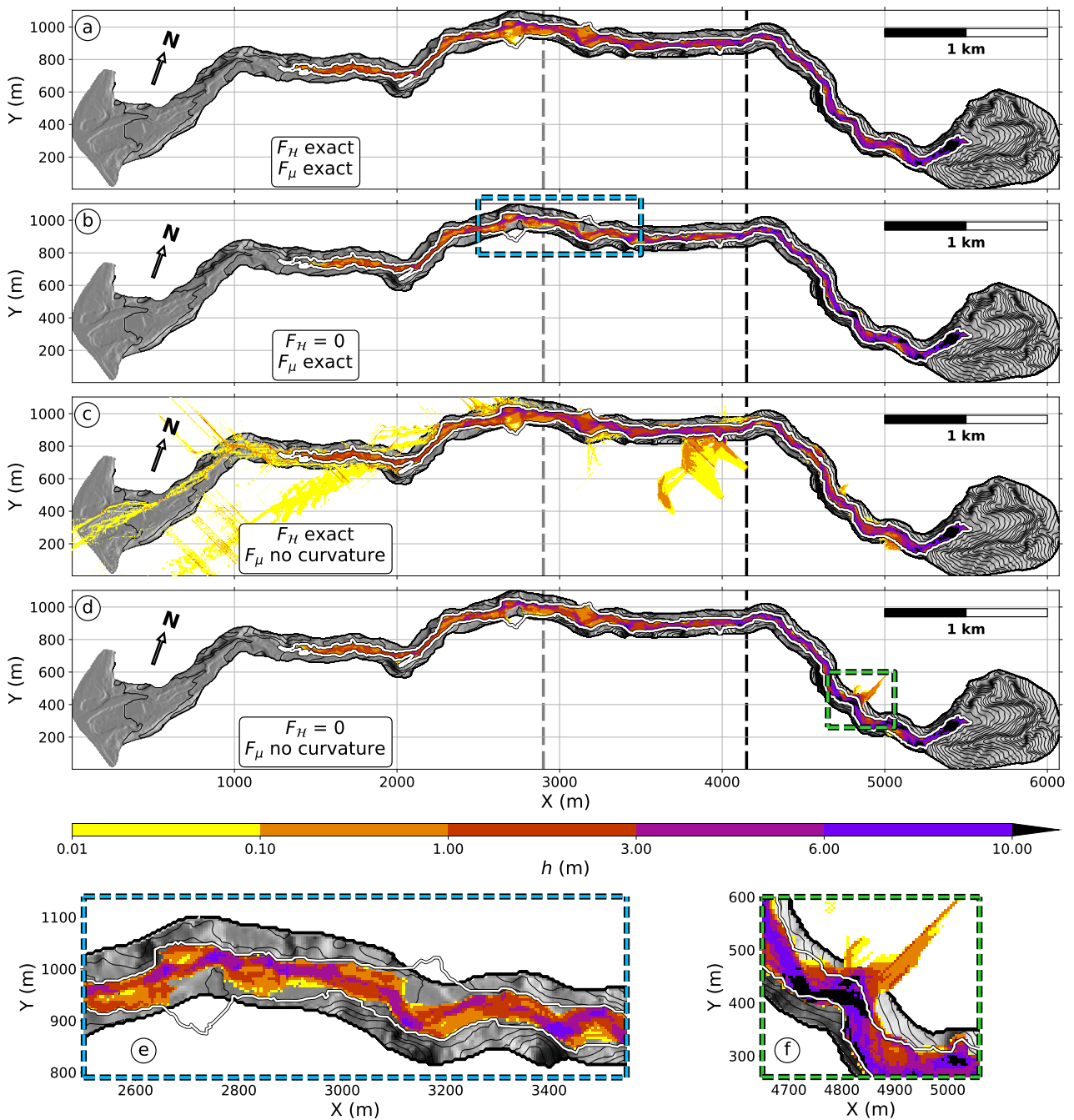
684 ity by 20 to 30% (Figure 11b, $F_{\mathcal{H}}$ exact, F_{μ} no curvature). This allows the flow to gain
 685 enough momentum to overrun flatter areas, whereas it remains stuck there in the reference
 686 case. To the contrary, neglecting the curvature force ($F_{\mathcal{H}} = 0$, F_{μ} exact) slows down the
 687 flow by 30 to 50%.

688 With the Voellmy rheology, the prominent factor impacting the flow dynamics is the
 689 curvature force (Figure 11d): without it, the flow needs up to 15% more time to travel the
 690 first 3 km. Further downstream, the delay between simulations is however constant (e.g.
 691 no more than 25 s with $\xi = 3500 \text{ m s}^{-2}$, Figure 11c), which indicates once more that
 692 curvature effects affect the flow mainly in the upper part of the river.

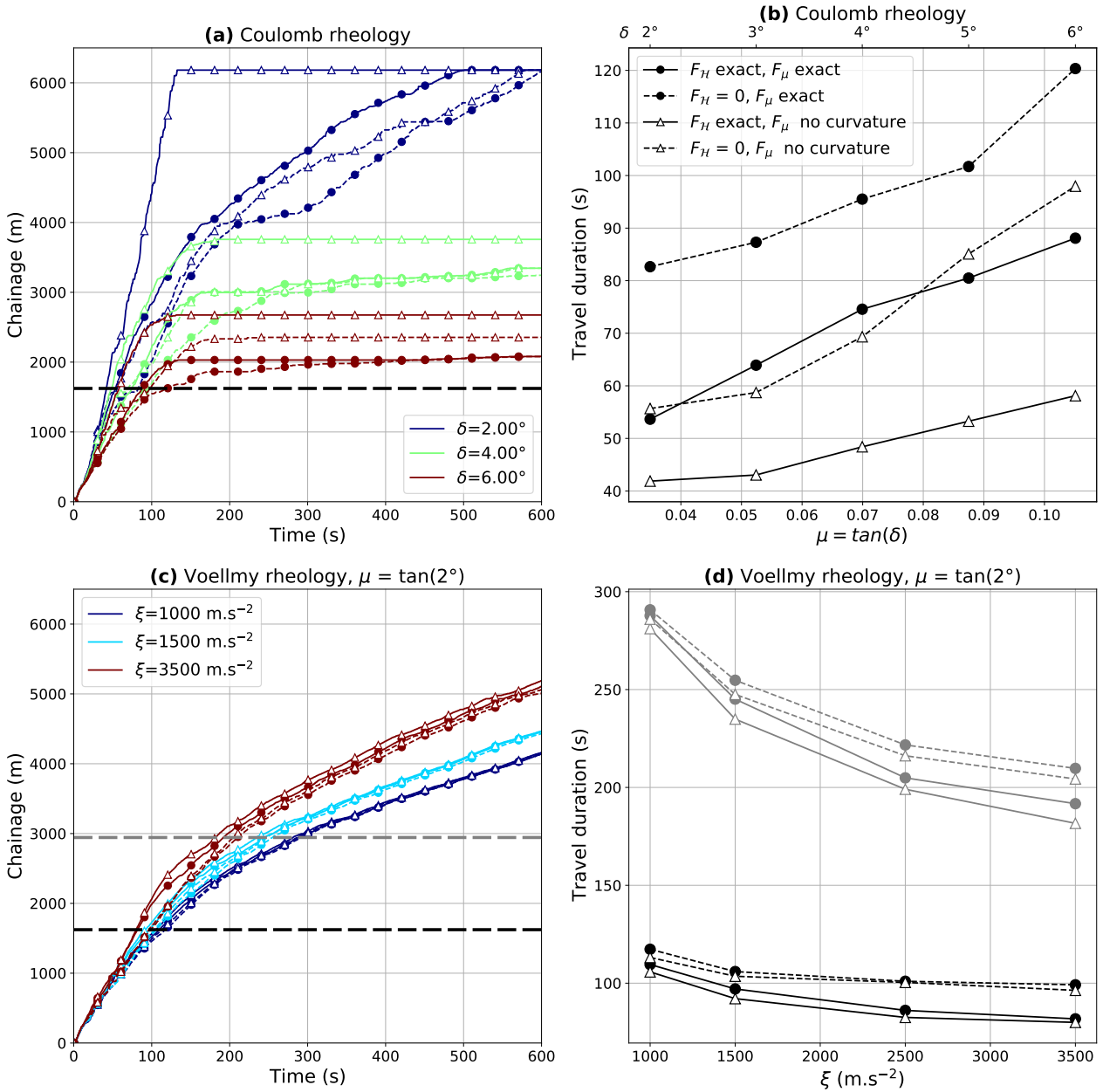
708 5.2 Debris avalanche on the Soufrière de Guadeloupe volcano

709 Thin-layer numerical models are also commonly used to model the dynamics and
 710 emplacement of debris and rock avalanches, which are not confined to one channel as
 711 for debris flows. They usually involve bigger volumes (e.g. several million cubic meters)
 712 and spread on steeper slopes at least at their onset [e.g. *Guthrie et al.*, 2012]. In this sec-
 713 tion, we investigate the importance of curvature effects in simulations reproducing such
 714 events, studying the example of the Soufrière de Guadeloupe volcano, in Guadeloupe
 715 (French Caribbean). This volcanic edifice has a strong record of destabilization events,
 716 with at least 9 debris avalanches over the past 9,000 years [*Boudon et al.*, 2007; *Legendre*,
 717 2012]. *Peruzzetto et al.* [2019] model the runout of a $90 \times 10^6 \text{ m}^3$ debris avalanche: this
 718 volume is consistent with the estimated volume ($80 \pm 40 \times 10^6 \text{ m}^3$) of the 1530 CE debris
 719 avalanche. In order to reach the sea 9 km away from the volcano like the 1530 CE event,
 720 the friction coefficient $\mu = \tan(7^\circ)$ had to be used (Figure 12a).

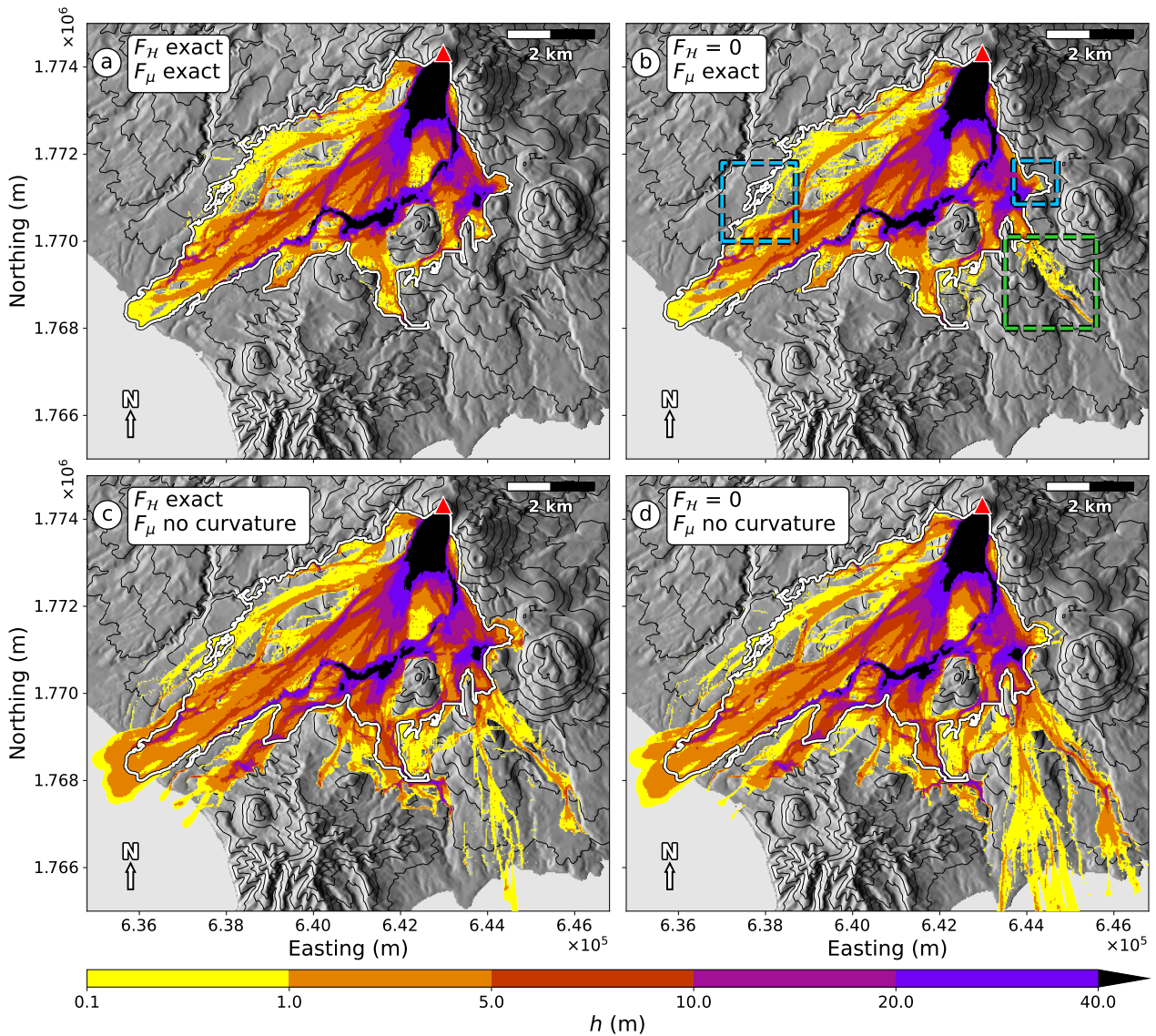
721 Using this friction coefficient and the same modeling set-up, we now model the de-
 722bris avalanche emplacement by neglecting the different curvature terms (Figures 12b to
 723 12f). The results of approximating the curvature are displayed in Supplementary Fig-
 724 ure 11, and maximum kinetic energies in Supplementary Figure 12. When the curvature
 725 force is neglected, the most prominent difference is an excessive travel distance to the
 726 south (more than 1.5 km, Figure 12b, green rectangle). In some areas, spreading is less
 727 important, but only slightly (a difference of less than 200 m, Figure 12b, blue rectangle).
 728 Neglecting the curvature in the friction induces the most significant deviation from the ref-
 729 erence simulation, with a generalized increase of the debris avalanche spreading (Figure
 730 12c and 12d). In particular, the debris avalanche reaches the sea south of the Soufrière
 731 volcano, which is not predicted in the reference case (Figure 12a). Such differences are
 732 critical for tsunami hazard assessment.



693 **Figure 10.** Maximum thickness of the flow simulated in the Prêcheur river with the Coulomb rheology and
 694 $\mu = \tan(3^\circ)$. Each plot (a to d) displays the result of the simulation when the curvature force is taken into ac-
 695 count (F_H exact) or neglected ($F_H = 0$) and when the curvature in the friction is exact (F_μ exact) or neglected
 696 (F_μ no curvature). The simulation results in the reference case, with exact curvature terms, is given in (a).
 697 The corresponding flow extent (white curve) is reported in all figures. Green dashed rectangles (respectively
 698 blue dashed rectangles) indicate areas where the spreading is greater (respectively lesser) in other simula-
 699 tions, in comparison to the reference simulation (a). Zooms on these areas are given in (e) and (f). The contour step
 700 is 20 m.



701 **Figure 11.** Simulations of debris flow in the Prêcheur river. Different situations are considered: when the
 702 curvature force is taken into account (F_H exact) or neglected ($F_H = 0$) and when the curvature in the fric-
 703 tion is exact (F_μ exact) or neglected (F_μ no curvature). (a) Flow front position with the Coulomb rheology.
 704 (b) Time needed for the flow to travel the first 1.6 km (black dashed line in (a)) with the Coulomb rheology, as
 705 a function of friction coefficient. (c) Flow front position with the Voellmy rheology and $\mu = \tan(2^\circ)$. (d) Time
 706 needed for the flow to travel the first 1.6 km (black dashed line in (c)) and 2.9 km (gray dashed line in (c))
 707 with the Voellmy rheology, as a function of turbulence coefficient.



733 **Figure 12.** Maximum thickness of a hypothetical $90 \times 10^6 \text{ m}^3$ debris avalanche on the Soufrière de Guade-
 734 loupe volcano (French Caribbean). Each plot (a to f) displays the result of the simulation when the curvature
 735 force is taken into account (F_H exact) or neglected ($F_H = 0$) and when the curvature in the friction is exact
 736 (F_μ exact) or neglected (F_μ no curvature). The simulation results in the reference case, with exact curvature
 737 terms, is given in (a) [Peruzzetto *et al.*, 2019]. The corresponding flow extent (white curve) is reported in all
 738 figures. Green dashed rectangles (respectively blue dashed rectangles) indicate areas where the spreading is
 739 greater (respectively lesser) in other simulations, in comparison to the reference simulation (a). The DEM is
 740 from IGN BDTopo, coordinates: WGS84, UTM20N. The contour interval is 100m.

6 Discussion

6.1 Importance of curvature effects for different rheologies

In our study, we derived curvature forces for the simplified case of a inviscid thin-layer flow. That is of course a simplification, as complex interactions between solid particles and between the solid and liquid phases can be expected (see *Delannay et al.* [2017] for a review). The formal derivation of SHALTOP equations requires, for instance, that the kinematic viscosity is small (see Supplementary Note 1), which can be questioned in practice for muddy debris flows. *Pastor et al.* [2004] and *Pastor et al.* [2009b] used the Bingham and Herschel-Bulkley theories to derive an implicit relation between the flow average velocity and the basal shear stress for simple shear flows on 1D topographies. Note that the resulting equations are similar to that of SHALTOP, provided we use an appropriate friction coefficient μ that depends on the thickness and on the flow velocity. A more comprehensive description of viscous flows is done by *Pudasaini and Mergili* [2019].

Historically, the first constitutive equations for 1D granular flows thin-layer models were linked to soil mechanics, with the introduction of an internal friction coefficient [*Savage and Hutter*, 1991; *Gray et al.*, 1999]. Some studies suggest that it is needed to model granular flows [*Hungr*, 1995; *Gray et al.*, 1999; *Pirulli et al.*, 2007]. This is, however, difficult to extend to complex 2D topographies, requiring simplifications [*Iverson and Denlinger*, 2001] or on the contrary the resolution of the complete stress state within the flow [*Denlinger and Iverson*, 2004]. Besides, *Gray et al.* [2003] show that a hydraulic approach without internal friction, as in our study, allows to reproduce accurately shock waves generated when granular materials flow around obstacles or over topography slope breaks.

Finally, the $\mu(I)$ -rheology has been increasingly used over past years to model dry granular flows [e.g. *GDR MiDi*, 2004; *Jop et al.*, 2006]. Formal derivations have been done to derive its depth-integrated version but for simple topographies only [e.g. *Gray and Edwards*, 2014; *Baker et al.*, 2016].

More generally, following the classification of thin-layer models done by *Luca et al.* [2009b], our study shows that curvature effects are important when there is limited resistance to shearing in the flow. However, we did not consider situations where resistance to shearing increases and/or stresses acting on topography perpendicular planes become significant (e.g., when an internal friction coefficient is used). In such cases, it is nevertheless difficult to concile both a fine description of the topography and of the rheology. For instance, the velocity profile in the normal variable is in general unknown and evolves with the flow [*Ionescu et al.*, 2015], such that assuming a velocity profile as in *Luca et al.* [2009b] may break down energy conservation. It becomes even more complex when we consider that the flow rheology can change during propagation [*Iverson*, 2003].

6.2 Importance of curvature effects in the Coulomb rheology

With the Coulomb rheology and without internal friction, our simulations show that curvature effects can be significant for fast flows (e.g. several m s^{-1}). For a given topography, the relative importance of the curvature force and of the curvature within friction strongly depends on the friction coefficient μ (see Table 3): when it increases, the friction force (and thus the curvature term within friction) prevails over the curvature term. In the extreme case $\mu = 0$, there is no friction and the curvature force has a strong influence on the flow dynamics. The transition between these two regimes occurs between $\mu = \tan(6^\circ)$ and $\mu = \tan(8^\circ)$ in our simulations on synthetic topographies for a slope $\theta = 10^\circ$. It is not clear how much these transition values depend on the flow path and on the topography itself: neglecting the curvature force in the Prêcheur river simulations entails a significant acceleration of the flow even for $\mu = \tan(2^\circ)$. The latter is however artificially compensated for when the curvature force is also neglected for friction coefficient below $\tan(4^\circ)$.

795 This artificial compensation of two errors with competing effects (accelerating and decel-
796 erating the flow) is fortuitous and not at all a generality.

797 We did not focus on the effect of approximating the curvature in the friction force
798 because it is actually straight-forward to implement the accurate curvature term. However,
799 approximating the curvature does results in significant differences. On synthetic topogra-
800 phies with friction angles above $\mu = \tan(5^\circ)$, the prominent error compared to the correct
801 simulation is obtained when the curvature within friction is approximated (e.g., see Sup-
802 plementary Figures 2, 7 and 8). However, on real topographies, the prominent error occurs
803 when it is neglected (see Supplementary Figure 9 and 10 for the Prêcheur river, and Sup-
804 plementary Figure 11 and 12 for Soufrière de Guadeloupe simulations). This difference
805 may stem from the roughness of the terrain (which has locally high curvatures) that is not
806 rendered in our smooth synthetic channel. Such local effects can then strongly affect the
807 simulations results globally, impacting for instance the travel distance.

808 Reproducing the laboratory experiment from *Iverson et al.* [2004] yields similar con-
809 clusions as what we observed for synthetic and real topographies. In this experiment, a
810 granular flow propagates in and irregular channel. We model it using friction coefficients
811 calibrated by *Lucas* [2010] ($\mu = \tan(23^\circ)$ in the channel and $\mu = \tan(26^\circ)$ elsewhere).
812 The channel is not significantly twisted: using the same notations as previously, we esti-
813 mate a non-dimensionalized bend curvature $\bar{\gamma} = 0.15$. In comparison, we had $\bar{\gamma} = 0.48$
814 for our synthetic topographies. Thus, the flow path is rather straight in the channel and
815 curvature effects are limited in the first 0.3 s of the simulations (see Supplementary Fig-
816 ure 14). However, the important slope break at the channel increases bottom friction when
817 curvature is taken into account, and removing curvature accelerates the flow (see Supple-
818 mentary Figure 15). Omitting the curvature force does change the final geometry of the
819 mass but to a lesser extent. Indeed, the flow velocity remains globally in the direction of
820 the topography slope. All these observations are in agreement with our results for syn-
821 thetic channels. Future comparisons could be carried out by modelling experimental flows
822 in twisted flumes, as in *Scheidl et al.* [2015].

823 Errors induced by inaccurate curvature description can be highly critical for model
824 calibration. For instance, without curvature in friction, higher friction angles are needed to
825 reproduce the previous experiments deposits (at least 3° higher, see Supplementary Figure
826 16). That could explain why, in the first JTC1 benchmarking exercise in 2007, SHALTOP
827 used in many examples a lower friction coefficient than other thin-layer models [*Hungr*
828 *et al.*, 2007].

829 **6.3 Limited influence of curvature effects in the Voellmy rheology**

830 In the Voellmy rheology, an empirical turbulence term proportional to the square
831 of velocity is added to the basal friction. It slows down the flow, but it also minimizes
832 the relative importance of curvature effects (that are also proportional to the square of ve-
833 locity) when there are sudden changes in the flow direction. As a result, there are only
834 slight changes in the flow runout when curvature effects are not accounted for. This is
835 clearly seen in our simulations, both on synthetic and real topographies. In Supplemen-
836 tary Note 5, we give an example of a back-analysis of a debris flow simulated with the
837 Voellmy rheology. Neglecting the curvature force influences only slightly the results.

838 However, *Salm* [1993] actually suggests that the turbulence coefficient ξ is propor-
839 tional to the mean distance between irregularities on the topography. This distance can be
840 seen as an estimate of the local topography radius of curvature. In the same perspective,
841 in their guide book to avalanche modeling, *Salm et al.* [1990] advise choosing higher tur-
842 bulence coefficients on rough topographies than on smooth topographies. More recently,
843 *Gruber and Bartelt* [2007] calibrated a spatially varying turbulence coefficient by back-
844 analysis of snow avalanches. As *Fischer et al.* [2012] point out, the resulting map of tur-
845 bulence coefficients is strongly correlated to the topography curvature. This suggests that

846 the Voellmy rheology, or at least the Voellmy rheology with a turbulence coefficient cor-
 847 related to the local topography curvature, might be a way of taking into account curvature
 848 effects empirically. However the resulting model cannot correctly reproduce the complex-
 849 ity of the interaction between the flow and the topography, in particular because the curva-
 850 ture force depends on the velocity direction and not only on its norm.

851 **6.4 Importance of local curvature effects for overflows and runup estimations**

852 For smooth topographies and channelized flows, we can expect some hazard indi-
 853 cators, such as the travel distance and the impacted area, to vary evenly when the simu-
 854 lation parameters span their variation ranges. This is no longer the case when the flow
 855 manages to overflow topographic barriers. Such non-linear behaviors and threshold effects
 856 are highly critical for hazard assessment and complicate hazard mapping [Mergili *et al.*,
 857 2018]. They cannot be described by simple laws relating, for instance, the travel distance
 858 to the initial unstable volume [e.g. Lucas *et al.*, 2014; Mitchell *et al.*, 2019].

859 In our study, when we disregarded the curvature force in the simulation of the debris
 860 avalanche on Soufrière de Guadeloupe volcano, part of the material managed to overrun
 861 a plateau and enter a ravine. It could then spread much further (about 1.5 km, see Fig-
 862 ure 12b). Such a behavior is however not systematic. For channelized flows, the curvature
 863 force tends to maintain the flow at the bottom of the channel, whereas it would otherwise
 864 bounce back and forth on its walls and potentially overflow the channel. However, in the
 865 long run, the curvature force allows the flow to move faster because it does not dissipate
 866 energy bouncing back and forth on the channel banks. If a sudden twist is encountered fur-
 867 ther down, the flow may in turn have enough energy to overflow the channel banks, which
 868 would not be the case without the curvature force. We could reproduce such a situation
 869 in the Prêcheur river, with the Coulomb rheology and $\mu = \tan(2^\circ)$ (see Supplementary
 870 Figure 13).

871 Local curvature effects are thus worth taking into account when considering debris
 872 flow runup against steep slopes [Iverson *et al.*, 2016] and on the outer bank of a channel
 873 bend [Scheidl *et al.*, 2015]. In the latter case, the runup (that is, the elevation difference
 874 between the inner and outer boundaries of the flow in the channel, as measured in the
 875 field) can be related to the flow velocity [Prochaska *et al.*, 2008; Scheidl *et al.*, 2015]. The
 876 runup and/or the deduced velocity can then be used to fit rheological parameters in thin-
 877 layer simulations. As they describe the dynamics of the flow in locations where we can
 878 expect strong curvature effects, the resulting best-fit parameters may depend significantly
 879 on whether or not these curvature effects are properly described in the model, even with
 880 the Voellmy rheology.

881 **7 Conclusion**

882 In this work, we show how an incorrect derivation of the thin-layer equations can
 883 lead to the omission of two curvature terms, originating from the expression of the pres-
 884 sure at the bottom of the flow. The first one, the curvature force, does not depend on the
 885 rheology and ensures that the flow velocity remains tangent to the topography. The second
 886 one appears in the bottom friction force (and thus only when frictional rheologies such
 887 as Coulomb or Voellmy are used). They are both proportional to the square of the flow
 888 velocity, but also depend on the velocity orientation and topography curvature tensor.

889 We have carried out simulations on synthetic and real topographies to highlight
 890 the influence of these curvature terms in thin-layer numerical simulations, with the code
 891 SHALTOP (see Table 3). The curvature terms are all the more important when the flow
 892 is fast (typically, several m s^{-1} to tens of m s^{-1}), that is for low friction coefficients and/or
 893 steep slopes.

894 For flows propagating in twisted channels modeled with the Coulomb rheology, the
895 curvature force tends to maintain the flow at the bottom of the channel. Thus, neglecting
896 it favors bouncing on the channel walls and reduces the propagation velocity. For instance,
897 in the case of the upper section of Prêcheur river where slopes are higher than $\theta = 7^\circ$,
898 omitting the curvature force in simulations reduces the average velocity of channelized
899 flow by 30%, for friction coefficients below $\mu = \tan(6^\circ)$. Simulated overflows then differ,
900 which is critical for hazard assessment.

901 Approximating the curvature in the friction force can break the rotational invariance
902 of the model and slow down the flow. Neglecting the curvature in friction decreases the
903 norm of the friction force and thus accelerates the flow, when the latter propagates from
904 steep to more gentle slopes. It results in the most important errors when the flow veloc-
905 ity is in the main slope direction, and more generally on real topographies. For instance,
906 in the case of the simulation of a debris avalanche on the Soufrière de Guadeloupe, travel
907 distances are increased by several hundred meters. We observe similar effects on a syn-
908 thetic channel with slope $\theta = 25^\circ$ and $\mu = \tan(15^\circ)$, with a 50% increase of the kinetic en-
909 ergy. Though such effects can sometimes be artificially compensated for by also neglect-
910 ing the curvature force, it is not at all systematic and thus both terms need to be properly
911 taken into account for correct model calibration.

912 Though we have focused on debris flows and debris avalanches modeling, our results
913 could apply to other geophysical flows, such as mountain river stream flows [*Borthwick and*
914 *Barber, 1992; Churuksaeva and Starchenko, 2015*] and concentrated and dilute pyroclastic
915 currents [*Komorowski et al., 2013; Kelfoun et al., 2017*]. Curvature effects may also be
916 important for modeling landslide-generated tsunamis, for which the thin-layer equations
917 must be integrated in the direction normal to the topography for the landslide, and in the
918 vertical direction for the fluid layer [*Ma et al., 2013; Delgado-Sánchez et al., 2019*].

919 Note that strong curvature effects may also be an inherent limitation of thin-layer
920 models. Indeed, curvature forces are particularly strong when the topography curvature is
921 high, as for instance in a narrow channel. However, in this case, the thin-layer assumption
922 may no longer be valid. In order to discriminate between real curvature effects and numer-
923 ical artifacts, comparisons with full 3D models where no approximations are done on the
924 layer thickness could be conducted. Yet such comparisons exercises may prove difficult
925 [e.g. *Pirulli et al., 2018*].

926 Of course, our results must also be considered in regard of the rheology uncertainty,
927 which is sometimes large. We believe future research should focus on both the develop-
928 ment of accurate physically-based rheologies with constrained realistic parameters, and on
929 methodologies to properly describe topography effects. Such studies are complementary
930 to, in turn, develop a model uniting both aspects.

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