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1 **Considerations about the applicability of the Reynolds equation for analyzing**
2 **high-speed near field levitation phenomena**

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9 **Abstract**

10 *This paper presents the results of investigations into the applicability of the Reynolds*
11 *equation for analyzing near field levitation (NFL) phenomena. Two separate*
12 *approaches were developed, experimentally verified, and applied to meet the research*
13 *objective. One was based on the Reynolds equation and the other was based on*
14 *general conservation equations for fluid flow solved using computational fluid dynamic*
15 *(CFD). Comparing the calculation results revealed that, for certain operating*
16 *conditions, differences in the predicted system parameters appear. It was proven that*
17 *both the Reynolds and Helmholtz numbers define border operating conditions for the*
18 *applicability of the Reynolds equation to analyze NFL problems.*

19 **Keywords:** Near field levitation, Reynolds equation, CFD, High frequencies

1 Nomenclature

a	ms^{-1}	speed of sound
e	m	oscillation amplitude
f	Hz	oscillation frequency
F_{aero}	N	aerodynamic force
g	ms^{-2}	gravitational acceleration
h	m	average levitation height
H	-	Helmholtz number
Kn	-	Knudsen number
m	kg	mass of the levitating object
M	kg mol^{-1}	molar mass
p	Pa	pressure
p_a	Pa	atmospheric pressure
r, β, z	M	cylindrical coordinates
R	$\text{J mol}^{-1} \text{K}^{-1}$	gas constant
R_d, L	m	radius (width) of the levitating object
Re	-	Reynolds number
t	s	time
T	K	temperature
\mathbf{U}	ms^{-1}	velocity vector
λ	m	acoustic wavelength
λ_g	m	mean free path of the gas
μ	Pa s	dynamic air viscosity
ρ	kg m^{-3}	density of the air
σ	-	squeeze number
τ	Nm^{-2}	stress tensor
ω	rad s^{-1}	angular frequency (pulsation of the vibration)

1. Introduction

The possibility to levitate objects offers an interesting alternative to eliminate friction and adhesive forces in micro-systems and allow accurate and controlled displacements of micro-objects [1,2]. Among all the possible technical solutions, Vandaele et al. [1] indicated that acoustic levitation is the most interesting solution because it is more versatile than other methods. The levitation is provided by a vibrating plate that modifies the air pressure distribution between the plate and the object, leading to a net force able to balance the weight of the object. Depending on the ratio of the air gap, h , to the acoustic wavelength, λ , two levitation modes can be identified [1]. If h is higher than λ , this is standing wave levitation; otherwise, it is near field levitation (NFL) or squeeze film levitation [2].

NFL, which provides much higher force [1], is the topic of this paper. One of the first experimental studies of squeeze film levitation was performed by Salbu in 1964 [3]. A piston-like motion of the bottom plate was imposed by an electromagnetic actuator, allowing Salbu to measure the net force value as a function of the average distance between the plates. The piston-like motion, which can be easily measured and theoretically modeled, was adopted in several later studies where the electromagnetic actuator was replaced by a more efficient piezoelectric actuator [4,5,6,7,8]. This solution can provide high levels of levitation force, estimated as up to 70 kNm^{-2} by Ueha et al. [6]. Even if, in many studies, the system was operated at its natural oscillation frequency (generally corresponding to several tenths of 1 kHz) to reduce the energy needed for actuating the bottom plate, a very stiff and massive plate is usually needed to ensure perfect motion of the top surface. To reduce the thickness of the bottom plate and allow lateral size increases, it was necessary to operate the system at its structural flexural mode. In this case, the bottom plate motion is more complex and controlled by structural dynamic behavior. This solution was adopted in many papers [6,9,10,11]. This approach has been extended to create journal bearings [12,13,14,15,16,17] or methods to move the levitated objects [18,19,20,21,22]. In addition, the vibrating motion of the bottom plate can lead to the flexural vibration of the top plate, which makes the problem more complex [23].

The airflow between the vibrating plate and the levitated object is governed by the general Navier-Stokes set of equations. Depending on the configuration, some physical contributions vanish, and the equations can be simplified. When the levitation distance, h , is small compared to the characteristic width of the object, L , viscous effects are dominant and inertia effects in the fluid are negligible. It is thus possible to use the Reynolds equation to describe the flow in the air gap [24]. This approach has been successfully used by many authors [3,8,25,26,27,28,29]. When the configuration is axisymmetrical and the air gap is a sine function of time, the Reynolds equation has an analytical solution involving Kelvin functions, as demonstrated by Crandall [25]. Crandall also showed that two regimes exist, depending on the value of what is

1 currently named the squeeze number: $\sigma = 12 \mu \omega R_d^2 / (p_a h^2)$ where μ is the viscosity
2 of air, ω is the vibration pulsation, R_d is the outer radius of the system, h is the average
3 gap, and p_a is the atmospheric pressure. At low squeeze numbers, the flow is mainly
4 viscous, leading to a high damping coefficient for the film and low stiffness. However,
5 at high squeeze numbers, the compressibility effect becomes dominant, damping is
6 low, and the stiffness is maximal [25,27]. In this last regime, the fluid behaves as an
7 enclosed mass of air that is compressed between the plates without any radial flow.
8 The gas film can be described by Boyle's law [3,29].

9 If it is assumed that the oscillating motion is fast enough so that no viscous fluid motion
10 is observed, the Navier-Stokes equations can be reduced to the Euler set of equations.
11 The pressure distribution can thus be obtained from acoustic theory [2,5,6]. If the lateral
12 size of the domain is large compared to the wavelength, λ , of acoustic waves, an
13 analytical solution of the pressure is obtained. It can be further simplified in the case
14 of near field acoustic levitation ($h \gg \lambda$) ([2,5,6]). Using this simplified solution, it can
15 be shown that the average levitation height, h , is proportional to the vibration amplitude,
16 e , of the bottom plate and inversely proportional to the square root of the mass, m , of
17 the levitated object. The same solution is obtained with the Reynolds equation at high
18 σ value or with Boyle's law [8].

19 Based on this last finding, it can be postulated that the Reynolds solution, which has
20 the advantage of being solved at low computation cost, can be used to solve all near
21 field problems. However, depending on the operating conditions, many authors found
22 that additional effects not considered in the Reynolds equation must be accounted for.
23 Andrews et al. [27] performed some tests at low ambient pressure (80 Torr) and
24 showed that the rarefaction effect, inducing slip of the air on the plates, must be
25 considered. Indeed, in this case, the mean free path of the gas, λ_g , is increased,
26 leading to high values of the Knudsen number, $Kn = \lambda_g / h$. Based on the Knudsen
27 definition, rarefaction effects can also be observed at low film thickness values, h [30].
28 Mohite et al. [30] also showed that fluid inertia can affect the behavior of the squeeze
29 film when the magnitude of the Reynolds number, $Re = \rho \omega h^2 / \mu$, is higher than 1. For
30 small Re values, Stolarski and Chai [31] verified that the Reynolds equation is relevant.
31 Li et al. [32] also included the effect of convective inertia in a modified Reynolds
32 equation. In addition, they considered the pressure drop occurring during fluid suction
33 in the contact due to the sudden section reduction at the inlet. They showed that this
34 effect becomes significant when the squeeze number increased. However, Minikes
35 and Bucher [33] showed that edge effects, manifested as a pressure difference at the
36 inlet, can be observed at low squeeze numbers due to flow interaction with the
37 oscillating plate when the plate is bigger than the levitated object. Finally, in the case
38 when one of the plates rotates, centrifugal inertia effects can be experienced [29]. This
39 last point is beyond the scope of this work. These additional effects can make the
40 Reynolds equation ineffective for solving the levitation problem, particularly when high

1 oscillating frequencies are considered. As previously discussed, high operating
2 frequencies are encountered in many applications where the system is used at its
3 natural frequency. However, the limit of using Reynolds has not been clearly identified
4 for this type of squeeze problem.

5 In this work, simulations were carried out with Navier-Stokes equations and the
6 Reynolds equation. The results were compared to find the limit of validity for the
7 Reynolds equation. Some deviations are observed at high oscillating frequencies and
8 are related to acoustic wave propagation in the contact. It was possible to define a
9 dimensionless number equivalent to the Helmholtz number to identify the limit of
10 validity for the Reynolds equation.

11 **2. Goal and scope of the research**

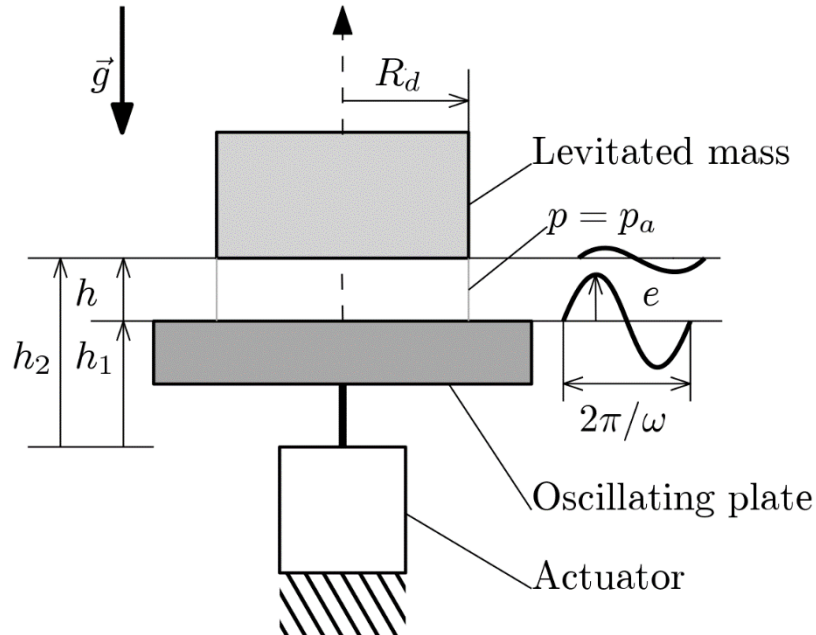
12 The Reynolds equation was derived from general fluid dynamics conservation
13 equations using a number of simplifying assumptions, one of them is neglecting fluid
14 inertia. The goal of this work is to determine the range in which the assumptions of the
15 Reynolds equation are acceptable for analyzing NFL problems. For this purpose,
16 calculations of the piston-like motion system were carried out using two separate
17 computational models that differ in their description of airflow in the air gap: a) a model
18 using general fluid dynamics conservation equations, b) a model using the Reynolds
19 equation. The developed models considered both time-varying flow in the air gap and
20 the movement of the levitated mass caused by aerodynamic force. Investigations were
21 carried out for object geometry and oscillation parameters accessible at the test stand.
22 Selected results of the experimental tests were also used to verify the correctness of
23 developed models.

24 The calculations were carried out using both theoretical models for the same operating
25 conditions and oscillation frequencies of the plate, assumed in a wide range. A
26 comparison of the calculated levitation parameters showed differences between the
27 results and determined the validity range of the results obtained from solving the
28 Reynolds equation for analyzing an NFL system. The research was carried out
29 assuming that oscillating mass and oscillating plate remain rigid during operation. That
30 simplification could influence the whole system behavior in the case when relatively
31 elastic elements were applied (thin and/or with a large diameter). However, despite of
32 this assumption, conclusions about applicability of different approaches for
33 investigations of the air flow in the gap, which is the goal of this paper, will be still valid.

34 **3. Problem description**

35 The principle of obtaining load capacity in an aerodynamic gas film, as a result of forced
36 oscillations using a piston-like motion system, is illustrated schematically in Fig. 1. An
37 oscillating plate, as a result of actuator operation, is nominally introduced into uniaxial
38 oscillations. Above the plate, the levitating mass (disk-shaped) is placed. Between

1 those elements, due to forced sinusoidal oscillations, an aerodynamic lubricating film
 2 is formed (on the principle of squeeze film effect). During system operation, pressure
 3 in the film exerts force on the levitated mass and causes its lift. As a result, a levitated
 4 mass is also introduced into oscillations, the amplitude of which depends on the
 5 frequency of the oscillating plate.



6
 7 Fig. 1. Configuration of the problem.

8 4. Methodology

9 Taking into account the operating principle of NFL, the aerodynamic film parameters
 10 (e.g. pressure field or film thickness) should be analyzed in the time domain. This
 11 applies both to the flow in the air gap and the displacement of the surfaces limiting it:
 12 the oscillating plate and levitated mass. In the described problem, oscillating plate
 13 displacement is imposed and known while the mass elevation distance and its
 14 oscillations are the result of forces acting on the object and could be calculated using
 15 the fundamental dynamics law of motion.

16 It was assumed that airflow was isothermal, laminar, without external forces acting on
 17 the fluid and air satisfies the ideal gas assumptions. Within the scope of this work, two
 18 different approaches for describing airflow were implemented: a) The computational
 19 fluid dynamics model (CFD model) based on the solution of the set of general
 20 conservation equations of the fluid flow, which for a Newtonian fluid, reduces to the
 21 Navier-Stokes set of equations, b) Reynolds model based on the solution of the
 22 Reynolds equation.

23 CFD model

24 The model was developed using the CFX package [34], which uses the finite volume
 25 method for discretizing and solving 3D conservation equations for transient flow of
 26 compressible fluids:

1 the continuity equation:
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \quad (1)$$

2 the momentum equations:
$$\frac{\partial(\rho \mathbf{U})}{\partial t} + \nabla \cdot (\rho \mathbf{U} \otimes \mathbf{U}) = -\nabla p + \nabla \cdot \boldsymbol{\tau} \quad (2)$$

3 which, together with the ideal gas equation of state:

4
$$\rho = \frac{Mp}{RT} \quad (3)$$

5 allows obtaining a solution to the air gap flow problem. Symbols in equations (1) – (3)
 6 mean: ρ - air density, t - time, \mathbf{U} - velocity vector, p - pressure, $\boldsymbol{\tau}$ - stress tensor, M - air
 7 molar mass ($28.96 \times 10^{-3} \text{ kg mol}^{-1}$), T - temperature, R - gas constant ($8.314 \text{ J mol}^{-1} \text{ K}^{-1}$).
 8 Since air is Newtonian fluid, stress tensor $\boldsymbol{\tau}$ (in equations (2)) is proportional to velocity
 9 gradients $\nabla \mathbf{U}$ and dynamic viscosity μ , according to Equation (4):

10
$$\boldsymbol{\tau} = \mu(\nabla \mathbf{U} + (\nabla \mathbf{U})^T) - \frac{2}{3} \delta \nabla \cdot \mathbf{U} \quad (4)$$

11 where: δ is the Identity matrix. Mathematical operators in equations (1) to (4) are: ' $\nabla \cdot$ '
 12 means divergence, ' $\mathbf{U} \otimes \mathbf{U}$ ' is the dyadic operator of velocity vectors \mathbf{U} and ∇ means
 13 gradient.

14 Taking advantage of axial symmetry, the developed discrete model was simplified so
 15 that it represents only a part of the air gap with symmetry boundary conditions at the
 16 side walls (Fig. 2). At the outer wall of the model, an opening boundary condition was
 17 defined with the value of total pressure equal to ambient pressure. At the bottom and
 18 top walls, a 'no-slip' boundary condition was imposed. The bottom wall represents the
 19 surface of the oscillating plate and its displacement was imposed on the vertical axis
 20 (z in Fig. 2) using 'mesh motion' capability, according to Equation (5):

21
$$h1 = -e \cdot \sin(2 \cdot \pi \cdot f \cdot t) \quad (5)$$

22 where: $h1$ is imposed position of the oscillation plate in vertical direction as a function
 23 of time t , e is oscillation amplitude, f is oscillation frequency.

24 The top wall of the CFD model represents the surface of the levitated mass. To include
 25 the interaction between the levitated object and air gap pressure, the Rigid Body solver
 26 was employed. Rigid Body is a simplified form of fluid-structure interaction (FSI)
 27 available in CFD software allowing to analyze the body response to flow forces under
 28 the assumption that the body does not change shape under fluid load [34,35].

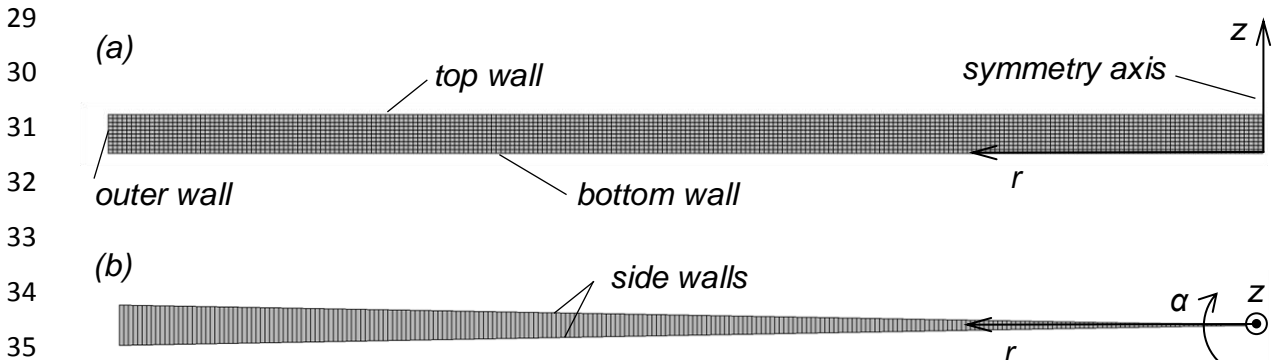


Fig. 2. Schematic view of the CFD mesh, (a) side view; (b) top view.

1 It was assumed, considering the symmetry of the analyzed problem, that movement of
2 the levitated mass was only possible in the vertical direction (1 DOF option).
3 Consequently, the top wall movement was obtained with the Rigid Body solver using
4 the solution of the dynamic equation of motion for one direction (using the Newmark
5 integration scheme):

$$6 \quad m\ddot{z} = F_{\text{aero}} + mg \quad (6)$$

7 where m is the mass of the levitated object, kg; \ddot{z} is the object's acceleration, ms^{-2} ; F_{aero}
8 is the force in the air gap due to the squeeze effect, N; and g is gravitational
9 acceleration, ms^{-2} .

10 Analyses were carried out using a mesh divided into 500 finite volumes in the air gap
11 radial direction, 20 in the film thickness direction and one in the circumferential
12 direction. Mesh parameters were determined using the results of a mesh
13 independence study. Angular size of the grid was assumed $\alpha_g = 1$, to get its adequate
14 quality. Calculations were started with the assumed initial air gap thickness (usually 5
15 μm). During calculations, the thickness of the air gap changed as a result of interaction
16 between structural and aerodynamic loads. Calculations were stopped after obtaining
17 a stable and repeatable oscillating movement of the upper surface (levitation mass).
18 Each oscillation period was divided into 400 time steps. To achieve stable elevation of
19 the levitated object and repeatable oscillations, one calculation task required,
20 depending on the case, from several to hundreds of analyzed periods of enforced
21 oscillation.

22 **Reynolds model**

23 The model using the solution of the Reynolds equation to describe flow in the air gap
24 of an NFL problem, which was applied in this study, was presented in detail in work
25 [8]. The equations are solved in a home-made code written in Fortran 2003 and based
26 on the Finite Volume Method. As discussed in the introduction, the Reynolds equation
27 is a simplification of the Navier-Stokes equations due to the thin film configuration
28 where only a part of the viscous contributions are preserved [24]. Therefore, the inertia
29 terms are neglected and the pressure is constant in the through film direction. The fluid
30 velocity profiles are thus parabolic and proportional to the radial pressure gradient. The
31 Reynolds model, similar to the CFD model, used the axial symmetry of the analyzed
32 system (for simplifying calculations) and the same set of boundary conditions and
33 assumptions for air behavior in the gap (ideal gas). In addition, movement of the
34 oscillation plate was imposed using Equation (5) and the levitated mass had one
35 degree of freedom, with position calculated using the dynamic law of motion with
36 Equation (6), as for the CFD model.

37 **Input data**

38 The geometrical and physical parameters of the system, as well as operating
39 conditions used in analyses, are shown in Table 1. The air dynamic viscosity was

1 assumed to be $\mu = 1.831 \times 10^{-5}$ Pas, gravitational acceleration was $g = 9.81 \text{ ms}^{-2}$ and
2 temperature was $T = 300 \text{ K}$.

3 Table 1. Problem input data.

parameter	value
mass, m	$19.28 \times 10^{-3} \text{ kg}$
mass radius, R_d	15 mm
oscillation amplitude, e	1 μm
oscillation frequency, f	2 – 60 kHz

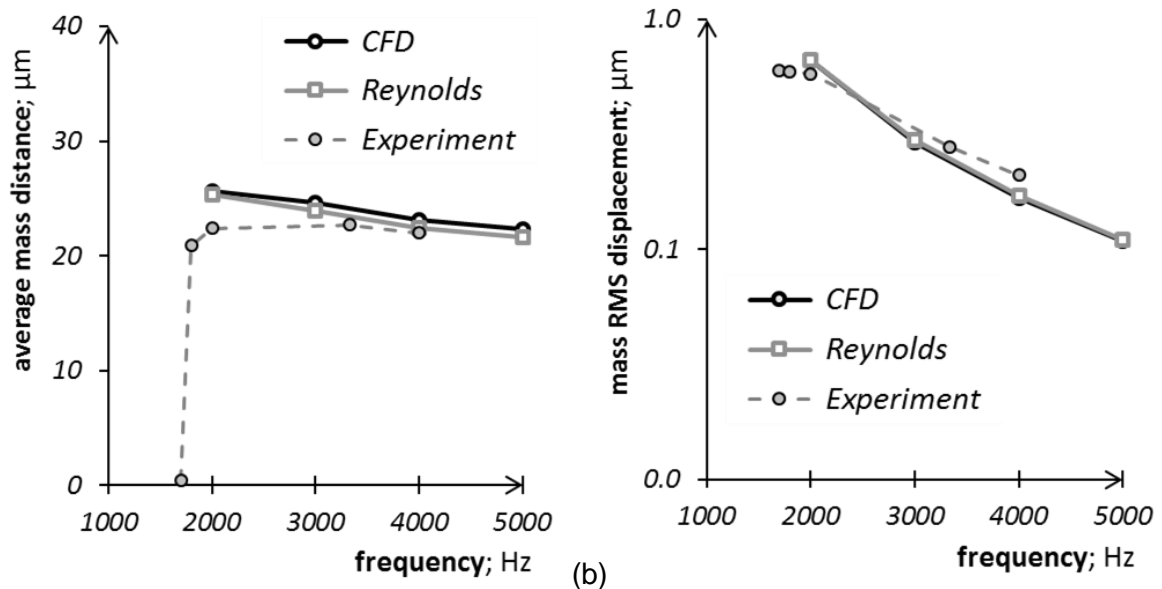
4 5. Results

5 Experimental validation

6 The correctness of the developed theoretical models was evaluated using the results
7 of experimental measurements obtained using a test stand in the piston-like-motion
8 NFL problem configuration. Details regarding the construction of the test stand were
9 presented in [8]. Due to the limitations of the test stand, it was only possible to carry
10 out measurements for oscillation frequencies up to 4 kHz. Test parameters were the
11 same as shown in Table 1. Each test was repeated three times. The maximum
12 difference in average mass distance between the different tests is 0.2 μm indicating an
13 excellent reproducibility of the experiments.

14 Fig. 3 shows a comparison of the measured and calculated values of average levitated
15 mass distance (a) and mass RMS (Root Mean Square) displacement (b). Similar
16 average levitated mass distances were calculated using both models (the maximum
17 relative difference was not larger than 3.5%). Measured values differ from calculated
18 ones, especially for low frequencies; however, for frequencies over 3 kHz, good
19 agreement was observed. Slightly better agreement observed for the Reynolds
20 approach results is rather accidental and is probably caused by uncertainty of
21 experimental data. Surfaces planarity-flatness errors or deviations from assumed
22 oscillating plate motions are possible examples of measurement inaccuracies sources.
23 Two initial points from experimental research show measured film thickness under
24 conditions of non-fully developed squeeze film lubrication. Direct contact between
25 cooperating surfaces took place for lowest tested frequencies. Such situation was not
26 possible to consider with the use of developed methodology for investigations of NFAL
27 system operation.

28 In the case of the RMS value of levitated mass displacement, which is a measure of
29 similarity of calculated and experimentally observed oscillations of the levitated object,
30 very good agreement was noticed for the whole analyzed range of frequencies.



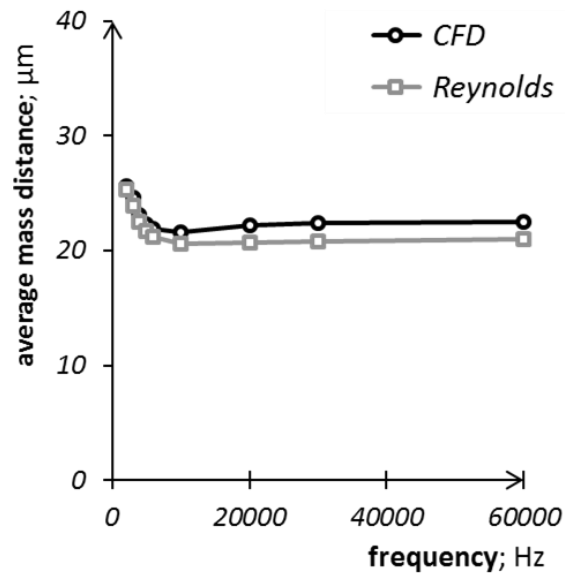
1 (a) (b)
 2 Fig. 3. Comparison of the results of calculations and measurements; (a) average mass
 3 distance, μm ; (b) mass RMS displacement, μm .

4 The comparison of the calculated and measured parameters of NFL proved the
 5 correctness of the results obtained from theoretical models and the differences could
 6 be explained mainly by the difficulties and uncertainties in experimental tests, such as
 7 flatness defects in the surfaces, non-perfect sinusoidal motion, and height
 8 measurement uncertainty.

9 Experimental validation of the theoretical results for high frequencies was not possible
 10 with the use of available equipment. Good agreement for low frequencies confirmed,
 11 that both models work well, and predicted parameters of NFAL system were very close
 12 to measured data. However, when try to expand that statement also for high
 13 frequencies, it could be supposed, that simplifications introduced to the Reynolds
 14 approach (e.g. assumption of parabolic velocity profile across gap or no inertia effects)
 15 can lead to not adequate results, because flow inertia has presumably strong influence
 16 on operating parameters of the system in such conditions.

17 Parametric study

18 Parametric studies were carried out for the frequency range of the oscillating plate,
 19 from 2 to 60 kHz. The calculated average mass distance is compared in Fig. 4.

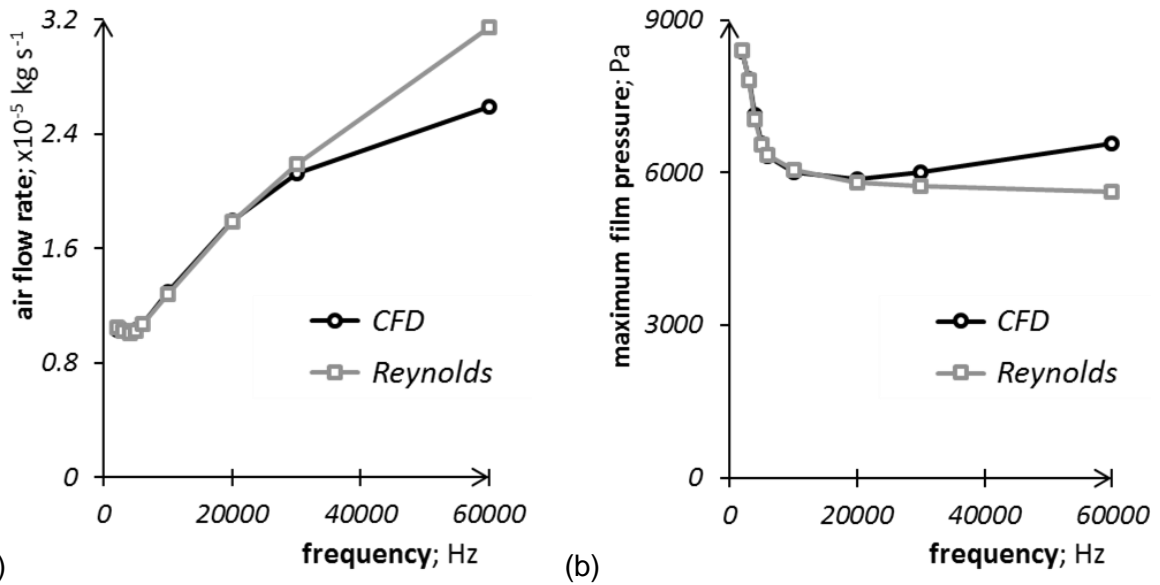


1
2 Fig. 4. Average mass distance, μm , as a function of oscillation frequency, comparison
3 between CFD and Reynolds model results.

4 The highest mass levitation height was obtained for the lowest analyzed frequency.
5 With increased oscillation frequency up to 10 kHz, mass lift distance decreased. At
6 higher excitation frequencies (> 10 kHz), the lifting height did not depend strongly on
7 the oscillation frequency. It is worth noting that results obtained from both theoretical
8 models showed the same trend of changes, whereas average mass lift distances
9 calculated using CFD were slightly higher than those obtained from the Reynolds
10 model. However, differences in results for frequencies < 6 kHz were relatively low
11 (about 3%). For higher oscillation frequencies, differences between the results of CFD
12 and Reynolds models were slightly higher (up to 8%).

13 In Fig. 5, the calculated maximum air flow rate at the gap border (inlet/outlet) (a) and
14 maximum air gap pressure (b) are compared. Similar to the average mass distance,
15 the trend of changes for air flow rate as a function of frequency, predicted using both
16 models, was the same (Fig. 5a). Except for the initial frequency range ($f < 4$ kHz), the
17 flow rate increased with increasing oscillation frequency; while for frequencies above
18 20 kHz, the calculated flow rate using the Reynolds model was higher than for CFD.

19 The trends of maximum pressure changes in the film (Fig. 5b) calculated by the CFD
20 and Reynolds models look slightly different. In the initial frequency range for both
21 models (< 20 kHz), the maximum pressure in the film decreased with increasing
22 frequency. For higher frequencies, this trend is also visible in the results obtained from
23 the Reynolds model; however, the calculated change in the maximum pressure value
24 was relatively small. The opposite trend was noticed for results obtained from the CFD
25 model, which for frequencies > 20 kHz, the calculated maximum pressure in the air film
26 increased with increases in the oscillation frequency.



1 (a) (b)
 2 Fig. 5. Air flow rate, $\times 10^{-5} \text{ kg s}^{-1}$; (a) and maximum film pressure, Pa (b) as a function
 3 of oscillation frequency, comparison between CFD and Reynolds model
 4 results.

5 In the case of analyzed quantities at frequencies up to 20 kHz, both models predicted
 6 nearly identical parameter values (Fig. 5). For higher frequencies ($> 20\text{kHz}$), a
 7 comparison of the results obtained from the CFD and Reynolds models shows visible
 8 differences, which increase with increasing oscillation plate frequency.

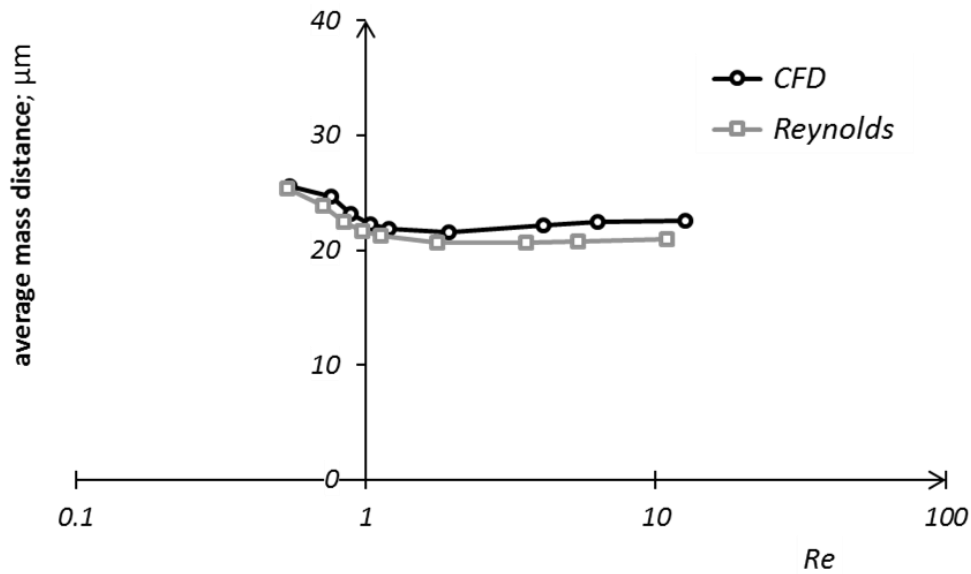
9 6. Discussion

10 The derivation of the Reynolds equation assumes that the inertia effects in the flow are
 11 negligible. Consequently, one of the main reasons for the deviation between the
 12 Reynolds and CFD solutions is the appearance of a significant inertia effect. Based on
 13 the work of Mohite et al. [30], the Reynolds equation for the problem can be defined
 14 as:

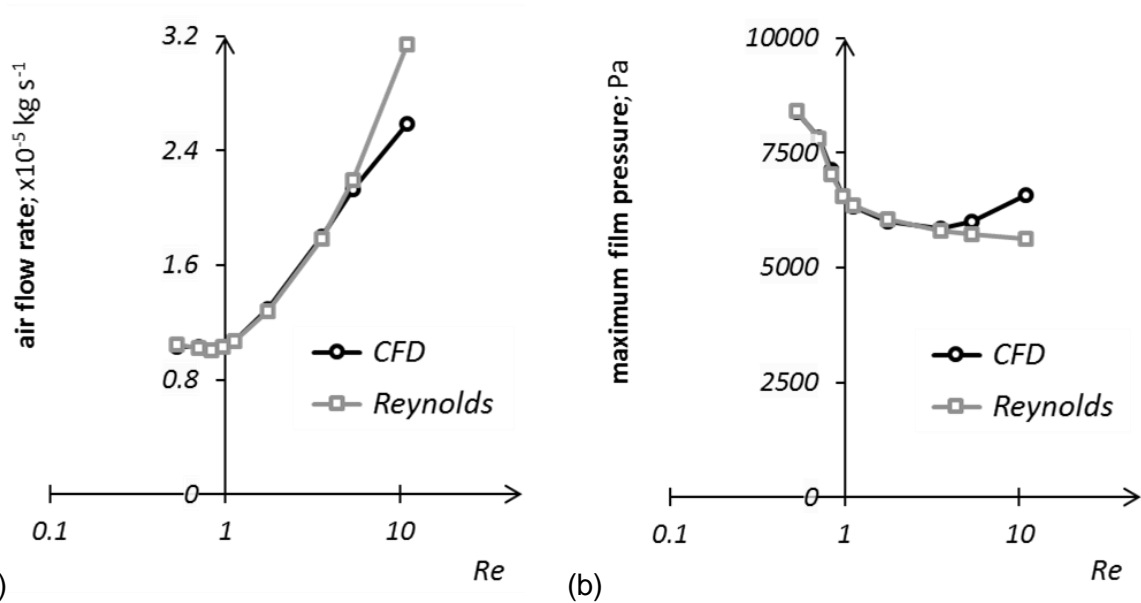
$$15 \quad Re = \frac{\rho \omega h^2}{\mu} \quad (7)$$

16 where h is the averaged levitation height in one period. Fig. 4 shows the levitation
 17 height calculated by both methods as a function of the oscillation frequency. It has
 18 been replotted as a function of Reynolds number in Fig. 6. It is clear from this graph
 19 that the deviation occurs when the Re becomes higher than 1, confirming that the
 20 deviation is due to inertia effects in the gas flow.

21 Doing the same analysis for the maximum pressure and air-flow presented in Fig. 5
 22 does not give satisfactory results, as can be seen in Fig. 7. Indeed, the deviation starts
 23 at Re much higher than 1. Another physical phenomenon should explain this
 24 difference. This point is discussed in the following part of this section.

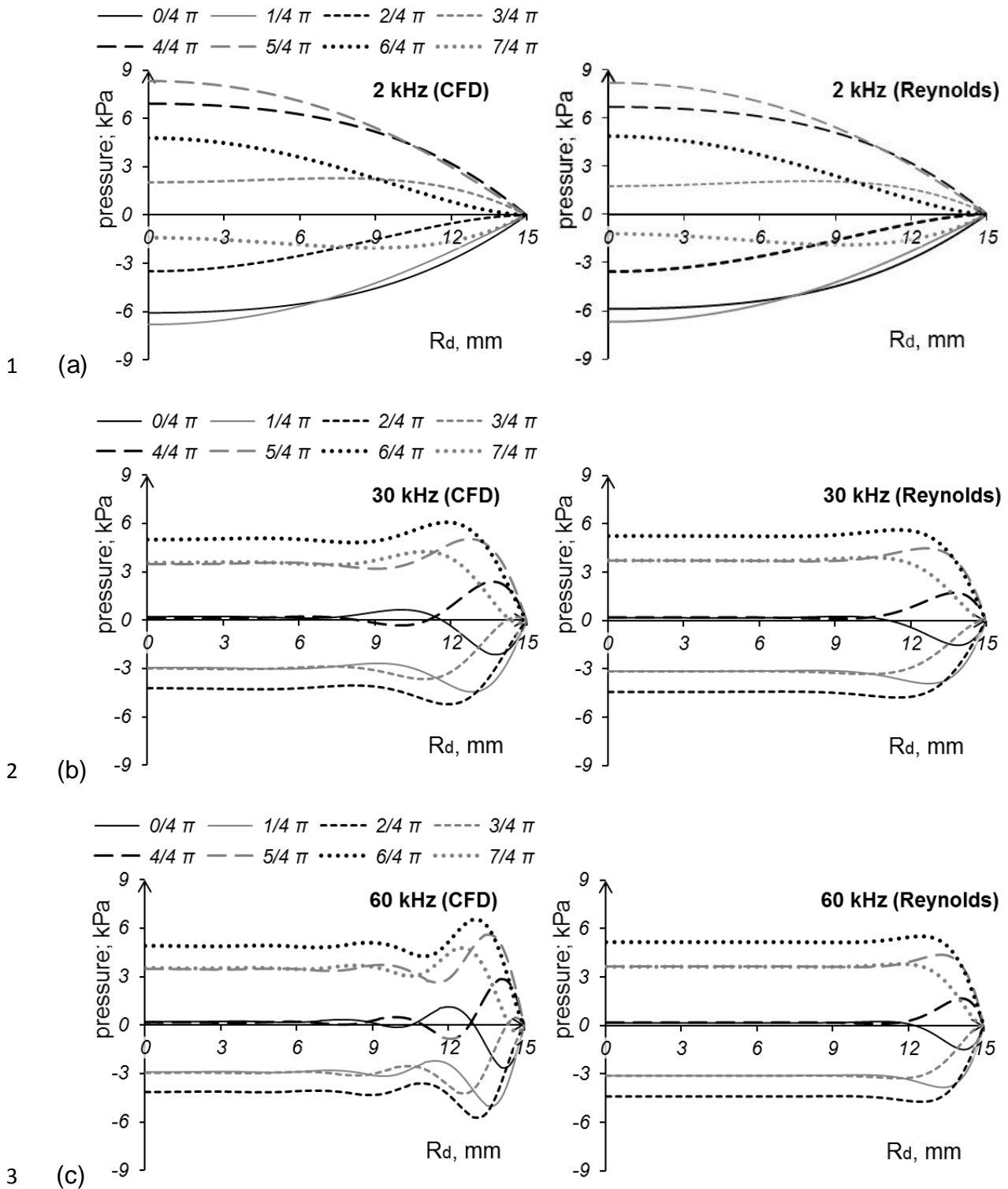


1
2 Fig. 6. Average mass distance, μm , as a function of Reynolds number, comparison
3 between CFD and Reynolds model results.



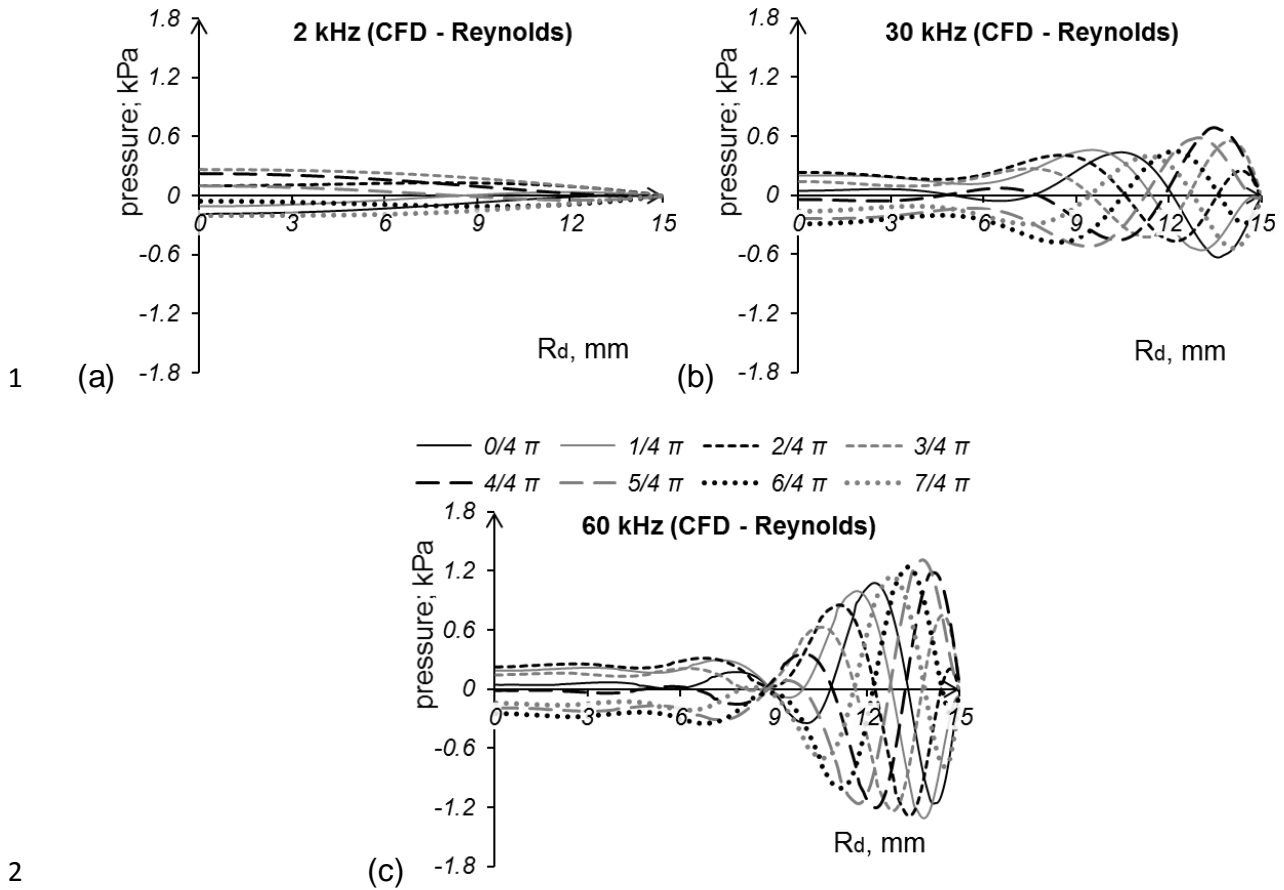
4 (a) (b)
5 Fig. 7. Air flow, $\times 10^{-5} \text{ kg s}^{-1}$; (a) and maximum pressure Pa; (b) as a function of
6 Reynolds number, comparison between CFD and Reynolds model results.

7 For a better understanding, pressure profiles in the air film along the radial direction
8 are presented at different oscillation time steps obtained at different vibrating frequency
9 values, f (see Fig. 8).



4 Fig. 8. Comparison of the calculated radial pressure profiles for selected time points of
 5 the oscillation period obtained from CFD and Reynolds models; (a) 2 kHz;
 6 (b) 30 kHz; (c) 60 kHz.

7 The difference between the pressure profiles from the NS and Reynolds methods is
 8 presented in Fig. 9.



1 (a) 2 kHz (CFD - Reynolds) (b) 30 kHz (CFD - Reynolds)

2 (c) 60 kHz (CFD - Reynolds)

3 Fig. 9. Difference in pressure profiles obtained from the CFD and Reynolds models;

4 (a) 2 kHz; (b) 30 kHz; (c) 60 kHz.

5 At low f value (2 kHz), the results of the two methods are very close, the difference

6 being less than 0.3 kPa. When the frequency is raised to 30 kHz, some differences

7 between CFD and Reynolds can be observed. Some waves are visible on the pressure

8 profiles obtained with the CFD solution. These waves are more pronounced at higher

9 f values, the difference reaching 1.2 kPa. These waves explained the difference in

10 maximum pressure and airflow between the CFD and Reynolds solution. These waves

11 at high frequencies could be due to acoustic waves propagation which cannot be

12 captured by the simplified Reynolds equation. Indeed, since the fluid inertia terms are

13 neglected in the Reynolds equation, it is not possible to simulate acoustic waves which

14 are the results of the fluid compressibility and fluid inertia interaction.

15 To analyze this assumption, the Helmholtz number of the flow, defined as the ratio of

16 the time necessary for acoustic waves to travel across the contact divided by the

17 oscillation period, is used. It is:

$$18 \quad H = \frac{R_d f}{a} \quad (8)$$

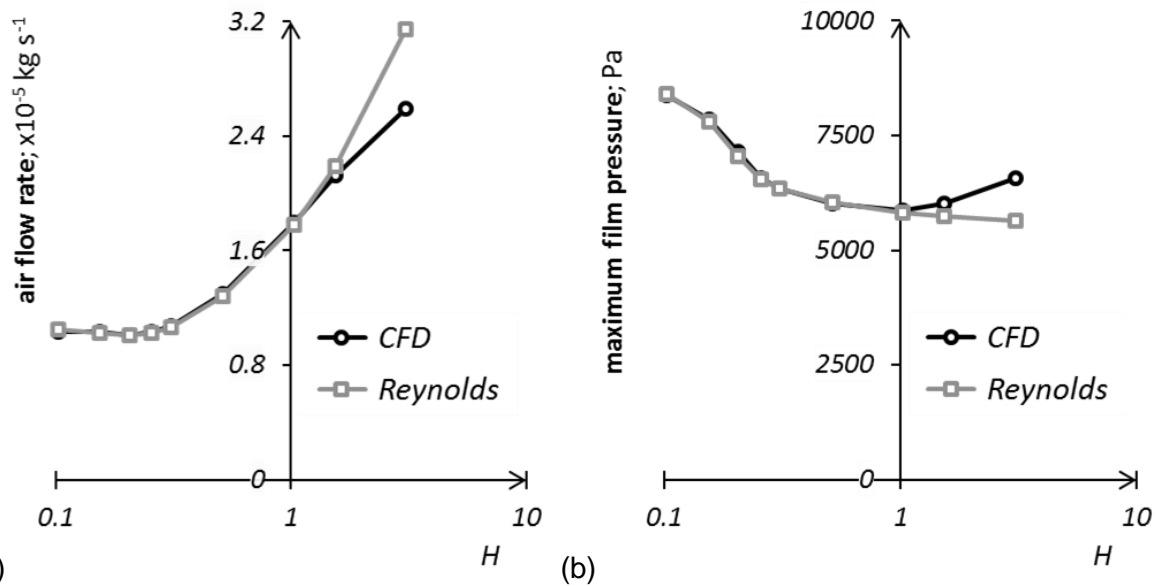
19 where a is the speed of sound. When Fig. 5 is modified to be presented as a function

20 of H , it appears that the deviation occurs when the Helmholtz number becomes higher

21 than 1, confirming our assumption (see Fig. 10). When the oscillation period is lower

22 than the time necessary for the pressure information to travel across the contact, a

1 change in behavior is observed. This change cannot be captured by the Reynolds
 2 equation.



3 (a) (b)
 4
 5 Fig. 10. Air flow, $\times 10^{-5} \text{ kg s}^{-1}$; (a) and maximum pressure, Pa, (b) as a function of
 6 Helmholtz number, comparison between CFD and Reynolds model results.

7 To analyze more deeply the effect of the acoustic waves propagation on the simulation
 8 results, air velocity profiles across the film thickness at $r = 14 \text{ mm}$ calculated with the
 9 use of Reynolds and CFD solutions were compared in Fig. 11. First of all, it can be
 10 seen that at H values much lower than 1, the velocity profiles obtained with both
 11 approaches were very close and with a parabolic shape (2 kHz case, Fig. 11a). When
 12 the frequency was increased to get H values higher than 1 (Fig. 11b and c), some
 13 significant differences can be observed. CFD solution showed that the velocity profiles
 14 are no more parabolic certainly because of the fluid acceleration which can be
 15 significant at high frequency. Secondly, the velocity calculated with the Reynolds
 16 solution is, at some particular time period, inverted when compared to the exact
 17 solution. This is due to the pressure gradients that are not correctly calculated with the
 18 Reynolds equation. Indeed, the pressure waves can provided pressure gradient
 19 inversions not captured with the Reynolds equation (see Fig. 8). This last part clearly
 20 confirms, that when the Helmholtz number is higher than 1, air flow in the NFAL system
 21 should be analyzed with the use of Navier-Stokes set of equations.

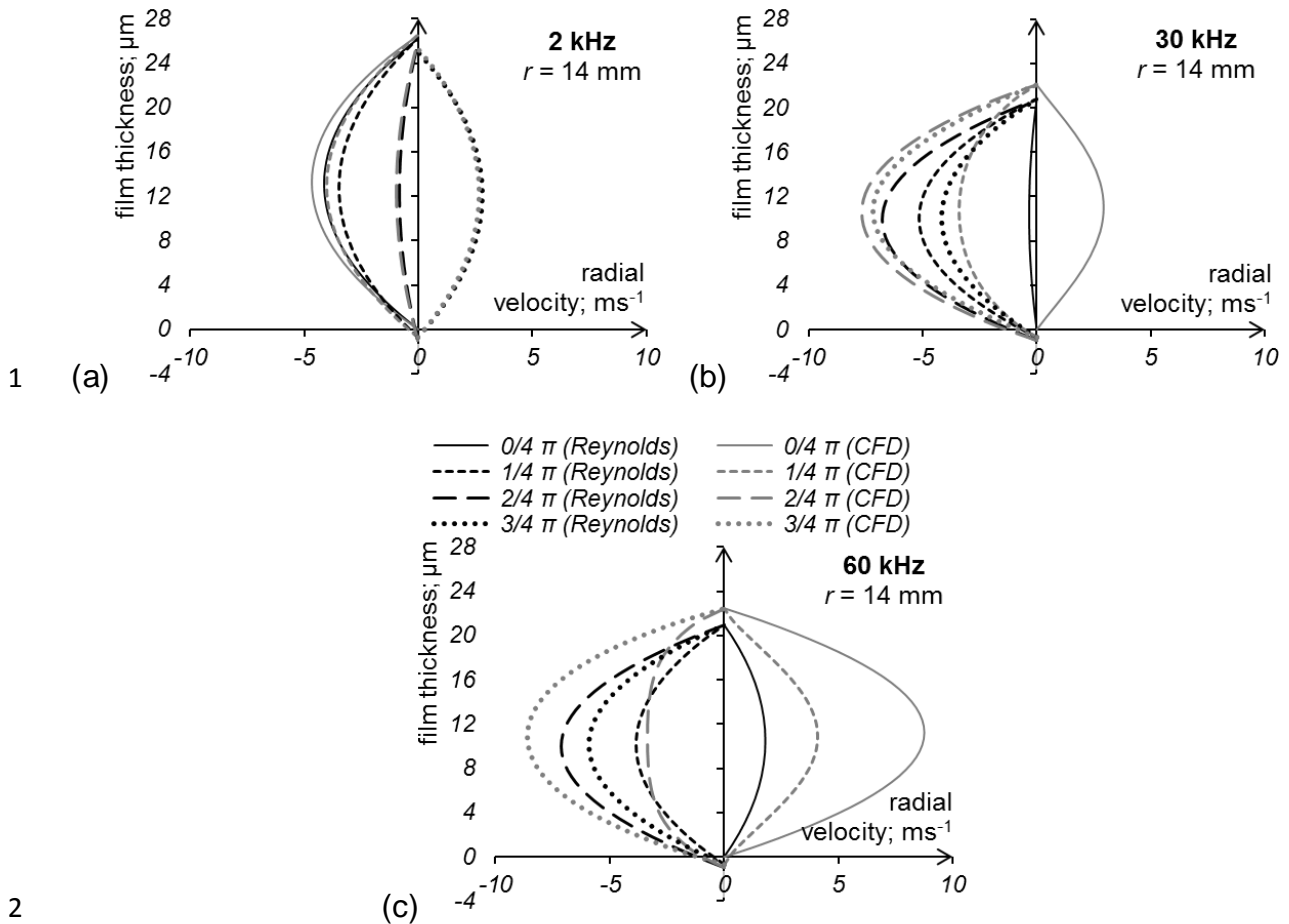


Fig. 11. Comparison of the calculated velocity profiles across air gap (at $r = 14$ mm) for selected time points of the oscillation period obtained from CFD and Reynolds models; (a) 2 kHz; (b) 30 kHz; (c) 60 kHz.

7. Summary and conclusion

The gas flow in an NFL system can be simulated by the Navier-Stokes approach, which is more general, and the Reynolds equation, which is a simplified approach. The latter has the advantage of a very low computation time.

These two methods were satisfactory compared to experiments performed on a levitated disk at low-frequency values (a few kHz). However, many near field levitation systems work at high frequency (several 10 kHz) for which the Reynolds equation is not necessarily valid. A comparison of the results obtained with the two methods was performed on a range of frequencies.

It was shown that, as expected, the Reynolds method deviates from the NS solution when the Reynolds number (Eq. 6) is higher than 1. A second deviation was observed at higher oscillation frequency. It appears when the plate oscillation period is shorter than the time necessary for pressure information to radially travel through the gas film. It exactly corresponds to a Helmholtz number (Eq. 7) of 1. Above this value, some acoustic pressure waves, which cannot be captured by the Reynolds equation, appear in the gas film.

1 It is thus necessary to check these two conditions before using the Reynolds equation
2 to simulate the gas flow during near field acoustic levitation.

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