A Contact Model Coupling Friction and Adhesion: Application to Pile/Soil Interface

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International Review of Civil Engineering (I.RE.C.E.), vol.9, N. 1, 2018

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Abstract – *In this paper, the behavior of the interface pile-soil is simulated using a unilateral contact model with the coupling of friction and adhesion. This model gives a continuous transition from complete adhesion to the classical Coulomb friction law with unilateral conditions. The model is implemented in the finite element code GYPTIS90 developed by Raous et al. An application is presented, for modeling soil/pile interface behavior, on the simulation of pull-out experiments of a pile. The identification of the model parameters is discussed and comparisons are presented between modeled pile behavior and that predicted from experimental results. The proposed model have achieved better results.*

Keywords*: Frictional contact; Adhesion; Soil-structure Interface; Piles; Finite elements*

Nomenclature

A, B	:	Elastic solids in contact
Ω_1, Ω_2	:	Domains of solids in contact
Γ_c	:	Contact surface
P	:	Candidate point on contact
P'	:	Target point
$\vec{n}$	:	Normal unit vector
$\vec{t}_1, \vec{t}_2$	:	Tangential unit vectors
x_n	:	Magnitude of the gap between the contact node and the target surface
R_n	:	Normal contact reaction (contact pressure)
R_t	:	Tangential contact reaction (friction force)
$\dot{u}$	:	Relative velocity
$\dot{u}_n$	:	Normal separation velocity
$\dot{\boldsymbol{u}}_{\boldsymbol{t}}$	:	Sliding velocity
u	:	Relative displacement
u_n	:	Normal relative displacement
u_t	:	Tangential relative displacement
μ	:	Friction coefficient
K_μ	:	Coulomb's cone
β	:	Adhesion intensity
Ψ	:	Free energy
Φ	:	Potential of dissipation
C_n	:	Initial normal stiffness of interface
C_t	:	Initial tangential stiffness of interface
w	:	Decohesion energy
α	:	Viscosity parameter
I_s	:	Indicator function of the specified sets S
u_c	:	Critical displacement
σ_c	:	Critical stress
E	:	Young's modulus
υ	:	Poisson's ratio
γ	:	Unit weight
ϕ_p	:	Pile diameter
L_p	:	Pile length
K_0	:	At-rest earth coefficient
σ_r^0	:	Initial radial stress
σ_θ^0	:	Initial orthoradial stress
φ_s	:	Angle of internal friction of soil

I. Introduction

In geotechnical engineering, a large number of structures lie in contact with soils such as retaining walls, reinforcements, tunnels and foundations. The behavior of these structures is a problem of soil-structure interaction where loads are transferred between the structures and the soils, principally through a thin layer of soil in contact with the structure called "interface". It is the seat of important localization of deformation and concentrations of significant constraints where failure is observed within this zone which serves to transmit stresses and strains. Generally, these loading conditions are simplified by considering prescribed loads, or prescribed displacements [1]-[4]. However, in a soil-structure interaction, the relative movement between the soil and the structure can occur. During loading, at the contacts surfaces (the soil-structure interface) we can have frictional sliding, as well as the separation and the closing of the surface, and the phenomena, cannot be modeled by simple prescribed boundary conditions. So, for the stability analysis and correct modelling of a geotechnical structure in contact with soil, it is important to take into account the specific behaviour of this interface zone or in other words, the characterization of the contact between the soil and the structural element.

Since the 1960s, considerable progress has been developed for studying the behavior of soil-structure interfaces. In this context, a large number of experimental, analytical and numerical models classified as either a stiffness or a constraint approach have been presented in the literature. These include the use of zero-thickness elements, thin layer elements, and hybrid methods [1]-[15].

Zero-thickness elements have been first used in geotechnical finite element simulations by Goodman for rock joints [5]. They are composed of 4 nodes with 8 degrees of freedom. In this method, it is considered as active only the connection between the two antagonist nodes. The interface has no thickness and it must only reproduce the properties of the soil-structure interface where a normal and tangential stiffness is used for modeling pressure transfer and friction. But it can be noted that these elements are not well appropriate for simulating pile insertion that involves large displacements, separation, and closure between the pile and the soil (unilateral contact) [1],[6]. To overcome the difficulties provided in the joint element approach, Desai and Zaman have proposed two-dimensional finite elements called "thin layer" elements. This model is based on elasto-plasticity theory [7][8]. The concept of the thin layer considered that the response of the interface should be dealt with by an appropriate constitutive model. The interface has a certain thickness and represents a filling material (joints in the rock massifs, zones of localization in the soil-structure interactions). The formulation of the thin layer element is based on the assumption that the interface can be replaced by an equivalent solid element with a small thickness and a special constitutive law derived from the theory of elastoplasticity. In the two-dimensional case, the most conventional isoparametric thin layer elements are at 8 nodes, and they have the relative displacements between the soil and the structure as degrees of freedom. The element has four Gauss integration points. The use of the thin layer element is discussed at length in the literature [3],[4],[6],[9],[10].

During the last decade, these two methods saw great progress and applications in various fields such as hydraulics, masonry structures, and especially in geotechnics in the modeling of pile-soil interactions [10]-[15].

The first uses of contact and friction in the modeling of soil-structure interfaces date back to the 1980s. In 1983, Katona presented a method based on node-to-node contact algorithm coupled with the Lagrangian multiplier method to simulate the interaction of a buried structure and the surrounding soil [16]. However, the method is used only in the case of linear problems involving small sliding between the soil-structure contact surfaces. Using a contact-friction algorithm based on the penalty method with a slide-line formulation, Mabsout *et al.* presented an interface model for studying the pile-soil interaction [17]. The pile is pre-bored in undrained clays, and the study takes into account only small sliding. To take into account the interface problems with the presence of significant slip, P. Villard proposed a hybrid approach by considering the contact area forming the interface as a predefined discontinuous zone characterized by relative sliding with friction [18]. In this method, the interaction between the antagonist nodes forming the interface is controlled by an imposed compatibility condition for each calculation iteration, which ensures the non-penetration condition. For friction, the penalty method is used by imposing a tangential stiffness [18].

Over the last decade, diverse works have been presented on the modeling of contact and friction problems in geotechnical engineering [19]-[24]. Recently, several researchers have adapted the Coulomb friction contact model to simulate the pile-soil interface behavior. We can cite the works of De Gennaro *et al.*,[25]; Sheng *et al.*, [26]-[28]; Fischer *et al.*, [29]; Said *et al.* [30]; Ninic *et al*. [31], Taleb and Berga [32].

In this work our emphasis is given on the behavior of the piles under axial loadings which is a typical example of problems involving an interface. The current study pertains to defining special cohesive model coupling contact, friction and adhesion as well as to formulating an appropriate interface constitutive law. The behaviour of the interface pile-soil is simulated using an interface model called RCCM, introduced by Raous, Cangemi, Cocou, and Monerie) [33],[34].

The RCCM model, presented in Section III, is based on a surface damage variable defined by Frémond, and called adhesion intensity [35], [36]. This variable is a scalar that takes these values between 1 and 0, which represents the state of adhesion (Zero: no adhesion and One : perfect adhesion).

The presented model describes a progressive los of adhesion, initially coupled with a friction mechanism. This coupling is achieved by introducing a compliance law that depends on this surface damage. That means that the transition from adhesion to Coulomb friction law with unilateral conditions is progressive. The local behavioral laws for this model are deduced from thermodynamic considerations, based on a material surface assumption for the contact zone. The model is implemented in the finite element code GYPTIS90 developed at LMA, Marseille [33], [37].

In this paper, the behavior of the pile-soil interface is simulated using the RCCM model. The pile is subjected to an axial pull-out load. Identification of the model parameters is discussed and comparisons are presented between modeled pile behavior and that predicted from experimental results.

II. Unilateral contact and Coulomb friction laws

In the following, before defining the RCCM model, it is useful to recall the laws of contact and friction most often used. After a kinematic description at the vicinity of a material particle candidate to the contact, the geometric and kinematic quantities retained to express

the contact problem are defined. For the sake of simplicity and under the hypothesis of small perturbations, we consider the contact of two elastic solids **A**, and **B** of domains respectively $\mathbf{\Omega_1}$ and $\mathbf{\Omega_2}$ whose common contact surface is $\mathbf{\Gamma_c}$ for some value of time (Fig. 1). By convention, we denote '**A**' contactor body and '**B**' as an obstacle. Let P be a point of **A** called the candidate on contact and P' is the target point defined by the normal projection of P on **B**, called an antagonist. P' will be the origin of the local coordinate system $(\vec{n},\vec{t}_1,\vec{t}_2)$, where $\vec{n}$ denotes the normal unit vector at point P' to the bodies, directed towards **A**, and T$(\vec{t}_1,\vec{t}_2)$ denotes the orthogonal plane to $\vec{n}$ in $\mathfrak{R}^3$.

The normal coordinate $\mathbf{x_n}$ denotes the magnitude of the gap between the contact node and the target surface (the shortest distance between the point P and the obstacle: x_n = PP'). When **A** comes into contact with **B**, the latter exerts a reaction of contact $\boldsymbol{R}$ at the point P' onto **A**. Hence, the solid **B** will be subjected to the reaction $-\boldsymbol{R}$ acting from **A** [33][38][39].

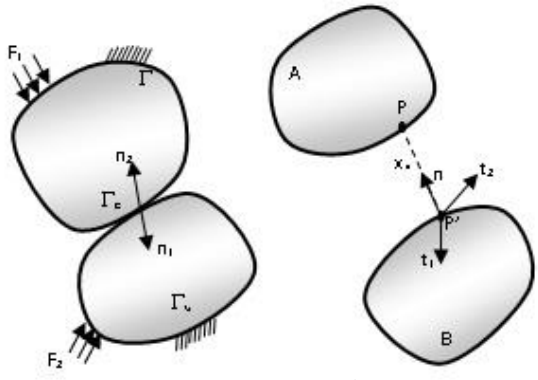

Fig. 1 : Kinematics of contact

The relative velocity $\dot{u}$ of the particle P with respect to its projection P' is defined by:

$$\dot{\boldsymbol{u}} = \dot{\boldsymbol{u}}_1 - \dot{\boldsymbol{u}}_2 \tag{1}$$

Where $\dot{u}_1$ and $\dot{u}_2$ being respectively, the instantaneous velocity of the particles of A and B. By projection on the local base $(\vec{n},\vec{t}_1,\vec{t}_2)$, the relative velocity and the contact reaction $\boldsymbol{R}$ can be decomposed in their normal and tangential components as follows [33][39]:

$$\dot{\boldsymbol{u}} = \dot{\boldsymbol{u}}_{\boldsymbol{t}} + \dot{u}_n \boldsymbol{n} \tag{2}$$

$$\boldsymbol{R} = \boldsymbol{R}_{\boldsymbol{t}} + R_n \boldsymbol{n} \tag{3}$$

where $\dot{u}_n$ is the normal separation velocity, $\dot{\boldsymbol{u}}_{\boldsymbol{t}}$, the sliding velocity, R_n the normal force (contact pressure), and $\boldsymbol{R_t}$ the friction force. The points belonging to the contact zone must satisfy the laws governing unilateral contact with friction. Such laws are generally expressed according to the variables x_n (the shortest distance between **A** and **B** x_n = PP'), the relative velocity $\dot{u}$, the contact force $\boldsymbol{R}$, and the characteristics of the surfaces in contact such as the coefficient of friction μ. Therefore, each point in contact must satisfy the conditions of unilateral contact, generally known as the Signorini condition, and which can be expressed as follows:

$$x_n \geq 0 \; ; \; R_n \geq 0 \; ; \; and \; R_n x_n = 0 \tag{4}$$

This expression summarizes the three contact conditions [33], [38], [39]:

- Kinematic (impenetrability $x_n \geq 0$),
- Static (non adhesion $R_n \geq 0$),
- Mechanical complementarity condition (non-contact: $x_n.R_n = 0$)

Taking into account the initial gap h_0 between the solids A and B, with:

$$x_n = h_0 + u_n \tag{5}$$

The condition of Signorini is written then:

$$u_n \leq 0 \; ; \; R_n \leq 0 \; ; \; and \; R_n u_n = 0 \tag{6}$$

Furthermore, the kinematic character of the contact problems requires a writing of the different laws in terms of velocity. Then, when the two bodies A and B are in contact, the unilateral contact law (Signorini's conditions) (Eq. 6) are written for $u_n = 0$ as:

$$\dot{u}_n \geq 0 \; ; \; R_n \geq 0 \; ; \; and \; R_n \dot{u}_n = 0 \tag{7}$$

The kinematic and static formulation of the contact must be completed by a tangential friction law to make the problem of contact well posed. The Coulomb model has been adopted where the limit shear is proportional to the normal pressure. In the case of an isotropic contact-friction, the Coulomb model friction law is written:

$$\begin{cases} \left\| \boldsymbol{R_t} \right\| \leq \mu . R_n & if \; \dot{u}_t = 0 \\ \left\| \boldsymbol{R_t} \right\| = -\mu . R_n \dfrac{\dot{u}_t}{\left\| \dot{u}_t \right\|} & if \; \dot{u}_t \neq 0 \end{cases} \tag{8}$$

Where $\boldsymbol{\mu}$ is the coefficient of friction, $\dot{u}_t$, $\boldsymbol{R_t}$ and R_n are respectively the tangential relative velocity, the normal and tangential reactions expressed in the local coordinate system. Usually, we defined the Coulomb isotropic friction cone K_μ for each point of contact by the expression:

$$K_\mu = \left\{ \boldsymbol{R} \in \mathfrak{R} \; such \; that \; \boldsymbol{f(R)} = \left\| \boldsymbol{R_t} \right\| - \mu . R_n \leq 0 \right\} \tag{9}$$

When the contact occurs, each pair of contact points belonging to the interface can be in one of the three states [38][39]:

$$\begin{cases} \textbf{a) contact with adhesion (sticking) :} \\ \quad R \in K_\mu ,\ \|\boldsymbol{R_t}\| \le \mu . R_n \ and\ \dot{u} = 0 \\ \textbf{b) contact with slip (sliding) :} \\ \quad R \in K_\mu , and \|\boldsymbol{R_t}\| = -\mu . R_n . \dfrac{\dot{u}_t}{\|\dot{u}_t\|}, \dot{u} \neq 0 \\ \textbf{c) non-contact } (\textbf{separation}) : \\ \quad R_n = 0 \ , and\ \dot{u}_n \ge 0. \end{cases} \tag{10}$$

Indeed, most algorithms dealing with the problems of contact and friction adopt the laws summarized by the expressions (10).

III. The RCCM model

Interface models are adopted in many civil and mechanical engineering problems to simulate different problems involving contact and decohesion phenomena. In order to simulate the behavior of complex interfaces, a model coupling adhesion, and unilateral contact has been proposed by Raous *et al.* in [33](the RCC model), and extended in [34](the RCCM model). The main idea of this cohesive model is to consider the damage of the interface and to describe a gradual loss of adhesion initially coupled with a friction mechanism. This loss of adhesion is described by a variable of damage introduced by Frémond [35],[36], called intensity of adhesion (noted ***β***), which is a surface damage variable that takes its values between 0 and 1 (0 is no adhesion and 1 is total adhesion). The introduction of an interface stiffness (noted ***C***) depending on ***β*** makes it possible to ensure a continuous passage from an adhesive state (initial adhesion), to a state of friction of Coulomb (final frictional sliding), with possible separation of the interface [34]. The RCC model (Raous- Cangémi-Cocou) has been first given in [33], and then extensively presented in [34]. It has been extended to the present form (RCCM model: Raous-Cocou-Cangémi-Monerie) by including progressive friction with the term $(1-\beta)$ when adhesion is lost (***β = 0)*** [34].

The RCCM model has been used with great success to study several problems in mechanics and civil engineering. For examples, we can cite composite materials (matrix-fibre interfaces)[33], delamination of coated bodies [34], delamination of glued assembling [40], the behavior of interfaces and cohesive masonry structures [41],[42], steel-concrete interfaces (pull out of reinforced concrete) [43]. In the field of geophysics, the RCCM model is used by FESTA and Henninger to simulate interface behavior along faults between tectonic plates in the nucleation of earthquakes [44].
For more details about the RCCM model, readers are referred to [45][46].

III.1 The model

The behavior of the interface is described, according to M. Frémond [35],[36], within the framework of the theory of generalized standard materials [47].

The contact zone is considered as a material surface which is supposed to have a specific thermodynamic behaviour [33],[35]. This hypothesis makes it possible to associate variables and thermodynamic forces with the contact surface. The relative displacements (u_n, u_t), and the adhesion intensity $\boldsymbol{\beta}$ are chosen as state variables. The behavior of the interface is characterized by a free surface energy Ψ depending on the retained state variables, and by a pseudo-potential dissipation Φ depending on the velocities of the state variables. The only envisaged dissipative mechanisms at the interface are the friction and the damage of the adhesive bonds. The local constitutive equations required for this model are deduced by choosing two specific forms for the free energy and the dissipation potential described by the expressions [33],[34]:
The free energy is the following:

$$\Psi(u_n, \boldsymbol{u_t}, \beta) = \frac{C_n}{2} u_n^2 \beta^2 + \frac{C_t}{2} \boldsymbol{u_t^2} \beta^2 - wh(\beta) + I_{\Re^+}(u_n) + I_{[0,1]}(\beta) \tag{11}$$

The potential of dissipation is chosen as follows:

$$\Phi(\dot{\boldsymbol{u}}_{\boldsymbol{t}}, \dot{\beta}, u_n, R_n, \beta) = \mu f(\beta) \left| R_n - C_n u_n \beta^2 \right| \|\dot{\boldsymbol{u}}_{\boldsymbol{t}}\| + \frac{\alpha}{2} (\dot{\beta})^2 + I_{c^-}(\dot{\beta}) \tag{12}$$

Where

- $\boldsymbol{C_n}$: and $\boldsymbol{C_t}$ are the initial stiffness of the interfacial link, homogeneous to a modulus of elasticity per unit length;
- $\boldsymbol{w}$: is the decohesion energy (an energy threshold from which the interface damage begins; the adhesive bonds break, and the friction is activated);
- $\boldsymbol{\mu}$: is the friction coefficient;
- $\boldsymbol{\alpha}$: is the viscosity associated to the evolution of the adhesion.

The introduction of the $\boldsymbol{I}_{\Re^+}$ and $\boldsymbol{I}_{[0,1]}$ indicator functions makes it possible to take into account the unilateral contact conditions and to constrain β to belong to the interval [0,1]. In Eq. (12), the indicator function I_{C^-} imposes the condition $\frac{\partial \beta}{\partial t} < 0$ which means that, the evolution of the interface can only go towards a separation, without any possibility of re-bonding, and the adhesion cannot be regenerated.

Introduced for the first time by Monerie [34], the term ***f(β)*** in the friction law with adhesion is a continuous decreasing function of ***β***. This function introduces a variable coefficient of friction depending on the damage of the adhesive bonds. It ensures that the coefficient of friction is equal to μ when the adhesion is broken, and null, when the interfacial bond is virgin, and adhesion is

complete (β = 1). That means that, the friction is progressively introduced when adhesion decreases which allows the gradual and continuous transition between a purely adhesive state and a Coulomb friction type behavior. Generally we use [34] :

$$f(\beta) = 1 - \beta \tag{13}$$

Taking advantage of the convex analysis tools, a partial sub-differentiation has been used to Eqs 10 and 11. That permits to write the state laws and the complementarity conditions in a form of differential inclusions. Finally, the interface behaviour is described by the following relations [33],[34]:

- Unilateral contact (Signorini conditions) with adhesion:

$$R_n - C_n u_n \beta^2 \geq 0;\ u_n \leq 0;\ (R_n - C_n u_n \beta^2).u_n = 0 \tag{14}$$

- Coulomb friction with adhesion:

$$\begin{cases} if\ \left\| \boldsymbol{R_t} - C_t \boldsymbol{u_t} \beta^2 \right\| < \mu(1-\beta)\left| R_n - C_n u_n \beta^2 \right| \Rightarrow \dot{\boldsymbol{u}}_{\boldsymbol{t}} = 0 \\ if\ \left\| \boldsymbol{R_t} - C_t \boldsymbol{u_t} \beta^2 \right\| = \mu(1-\beta)\left| R_n - C_n u_n \beta^2 \right| \\ \qquad \Rightarrow \exists \lambda \geq 0,\ \dot{\boldsymbol{u}}_{\boldsymbol{t}} = -\lambda \dfrac{\left(\boldsymbol{R_t} - C_t \boldsymbol{u_t} \beta^2 \right)}{\left\| \boldsymbol{R_t} - C_t \boldsymbol{u_t} \beta^2 \right\|} \end{cases} \tag{15}$$

- Evolution of the adhesion intensity:

$$\begin{cases} \alpha \dot{\beta} = -(w - (C_n u_n^2 + C_t \boldsymbol{u_t}^2)\beta)^- & if\ \beta \in [0,1[\\ \alpha \dot{\beta} \leq -(w - (C_n u_n^2 + C_t \boldsymbol{u_t}^2)\beta)^- & if\ \beta = 1 \end{cases} \tag{16}$$

Where $(x)^- = \max(0;-x); \forall x \in \Re$, denotes the negative part of x.

In this interface model, Eq.(14) gives the unilateral contact with adhesion, Eq. (15) gives the Coulomb friction with adhesion and Eq. (16) gives the evolution of the adhesion intensity $\boldsymbol{\beta}$. It is interesting to note that, when $\boldsymbol{\beta}$ vanishes, the adhesion is completely broken, and one finds the classic law Eqs.(10) of Signorini with friction of Coulomb.

IV. Simulation of the pile-soil interface

The simulation of pull-out experiments of a pile reported by Franck R. is presented [48]-[52]. The test data are first described, then experimental and numerical results are compared and discussed.

IV.1 The experiment

It is a series of static pulling out tests of two full-scale model piles, carried out at two sites: CRAN and PLANCOET [48]-[50]. They were part of a more comprehensive program run by the Saint-Brieuc LPC for the Institut Français du Pétrole (IFP). The CRAN site consists of about 18 m of alluvial fine deposits of marine origin. The PLANCOET site consists of about 13 m of silts, loose sands, and silty clays, also of marine origin. The piles are driven closed-end tubes 27.3cm in diameter, 6.3mm thick and of length: 17m in CRAN soils, 13m in PLANCOET soils [48],[49].
They were provided with strain gauge measurement levels, every 90 cm which allowed determination of experimental unit shaft friction curves during the pulling out tests. All piles tested are subjected to axial loading.

IV.2 Numerical modelling

Numerical simulations were performed using the soft-ware GYPTIS90 developed by Raous *et al*. (LMA Marseille) [37]. In the figures 2 and 3, the geometry of the problem, and the finite element mesh used for the simulations of the pile test are shown.

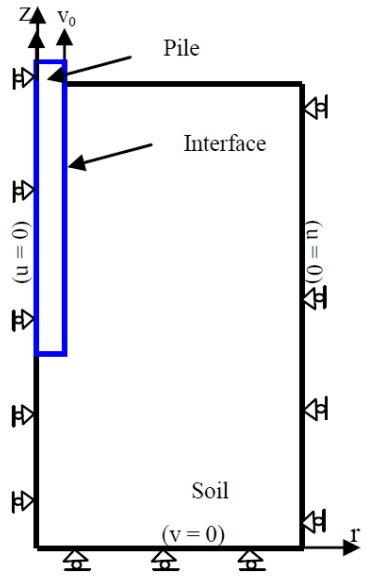

Fig. 2. Geometry and boundary conditions

An axisymmetric model has been admitted since the piles treated are cylindrical with a circular cross-section and axially loaded (the axis of symmetry, coinciding with the pile axis)[50],[54],[55]. The limits of the investigated domain are fixed more than 3.5 times the pile length (Lp), in the vertical direction and at least 2 times the pile length in the horizontal direction (Fig. 2). The discretization of both soil and pile is carried out using three-node isoparametric plane strain elements [50],[54].

At the neighborhood of the pile-soil interface, the mesh has been refined to avoid the influence of stress concentration on the pile response (Fig. 3)[54],[55].

On the vertical mesh boundaries, the horizontal displacements are blocked (u = 0), whereas, on the bottom boundary, the vertical displacements are set to

zero (v = 0). In the left side the symmetry condition is considered.The interface consists of 33 duplicated contact nodes.

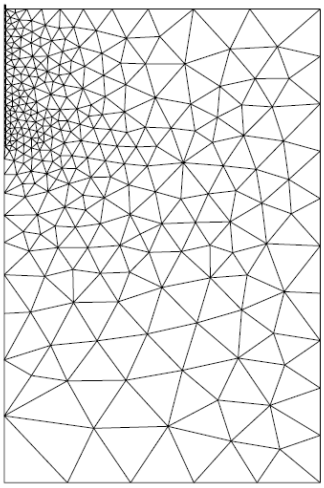

Fig. 3: Finite element mesh

The mechanical characteristics used in the simulation are given in Table 1. More details can be seen in [48].

TABLE 1.
PARAMETERS USED FOR THE TEST SIMULATIONS

		Parameters		
		Young's Modulus E (kN/m²)	Poisson's ratio υ	Unit weight γ (kN/m³)
Soil	Cran	3.E+04	0.33	20
	Plancöet	3.E+04	0.33	20
Pile	Cran ϕ_P = 27cm L_P = 17m	3.E+06	0.3	20
	Plancöet ϕ_P = 27cm L_P = 13m	3.E+07	0.3	20

The elastic behavior is assumed. This choice is based on the remarks reported by Frank and Barbas, following comparative analyses in elastic and elastoplastic soils [48],[51].

During the pile driving process, the soil adjacent to the pile tends to densify by displacement, resulting in a rearrangement of the soil structure around the pile. This process generates stresses and deformations in the surrounding soil in contact. The volume of moved soil is equal to the volume of driven pile [54],[56],[57]. This initial soil stress is important and determines the final solution. To reproduce the initial conditions in terms of effective stresses in the soil mass before loading, the problem is analyzed in three phases [54]. First, the unit weight of the pile and soil are applied. For the in-situ soil condition prior installation, the soil is considered to be normally consolidated and the at-rest earth coefficient K_0 is taken to be 0.43 [48]-[50]. This defines the geostatic initialization of the state of stress ($\sigma^\circ_r = \sigma^\circ_\theta = K_0.\gamma.z$) where σ°_r ,σ°_θ are respectively the radial and the orthoradial stress, and γ, the unit weight of the soil [48],[49].

In the second phase, the initial normal stresses due to the pile installation are taken into account by applying a prescribed displacement on the pile shaft [55][56]. This imposed displacement to be chosen on the one hand to recover the phenomenon of soil remolding around the pile after driving, and on the other hand, related to the friction coefficient of such a way of approaching the experimental limit of lateral friction. It should be noted here that, generally in the FEM, it is very difficult to simulate correctly the effects of the pile installation. The initials conditions induced displacements field which is then canceled and the obtained initial stress state is introduced with the first loading step at the head of the piles.

Finally, Computations are performed incrementally by progressively applying the tensile loads. The pull-out loading test was simulated by applying a total displacement v_0 at the pile top and achieved in 50 increments (v_0 = 15mm on the Plancöet pile and 6mm for the Cran pile) [48]-[50].

The obtained numerical results (load-displacement curves along the pile shaft) are then compared with available measurement results. The experimental data are plotted by the squares on all the following figures. To simulate the interface behavior, two model are tested. Firstly, the interface behavior is simulated by means the classical unilateral contact law with Coulomb friction(Eqs. 10) and zero cohesion (Fig. 4). The analysis of the load-displacement curves does not show good agreement between numerical and experimental curves.

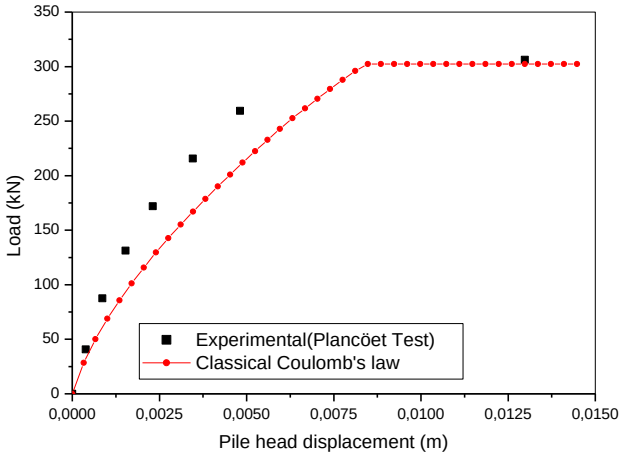

Fig 4: Results with classical Coulomb's law

In the second study, we have used the RCCM model, which needs three parameters: the decohesion energy (*w*), the friction coefficient (μ) and the initial normal (resp. tangential) stiffness of the interface $\mathbf{C_n}$ (resp. $\mathbf{C_t}$).

The model parameters are identified and adjusted by comparing the corresponding experimental curve and results obtained by finite element computations. The interface is analyzed by considering the shear behavior.

We suppose initially that we have a complete adhesion ($\beta = 1$) and zero displacement ($u_n = u_t = 0$).

IV. 3 Identification of the interface model parameters

Let us give now some detail on the identification phase of different parameters of the RCCM model. The method of identification is based on estimates and calculations carried out a priori on these parameters, conducted on the corresponding experimental curve. The range of values is thus defined for the coefficients ***C***, ***w*** and $\boldsymbol{\mu}$, where the precise identification of this parameters will be conducted. These values are adjusted by a series of calculations carried out with close values, and by minimizing the gap between the numerical simulation with the model and the experimental curves [33],[34].

Firstly, the initial stiffness $\boldsymbol{C_n}$ and $\boldsymbol{C_t}$ will be identified. It is determined from the initial rising branch (the initial linear behavior) of the experimental curve. During loading, the normal elasticity will not be involved and the normal component u_n is not activated. That means that the value of the normal stiffness $\boldsymbol{C_n}$ has no significant effects. Thus, the kinematic behavior of the interface will be governed only by the relative tangential displacement $\boldsymbol{u_t}$. Therefore only the tangential stiffness $\boldsymbol{C_t}$ is identified and the same contact stiffness is considered in the normal direction ($\boldsymbol{C_n}$).

One of the main mechanical characteristics of the RCCM model is that the damage of the interface occurs only when the elastic energy exceeds a critical value ***w*** (decohesion energy) [33],[34]. As long as that threshold is not reached, adhesion stays to be complete and the behavior of the interface is elastic with the initial stiffness $\boldsymbol{C_n}$ and $\boldsymbol{C_t}$. Once the elastic part of the experimental curves is adjusted, the decohesion energy ***w*** is obtained by considering the stress limit of elastic behavior $\sigma_0 = \sqrt{w.C}$ where ***C*** is the initial stiffness previously calculated [33],[34].

In the proposed model, the adhesion and the friction are coupled. Thus allows introducing the friction progressively when adhesion decreases by means the function $f(\beta)$, and ensuring a continuous transition between initial adhesion and final frictional sliding.

In the literature, according to [48]-[50], the value of the friction coefficient can be taken as: $\mu = \tan(2/3\varphi_s)$ or $\mu = \tan(3/4\varphi_s)$, where $\boldsymbol{\varphi_s}$ is the angle of internal friction of soil. The precise identification of the coefficient will be conducted in this range of values. In this work, we have $\varphi_s = 35°$ [48]-[50].

Pull-out test simulation results of the PLANCOET test and CRAN test, have been presented on Figs 5 and 6. The parameters used in these simulations are displayed in Table 2. In this study we have neglected the viscosity effect.

Hence, α will be taken equal to zero .

TABLE 2.

PARAMETERS OF THE RCCM MODEL

		Parameters		
		Initial Stifness C C (kN/m^3)	Decohesion Energy w (J/m^2)	Friction coefficient
Test	Cran	2500	3.E-03	0.46
	Plancöet	10500	3.E-03	0.46

As it can be observed in Figs 5 and 6, a satisfactory level of agreement between simulation and test results may be noted, where the relative error does not exceed 5%.

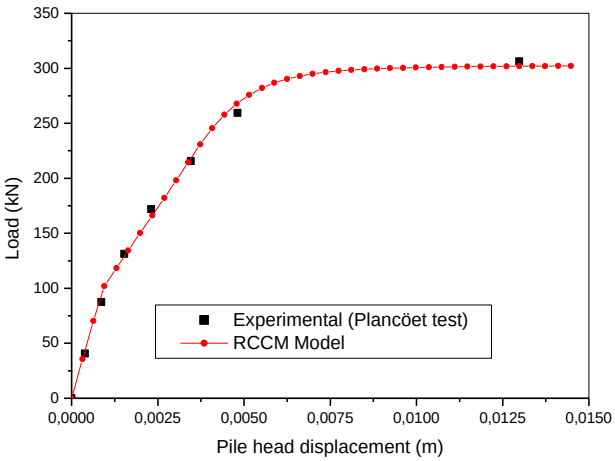

Fig. 5: Simulation of the Plancöet test

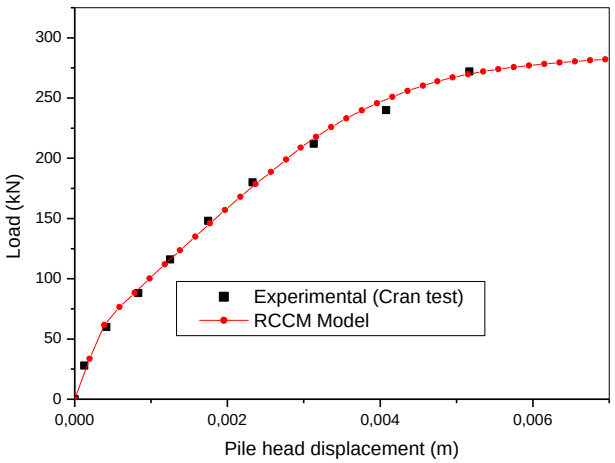

Fig. 6: Simulation of the Cran test

The specificity of the RCCM model is the coupling of the tangential adhesive behavior to the friction. To describe more precisely this coupling, we consider the shear behavior of the interface pile-soil in the case of Plancöet test (see Fig. 5). Initially, the pile-soil system is considered in a state of total adhesion ($\beta = 1$) with zero displacement ($u_n = 0$, $u_t = 0$).

Under the confinement effect, resulting from the pile installation, a normal contact reaction R_n is generated at the interface [57]-[59] and the sliding limit is ($\mu\|R_n\|$) because $u_n = 0$. As long as the norm of the tangential

force ($\|\boldsymbol{R}_t\|$) is smaller than this limit, the contact nodes forming the interface have a status of contact with sticking expressed by the relation Eq. 15. Sliding does not occur ($\boldsymbol{u}_t$ = 0) and $\dot{\boldsymbol{u}}_t = \boldsymbol{0}$. During the pulling out loading, and once the sliding limit is reached, a tangential displacement is triggered and the pile begins to slip. At this stage, an adhesive resistance ($R_{adh} = \int_{\Gamma_c} C\beta^2 u d\Gamma_c$) is activated, and we have a linear elastic behavior characterized by a slope $\boldsymbol{C}_t$: ($R_t = C_t.u_t.\beta^2$). By continuing the pulling out of the pile, the tangential displacement ($\boldsymbol{u}_t$) becomes sufficiently important, and therefore the elastic energy ($(C_n u_n^2 + C_t \boldsymbol{u}_t^2)\beta$)) exceeds the limit of the decohesion energy $\boldsymbol{w}$, which means that the adhesion limit is reached (Eq. 15). Beyond this threshold, the adhesive bonds break and the damage of the interface begins. This results in a decrease in the adhesion intensity β ($\dot{\beta} \le 0$) (Eq. 16), and the friction begins to act (when β tends towards zero, μ (1-β) tends gradually to μ). We will have a progressive reduction of adhesive reactions ($R_{adh} = \int_{\Gamma_c} C.\beta^2.u\, d\Gamma_c$ tends towards zero) until their complete disappearance. Once the interface is totally damaged and the adhesion is completely broken, the adhesion intensity vanishes (β = 0), and the interface behavior obeys the classical Coulomb's friction law presented by Eqs 7,8,10.

IV.4 Influence of RCCM model parameters

Following the identification, we carried out a parametric study of the coefficients C_n, C_t, and w of the proposed interface model in order to examine the sensitivity of the numerical results to the different parameters and better understand their influence on the global behavior of the pile-soil interface while studying load-displacement curves. This relatively classical study consists in varying one of the parameters when the others are fixed. A similar study can be found in [60].

a. Influence of the stiffness of the interface

On Fig. 7 we reported the different curves obtained by varying the initial normal (resp. tangential) stiffness of the interface $\boldsymbol{C_n}$ (resp. $\boldsymbol{C_t}$). It is clear from these curves, that a progression from $\boldsymbol{C}$ to increasingly larger values, generates an increase in the constraint of rupture. This means that the behavior of the interface becomes more brittle, and cracking will be sharper and more advanced (change of slope).
According to the RCCM model, the critical stress and displacement thresholds are given as a function of $\boldsymbol{C}$ and w [33]-[34]:

- Critical stress $\sigma_c = \sqrt{w.C}$
- Critical displacement $u_c = \sqrt{w / C}$

As long as the critical value u_c of the slip is not reached, the critical stress increases in a linear way. Then, for higher values of the stiffness of interface "$\boldsymbol{C}$", $\boldsymbol{\sigma_c}$ increases and $\boldsymbol{u_c}$ decreases, which leads to a brutal decohesion, localized over a reduced length.

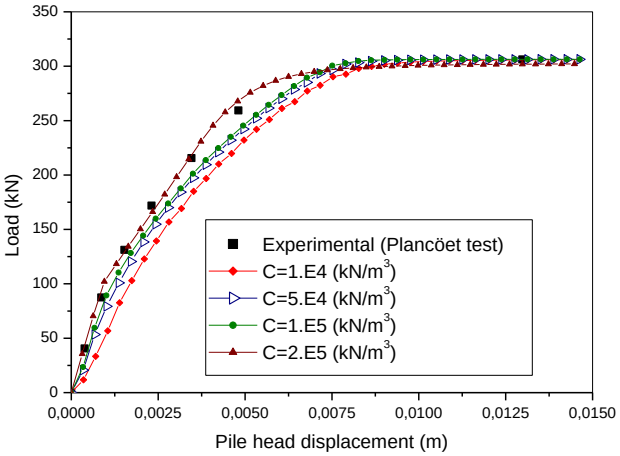

Fig. 7 : Influence of the initial stiffness C_n, C_t

b. Influence of the decohesion energy w

For this simulation, we keep the same values of the parameters of the interface law, as shown in Table 2. However, we can only vary the $\boldsymbol{w}$ parameter of the decohesion energy Fig. 8 shows that, the increase of $\boldsymbol{w}$ delays the occurrence of the global stiffness change, without affecting the rest of the evolution (parallel slopes). In other words, the evolution of the crack is identical whatever the values of the threshold energy, but it is carried out, at an increasing force level with w. In fact, the behavior before decohesion is linear elastic and characterized by the value of the coefficient of initial stiffness $\boldsymbol{C}$ (β = 1).

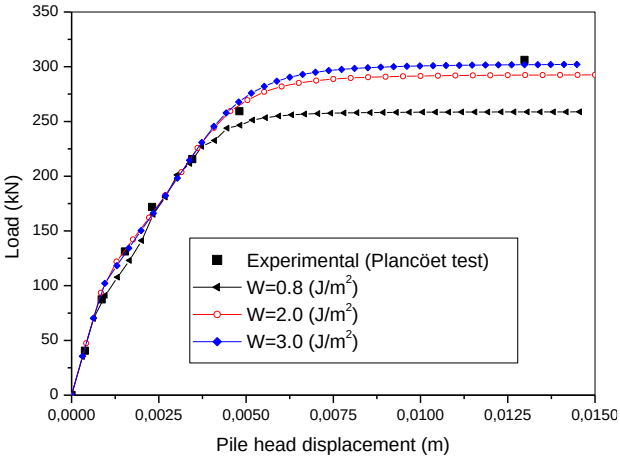

Fig. 8 : Influence of the decohesion energy (w)

Damage to the pile-soil interface begins when the energy stored in the contact surface reaches the threshold $\boldsymbol{w}$, that is, when: $w = (C_n u_n^2 + C_t \boldsymbol{u}_t^2)\beta$. Then, an increase

in the decohesion energy generates a thrust of the critical stress σ_c and the critical displacement u_c. This leads, of course, to a delay in interface fissuration.

c. Influence of the coefficient of friction μ

We have also examined the influence of the variation of the coefficient of friction μ on the behavior of the interface (Fig. 9). It can be noted that, when the coefficient of friction μ increases, the decohesion threshold is slightly pushed back. However, the high values of μ essentially contribute to increasing the slope of the curve, a sign of an increasingly difficult sliding.

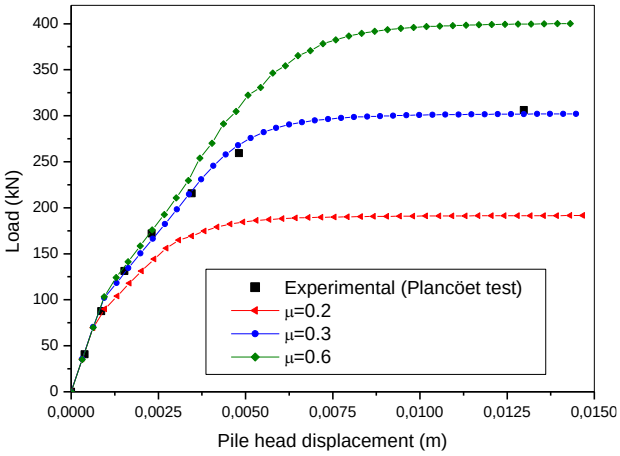

Fig. 9 : Influence of the friction coefficient μ (for α = 0)

V. Conclusion

In this paper, we have presented a numerical simulation of the pile-soil system subjected to traction loading, by using an interface model (the RCCM model), initially developed by the group of Michel Raous at LMA (Marseille) [33]-[34]. It is, therefore, a model that takes into account variations in the stiffness of the interface by means of a parameter of damage, whose variations are controlled by an energy threshold (decohesion energy w), and jumps of displacements at the interface. A numerical method is deduced and the model was validated on experimental results relative to a number of available tests obtained by Frank R. *et al*. [48]-[50].

The comparisons drawn between experimental results and numerical simulations, observed as regards the load-vertical displacement curves at the head of the piles for the pull-out test have revealed the satisfactory interface model response for this test. Globally, we can deduce from these numerical results that the RCCM model describes well the shear behavior studied and governs the sensitive behavior of the interface. An important result of this study is that taking into account an elastic behavior for the soil may not affect the results if the pile-soil interface is modeled realistically. This result is in agreement with the remarks reported by Frank and Boulon [48],[53]. This highlights the ability of the chosen model to take into account the real mechanical behavior of the interface.

The presented study is more or less incomplete but is already an initial approach for future developments. However, several points remain to be dealt with: the laws of behavior of natural soils, pile installation effect, and the initial state of stresses in the soil. These points are the subject of a study in progress using a bipotential formulation coupling contact, friction and adhesion recently published by Terfaya, Berga and Raous [39].

Acknowledgements

We are grateful to Professor FRANK Roger, ENPC, National School of Bridges and Highways, Geotechnical Team CERMES, France, for providing the documents containing the experimental results given in [48],[49].

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