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Introduction

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► **To cite this version:**

Moreno Andreatta, Emmanuel Amiot, Jason Yust. Introduction. Journal of Mathematics and Music, 2020, Special Issue “ Geometry and Topology in Music ”, 14 (2), pp. 107-113. 10.1080/17459737.2020.1793528 . hal-03031203

HAL Id: hal-03031203

<https://hal.science/hal-03031203v1>

Submitted on 30 Nov 2020

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Andreatta M., E. Amiot et J. Yust Eds. (2020), "Introduction", *Journal of Mathematics and Music*, Special Issue « Geometry and Topology in Music », vol. 14, n° 2, p. 107-113.

INTRODUCTION

This special issue focuses on some *geometrical* and *topological* aspects of the *representation* and *formalisation* of musical *structures* and *processes*. There are six words in the previous sentence which have been emphasized since they could work as explanatory guidelines for the present issue. More generally, they seem to offer to the “working” musicologist and music-theorist some conceptual and operational principles for approaching contemporary “mathemusical” research from a more philosophical and epistemological perspective. Shedding some light to this double underlying dimension may constitute a fruitful exercise, so we hope, for the reader of the *Journal of Mathematics and Music* and of the present issue in particular. Let us start with *geometry*, one of the oldest fields of mathematics and whose history accompanied the discussions on the relations between music and mathematics since the time of Greek philosophy. Geometry has a central place in the Pythagorean tradition, and particularly in Plato’s philosophy, which is well expressed by the famous quotation that was supposed to be engraved at the door of the Plato’s Academy in Athens: “Let no one ignorant of geometry enter.” In Plato’s perspective, geometry is surely far from what it should be according to its etymological meaning, which reduces the scope of the discipline to an “earth measure.” Plato himself found such a name totally ridiculous, as he says in the *Epinomis*, the short dialogue that serves as an appendix to Plato’s *Laws*.¹ As it has been pointed out by several scholars and commentators of Plato, starting from the great and influential music theorist Aristoxenus of Tarentum, geometry is first of all a science of abstraction enabling to transcend the level of sensible experience and reach the world of Ideas. One is immediately reminded of the experiment described in the *Meno*, where the slave boy discovers the existence of truths via geometrical thinking, as in the case of doubling the surface of a square.² Geometry is therefore intrinsically linked to the visual way in which concepts are represented and, eventually, drawn, as it is largely shown by Euclid in his *Elements*.³ The influence of this geometrical thinking goes until the twentieth century, as one may observe by considering, for example, David Hilbert’s debt to Euclid as documented in his *Grundlagen der Geometrie* (1899).

It can be easily shown that this emphasis on the axiomatization of geometry, which is prevalent in Hilbert’s work, has a direct influence in the way in which twentieth century composers, starting from Ernst Krenek (1937), approached the problem of the theoretical foundations in music via formalization.⁴ A turning point towards a full axiomatic thinking in geometry, with extremely significant musical consequences, is symbolized by the way in which geometry is progressively approached via an algebraic perspective. Starting from Felix Klein’s *Erlangen Program* (Klein 1893), the way of conceiving the geometry of a space is in fact intrinsically linked to the way in which transformation groups act on such a space. With the entry of abstract group theory into play we assist to a progressive shift from the purely representational nature of geometrical entities to their truly operational power which has a big influence on the way in which music theorists conceptualize the musical phenomena.⁵ This little historical digression may offer some elements to understand the complex link one can establish between the geometrical properties of musical structures and their underlying formalization which accompanied the evolution of mathematical music theory from a simply representational to an operational discipline capable of approaching in a powerful

way the dynamic character of musical processes. *Topology* – let us now focus on this term – precisely enters the picture as a subsidiary discipline when the focus on *musical structures* is substituted or at least accompanied by a new attention of music theorists and analysts towards *musical processes*, i.e. theoretical constructions in which time plays a fundamental role. The extensive use of topological techniques in the mathematical modeling of musical structures and processes is a relatively new phenomenon, although one may already find many examples of genuine topological constructions in Guerino Mazzola's *Geometrie der Töne* (1990). The foundational basis of the concept of “global compositions,” as opposed to the “local” ones (originally considered as finite non-empty subsets of a module over a ring), is the mathematical structure of Riemannian manifolds allowing the music theorist to make use of several topological constructions. The reader familiar with this approach in mathematical music theory will easily recognize some traditional visualizations of musical structures, such as the Möbius strip⁶ and, more generally, the nerve of a n -dimensional simplicial complex as representing the complexity of the specific covering that the analyst will associate to a given global composition. Interestingly, there is already in this approach an interplay between geometry and combinatorial topology which shows that there is a shift from the simple characterization of (static) musical structures to (dynamic) musical processes, such as modulation, whose properties are now described in a topological way.⁷ More recently, it became clear to music theorists that some of most popular constructions in neo-Riemannian and, more generally, transformational music analysis do intrinsically possess a topological structure. The most celebrated example is probably the *Tonnetz*, originally mostly studied from a graph-theoretical and algebraic perspective, and successively apprehended from a more topological way, enabling the complete characterization of their different instances (or *Tonnetze*) via the study of the associated algebraic invariants, such as Betti numbers (Bigo, Giavitto, and Spicher 2011; Catanzaro 2011; Bigo 2013). This interplay between geometry, algebra and topology provides nowadays a better understanding of the way in which generalized musical spaces can be represented and formalized from a very elegant and computationally powerful perspective.⁸ Each of the three contributions of the present issue can be considered as offering to the reader a specific perspective on this complex interplay starting from different but intrinsically linked music-theoretical problems.

“Why Topology?” by Dmitri Tymoczko makes use of topological techniques to formalize voice leadings as paths in n -dimensional geometric spaces. The approach is an extension of the conceptual constructions that constitute the foundation of Tymoczko's geometrical theory of music as presented and discussed in details in his book *A Geometry of Music* (Tymoczko 2011). Previous work by the author also uses topological techniques but without the detailed argumentation for the value of topology in contemporary mathematics and music research.⁹ Higher dimensional geometric spaces whose points are generic n -notes chords are *orbifolds* in a technical mathematical sense, i.e. quotient spaces obtained by the action of the permutation (or symmetric) group of $n!$ elements on the n -dimensional torus and possess therefore non-trivial topological properties. The present paper focuses on the interpretation of voice-leading properties via the fundamental group and can therefore offer new insights to the analyst to fully understand the author's previous music-theoretical constructions. Moreover, it shows that not only voice leading but, more generally, transformational constructions in music analysis are linked to the fundamental concept of homotopy equivalence, which again puts algebra – and group theory in particular – into play. Although homotopy theory has become very popular in recent years thanks to Guerino Mazzola's gesture theory,

the approach described in Tymoczko's paper is fundamentally different since it provides the theoretical background to fully understand the topological properties of different harmonic spaces.

While Tymoczko's voice-leading geometries have led to many significant analytical and music-theoretic applications, as is in evidence of *Geometry of Music*, to some extent there has always been a disconnect between the higher dimensional orbifolds defined in Callender, Quinn, and Tymoczko 2008 and music analysis and composition, because as the spaces get more complex we lose the benefit of direct spatial intuition. In previous work (such as Tymoczko 2011) spatial intuition has often been recovered by simplifying chord spaces to lattices on a limited number of chord-types, which renders their more interesting topological features irrelevant. In the present work, Tymoczko instead uses homotopic equivalence to simplify the spaces. Remarkably, he is able to reduce these topological features to simple two-dimensional representations, making them easily surveyable and available to analytical intuition, regardless of the chord cardinality and resulting complexity of the original space. He introduces two kinds of simplifications: one is the circle of pitch-class sums, which has the topology of an annulus or circle, regardless of cardinality. The homotopy of this space corresponds to total voice-leading ascent and descent. The other simplification is a homotopic classification of set-class spaces (which factor out transposition or transposition or inversion). The analytical applications in Tymoczko's article begin to reveal the wealth of musical interest in the interactions between features of a voice leading that we usually are tempted to "reduce out," such as voice crossings and transpositions along the chord. The article also introduces a principled generalization of neo-Riemannian transformations to any chord type, as voice leadings to its inversions.

The following paper by Jason Yust, entitled "Generalized *Tonnetze* and *Zeitnetze*, and the Topology of Music Concepts," also focuses on the interplay between the topological structure of the generalized *Tonnetz* seen as a n -dimensional simplicial complex and its possible graph-theoretical and geometric realizations. The notion of a generalized n -chord *Tonnetz* can be articulated in fact at different levels of abstraction, ranging from the traditional presentation as a lattice of chords relating by maximal intersection to more sophisticated representations, embedding this structure on the powerful theory of Fourier phase spaces. This further interpretation adds a new subfield of mathematics to the overall picture, also showing the natural bridges between geometry, algebra, and (mathematical) functional analysis. Many music-theoretical problems, from the construction of rhythmic tiling canons to the classification of homometric musical structures,¹⁰ have been shown to be deeply rooted into Fourier theory, but adequate understanding of the topological implications of such a formalism still constitutes an open research area in mathematics and music. A first crucial achievement has been the characterization of the geometrical structure of generalized *Tonnetze* as toroidal representations in the spaces of Fourier phases, as proposed by Emmanuel Amiot (2013) and further developed by Jason Yust in his critical discussion on the relations between the geometric extensions of Generalized *Tonnetze* and the Fourier phase Spaces (Yust 2018). The present article pushes the previous conclusions further by focusing on the implications of geometrical *Tonnetze* interpreted as embedding a simplicial *Tonnetze* in some geometric space. Yust's article complements Tymoczko's in showing how a different theoretical emphasis, on shared pitch-class content between chords rather than the distances moved by voices, leads in the direction of different kinds of topologies. Despite the different mathematic foundations of their spaces (Fourier phase vs. voice-leading orbifold), the two theories arrive at surprisingly similar constructions in places, with some

of Yust's foldings producing spaces topologically equivalent to Tymoczko's pitch-class sum annuli. Despite appearances however, a closer examination reveals that a path or homotopy-class of paths in the two kinds of space actually have distinct musical meanings. The two approaches also lead to interestingly different generalizations of neo-Riemannian transformations, with Tymoczko focusing on the idea of inversive voice leadings while Yust emphasizes maximal intersection.

There is much music-theoretical and mathematical potential in using this new geometric approach, as the reader will see by following Yust's systematic presentation of a panoply of generated collections. These include dyadic *Tonnetze* (that reduce to the traditional chromatic and fifths-generated circular representations of the equal-tempered system), trichordal *Tonnetze* and higher-dimensional *Tonnetze*. All these geometric structures are first defined in toroidal spaces before applying higher-dimensional foldings which shed a new light on different n -chord generated spaces. To this family belong, for example, the *Tonnetz* of chromatic tetrachords and of different 4-note chords and also *Tonnetze* of ninth chords, which are rare in the literature. Special attention is given to *Tonnetze* of diminished seventh chords, which enables the author to stress the fact that an object that is trivial from a graph-theoretical and simplicial perspective can still be interesting from a geometrical and music-theoretical perspective. In addition to its elegant and powerful character, taking phase spaces as a geometrical realization of *Tonnetze* has dramatic consequences in the way in which traditional neo-Riemannian harmonic constructions can be generalized and applied to other musical parameters, and in particular to rhythm. Building bridges between harmonic and rhythmic spaces has a long tradition in music theory and composition, as the history of serialism clearly shows. Many other attempts at providing a general theory on the structural relations between harmonic and rhythmic domains have marked the development of mathematical music theory, from Mazzola's formalization of the two domains as local/global compositions based on the same underlying module structure to more perceptual-based approaches, such as Jeff Pressing's foundational studies on the cognitive isomorphism between pitch and rhythm in ethnomusicology (Pressing 1983). In defining a "*Zeitnetz*" in analogy with the *Tonnetz*, the paper also suggests new very interesting connections between research in mathematical music theory and problems raised by scholars working on different areas, in particular cognitive and empirical musicology.

The final paper by Mattia G. Bergomi and Adriano Baratè, entitled "Homological persistence in time series: an application to music classification", provides a third perspective on the interplay between geometry, topology and algebra focusing on the problem of automatic stylistic music analysis. Based on persistence homology as a specific domain in the field of topological data analysis, the paper shows not only the possibility but, for some extent, the necessity of establishing a fruitful dialogue between two apparently orthogonal research traditions in the field of application of mathematical models to music analysis: mathematical music theory and music information research (MIR). Despite efforts to bring together researchers belonging to the *Society for Mathematics and Computation in Music* with those working in the area of the *Society for Music Information Retrieval*, it is not difficult to recognize that there are deep methodological and theoretical differences between these two major orientations in the field of computational musicology.¹¹ The paper by Bergomi and Baratè begins to fill the gap by applying a panoply of topological, geometrical, and algebraic techniques to the domain of automatic stylistic music classification. Providing models and tools for the automatic stylistic classification is probably one of the most active areas in the field of music information research. Moreover, the authors' focus on time series

suggests that topology not only applies to the formalization of musical structures but clearly deals with the modeling of musical processes (or, what they call “time-varying systems”). Using persistence homology as the main conceptual and theoretical framework for approaching style analysis is a relatively recent and very promising approach. The approach originates in one of the authors’ dissertation ([Bergomi 2015](#)) which gave rise to different research directions exploring the interplay between geometry, topology, and, more recently, category theory.¹² The present paper clearly shows the interplay between static topological representations of musical structures and dynamic geometric formalizations of these topological features. In the former case, the authors use persistent homology as the main topological framework to represent the inner structure of musical object via a collection of persistent diagrams. These static representations are then studied dynamically by formalizing their temporal evolutions as time series that can be compared through dynamic time warping. This very rich mixture of topological and geometrical techniques in the representation and formalization of musical structures and processes, together with their implementation in computer-aided analytical systems, also enable the authors to offer a new and complementary computational perspective on the generalized *Tonnetz* with respect to the one presented in the first two articles. It also starts with the same simplicial complex interpretation of the *Tonnetz* as described in Yust’s article, but instead of focusing on their subcomplexes generated by a sequence of pitch classes (as extensively studied in [Bigo 2013](#) and [Bigo et al. 2013](#)) it proposes to take into account additional music information in order to break the isotropic character of the simplicial complex. The new geometric space in which musical structures and processes are represented is therefore a deformed *Tonnetz*, i.e. a non-isotropic *Tonnetz* exhibiting variable geometric features which can be retrieved via ad-hoc filtration functions. This enables the authors to approach automatic stylistic music classification in a geometric structural way, which is very original in the field of music information research, where statistics and machine learning constitute the main theoretical framework for retrieval purposes. The tests that have been performed by the author on different data sets, and which are discussed in the final part of the article, show very promising results that shed new lights on the nature of musical style.

We hope that this short introduction provides the reader with enough elements to appreciate not only the singular character of each of the three papers published in this special issue but their deep and fruitful intersections. We insisted several times on the necessity of building bridges between different music-theoretical and computational approaches; but before establishing connections between research groups having different aims and methodological criteria, it is important to reinforce the synergies within the members of the same community. Research carried out by members of the Society for Mathematics and Computation in Music has spread in a multitude of different directions, as the readers of our Journal can easily see. Whereas each of the three contributions offers distinct perspectives on the same music-theoretical constructions, all of them together provide a comprehensive picture of the interplay between geometry and topology which is surely more than the sum of the three local perspectives. Probably the time for a synthetic view on all these different approaches has not yet come, but the reader has the opportunity to build his or her own conceptual bridges between geometry and topology and their mutual contribution to music representation and formalization.

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NOTES

¹ This and other very interesting aspects of the role of geometry in Plato's dialogues are developed by Athanase Papadopoulos in his forthcoming essay devoted to the relation between music, mathematics and astronomy starting from Plato's *Timaeus* ([Papadopoulos 2020](#)).

² There are several editions available of Plato's dialogues, including the two mentioned ones. See, for example, Plato's collected work edited by John M. Cooper and Douglas S. Hutchinson ([Plato 1997](#)).

³ The thirteen books of Euclid's *Elements* are available online at Clay Mathematics Institute Historical Archive (<https://www.claymath.org/library/historical/euclid/>)

⁴ See [Andreatta \(2003\)](#) for a first attempt at reconstructing this legacy from both an European and American perspective.

⁵ This dialectical principle between the *objectal* and *operational* nature of the reality has been deeply studied by the French philosopher and epistemologist Gilles-Gaston Granger who took this duality as the foundational basis for the very notion of "concept" in philosophy ([Granger 1947/1994](#)).

⁶ Which the author also calls "Harmonisches Band" to emphasize its truly music-theoretical origin, that he bases on Arnold Schoenberg's writing on the theory of harmony ([Schoenberg 1911](#)).

⁷ There are other examples that could be considered as genuinely "topological" within the same approach in mathematical music theory. We only mention two of them, both dealing with the formalization of musical motifs as combinatorial and topological structures: [Netske \(2004\)](#) and [Buteau and Mazzola \(2008\)](#).

⁸ We could also add to the picture the categorical perspective which is intrinsically linked to the interplay between geometry and algebra and which can also shed some new light on existing topological approaches not only in music theory and analysis but also in performance studied like in the case of a category and topos-based theory of musical gestures. See in particular the third volume of the revised version of *Topos of Music* ([Mazzola 2017](#)). The reader who might be interested in going more deeply into some philosophical and epistemological aspects of the categorical perspectives on the interplay between geometry, algebra, and topology, will find a good starting point in Jean-Pierre Marquis' monograph *From a Geometrical Point of View* ([Marquis 2009](#)).

⁹ See, in particular, [Callender, Quinn, and Tymoczko \(2008\)](#) for a more detailed presentation of the technical aspects of a geometrical approach in music theory, also including the introduction of orbifold structures into music formalization.

¹⁰ Summarized in [Amiot \(2016\)](#).

¹¹ As pointed out by Anja Volk and Aline Honingh in their introduction of the special issue of the Journal of Mathematics and Music devoted to "Mathematical and computational approaches to music" ([Volk and Honingh 2012](#)), this gap was already acknowledged by Thomas Noll and Robert Peck in the first issue of the Journal of Mathematics and Music in 2007 ([Noll and Peck 2007](#)).

¹² See, in particular, [Bergomi, Baratè, and Di Fabio \(2016\)](#) for a topological perspective and [Bergomi et al. \(2019\)](#) for the first attempts at going beyond a topological approach in the study of a category theory-based persistent homology. Some extensions of the original framework have also been intensively studied within the SMIR (Structural Music Information Research) project. This research project is a permanent interdisciplinary research axis supported by CNRS and led by Moreno Andreatta at the University of Strasbourg / IRMA (Institut de Recherche Mathématique Avancée), in a formal collaboration with IRCAM. Research topics include Mathematical Morphology, Formal Concept Analysis, Persistent Homology and automatic classification of musical styles, Category theory and transformational (computer-aided) music analysis as well as the study of musically-driven open conjectures in mathematics and the epistemological and cognitive implications of contemporary mathematical research. See [Andreatta \(2018\)](#) for a short overview of the different research topics included in the SMIR project.