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On Two Computational Models of the Pitch-Rhythm Correspondence

A Focus on Milton Babbitt's and Iannis Xenakis's Theoretical Constructions

According to a well-established tradition in music theory and systematic musicology, pitch space and the rhythmic domain share the same mathematical underlying structure, i.e. they are ›isomorphic‹. This algebraic relation allows the transfer of pitch-based operations and transformations into the rhythmic domain by interpreting the octave identification as a periodicity in the time axis and the pitch interval as a temporal distance between two successive onsets. As claimed by Godfried Toussaint in his book devoted to the geometric aspects of musical rhythm, »It is well known that there exists an isomorphic relation between pitch and rhythm, which several authors have pointed out from time to time.«¹ Therefore, he observes, there is no need to develop further this relationship, which explains the fact that there is no theoretical statement about the pitch-rhythm correspondence in the entire book.²

Historically, although one may find some attempts at establishing formal relationships between the pitch and time domains in the theoretical writings by

1 Toussaint 2013, p. xiii.

2 This assumption is far from being universally accepted, as the following quotation by music theorist Justin London clearly shows. In an article entitled *Some Non-Isomorphisms between Pitch and Time*, he observes that pitch space and the rhythmic domain are essentially non-isomorphic. The lack of isomorphism derives from the observation that »there are no temporal analogs to octave and enharmonic equivalence and that there are no tonal analogs to various limits on our temporal perception and acuity« (London 2002, p. 128). This critical perspective on the pitch-time correspondence offers a nice counterpoint to the main constructions we will discuss in detail in the paper. Our approach follows Jeff Pressing's assumption of the existence of a cognitive isomorphism between pitch and rhythm (Pressing 1983), to which we add an algebraic perspective derived by Milton Babbitt's and Iannis Xenakis's theoretical construction. For a more advanced mathematical model on the Pitch/Rhythm correspondence, see Anatol Vieru's treatise *Cartea modurilor* (1980) and its algebraic formalization by Dan Vuza (1985).

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Joseph Schillinger (1941), the pitch-rhythm correspondence becomes explicitly an algebraic isomorphism within Milton Babbitt's formalization of the twelve-tone system. Babbitt's PhD thesis, submitted in 1946 and finally approved in 1992, contains the germs of this formal correspondence, which is rooted in the set-theoretical as well as permutational character of the system.³ But the idea of a technical correspondence between the pitch and the time domains was formalized for the first time in the early 1960s in an article entitled *Twelve-Tone Rhythmic Structure and the Electronic Medium*.⁴ In order to approach the time dimension from a pitch-theoretical perspective, Babbitt proposes two concepts that make use, in different ways, of the »immanently temporal nature of the twelve-tone pitch-class system.«⁵ These two concepts express some axiomatic properties that are valid, according to Babbitt, for all temporal relations between two kinds of musical structure: the durational row and the time-point system.

The durational row

Babbitt uses this technique in his *Three compositions for piano* (1947), which is, from the American perspective, the first piece belonging to integral serialism.⁶ Example 1 shows the rhythmic pattern P used by the composer in his three compositions. The numerical values assigned to the durational row in Example 1 assume the sixteenth note as the minimal durational unit.

3 See Babbitt 1946/1992.

4 Babbitt 1962.

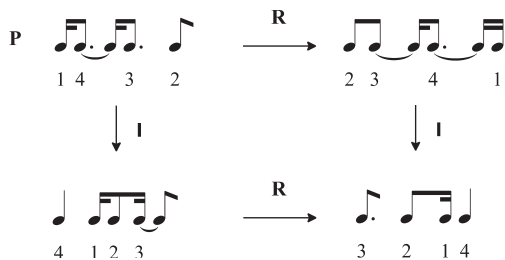
5 See Babbitt 1962, p. 151.

6 The European perspective prefers to »ignore« this historical antecedent and focus on the central place occupied by Olivier Messiaen and his total-serialized non-dodecaphonic piece *Mode de valeurs et d'intensités* (1949). This piece had indeed a considerable influence on a generation of composers as the germ of thought in Europe about the generalized series. Nevertheless, as was suggested some years ago, when a French-American Musicological Meeting was organized at IRCAM on the set-theoretical tradition from both American and European perspectives, there is a direct influence by Milton Babbitt on the very conception of this third piece from the *Quatre études de rythme*. In fact, Milton Babbitt attended Olivier Messiaen's composition classes at Tanglewood in 1948, that is, one year prior the creation of the piece *Mode de valeurs et d'intensités*, which explains why the first two *Études de rythme* were composed in the United States, as the frontispiece of the partition indicates. The reference »Darmstadt – 1949« on the score of *Mode de valeurs et d'intensités* therefore confirms that the piece was indeed composed after Messiaen and Babbitt met. See Andreatta et al. 2008 for a discussion of the set-theoretical tradition from a larger and trans-national perspective.

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Example 1: Rhythmic pattern of M. Babbitt's *Three compositions for piano* (1947)

It is now possible to apply to this rhythmic pattern the three usual twelve-tone operations – that is, inversion, retrograde and inverted retrograde. In doing so, one has to attend to the way in which the analogy is constructed between the inversion operation in the pitch-domain and the rhythmic inversion applied to the durational row. The major difference concerns the value according to which the complement (that is, the difference) is calculated (modulo 5 instead of the usual complement modulo 12, as in the pitch domain). The durational row $P=(1, 4, 3, 2)$ is therefore transformed in the structure $(n-1, n-4, n-3, n-2)$ with $n=5$, which leads to the inversion $I(P) = (4, 1, 2, 3)$. As in the case of the traditional ›twelve-tone group‹, the operation P plays the role of the identity, so that one can simply write I instead of $I(P)$. Analogously, the retrograde of a durational row reverses the order of their elements, which leads to $R(P)=(2, 3, 4, 1)$, and to the inversional retrograde $RI(P) = (3, 2, 1, 4)$. The four ›twelve-tone‹ forms of the rhythmic pattern P are represented in Example 2.



Example 2: The four ›twelve-tone‹ forms of a rhythmic pattern

As in the case of pitch space, the same algebraic structure (formally known as ›the Klein group‹) acts by permuting the elements of the original rhythmic pattern. It therefore represents an algebraic tool whose structural component, with respect to combinatorial techniques used by composers of the same period, represents a significant progress in the music-theoretical domain.⁷ Nevertheless, as pointed out by several music theorists, the durational row technique is not a satisfactory answer to the problem of pitch-rhythm combinatorics since the traditional twelve-tone operations destroy the perceptual character of the temporal intervals. This simple consideration would be enough to justify Babbitt's elaboration of a second concept that will remain one of his most original inventions: the time-point system.

⁷ See Fritts 2000 for a short introduction to the concept of group and group actions in Babbitt's compositional techniques, and more generally, in 20th century music.

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The time-point system

This concept takes its origin from the interpretation of the intervallic distances between different pitches of a twelve-tone row in terms of durations. In the explication given by Babbitt, a »pitch number is interpretable as the point of initiation of a temporal event, that is, as a time-point number.«⁸ In order to »project« a series of pitches onto the rhythmic domain, Babbitt uses the concept of »modulus«, a time-span divided into 12 minimal units representing the twelve classes of time-points.

A juxtaposition of several moduli represents the temporal grid onto which the twelve pitch-classes will be projected. A twelve-tone series can therefore be interpreted rhythmically in different ways, since a pitch-class integer can be projected in different ways within the temporal grid.⁹ The following musical example shows how a twelve-tone row can be realized rhythmically by taking the thirty-second note as a minimal durational unit.

Example 3: The rhythmic realization of a twelve-tone row via the technique of the time-point system

Note that the time-point system enables the composer to establish a much more natural and perceptually intuitive correspondence between the twelve-tone operations in the pitch realm and their rhythmic analogs. To these considerations, Babbitt adds a remark concerning the possibility of applying combinatoriality to the time-point system, which enables the construction of sometimes very complex temporal patterns, such as rhythmic canons by inversion whose »total rhythmic progression is in disjunct aggregates.«¹⁰

From a theoretical point of view, Milton Babbitt enumerates eleven qualitative properties based on the order relation (indicated with the mathematical symbol

8 Babbitt 1962, p. 162.

9 See Mead 1994 for a comprehensive introduction to the music of Milton Babbitt and his time-point system.

10 Babbitt 1962, p. 170.

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<). In contrast to the modular congruence relation, upon which the twelve-tone system is based, this relation is not, mathematically speaking, an equivalence relation, since neither the *reflexive* property nor the *symmetric* properties hold. (In other words, there is no musical event x for which $x < x$ and the relation $x < y$ never implies that $y < x$. The only property that is shared with the equivalence relation is *transitivity*, since given three musical events x , y , and z , the two relations $x < y$ and $y < z$ imply $x < z$.)

The axiomatics proposed by Milton Babbitt reflects a major concern of many music theorists of the 1960s, in particular Michael Kassler (1967) and Benjamin Boretz (1969/1995), who also abandon mathematical equivalence in order to focus on order relations between musical structures. This is exactly Iannis Xenakis's starting point for a musical axiomatics that is inspired by Giuseppe Peano's description of natural numbers and that directly leads to Sieve Theory.¹¹

The pitch-time correspondence in Xenakis's Sieve Theory

In order to understand the algebraic nature of pitch-time correspondence in Xenakis's writings, it is important to provide a short introduction to his algebraic thought. Almost contemporaneously with the emergence of an algebraic perspective in the American set-theoretical tradition, there appear in the 1960s the first signs of Xenakis's consciousness of the algebraic nature of the musical system.

There is an evident ›conceptual‹ proximity between Xenakis's and Babbitt's music-theoretical thought.¹² This proximity is particularly striking if one considers the insistence with which Babbitt speaks about the set-theoretical, and at the same time, algebraic character of musical structures. As in the case of Babbitt, Xenakis postulates in his writings the existence of an interplay between set-theoretical constructions and algebraic formalization that resides in the abstract nature of modern mathematics. This reflection finds a well-defined place in his book entitled *Musiques Formelles* (1963/1981), where the search for »new formal

11 See the article by John Rahn entitled *Logic, Set Theory, Music Theory* (Rahn 1979/2001) for a different use of order relation in the formalization of music-theoretical properties.

12 See Andreatta 2003 for an account of such a ›conceptual‹ proximity starting from a mathematical investigation of algebraic methods in 20th century music and musicology. See also Schaub 2009 for a critical evaluation of the relations between Xenakis's and Babbitt's theoretical, compositional, and analytical constructions.

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principles in music composition« is deeply related to the concepts of abstraction and formalization in mathematics.

Following Hilbert's and Peano's axiomatic approach in mathematics, Xenakis proposes to define a new category in contemporary musical thought: ›symbolic music‹. In fact, according to Xenakis, »formalization and axiomatization constitute [...] a procedural guide, better suited to modern thought.«¹³ This position is very close to what Milton Babbitt proposes within the American set-theoretical tradition, especially with respect to the relationship between scientific language and music theory, most notably in his article *The Structure and the Function of Music Theory*.¹⁴ The central notion around which Xenakis defends the axiomatic approach is, as in Babbitt's case, that of the interval. But in contrast to the American music-theorist, Xenakis does not need to use the notion of congruence modulo 12 as a necessary condition for defining the concept of interval. According to Xenakis, the equal-tempered system is just one particular case with respect to a much more general phenomenon that also concerns other musical dimensions such as intensities and durations. This clearly shows why Xenakis's Sieve Theory, which would be conceived some years later, has a much more generalizing power than Babbitt's interval theory. As stated by Xenakis:

In music, the question of symmetries (spatial identities) or of periodicities (identities in time) plays a fundamental role at all levels, from the sample in sound synthesis by computer to the architecture of a piece. It is thus necessary to formulate a theory permitting the construction of symmetries which are as complex as one might want, and inversely, to retrieve from a given series of events or objects in space or time the symmetries that constitute the series. We shall call these series »sieves«.¹⁵

Interestingly, the point of departure for Xenakis's Sieve Theory is the same as Babbitt's, having to do with the mathematical concept of ›total order‹; such an order obliges any couple of elements a and b of a set to have either $a < b$ or $b < a$. This is the case, for example, of the set of natural numbers \mathbb{N} , of positive or negative integers \mathbb{Z} , of rational numbers \mathbb{Q} , or real numbers \mathbb{R} , with the usual ›total order‹ relation (indicated with the symbol $<$). Note that Xenakis makes no attempt to explicitly develop a general algebraic theory based on ordered structures, although one may find some suggestions on how to generalize the sieve-theoretical constructions with the help of partial ordered structures such as

¹³ Xenakis 1963/1981, p. 212. The English translation is quoted from Xenakis 1992, p. 178.

¹⁴ Babbitt 1965.

¹⁵ Xenakis 1992, p. 268.

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trees and networks. As pointed out in the revised and enlarged English-language version of his doctoral thesis entitled *Arts/Sciences: Alloys*, Sieve Theory is very general since it can be applied to any musical parameter possessing a total order structure (such as intensities, densities of events, degrees of order/disorder, attacks, etc.).¹⁶ Xenakis also predicts the explosion of interest and applications of this approach in the following years thanks to its computational character, which makes the theory suitable for computer-aided investigations. He also points out the necessary generalization of the total-ordered structures towards partial-ordered structures, such as those which are introduced in the study of timbre and that make use of lattices and networks.

History has validated Xenakis's predictions in two ways: first, through the development of computer-aided environments in which Sieve Theory has been implemented as a tool for computational music analysis and composition, and second, through the development of partially ordered structures in the last thirty years. In the first case, I would like to mention the use of Sieve Theory by two leading figures of French computational musicology, André Riotte and Marcel Mesnage. They both not only implemented Xenakis's original model in a computational environment, the *Morphoscope*,¹⁷ but they suggested how to generalize the initial pitch-theoretical model in order to obtain complex rhythmic structures. The generalization consists in taking as the underlying support space not only a ›regular‹ rhythmic space, but any kind of rhythmic pattern that will be ›sieved‹ in such a way that the different elementary sieves will produce complex results by the use of classical set-theoretical operations (such as union, intersection, or complementation). Example 4 shows a possible generalization of Xenakis's Sieve Theory starting with an irregular underlying rhythmic space (called the *structure métrique*).

¹⁶ See Xenakis 1985, p. 108.

¹⁷ Several articles devoted to this programming language are contained in the two volumes collection of writings by André Riotte et Marcel Mesnage entitled *Formalismes et modèles musicaux* (Riotte/Mesnage 2006).

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Structure métrique	
1 ₀	
2 ₀	
3 ₀	
5 ₀	
	2 ₀ U3 ₀ U5 ₀

Example 4: A rhythmic pattern obtained by the union of elementary sieves having as a support space a non-regular metric grid (quoted from the first volume of Riotte and Mesnage 2006, p. 83)

Pieces making use of this generalized approach to Sieve Theory include *La Bibliothèque de Babel* (1985) by André Riotte, for reciting voice, two solo voices, two choirs, a quintet of brass instruments, percussion, and an orchestra of wind instruments; *Color* (1986) by Claudy Malherbe, for 18 instruments; and *Partitions-gouffres* (1986), an algorithmic piece by André Riotte for four percussionists.¹⁸

As suggested earlier, Xenakis's introduction of total-order relations in the formalization of musical structure provides a first axiomatization of musical scales, which is nothing less than a translation into the musical domain of Peano's formalization of integer numbers. More precisely, Peano's axioms are translated into the musical domain with the help of three basic concepts: the origin (or stop), the set of elements D (or durations), and the relation ›successor of an element n ‹, which is symbolized by the symbol n' . The six ›first propositions‹, as Xenakis calls them, take the following form:¹⁹

1. The origin (or stop) is an element of the set of durations D .
2. If stop n is a duration, then its successor n' is also an element of D .
3. If stops n and m are elements of D , then the respective successors n' and m' are identical if the two stops n and m are identical.²⁰
4. If stop n is a duration, it will be different from stop O at the origin.
5. If elements belonging to D have a special property P , such that the origin also has it, and if, for every element n of D having this property its successor n' has it also, all the elements of D will have the property P .

¹⁸ For a wider perspective on algorithmic composition, including sieve-theoretical techniques, see Andreatta (2013).

¹⁹ The list of six axioms is adapted from Xenakis's *Formalized Music* (Xenakis 1992, p. 195).

²⁰ By ›iff‹ (read ›if and only if‹) is denoted the necessary and sufficient condition for a property to be true.

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As in the case of Milton Babbitt's durational row or time-point system, Xenakis's axiomatics is derived from an underlying formalization that originally applied to the pitch domain. Interestingly, if the set-up of Sieve Theory in the pitch domain goes back to the 1960s (Xenakis, 1965), the transfer of Peano's axiomatics to the rhythmic domain, at least from a theoretical point of view, only occurs in the 1980s. Surprisingly, the theoretical formalization is anticipated in several compositions by Xenakis, such as *Persephassa* (for a sextet of percussionists) or *Psappha* (for multi-percussion solo), dating respectively from 1969 and 1975, in which the rhythmic structures are explicitly obtained by application of sieve-theoretical constructions in the rhythmic domain. The article in which Xenakis formalizes for the first time the pitch-rhythm correspondence with the tools belonging to Sieve Theory is *Redécouvrir le temps* (Xenakis 1988/1996), in which Xenakis's previous work on the categories ›on time‹/›outside-of-time‹ (*en temps/s/hors temps*) provides the general framework for understanding temporality. In Xenakis's words, »any temporal scheme [...] is an ›outside-of-time‹ representation of the temporal flow within which the phenomena take place.«²¹ The axiomatics of temporal structures allows for a better understanding of the notion of separability between temporal events, which can therefore be »assimilated to *attacks-point* within the time flow.«²² Note the similarity between Xenakis quotation and Milton Babbitt's time-point system, the only conceptual difference being Xenakis's rejection of the primacy of the octave. This makes Sieve Theory an extremely powerful tool for computational music analysis, as amply demonstrated by André Riotte and Marcel Mesnage in their theoretical and analytical writings.²³

A comparison of attacks-points naturally leads to the notion of distance, which becomes operational precisely through the use of Sieve Theory, provided that the sieve 1_0 is identified with the most elementary rhythmic notion, i.e. the regular rhythm. As in the case of pitch, by using the three main set-theoretical operations which are the union, the intersection, and the complement, »one can build very complex rhythmic architectures which can simulate the pseudo-aleatoric distribution of points in a line, if the period is long enough.«²⁴ This clearly shows the possibility of recovering probabilistic and statistical music processes from a deterministic approach guided by set-theoretical construction,

21 Xenakis 1988/1996, p. 40.

22 Xenakis 1988/1996, p. 41.

23 Riotte/Mesnage 2006.

24 Xenakis 1988/1996, p. 43.

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and therefore the primacy, from a theoretical point of view, of the category of ›Symbolic Music‹ with respect to the category of ›Stochastic Music‹. The pitch-time correspondence definitely plays a major role in this structural perspective.

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