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Making sense of Born's rule $p_\alpha = \|\Psi_\alpha\|^2$ with the many-worlds interpretation

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Abstract This work is an attempt to justify Born's rule within the framework of the many-worlds interpretation proposed by Everett. Here, we develop a unitary model of many-minds based on the work of Albert and Loewer (Synthese **77**, 195 (1988)). At the difference of the model of Albert and Loewer ours is not genuinely stochastic and dualistic and also involves some classical-like randomness based on ignorance about initial conditions of the Universe. We also compare the present method for recovering Born's rule with previous works based on Decision theory *a la* Deutsch Wallace and envariance *a la* Zurek and found that all these approaches are strongly related to each other.

Keywords Everett · many-minds · probability · Born's rule

1 Introduction and motivation

The many-worlds interpretation (MWI) proposed by H. Everett in 1957 [1–3] is remarkable attempt for providing an ontology to quantum mechanics. However, the MWI has some unwarranted features concerning the role of probability plaguing early attempts for recovering the standard quantum predictions (aka the famous Born rule). During the last two decades new proposals have been discussed in order to make sense of quantum probability in the MWI by using subjectivist concepts such as self-locating uncertainty, degrees of belief, Laplacian principle of indifference, 'envariance' and so on and so forth. While in my opinion none of these 'Bayesian' and 'decision-theoretic' based scenarios have successfully and objectively recovered the Born rule these ideas nevertheless provide clues which strongly motivated the present work. I actually believe that it is perhaps not so impossible to save the unitary dream of Everett. However, to realize that goal we are going to resign something else

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about our preconceptions in physics and the question at the end would be to know if we are ready to accept that possibility (the issue will be let open at the end of this article).

As I am going to show in this very speculative work every thing is related to the meaning we try to give in the MWI to quantum measurements i.e., as these are experienced and memorized by the observers. The great discovery of Everett was to treat the observer quantum mechanically as a memory device or automaton in order to develop a self-consistent MWI. This strategy in turn had a huge impact for how the observer is going to interpret and make sense of probabilities. Here, I will try to emphasize the role of some old and probably out of fashions ideas about minds and brain elaborated in the late 1980's and early 1990's in connection with the MWI: i.e., the so called many-minds interpretation (MMI) of Albert and Loewer [4,5]. Based on a new unitary formulation of the MMI I will try to convince suspicious readers that we can develop a completely self-consistent unitary toy model for a quantum observer that recovers Born's probabilities in the MWI.

The paper is structured as follow: First, in Sections 2, 3 and 4 we will review the probability conundrum in the MWI by summarizing the main past attempts for solving the problem. We will start with the original Everett interpretation in Section 2, discuss the recent ontological modifications defining 'many-Bohmian paths' in Section 3 and also present the many-minds interpretation which plays a key role in our work in Section 4. Second, in Sections 5 we will discuss the quantitative problem of justifying Born's rule in the MWI and focus our analysis on the work made by Deutsch [6], Wallace [7] and Zurek [8] in the recent years using Bayesian deductions. Finally, in Section 6 we will propose a scenario for making sense of quantum probability inspired of recent works by Deutsch [6], Wallace [7] Zurek [8], Sebens and Carroll [9] and McQueen and Vaidman [10]. The proposal strongly differs from the previous attempts by introducing classical ignorance of the initial conditions linked with a idealized and unitary version of the MMI. In other words, we will show that by taking into account the problem of the mind observer described in the context of the all-unitary MWI together with some ingredients of randomness and uncertainty coming from quantum entanglement with the local environment could help us to decipher the still ambiguous concept of 'self-locating uncertainty'. Ultimately, we believe to be able to justify and recovering the Born rule with this sketchy model. While the model proposed here is highly speculative we hope this could ultimately prompt future works for obtaining more satisfying and realistic models of the observer 'minds' in the MWI.

2 Everett and the meta-theorem: the incoherence problem

The MWI has been the subject of intense and recurrent debates concerning the meaning and role to be given to quantum probabilities in this theory. Indeed, being a purely deterministic theory admitting strict unitary Schrodinger

evolution as the only rule, the usual 'collapse' probability formula

$$\mathcal{P}_\alpha^{(\mathrm{Born})} = \|\Psi_\alpha\|^2 = \|\langle\alpha|\Psi\rangle\|^2 \tag{1}$$

for an outcome α–also known as Born's law– and which is at the core of quantum measurements, pains to find a rational justification and interpretation in the MWI.

Everett [1–3], starting with a version of what is nowadays known as Gleason's theorem [11] discovered independently by him, introduced an additive measure $\mathcal{M}(\|\Psi_\alpha\|^2) \equiv \|\Psi_\alpha\|^2$ and subsequently identified it with a probability for the outcome α in the Hilbert space. Moreover, identifying the Gleason-Everett measure $\mathcal{M}(\|\Psi_\alpha\|^2)$ to a probability seems *apriori* paradoxical since the MWI doesn't contain chancy events or randomness which could allow us to speak about probability for the outcomes of being this or that. In the MWI all events occur in parallel and the only certainty is that, after an experiment, we will end-up with a probability $\mathcal{P} = 1$ in a superposed quantum state including many branches. In order to solve this contradiction and establish Born's rule Everett based his reasoning on the law of large numbers and the notion of 'typicality' advocated by Boltzmann for statistical mechanics [12] (this notion is also assumed by some 'Bohmians' [13]). Indeed, as it is explained in Everett's PhD thesis [2] (and as it was more rigorously demonstrated by Hartle [14], DeWitt and Graham [3,15] and several others[16,17]), a long-run experiment reproducing a multinomial sequence leads in the infinite limit, i.e., by a direct application of the law of large numbers, to the empirical statistical Born's rule. In other words, considering

$$|\Psi\rangle = \sum_\alpha \Psi_\alpha|\alpha\rangle \tag{2}$$

a quantum state with outcomes labeled by α which is analyzed in a multi-gates Stern-Gerlach experiment and taking a long-run sequence of the same experiment (i.e., by using a tensor product state like $|\Psi_N\rangle\otimes := |\Psi^{(1)}\rangle \otimes ... \otimes |\Psi^{(N)}\rangle$) Everett was able to obtain the relation

$$\mathcal{M}(\|\Psi_\alpha\|^2) = \lim_{N\to+\infty} \frac{N_\alpha}{N} := \mathcal{P}_\alpha \tag{3}$$

i.e., he was able to identify his measure to the relative frequency of occurrence $\frac{N_\alpha}{N}$, where N_α is the number of times the outcome α occurred in the long-run sequence with $N \to +\infty$ repetitions. Everett here used a standard frequentist definition of probability relying on an infinite ensemble but he also motivated his reasoning with Bayesian and epistemic concepts. In [2] he wrote:

> *We are then led to the novel situation in which the formal theory is objectively continuous and causal, while subjectively discontinuous and probabilistic.* [2], p. 9.

To understand his reasoning suppose, that $h := [\alpha_1,... \alpha_N]$ is a sequence of outcome, i.e., an 'history'[1]. By subjective probability Everett actually meant something which is directly measurable by the observer in the branch h where (s)he is located, i.e., completely ignoring the existence of the other decohered branches. In other words, for Everett the natural subjective probability is the limit frequency $\frac{N_\alpha(h)}{N}$ where $N_\alpha(h)$ is the number of times the event α was repeated in the history h. In the $N \to +\infty$ limit Everett shows that the total measure $\delta\mathcal{M}$ associated with histories h not fulfilling Born's rule is 'overwhelmingly' smaller than the measure associated with the set of histories satisfying Born's rule. At the limit $N \to +\infty$ the fraction goes to zero [2]. Moreover, a typical history $\bar{h}$ seen by a typical observer entangled with the system will confirm the record $\frac{N_\alpha(\bar{h})}{N} \simeq \|\Psi_\alpha\|^2$ with a error which is going like $\Delta N_\alpha/N_\alpha(\bar{h}) \simeq \frac{1}{\sqrt{N}}\sqrt{\frac{1-\|\Psi_\alpha\|^2}{\|\Psi_\alpha\|^2}} \to 0$. Therefore, as we say the Born rule is 'typical' in the Boltzmann sense since the overwhelming majority of the

[1] The notion of history used here is reminiscent of GellMann and Hartle work in the context of the consistent/decoherent histories interpretation [18]. Here, we use history for either a chronological series or for describing a large ensemble of N identical subsystems at a given time. The results would be the same since the subsystems are factorized and non-interacting.

[2] If we use the sequence $|\Psi_N\rangle = |\Psi^{(1)}\rangle \otimes ... \otimes |\Psi^{(N)}\rangle = \otimes_{i=1}^N |\Psi^{(i)}\rangle$ with $|\Psi^{(i)}\rangle = \sum_\alpha \Psi_\alpha |\alpha^{(i)}\rangle$ we can define a frequency operator as

$$\hat{Q}_\alpha = \sum_{i=1}^N \frac{\hat{\Pi}_\alpha^{(i)}}{N} \tag{4}$$

with the projectors $\hat{\Pi}_\alpha^{(i)} = |\alpha^{(i)}\rangle\langle\alpha^{(i)}|$ associated with the eigenvalue α for the ith subsystem. We can expand the total state $|\Psi_N\rangle$ as a sum over the different histories $h = [\alpha_1, ..., \alpha_N]$, i.e., $|\Psi_N\rangle = \sum_h |\Psi(h)\rangle$ with the history quantum state:

$$|\Psi(h)\rangle = \bigotimes_{i=1}^N \hat{\Pi}_{\alpha_i}^{(i)} |\Psi_N\rangle = \Pi_{i=1}^N \Psi_\alpha^{N_\alpha(h)} \bigotimes_{i=1}^N |\alpha_i^{(i)}\rangle \tag{5}$$

where $N_\alpha(h)$ is the number of times the outcome α occurs for the specific history h. Applying $\hat{Q}_\alpha$ on $|\Psi(h)\rangle$ leads directly to

$$\hat{Q}_\alpha |\Psi(h)\rangle = \sum_{i=1}^N \frac{\delta_{\alpha,\alpha_i}}{N} |\Psi(h)\rangle = \frac{N_\alpha(h)}{N} |\Psi(h)\rangle \tag{6}$$

where $\sum_{i=1}^N \frac{\delta_{\alpha,\alpha_i}}{N} = \frac{N_\alpha(h)}{N}$ appears as an eigenvalue. From the point of view of the observer memory having access to only one of the various histories h the number $N_\alpha(h)$ is all what is empirically and 'subjectively' available. However, for comparing the various histories and objectively define a criterion for the 'likelihood' we still need the measure $\||\Psi(h)\rangle\|^2$. For example the average on the whole ensemble leads to $\langle\Psi_N|\hat{Q}_\alpha|\Psi_N\rangle = \sum_h \langle\Psi(h)|\hat{Q}_\alpha|\Psi(h)\rangle = \sum_h \frac{N_\alpha(h)}{N} \||\Psi(h)\rangle\|^2 = \|\Psi_\alpha\|^2$ which is the standard quantum result. This could be made even more precise by reintroducing the notion of typicality in the $N \to +\infty$ limit. Then a typical history $\bar{h}$ seen by a typical observer will confirm the record $\frac{N_\alpha(\bar{h})}{N} \simeq \|\Psi_\alpha\|^2$ with a error which is going like $\Delta N_\alpha/N_\alpha(\bar{h}) \simeq \frac{1}{\sqrt{N}}\sqrt{\frac{1-\|\Psi_\alpha\|^2}{\|\Psi_\alpha\|^2}} \to 0$.

history space (i.e. weighted with Gleason-Everett's measure $\mathcal{M}$) is filled by terms satisfying the probability rule of quantum mechanics. This great result has been called the meta-theorem by DeWitt [15] but, as previously said, it was already discussed by Everett (this notion of 'almost all except for a set of measure nearly equals to zero' was considered by Everett as the core of his thesis). The theorem relies critically on the definition of an actually infinite sequence which is never encountered in the lab and therefore the deduction was often criticized for being circular [19–21]. However, this problem is not so fundamental and is actually generic of the application of the law of large numbers in statistical mechanics through the introduction of collectives or Gibbs ensembles (recently some attempts have been made for making sense of such an infinite sequence $N \to +\infty$ in the MWI, i.e., by linking the problem with the notion of Multiverses used in cosmology [22]).

Moreover, the real issue in Everett reasoning concerns the status of the Gleason-Everett measure $\mathcal{M}(\|\Psi_\alpha\|^2)$ for this quantum branching. To paraphrase Wallace [23] what is only proven by the Everett 'law of large numbers' is that relative frequency tends to weight with high weight... Therefore justifying the choice $\mathcal{M}(\|\Psi_\alpha\|^2) \equiv \|\Psi_\alpha\|^2$ is central in order to avoid circularity. Yet, it is known in the context of the pilot-wave interpretation (PWI) [24,25], i.e., in de Broglie Bohm (aka Bohmian) mechanics, that Everett's choice for the measure is far from being univocal (this point was already stressed by Pauli's [26] as an objection to Bohm's theory in the 1950's and it becomes the core of the work of Valentini [27] about quantum non-equilibrium in the PWI). Moreover, changing the measure also changes the notion of typicality and the convergence to a different probability rule. This problem also occurs in the context of the MWI and the Everett Gleason weight is clearly not the unique possibility for defining a probability measure. Importantly, in the PWI the ontology of the theory concerns (at least in the non-relativistic regime) the position particles X_t in the configuration space at time t. This distribution of particles defines an additional ontological structure which is absent in the MWI which is all about a wave ontology. This is the central difference between the two approaches and it creates great difficulties for the MWI. Indeed, as we explained in a previous article [28], the MWI ontology doesn't contain any fine-graining (like particle positions in the Bohmian or collapse approaches) which could help us to decide which measure $\mathcal{M}$ is more physical. The most natural weight in the context of the MWI is the simple branch counting but it is known since Graham [3] that this measure will generally conflict with Born's rule [3].

One of the central problem, as analyzed by Albert and others [30,20,31], is that the observer in general ignores all about the weight $\||\Psi(h)\rangle\|^2$, i.e., about $\mathcal{M}(h) = \Pi_\alpha \mathcal{M}(\|\Psi_\alpha\|^2)^{N_\alpha}$ which is not directly measured. As a matter of fact, there is no clue proving or convincing the observer that it should make sense to weight the branch h in which (s)he is living with $\mathcal{M}(h)$ or with something else.

[3] In particular a simple branch counting is not time invariant [7,29]. Additionally it requires a preferred basis which must be choise perhaps in relation with decoherence or the observer memory states.

The only 'good' choice is the simple branch counting but this is not agreeing with Born's rule. For instance, consider an observer (named Alex) participating to the unitary evolution of a quantum measurement with N repetitions. We have:

$$|\Psi_N\rangle \otimes |\mathrm{Alex}_0, E_0\rangle \rightarrow \sum_h |\Psi(h)\rangle \otimes |\mathrm{Alex}_h, E_h\rangle \tag{7}$$

where Alex_h is Alex_0 successor having a memory of the particular $h-$history in the whole sum and where $|\Psi_N\rangle\otimes = \sum_h |\Psi(h)\rangle$ is the sum of the quantum histories h. However, it is clear that by including Alex in the unitary evolution we didn't progress but only pushed back the problem to a larger and larger system. Indeed, after the measurement Alex is still not aware of the weight of his own history h. He only knows the 'patterns' (to use the semantics of Wallace [7], p. 50) of his particular history h, i.e., he knows the experimental frequencies $N_\alpha(h)/N$ for his particular history h. Again, the only natural choice seems to be a simple branch counting which has an objective and absolute meaning here since a preferred basis associated with the observer memories is defined.

In the recent years this persistent difficulty about defining probability in the MWI has been called the *incoherence problem* and it still plagues any serious discussion about probability in this theory. In this optics the Gleason-Everett measure is at best interpreted as an intensity of the ontological state: 'a measure of existence' as it is often called by Vaidman [32], and the physical interpretation is still contentious after 60 years since Everett's work. Vaidman [32–34] and Tappenden [35] for instance, propose to introduce Born's rule as an added postulate for assigning a degree of *subjective location uncertainty* to the observer in her/his history h (Greaves [36] also speaks about caring measure but the meaning is actually a bit different for both authors). However, the exact, physical and empirical meaning of the word probability used in this interpretation has been strongly criticized by Albert [30], Kent [20] and Maudlin [37] (see also the discussions in [38])[4]. As emphasized by Albert [30] this notion of subjective self-locating uncertainty contrasts and conflicts with the absence of objective uncertainty in the MWI (indeed everything is unitary so there is nothing of fundamentally uncertain). It is only by confusing these two notions that one could hope to see a virtual solution where actually there is none. Indeed, the subjective self-locating uncertainty is an internal 'pattern' [7] of the observer history h. There is no reason *per se* to introduce an objective 'caring measure' [36] or a 'measure of existence' [32,33] in order to weight this subjective notion. This confusion between what is objective and absolute in the one side and subjective and relative in the other side has been however elevated to the level of an axiom under the name *probability postulate*

[4] For similar reasons (and many others that we will not review) J.S. Bell [39], p. 192 dubbed the MWI a 'romantic counterpart of the pilot wave picture' since despite all its glamorous aspects at first sight it can not be developed into a net and sharp theoretical framework avoiding internal physical contradictions.

or Born-Vaidman rule [35,34]. However, this desperate strategy only reveals some intrinsic weakness in the current MWI.

3 Many-Bohmian paths?

Over the years several solutions for saving or making sense of the MWI have been attempted. One of the method is to modify the ontology of the MWI by adding a chancy or random aspect in the theory. By doing that we surely pervade the original all-is-unitary approach proposed by Everett. Initially, such a modification was suggested by Deutsch in 1985 [42] and it has be more lengthy developed in the recent years by Tipler [43,44], Valentini [38], p. 510 and many others [45–47] (with different subtle differences) forming what we could call a 'many-Bohmian' path interpretation (even though doing an emphasis on the de Broglie-Bohm mechanics is not necessary for developing such kinds of modified MWI). In those theories the system is following a well defined path or trajectory which could be deterministic (in agreement with Bohm's hidden-variables approach) or stochastic depending of the model considered. For example one could develop a MWI based on the consistent or decoherent histories interpretation originally proposed by Griffiths [48], Omnes [49], Gell-Mann and Hartle [18]. While in this view the stochastic paths followed by the system are (like in the PWI) univocally defined one could easily extend the framework to obtain a many-worlds version of the consistent histories approach in which all consistent histories exist simultaneously in parallel worlds. The approach is thus essentially dualistic (guiding ψ-wave + particle or field) in opposition to the monism advocated by Everett. However, unlike in the PWI where the system selects only one path (for example for traveling inside an interferometer) here the system travels though all paths simultaneously (a bit like in Feynman sum-over-paths formalism). In a sense, whereas in Everett's original it is fair to speak about a process of 'fission' here (at least in the case of a many-Bohmian path interpretation) it is better to speak about 'diverging' (but preexisting) paths. Moreover, all those parallel paths are mostly 'ignoring' each other, i.e., up to quantum-like interference effects induced by the guiding wave-function Ψ. Thanks to decoherence the interference effects are generally hidden at the macroscopic scale. Despite, the fact that such a solution clearly dissolves the above mentioned incoherence conundrum most proponents of the MWI are not ready to use such a desperate remedy which, indeed, would be equivalent to resign or undermine the initial goal of Everett, i.e., avoiding the introduction of hidden-variables à la Bohm.

4 About many-minds

A very different strategy, which goes back to the late Zeh in the 1970's [50] and was subsequently developed by Albert and Loewer in 1988 [4,5,54], is the the so called many-minds interpretation (MMI) which we will shortly describe below.

In brief, the idea is to include the role of states of consciousness or awareness into the quantum game. At the difference of older attempts in the same vein such as the von Neunmann [51], London and Bauer [52] and Wigner [53] approaches, the MMI involves several mind states $\mathcal{O}^{(1)}, \mathcal{O}^{(2)}, ...$ associated with a single observer. In the approach advocated by Albert and Loewer such mind states are not obeying to the unitary Schrödinger equation but are nevertheless guided by solutions Ψ_t of such an equation. Again, in complete analogy with the PWI the mind states associated with the brain structure surf on the Ψ_t associated with the entangled wave-functions coupling the observer to the measurement apparatus and the quantum object under studies. By surfing on the pilot-wave the many mind states $\mathcal{O}^{(i)}$, which are associated with a given observer and which are unaware from each other, are stochastically driven into the distinct grooves and channels associated with the wave function splitting during the measurement. If the wave function for the observed system reads as before $|\Psi\rangle = \sum_\alpha \Psi_\alpha |\alpha\rangle$ the MMI of Albert and Loewer postulates from the start that a fraction $\mathcal{P}_\alpha = \|\Psi_\alpha\|^2$ of mind states given by Born's rule is stochastically driven in the groove, i.e., world corresponding to the outcome α. Consider for example a simple non-symmetric beam splitter experiment where the quantum state of let say a single photon or electron evolves as

$$|\Psi_0\rangle \to |\Psi_t\rangle = \sqrt{\frac{1}{3}}|\uparrow\rangle + \sqrt{\frac{2}{3}}|\downarrow\rangle \tag{8}$$

where $\uparrow$ and $\downarrow$ describes the two states of the single particle transmitted or reflected by the beam splitter. In a more realistic way of describing the experiment we must include an observer (Alex), and an experimental environment (E) into the unitary evolution reading now:

$$|\Psi_0\rangle \otimes |E_0, \mathrm{Alex}_0\rangle \to \sqrt{\frac{1}{3}}|\uparrow\rangle \otimes |E_\uparrow, \mathrm{Alex}_\uparrow\rangle + \sqrt{\frac{2}{3}}|\downarrow\rangle \otimes |E_\downarrow, \mathrm{Alex}_\downarrow\rangle. \tag{9}$$

Here, the observer has a memory or record of the experimental outcome as indicated by the $\uparrow\downarrow$ label. In the MMI proposed by Albert and Loewer we add mind states moving stochastically. For example with one single mind state we have either

$$\begin{aligned}|\Psi_0\rangle \otimes |E_0, \mathrm{Alex}_0(\mathcal{O}_0^{(1)})\rangle \to \sqrt{\frac{1}{3}}|\uparrow\rangle \otimes |E_\uparrow, \mathrm{Alex}_\uparrow(\mathcal{O}_\uparrow^{(1)})\rangle \\ + \sqrt{\frac{2}{3}}|\downarrow\rangle \otimes |E_\downarrow, \mathrm{Alex}_\downarrow\rangle\end{aligned} \tag{10}$$

if the mind state move randomly to the brain of Alex seeing the $\uparrow$-photon or alternatively

$$\begin{aligned}|\Psi_0\rangle \otimes |E_0, \mathrm{Alex}_0(\mathcal{O}_0^{(1)})\rangle \to \sqrt{\frac{1}{3}}|E_\uparrow, \uparrow\rangle \otimes |\mathrm{Alex}_\uparrow\rangle \\ + \sqrt{\frac{2}{3}}|\downarrow\rangle \otimes |E_\downarrow, \mathrm{Alex}_\downarrow(\mathcal{O}_\downarrow^{(1)})\rangle\end{aligned} \tag{11}$$

if the mind state moves along the second groove or branch of the wave-function. In order to avoid having endless discussions about various issues raised by 'mindless-Hulk' kinds of observers (e.g., if a mindless Alex state is discussing with a second observer) Albert and Loewer suggested the introduction of several mind states existing in parallel in the observer brain and also moving randomly. For example with two mind states $\mathcal{O}^{(1)}$ and $\mathcal{O}^{(2)}$ the initial quantum state reads as $|\Psi_0\rangle \otimes |E_0, \mathrm{Alex}_0(\mathcal{O}_0^{(1)}, \mathcal{O}_0^{(2)})\rangle$ and it will evolves in one of the four following alternatives:

$$\begin{aligned}
&\sqrt{\frac{1}{3}}|\uparrow\rangle \otimes |E_\uparrow, \mathrm{Alex}_\uparrow(\mathcal{O}_\uparrow^{(1)}, \mathcal{O}_\uparrow^{(2)})\rangle + \sqrt{\frac{2}{3}}|\downarrow\rangle \otimes |E_\downarrow, \mathrm{Alex}_\downarrow\rangle,\\
&\sqrt{\frac{1}{3}}|\uparrow\rangle \otimes |E_\uparrow, \mathrm{Alex}_\uparrow(\mathcal{O}_\uparrow^{(1)})\rangle + \sqrt{\frac{2}{3}}|\downarrow\rangle \otimes |E_\downarrow, \mathrm{Alex}_\downarrow(\mathcal{O}_\downarrow^{(2)})\rangle,\\
&\sqrt{\frac{1}{3}}|\uparrow\rangle \otimes |E_\uparrow, \mathrm{Alex}_\uparrow(\mathcal{O}_\uparrow^{(2)})\rangle + \sqrt{\frac{2}{3}}|\downarrow\rangle \otimes |E_\downarrow, \mathrm{Alex}_\downarrow(\mathcal{O}_\downarrow^{(1)})\rangle,\\
&\sqrt{\frac{1}{3}}|\uparrow\rangle \otimes |E_\uparrow, \mathrm{Alex}_\uparrow\rangle + \sqrt{\frac{2}{3}}|\downarrow\rangle \otimes |E_\downarrow, \mathrm{Alex}_\downarrow(\mathcal{O}_\downarrow^{(1)}, \mathcal{O}_\downarrow^{(2)})\rangle.
\end{aligned} \tag{12}$$

It is clear that if we have N mind states $\mathcal{O}^{(1)}, ..., \mathcal{O}^{(N)}$ we have now 2^N combinations. If additionally the probabilities for each mind state $\mathcal{O}^{(i)}$ to go to the $\uparrow$ and $\downarrow$ branches are given by Born rule, i.e., $\mathcal{P}_\uparrow = \frac{1}{3}$ and $\mathcal{P}_\downarrow = \frac{2}{3}$ then the probability for having $N_\uparrow$ mind states in the upper branch and $N_\downarrow = N - N_\uparrow$ mind states in the lower branch is given by the binomial formula:

$$\mathcal{P}(N_\uparrow, N_\downarrow) = \frac{N!}{N_\uparrow! N_\downarrow!} \mathcal{P}_\uparrow^{N_\uparrow} \mathcal{P}_\downarrow^{N_\downarrow}. \tag{13}$$

By maximizing this probability in the $N \to +\infty$ limit we obtain

$$\mathcal{P}_\uparrow \simeq \frac{\tilde{N}_\uparrow}{N}, \mathcal{P}_\downarrow \simeq \frac{\tilde{N}_\downarrow}{N} \tag{14}$$

where $\tilde{N}_\uparrow$ and $\tilde{N}_\downarrow$ are the typical number of mind states in the upper and lower branch respectively. In this MMI the relative fluctuation to this optimum (written $\frac{\Delta N_\uparrow}{\tilde{N}_\uparrow} = \frac{1}{\sqrt{N}}\sqrt{\frac{\mathcal{P}_\downarrow}{\mathcal{P}_\uparrow}}$) goes to zero as $N \to +\infty$ and the probability to have maverick branches without mind state tend to vanish as well (which is a good feature of course). The previous example can easily be generalized to a quantum state like $|\Psi_t\rangle = \sum_\alpha \Psi_\alpha |\alpha\rangle$ which (together with the environment+observer) evolves as

$$|\Psi_0\rangle \otimes |E_0, \mathrm{Alex}_0\rangle \to \sum_\alpha \Psi_\alpha |\alpha\rangle \otimes |E_\alpha, \mathrm{Alex}_\alpha\rangle. \tag{15}$$

After the inclusion of N observer mind states and after redoing the previous reasoning we have a multinomial probability for having the set of $\{N_\alpha\}$ mind states (i.e., with $N = \sum_\alpha N_\alpha$):

$$\mathcal{P}(\{N_\alpha\}) = \frac{N!}{\Pi_\alpha N_\alpha!} \Pi_\alpha \mathcal{P}_\alpha^{N_\alpha} \tag{16}$$

which is leading again to the relative frequency of mind states $\mathcal{P}_\alpha := \|\Psi_\alpha\|^2 \simeq \frac{\tilde{N}_\alpha}{N}$ for the typical configuration in agreement with Born's rule (the fluctuation reads now $\frac{\Delta N_\alpha}{\tilde{N}_\alpha} = \frac{1}{\sqrt{N}}\sqrt{\frac{1-\mathcal{P}_\alpha}{\mathcal{P}_\alpha}}$ that is also vanishing in the $N \to +\infty$ limit). The MMI by keeping the psycho-physical parallelism at the statistical average level (i.e., as a very good approximation in the $N \to +\infty$ limit) is thus remarkably able to reproduce standard quantum mechanical results.

However, even if the introduction of the observer memory and mind state has a old and respectable tradition in quantum mechanics interpretation (it was also playing a role in the work of Everett himself) it is fair to say that such an odd approach has been watched with suspicion by many in part because the theory is dualistic in spirit (separating minds from the rest of the unitary evolution in the Universe), i.e., it breaks down the functionalist or psycho-physical parallelism which is generally accepted (see e.g. von Neumann [51]) for discussing quantum mechanics of the observer. Furthermore, a multiplicity of minds is required for solving the 'mindless-hulk' problem at the price of introducing a form of schizophrenic many-worlds. It has been attempted by some philosophers to eliminate this unwarranted feature by reinstating psycho-physical realism at the mind level (see Lockwood [55,56] and Donald [57–59]). In other words, it has been proposed to re-establish the supervenience of the mind state $\mathcal{O}$ on the brain state described quantum mechanically. The main difficulty with this new amendment of the MWI (see the interesting discussion following [56]: [60–66], see also [67]) is that the mind now becomes a deterministic function of the wave function state Ψ_t, i.e., $\mathcal{O}(\Psi_t)$ (instead of having $\Psi_t(\{\mathcal{O}^{(i)}\})$ in agreement with the theory of Albert and Loewer [4,63]). While this would seem a natural property in a quantum Universe the proponents of the MMI and MWI following this path have not yet been able to justify the Born's rule unambiguously: Clearly, eliminating stochasticity or hidden-variables altogether also unfortunately removes the nice remedy introduced to solve the incoherence problem. In Section 6 we will develop a unitary version of the MMI which is free from those contradictions.

5 Subjective versus objective probabilities: the quantitative problem

Despite all what has been written before concerning the incoherence problem one should emphasize that proponents of the MWI in general prefer to split the whole problem into two. The first one, that we called the incoherence issue is related to the existence of probability, the second the *quantitative problem* connects with the specific mathematical and physical recovering of the Born rule in the MWI. Notwithstanding that the incoherence problem has been solved (which as we emphasized is extremely difficult to believe without modifying Everett's ontology) the quantitative problem is fundamentally interesting by itself and motivated most researches in the last decades.

Here, I would like to discuss the issue a little by giving a brief introduction

to the remarkable works of Deutsch [6], Wallace [7] and Zurek [8] and to more recent works by Carroll, Sebens [9] and Vaidman [10]. D. Deutsch in his seminal article started with decision-theoretic axiomatic and wanted to derive the Born rule from non probabilistic axioms of quantum mechanics. As he wrote:

> *Thus we see that quantum theory permits what philosophy would hitherto have regarded as a formal impossibility, akin to 'deriving and ought to from an is', namely deriving a probability statement from a factual statement. This could be called deriving a 'tends to' from a 'does'.* [6]

Clearly this is a very strong claim which is touching both sides of the difficulty, i.e., the incoherence and quantitative issues. Deutsch's proof has been attacked on philosophical and mathematical grounds (see for example [68, 69]). The incoherence problem will not be further commented [5]. The formal part of the proof used the notion of Value function $\mathcal{V}_\Psi$ and utility assigned to a quantum 'game', i.e., a quantum experiment. The semantics of classical decision-theory leads to the definition $\mathcal{V}_\Psi(\{x_\alpha\}) = \sum_\alpha x_\alpha \mathcal{P}_\alpha$ where x_α are eigenvalues of the Hermitian operator $\hat{X}^{(S)} = \sum_\alpha x_\alpha \hat{\Pi}_\alpha^{(S)}$ acting on the quantum state $|\Psi^{(S)}\rangle = \sum_\alpha \Psi_\alpha |\alpha^{(S)}\rangle$ associated with system S. Assuming a set of decision-theoretic axioms which are non intrinsically probabilistic Deutsch built the probability function $\mathcal{P}_\alpha := \|\Psi_\alpha\|^2$ which is identical to Born's rule. The set of axioms was criticized in particular by Barnum et al. [68] who emphasized the existence of an additional permutation symmetry in the derivation (this is strongly connected to the role of entanglement between S and the observer as we show below). This prompted further important works by Wallace and Saunders [70–74] (see also [36] and [38] p. 181 and p. 227) who progressively clarified the whole analysis. It leads Wallace to his simple elegant proof [7] which expurgates the reasoning of unwarranted technical sophistication present in the original derivations. Remarkably, in the mean time Zurek [75,76,8,77] proposed an alternative proof of Born's rule based on *envariance* a neologism for environment-assisted invariance a purely quantum symmetry based on entanglement of a system with its environment. What is however key here is that Wallace and Zurek proofs are actually isomorphic to one another. I am going to resume briefly Zurek's proof which is capital for my own deduction and then go to Wallace's semantics.

Zurek starts with a Schmidt symmetric quantum state

$$|\Psi^{(SE)}\rangle = \sqrt{\frac{1}{N}} \sum_{\alpha \in \Delta} |\alpha^{(S)}\rangle \otimes |\varepsilon_\alpha^{(E)}\rangle \tag{17}$$

where S denotes the system and E its environment (the basis vectors are orthogonal). The label of the α-mode belongs to a set Δ with cardinality N. Zurek introduces swapping operators acting locally on S and reading $\hat{U}^{(S)}(\alpha \leftrightarrow$

[5] The claim has a long tradition in the MWI community. DeWitt for example famously wrote 'The mathematical formalism of the quantum theory is capable of yielding its own interpretation' [15].

$\beta) = |\alpha^{(S)}\rangle\langle\beta^{(S)}| + H.c.$ with similar operators $\hat{U}^{(E)}(\alpha \leftrightarrow \beta)$ for the environment ($\hat{U}^{(S,E)}(\alpha \leftrightarrow \beta)|\Psi^{(SE)}\rangle$ is orthogonal to $|\Psi^{(SE)}\rangle$). Now, as emphasized in [75] applying successively a swap on S and a counterswap on E let the state invariant, i.e.,

$$\hat{U}^{(E)}(\alpha \leftrightarrow \beta)\hat{U}^{(S)}(\alpha \leftrightarrow \beta)|\Psi^{(SE)}\rangle = |\Psi^{(SE)}\rangle. \tag{18}$$

It is a matter of fact (e.g., from no-signaling theorem even though this is not essential [78]) that a local action on S should have no-effect on E and therefore assigning *a priori* probability $\mathcal{P}_\Psi(\alpha^{(S)}, \varepsilon_\alpha^{(E)})$ to the branch $|\alpha^{(S)}\rangle \otimes |\varepsilon_\alpha^{(E)}\rangle$ in Eq. 17 we must have after application of $\hat{U}^{(S)}(\alpha \leftrightarrow \beta)$ on $|\Psi^{(SE)}\rangle$ and by application of Laplace principle of indifference the symmetry relation:

$$\begin{aligned} \mathcal{P}_\Psi(\alpha^{(S)}, \varepsilon_\alpha^{(E)}) &= \mathcal{P}_{\hat{U}^{(S)}\Psi}(\beta^{(S)}, \varepsilon_\alpha^{(E)}) \\ \mathcal{P}_\Psi(\beta^{(S)}, \varepsilon_\beta^{(E)}) &= \mathcal{P}_{\hat{U}^{(S)}\Psi}(\alpha^{(S)}, \varepsilon_\beta^{(E)}). \end{aligned} \tag{19}$$

Here, we have the strong correlations $\mathcal{P}_\Psi(\varepsilon_\alpha^{(E)}|\alpha^{(S)}) = 1$, $\mathcal{P}_{\hat{U}^{(S)}\Psi}(\varepsilon_\alpha^{(E)}|\beta^{(S)}) = 1$ and thus Eq. 18 actually reads

$$\begin{aligned} \mathcal{P}_\Psi(\varepsilon_\alpha^{(E)}) &= \mathcal{P}_{\hat{U}^{(S)}\Psi}(\varepsilon_\alpha^{(E)}) \\ \mathcal{P}_\Psi(\varepsilon_\beta^{(E)}) &= \mathcal{P}_{\hat{U}^{(S)}\Psi}(\varepsilon_\beta^{(E)}) \end{aligned} \tag{20}$$

which is a statement of Laplacian indifference for the subsystem E about what is occurring at S. By the same token a subsequent application of $\hat{U}^{(E)}(\alpha \leftrightarrow \beta)$ yields

$$\begin{aligned} \mathcal{P}_{\hat{U}^{(S)}\Psi}(\beta^{(S)}, \varepsilon_\alpha^{(E)}) &= \mathcal{P}_{\hat{U}^{(E)}\hat{U}^{(S)}\Psi}(\beta^{(S)}, \varepsilon_\beta^{(E)}) \\ \mathcal{P}_{\hat{U}^{(S)}\Psi}(\alpha^{(S)}, \varepsilon_\beta^{(E)}) &= \mathcal{P}_{\hat{U}^{(E)}\hat{U}^{(S)}\Psi}(\alpha^{(S)}, \varepsilon_\alpha^{(E)}). \end{aligned} \tag{21}$$

The basis of the reasoning is that in Eq. 19 an hypothetical observer attached to E is indifferent to what is occurring at S (i.e., a swap) whereas in Eq. 21 an hypothetical observer attached to S is indifferent about the counterswap acting on E [75]. Moreover, this indifference is both subjective (degree of belief $\mathcal{C}$) and objective (physical probability $\mathcal{P} \equiv \mathcal{C}$) [6] and defined by some more objective and physical properties of the system. Therefore, here rational agents should conform their credence to physical probabilities. The objectivity is here linked to the Schmidt form of the state which makes the phases of the different branches locally inoperative to S or E (i.e., we have locally a 'mixture'). This would not occur without entanglement because interference between branches

[6] The general idea behind a subjectivist approach of probability is to define a degree of belief or credence $\mathcal{C}_\alpha$ for the occurrence of the outcome α. Moreover, following the philosopher D. Lewis we might use the so called '*principal principle*' for equaling this subjective likelihood to an objective weight playing also the role of probability $\mathcal{P}_\alpha$. More precisely, the credence assigned to the realization of the outcome α and conditioned on the knowledge of the objective probability $\mathcal{P}_\alpha$ equals $\mathcal{P}_\alpha$: i.e., $\mathcal{C}(\alpha|\mathcal{P}_\alpha) \equiv \mathcal{P}_\alpha$.

are in principle possible so that a swap would break the symmetry [75]. Regrouping Eqs. 19 and 21 and using the fundamental global envariance Eq. 18 imply directly

$$\mathcal{P}_\Psi(\alpha^{(S)}, \varepsilon_\alpha^{(E)}) = \mathcal{P}_\Psi(\beta^{(S)}, \varepsilon_\beta^{(E)}). \tag{22}$$

Moreover, the pair of modes α and β was arbitrary in the set Δ and consequently by generalizing to every pairs we deduce the equiprobability condition reading $\mathcal{P}_\Psi(\alpha^{(S)}, \varepsilon_\alpha^{(E)}) = Const.$. Finally, by normalization we have Born's rule for this special state $|\Psi^{(SE)}\rangle$, i.e.,

$$\mathcal{P}_\Psi(\alpha^{(S)}, \varepsilon_\alpha^{(E)}) = \frac{1}{N} = \|\langle \alpha^{(S)}, \varepsilon_\alpha^{(E)} | \Psi^{(SE)} \rangle\|^2. \tag{23}$$

What is remarkable about this reasoning is its simplicity relying only on quantum symmetries. As stated by Zurek envariance results 'from coexistence between perfect knowledge of the whole and complete ignorance of the parts' [75]. Contrarily to classical Laplace indifference based on ignorance about information which could be in principle recorded and recovered here the indifference is more fundamental and linked to the entanglement of the system [8]. Indeed, there is not hidden-variable in this approach and nothing more fundamental to find out that the quantum symmetry of the system under swap and counterswap which are local operations acting on S or E.

This point was also emphasized by Wallace who explained that there is perfect symmetry between the outcomes and thus that the ignorance considered here must be genuinely quantum [7]. A quantum gambler (observer) attached to S or E acting on her/his own subsystem will bet rationally on the different outcomes by using Laplace's indifference as explained previously. Therefore, the decision-theoretic scenario proposed by Wallace and Deutsch reduces to the one made by Zurek (it is interesting to point out that the hidden symmetry contained in Deutsch's proof and which was discovered by Barnum et al. [68] is precisely envariance). For the seek of clarity I postpone to the Appendix 8 a 'derivation' of Wallace's proof using the semantic and logic of Zurek formalism.

At that stage, it should be clear that the various proofs assumed more that claimed. Specifically, with Albert we agree that:

> *The questions to which this program is addressed are questions of what we would do if we believed that the fission hypothesis were correct. But the question at issue here is precisely whether to believe that the fission hypothesis is correct!* [30]

This remark is related to th incoherence problem. However concerning the quantitative derivation presented before Zurek assumed already many physical properties which look at first innocuous but actually play a crucial role. Indeed, the indifference postulate is motivated by some bare facts concerning locality i.e., facts about the irrelevance of swap on a subsystem S (or E) on the physical properties of the subsystem E (or S). This is reminiscent either

from non-signaling (as already briefly alluded and discussed in [78]) or from a 'natural' postulate concerning 'knowledge about the whole versus ignorance of the parts'[75]. Actually, this axiom hides the notion of mixture and reduced density matrix which already assumes the notion of probability to be derived. Still, this issue is not so harmful if we consider only the quantitative problem independently of the incoherence one. From this perspective Zurek's axiom only told that beyond assuming the mere existence of probability (which is the point stressed by Albert) one must additionally postulate the 'strong'[7] symmetry which from standard mechanics reads (with pedantic notations):

$$\langle \hat{\Pi}_{\alpha}^{(S)} \otimes \hat{\Pi}_{\varepsilon_\alpha}^{(E)} \rangle_{\Psi} = \langle \hat{\Pi}_{\hat{U}^{(S)}\alpha}^{(S)} \otimes \hat{\Pi}_{\varepsilon_\alpha}^{(E)} \rangle_{\hat{U}^{(S)}\Psi} \tag{24}$$

where $\hat{\Pi}_{\hat{U}^{(S)}\alpha}^{(S)}$ is a 'causal' notation for the projector $\hat{U}^{(S)}|\alpha^{(S)}\rangle\langle\alpha^{(S)}|\,\hat{U}^{(S)\,\dagger} = |\beta^{(S)}\rangle\langle\beta^{(S)}|$. This just leads to Eq. 19 and similar expressions could be used to obtain Eq. 21. These strong symmetries naturally allow us to recover to equiprobability which is indeed the reasoning of Zurek. Therefore, while in the orthodox interpretation Eq. 24 follows from the symmetries of the Schmidt quantum state Eq. 17 together with the already assumed Born's rule (i.e., here equiprobability), in the axiomatic of Zurek it is enough to use Eqs. 19, 21 to recovering equiprobability (i.e., Born's rule) and thus avoiding circularity.

The previous analysis focused on the simple equiprobable case where $|\Psi^{(SE)}\rangle$ is given by Eqs. 17. For generalizing to any Schmidt state Zurek used a 'trick' he introduced in 1998 [79] (which was rediscovered by Deutsch [6]) consisting in applying a fine graining procedure. We start with a S state $|\Psi^{(S)}\rangle = \sum_a \sqrt{\mathcal{P}_a}|a^{(S)}\rangle$ where $\mathcal{P}_a = \frac{N_a}{N}$ is a rational number. Entanglement with the environment E leads to

$$|\Phi^{(SE)}\rangle = \sum_a \sqrt{\mathcal{P}_a}|a^{(S)}\rangle \otimes |e_a^{(E)}\rangle \tag{25}$$

We thus introduce the new vectors

$$|a^{(S)}\rangle = \frac{1}{\sqrt{N_a}} \sum_{\alpha\in\Delta_a} |\alpha^{(S)}\rangle \tag{26}$$

and where the cardinality of Δ_a equals N_a (we have also $\Delta_a \cap \Delta_b = 0$ if $a \neq b$ and $\cup_a \Delta_a = \Delta$ with Δ the set of all vectors $|\alpha^{(S)}\rangle$ with cardinality N). We have thus

$$|\Phi^{(SE)}\rangle = \frac{1}{\sqrt{N}} \sum_{\alpha\in\Delta} |\alpha^{(S)}\rangle \otimes |e_{a_\alpha}^{(E)}\rangle \tag{27}$$

[7] Such conditions are clearly stronger that mere no-signaling which only requires $\langle \hat{\mathcal{O}}^{(E)}\rangle_{\Psi} = \langle \hat{\mathcal{O}}^{(E)}\rangle_{\hat{U}^{(E)}\Psi}$ where $\hat{\mathcal{O}}^{(E)}$ is any local Hermitian operator acting on E solely and $\hat{U}^{(E)}$ is any unitary transformation acting on the environement E (a similar equation with the role of E and S reverted also holds true).

where $a_\alpha = a$ if $\alpha \in \Delta_a$. The last step consists in a global transformation in the SE system reading $|\alpha^{(S)}\rangle \otimes |e^{(E)}_{a_\alpha}\rangle \to |\alpha^{(S)}\rangle \otimes |\varepsilon^{(E)}_\alpha\rangle$ with $|\varepsilon^{(E)}_\alpha\rangle$ a new environmental basis. We finally obtain

$$|\Psi^{(SE)}\rangle = \frac{1}{\sqrt{N}} \sum_{\alpha\in\Delta} |\alpha^{(S)}\rangle \otimes |\varepsilon^{(E)}_\alpha\rangle \tag{28}$$

which is a Schmidt symmetric state identical to Eq. 17. Therefore, Eq. 29 obtains and we finally get by additivity and application Laplacian indifference:

$$\mathcal{P}_\Phi(a^{(S)}, e^{(E)}_a) = \sum_{\alpha\in\Delta_a} \mathcal{P}_\Psi(\alpha^{(S)}, \varepsilon^{(E)}_\alpha) = \frac{N_a}{N} = \|\langle a^{(S)}, e^{(E)}_a|\Phi^{(SE)}\rangle\|^2. \tag{29}$$

Continuity establishes the generality of the result for the case where $\mathcal{P}_a$ is a real number [75,76,8].

The most critical part of this proof is the fine graining procedure which can easily be implemented with beam splitters and unitary gates as shown for example by Vaidman [10,34]. However, observe first that this trick requires to have high dimensionality of the Hilbert space for the S subsystem (which is in general true). Second, there is here a form of conspiratorial preparation. Why indeed should the distinct beams $|a^{(S)}\rangle$ (which could be located in remote regions of space) be separated in such a way (i.e., Eq. 26) to have equiprobability at the end? Such choice is clearly motivated by the desire to rely on a simple branch counting argumentation to define histories for the observers. Indeed, back to 1985, this solution indeed would naturally avoid the problem existing with the original Deutsch approach [42] to postulate a density of worlds proportional to $\|\Psi_\alpha\rangle\|^2$, i.e., different from a naive branch-counting reasoning. In return, the price to be paid consists in dividing the intensity of the original beams $\mathcal{P}_a$ into many sub-beams of equal intensity $1/N$: a procedure which much be defined in advance by an agent knowing the full properties of the entangled SE system.

Zurek's fine-graining trick has been accepted by all proponents of the decision-theoretic approach [6,7] as well as by other more recent deductions like the one by Carroll and Sebens [9], and the one by McQueen and Vaidman [10,34] which all strongly rely on the methodology offered by Zurek with envariance and also involve the notion of self-location uncertainty. In particular, Carroll and Sebens [9] developed a narrative in which an external observer interacting with a SE system like the one described by $|\Psi^{(SE)}\rangle$ in Eq. 17 is going to assign probabilities to the various outcomes depending on the subsystem S or E (s)he is considering and whether or not the unitary swap $\hat{U}^{(S)}$ or counterswap $\hat{U}^{(E)}$ operation are applied. This interesting narrative (based on a principle named Epistemic Separability Principle or ESP) leads directly through Zurek's and Wallace's argument to the equiprobability condition and then ultimately recovers Born's rule as explained above. We emphasize, that there is a disagreement between Carroll and Sebens [9] on the one side, and Kent [80], McQueen and Vaidman [10,34] on the other side concerning the role of self-location uncertainty in this analysis (see also [30]).

6 A deterministic and quantum version of the many-minds interpretation

It is part of the MWI rhetoric and mythology that the introduction of probability is not worse (and perhaps not better) than it is in other interpretations of quantum mechanics or even in other fields of physical science. The claim goes back to Everett [1] who saw his measure-theoretic deduction as good as the one used in classical statistical physics. More recently, Papinau [65,67] and Wallace [7] repeated the same claim that probabilities are very obscure concepts and that the MWI is not in a worst position than for example GRW collapse of Bohmian models are for discussing randomness and chances. Wallace [70] and Zurek [75] following Deutsch [6], went further by claiming that the genuinely quantum Laplacian indifference, i.e., related to envariance and self-locating uncertainty, provides within the MWI framework a even better basis for a clean foundation of probability than in collapse or Bohmian approaches which are all relying on a purely frequentist approach, i.e., based on infinity sequences.

As we saw there are serious reasons to doubt about the validity of such strong claims. First, in collapse interpretations such as GRW or in the Copenhagen interpretation the notion of infinite sequences is not problematic and positing a frequency law like $\mathcal{P}_\alpha \equiv \lim_{N\to+\infty} \frac{N_\alpha}{N}$ means that the systems know stochastically, i.e., on the long run how to behave: there is no law at the individual level in a purely stochastic Universe. Second, the MMI of Albert and Loewer [4] is also based on a stochastic approach to probability and the model is self-consistent even though strongly dualistic.

The same is true for the PWI where probability arizes from ignorance and lack of information on the initial conditions of the whole Universe. In the PWI, which like classical mechanics is fully deterministic, one must impose an 'equivariant' Gleason distribution of particles and fields at one time in order to recover Born's rule at any other times. Like for classical statistical mechanics there are persistent debates about the probabilistic foundations of the PWI but these debates are not about incoherence *per se* (which is actually irrelevant in the de Broglie Bohm framework) but are instead focused on the uniqueness of Born's rule and on the status of quantum equilibrium versus quantum non-equilibrium particle distributions [13,27]. The problems are very similar to those existing in statistical thermodynamics for justifying microcanonical and canonical ensembles and for describing the tendency to reach thermal equilibrium. In particular, we emphasize that there exists what could be called a minimalist PWI advocated by Bell [39] p. 129 and Goldstein Durr and Zanghì [13] and where the Boltzmanian notion of typicality plays a central role for recovering Born's rule. This approach starts with the same methodology as in Everett's work [2], i.e., by introducing the preferred Gleason measure $\mathcal{M}(h)$ assigned to histories h (here defined in the coordinate configuration space for point-like particles or field variables for continuous Bosonic fields). From the weak law of large numbers we deduce, like in the MWI, that Born's rule holds with a near-unit Gleason weight in the limit $N \to +\infty$. The main difference

with the MWI is that here we have a definition of what is actual or not in this dualistic theory. The particles and fields can not have two or more configuration at once: a typical history $\bar{h}$ and only one is actually realized and this indepednelty of the application of the probability calculus and the law of large numbers. Therefore, the law of large number only indicates that such an idealized Gibbs ensemble can be defined without however proving its unicity. This is a indeed a measure dependent problem (associated with the choice of the Universe initial conditions) and constitutes the recurrent issue debated by Valentini on the one side [27] and Goldstein Durr and Zanghì on the other side [13]. Moreover, at the difference of the MWI we can precisely and univocally define what we mean by an actual configuration and this even for a number N finite. Indeed, taking as an example the non-relativistic de Broglie-Bohm dynamics for a system of N electrons we can write the actual density of Bohmian electrons at the spatial point $\mathbf{q} \in \mathbb{R}^3$:

$$\rho_N(\mathbf{q}, t) = \frac{1}{N} \sum_{i=1}^{i=N} \delta^3(\mathbf{q}_\Psi^{(i)}(t) - \mathbf{q}) \simeq \|\Psi(\mathbf{q}, t)\|^2 \tag{30}$$

with $\mathbf{q}_\Psi^{(i)} \in \mathbb{R}^3$ some 'typical' Bohmian paths for the electrons. The second approximate equality means that Born's rule is accurately valid for this 'history'[8] and the Gleason measure provides a quantitative figure of merit for that accuracy in the regime $N \gg 1$ (importantly, in this regime changing the measure for a non equivariant one would select a different set of typical histories in which Born's rule will not hold).

We will not discuss further the intricacies of probabilities and the PWI here (for a 'balanced' review of the general problem see [81]). Here, we will instead consider Bohmian and classical statistical mechanics as a motivation for new models applied to the MWI and MMI. In the following we will develop a very speculative although mathematically precise toy model for the MMI.

We start with the Albert and Loewer MMI [4] and go back to the example of Eq.9. However, now we replace Alex by some collective excitation of a memory device which we write $|E_0, \mathcal{O}_0^{(1)}\rangle$ before the interaction. $\mathcal{O}^{(i)}$ (with here $i = 1$) is going to play the role of a single mind in the MMI. Moreover, We first consider a symmetric beam splitter and during the measurement operation we postulate the unitary evolution:

$$\begin{aligned} |\Psi_0\rangle \otimes |E_0, \mathcal{O}_0^{(1)}\rangle \otimes |\spadesuit^{(1)}\rangle &\to \sqrt{\frac{1}{2}} \left(|\uparrow\rangle \otimes |E_\uparrow, \mathcal{O}_\uparrow^{(1)}\rangle + |\downarrow\rangle \otimes |E_\downarrow, \emptyset^{(1)}\rangle \right) \otimes |\spadesuit^{(1)}\rangle, \\ |\Psi_0\rangle \otimes |E_0, \mathcal{O}_0^{(1)}\rangle \otimes |\heartsuit^{(1)}\rangle &\to \sqrt{\frac{1}{2}} \left(|\uparrow\rangle \otimes |E_\uparrow, \emptyset^{(1)}\rangle + |\downarrow\rangle \otimes |E_\downarrow, \mathcal{O}_\downarrow^{(1)}\rangle \right) \otimes |\heartsuit^{(1)}\rangle, \end{aligned} \tag{31}$$

[8] This relation is equivalent to the frequency relation $\frac{N_\alpha(h)}{N} \simeq \|\Psi_\alpha\|^2$ defined for some histories h and which is valid even for N finite. Note that the accuracy increases with N since the highly discrete sum of Dirac peaks approaches a continuous fluid with density $\|\Psi(\mathbf{q}, t)\|^2$. A faster convergence is obtained by limiting our analysis to coarse grained probability functions in some elementary but finite spatial cells.

where $|\spadesuit^{(1)}\rangle$ and $|\heartsuit^{(1)}\rangle$ are two orthogonal normalized states of a single qubit taking part to the interaction. The exponent $i = 1$ (here $i = 1$) labels the qubit 'familiy' which is related to $\mathcal{O}^{(i)}$. Here we start with only one family but we will have to introduce as many families $i = 1, ..., M$ as we have observer minds $\mathcal{O}^{(i)}$ (see Eqs. 35-37 below). The $|E_{\downarrow}, \emptyset^{(1)}\rangle$ is a specific quantum state of the brain where the memory associated with the mind has been lost or destroyed, i.e., this is a kind of ground state. Yet, the brain and environment $E_{\downarrow}$ could keep some persistent information about the result $\downarrow$ (this is indicated by the state $E_{\downarrow}$). A similar comment could be done for $|E_{\uparrow}, \emptyset^{(1)}\rangle$. Note, that every thing is deterministic and unitary: depending on the value of the qubit $|\spadesuit^{(1)}\rangle$ or $|\heartsuit^{(1)}\rangle$ the evolution follows one or the other of the two alternatives. Now, comes the trick we could easily introduce some deterministic ignorance on the state of the qubit decided at, let say the beginning of the Universe. Consider then that we have a large ensemble of M such qubits in a product state like $|\spadesuit^{(1,1)}\rangle \otimes ... \otimes |\heartsuit^{(1,M)}\rangle$ and where the number of $\heartsuit^{(1)}$ equals the number of $\spadesuit^{(1)}$, i.e., $\tilde{M}_{\spadesuit^{(1)}} \simeq \tilde{M}_{\heartsuit^{(1)}}$ (the second exponent labels the different copies of the qubit of the family $i = 1$). We have here a 'random' but classical distribution of the two states. By doing and redoing the same experiment the observer (or her/his friend obeying the unitary evolution given by Eq. 37) will interact with one exemplar of the product state (i.e., $\heartsuit^{(1,j)}$ if the jth qubit is a heart or $\spadesuit^{(1,j)}$ if it is a spad). Therefore, for an observer mind $\mathcal{O}^{(1,j)}$ taken in the ensemble we can objectively define the probability for interacting with a spad or a heart as:

$$\mathcal{P}_{\spadesuit^{(1)}} := \frac{\tilde{M}_{\spadesuit^{(1)}}}{M} \simeq \frac{1}{2}, \ \mathcal{P}_{\heartsuit^{(1)}} := \frac{\tilde{M}_{\heartsuit^{(1)}}}{M} \simeq \frac{1}{2}. \tag{32}$$

This probability law could be justified like in classical or Bohmian mechanics by using a typicality approach with equal measures for the two outcomes and by application of the Bernoulli/Laplace law of large numbers in the limit $M \to +\infty$ [9]. Moreover, from Eq. 37 we have

$$\mathcal{P}_{\spadesuit^{(1)}} = \mathcal{P}(\mathcal{O}^{(1)}_{\uparrow}) \simeq \frac{1}{2}, \ \mathcal{P}_{\heartsuit^{(1)}} = \mathcal{P}(\mathcal{O}^{(1)}_{\downarrow}) \simeq \frac{1}{2}. \tag{33}$$

meaning that the probability (i.e., the relative frequency) for the observer mind $\mathcal{O}^{(1)}$ taken in the ensemble to memorize the $\uparrow$ (or $\downarrow$) quantum state is one half. This defines the pivotal result of our deduction for a single mind.

In a second stage we can easily extend the pivotal result to an ensemble of many minds. Consider for example two minds (the generalization being

[9] We emphasize that the typicality reasoning is here classical unlike the one of Everett. Considering the different histories $|h^{(1)}\rangle := \bigotimes_{k=1}^{M} |s^{(1,k)}_{1,k}\rangle$ (with $s_{1,k}$ = spade or heart) we need a density matrix $\hat{\rho}^{(1)} = \sum_{h^{(1)}} \mathcal{M}(h^{(1)})|h^{(1)}\rangle\langle h^{(1)}|$ with the measure $\mathcal{M}(h^{(1)}) = \frac{1}{2^M}$. Introducing the frequency operator $\hat{Q}_{s^{(1)}} = \sum_{k=1}^{M} \frac{\hat{\Pi}^{(k)}_{s^{(1)}}}{M}$ (see Eq.4 in footnote 3) we have $\text{Tr}[\hat{Q}_{s^{(1)}}|h^{(1)}\rangle\langle h^{(1)}|] = \frac{M_{s^{(1)}}(h^{(1)})}{M}$ which for typical histories gives us: $\frac{M_{s^{(1)}}(\bar{h}^{(1)})}{M} \simeq \frac{1}{2}$. For the whole ensemble we have also $\text{Tr}[\hat{Q}_{s^{(1)}}\hat{\rho}^{(1)}] = \frac{1}{2}$.

obvious for N minds as we see below). We suppose the initial state in Eq. 37 transformed into $|\Psi_0\rangle \otimes |E_0, \mathcal{O}_0^{(1)}, \mathcal{O}_0^{(2)}\rangle \otimes |s^{(1)}\rangle \otimes |s'^{(2)}\rangle$ where $\mathcal{O}_0^{(1)}$ and $\mathcal{O}_0^{(2)}$ are two mind states (collective excitations) unaware of each other and $|s^{(1)}\rangle \otimes |s'^{(2)}\rangle$ some spin states with s = spade or heart, and s' = spade or heart. After the interaction we obtain four possible outcomes:

$$|\Psi_0\rangle \otimes |E_0, \mathcal{O}_0^{(1)}, \mathcal{O}_0^{(2)}\rangle \otimes |\spadesuit^{(1)}\rangle \otimes |\spadesuit^{(2)}\rangle \rightarrow$$
$$\sqrt{\frac{1}{2}}\Big(|\uparrow\rangle \otimes |E_\uparrow, \mathcal{O}_\uparrow^{(1)}, \mathcal{O}_\uparrow^{(2)}\rangle + |\downarrow\rangle \otimes |E_\downarrow, \emptyset^{(1)}, \emptyset^{(2)}\rangle\Big) \otimes |\spadesuit^{(1)}\rangle \otimes |\spadesuit^{(2)}\rangle, \quad (34)$$

$$|\Psi_0\rangle \otimes |E_0, \mathcal{O}_0^{(1)}, \mathcal{O}_0^{(2)}\rangle \otimes |\spadesuit^{(1)}\rangle \otimes |\heartsuit^{(2)}\rangle \rightarrow$$
$$\sqrt{\frac{1}{2}}\Big(|\uparrow\rangle \otimes |E_\uparrow, \mathcal{O}_\uparrow^{(1)}, \emptyset^{(2)}\rangle + |\downarrow\rangle \otimes |E_\downarrow, \emptyset^{(1)}, \mathcal{O}_\downarrow^{(2)}\rangle\Big) \otimes |\spadesuit^{(1)}\rangle \otimes |\heartsuit^{(2)}\rangle, \quad (35)$$

$$|\Psi_0\rangle \otimes |E_0, \mathcal{O}_0^{(1)}, \mathcal{O}_0^{(2)}\rangle \otimes |\heartsuit^{(1)}\rangle \otimes |\spadesuit^{(2)}\rangle \rightarrow$$
$$\sqrt{\frac{1}{2}}\Big(|\uparrow\rangle \otimes |E_\uparrow, \emptyset^{(1)}, \mathcal{O}_\uparrow^{(2)}\rangle + |\downarrow\rangle \otimes |E_\downarrow, \mathcal{O}_\downarrow^{(1)}, \emptyset^{(2)}\rangle\Big) \otimes |\heartsuit^{(1)}\rangle \otimes |\spadesuit^{(2)}\rangle, \quad (36)$$

$$|\Psi_0\rangle \otimes |E_0, \mathcal{O}_0^{(1)}, \mathcal{O}_0^{(2)}\rangle \otimes |\heartsuit^{(1)}\rangle \otimes |\heartsuit^{(2)}\rangle \rightarrow$$
$$\sqrt{\frac{1}{2}}\Big(|\uparrow\rangle \otimes |E_\uparrow, \emptyset^{(1)}, \emptyset^{(2)}\rangle + |\downarrow\rangle \otimes |E_\downarrow, \mathcal{O}_\downarrow^{(1)}, \mathcal{O}_\downarrow^{(2)}\rangle\Big) \otimes |\heartsuit^{(1)}\rangle \otimes |\heartsuit^{(2)}\rangle. \quad (37)$$

This situation clearly mirrors the result discussed in Section 4 surrounding Eq. 12. Now, like for the single mind problem we can introduce product states $\bigotimes_{k=1}^{M}(|s_{1,k}^{(1,k)}\rangle \otimes |s_{2,k}^{(2,k)}\rangle)$ with $s_{i,k}$ = spade or heart. Again we suppose a 'typical' random distribution of spades and hearts such that in the product state the population is equally distributed. We have a kind of '*Stosszahlansatz*' (molecular chaos hypothesis) discussed in Boltzmann statistical mechanics and which involves an hypothesis about the absence of correlation (i.e. independence) between the $|s_{1,k}^{(1,k)}\rangle$ and $|s_{2,k}^{(2,k)}\rangle$. Therefore, the probabilities read

$$\mathcal{P}_{s^{(1)},s'^{(2)}} = \mathcal{P}_{s^{(1)}}\mathcal{P}_{s'^{(2)}} \simeq \frac{1}{4}, \quad (38)$$

and with the observer mind states:

$$\mathcal{P}_{\spadesuit^{(1)},\spadesuit^{(2)}} = \mathcal{P}(\mathcal{O}_\uparrow^{(1)}, \mathcal{O}_\uparrow^{(2)}),\ \mathcal{P}_{\spadesuit^{(1)},\heartsuit^{(2)}} = \mathcal{P}(\mathcal{O}_\uparrow^{(1)}, \mathcal{O}_\downarrow^{(2)})$$
$$\mathcal{P}_{\heartsuit^{(1)},\spadesuit^{(2)}} = \mathcal{P}(\mathcal{O}_\downarrow^{(1)}, \mathcal{O}_\uparrow^{(2)}),\ \mathcal{P}_{\heartsuit^{(1)},\heartsuit^{(2)}} = \mathcal{P}(\mathcal{O}_\downarrow^{(1)}, \mathcal{O}_\downarrow^{(2)}) \quad (39)$$

The previous formalism can be generalized to N minds by introducing states like $|E_0, \mathcal{O}_0^{(1)}, ..., \mathcal{O}_0^{(N)}\rangle$ and using product spin-states $\bigotimes_{k=1}^{M}\bigotimes_{i=1}^{N}|s_{i,k}^{(i,k)}\rangle$. The unitary evolution together with Stosszahlansatz allow us to define the probability

$$\mathcal{P}(N_\uparrow, N_\downarrow) = \frac{N!}{N_\uparrow! N_\downarrow!}\mathcal{P}(\mathcal{O}_\uparrow)^{N_\uparrow}\mathcal{P}(\mathcal{O}_\downarrow)^{N_\downarrow}. \quad (40)$$

where $N_\uparrow$ and $N_\downarrow$ are the number of observer mind states $\mathcal{O}_\uparrow$ and $\mathcal{O}_\downarrow$ in the two branches $|\uparrow\rangle$ and $|\downarrow\rangle$ respectively. This relation is obviously the same as

Eq. 13 in the MMI of Albert and Loewer. Therefore, by a direct application of the law of large numbers we get:

$$\lim_{N\to+\infty}\frac{N_\uparrow}{N}=\mathcal{P}(\mathcal{O}_\uparrow)=\frac{1}{2}$$
$$\lim_{N\to+\infty}\frac{N_\downarrow}{N}=\mathcal{P}(\mathcal{O}_\downarrow)=\frac{1}{2} \tag{41}$$

which justifies Born's rule for the very particular and simple quantum state evolution $|\Psi_0\rangle\to\sqrt{\frac{1}{2}}\,(|\uparrow\rangle+|\downarrow\rangle)$.

In order to further generalize the previous procedure we can use the methodology and tricks of Zurek [79,75,8]. For this we consider the time evolution $|\Psi_0\rangle\otimes|E_0\rangle\to\sqrt{\frac{1}{T}}\sum_{\alpha\in\Delta}|\alpha\rangle\otimes|E_\alpha\rangle$ for the symmetric Schmidt state (T is an integer and the cardinality of Δ). In presence of the observer and environment we admit now instead of Eq. 37 for a single mind:

$$|\Psi_0\rangle\otimes|E_0,\mathcal{O}_0^{(1)}\rangle\otimes|\beta^{(1)}\rangle\to\sqrt{\frac{1}{T}}\sum_{\alpha\in\Delta,\alpha\neq\beta}|\alpha\rangle\otimes|E_\alpha,\emptyset^{(1)}\rangle\otimes|\beta^{(1)}\rangle$$
$$+\sqrt{\frac{1}{T}}|\beta\rangle\otimes|E_\beta,\mathcal{O}_\beta^{(1)}\rangle\otimes|\beta^{(1)}\rangle \tag{42}$$

where we introduced a $T-$level quantum system $|s^{(1)}\rangle$ (i.e., with $s\in\Delta$) to root the system. If $s=\beta$ the mind state $\mathcal{O}_0^{(1)}$ evolves into the $\mathcal{O}_\beta^{(1)}$ channel and the other channels are empty. Naturally, we have T relations like Eq. 42 corresponding to the T different levels $|s^{(1)}\rangle$ and to the T different branches. In complete analogy with the previous case we define probabilities for the spin states and the related mind states $\mathcal{O}_\beta^{(i)}$ as:

$$\mathcal{P}_{\beta^{(i)}}=\mathcal{P}(\mathcal{O}_\beta^{(i)})\simeq\frac{1}{T} \tag{43}$$

We have from independence and the Stosszahlansatz for two mind states:

$$\mathcal{P}_{\alpha^{(i)},\beta^{(j)}}=\mathcal{P}(\mathcal{O}_\alpha^{(i)},\mathcal{O}_\beta^{(i)})=\mathcal{P}_{\alpha^{(i)}}\mathcal{P}_{\beta^{(j)}}=\mathcal{P}(\mathcal{O}_\alpha^{(i)})\mathcal{P}(\mathcal{O}_\beta^{(j)})\simeq\frac{1}{T^2} \tag{44}$$

Finally, for a many-minds system (with N mind states $\mathcal{O}^{(1)},...,\mathcal{O}^{(N)}$) we can define in analogy with Eq. 40 the multinomial probability for having the distribution $\{N_\alpha\}$ where N_α is the number of mind states $\mathcal{O}_\alpha$ in the branch $|\alpha\rangle$ (with the constraint $\sum_{\alpha\in\Delta}N_\alpha=N$):

$$\mathcal{P}(\{N_\alpha\})=\frac{N!}{\Pi_\alpha N_\alpha!}\Pi_\alpha\mathcal{P}(\mathcal{O}_\alpha)^{N_\alpha}\simeq\frac{N!}{\Pi_\alpha N_\alpha!}\frac{1}{T^N}. \tag{45}$$

In the typical regime we deduce from the law of large numbers the relation

$$\lim_{N\to+\infty}\frac{N_\alpha}{N}=\mathcal{P}(\mathcal{O}_\alpha)=\frac{1}{T} \tag{46}$$

which recovers Born's rule for the state $\sqrt{\frac{1}{T}}\sum_{\alpha\in\Delta}|\alpha\rangle$.

The last step of our deduction follows strictly the fine graining trick of Zurek discussed in Section 5. Indeed, starting with a quantum state $|\Psi_0\rangle = \sum_a \sqrt{\mathcal{P}_a}|a\rangle$ with $\mathcal{P}_a = \frac{T_a}{T}$ a rational number we redo the steps from Eq.25 to Eq. 28 and obtain after entanglement with a specifically designed environment the Schmidt state $|\Psi_0\rangle\otimes|E_0\rangle \to \sqrt{\frac{1}{T}}\sum_{\alpha\in\Delta}|\alpha\rangle\otimes|E_\alpha\rangle$. The rest of the reasoning is similar to the one leading to Eq. 45 and Eq. 46: we deduce the probability

$$\mathrm{Lim}_{N\to+\infty}\sum_{\alpha\in\Delta_a}\frac{N_\alpha}{N} = \sum_{\alpha\in\Delta_a}\mathcal{P}(\mathcal{O}_\alpha) = \frac{T_a}{T} := \mathcal{P}_a \tag{47}$$

where Δ_a is the subset of Δ associated with the quantum state $|a\rangle$ (see Eq. 26). This closes our derivation of Born's rule.

I emphasize, that in our model the role of the environment is central since it allows us to define unambiguously decohered Worlds evolving independently. Therefore, the theory agrees for all practical purposes with the standard quantum mechanical interpretation of decoherence. Like in Zurek existential interpretation [79,76] the unitary evolution is all what is required for the theory to hold. However, here Born's rule results as a contingent consequence of the dynamic relying on the Stosszahlansatz hypothesis. Like in Everett's (but unlike in Zurek's) work the role of the observer is here central since it is only from his/her perspective that the notion of probability makes physical sense. Also, we stress that the theory we propose here is naturally generalized to systems of several observers with huge numbers of minds $M \to +\infty$. This condition is mandatory as stressed by Albert and Loewer in order to recover a common experience agreement between the separate perspective of the various observers (i.e. in order to avoid the mind-less Hulk dilemma discussed in [4,5]). Finally we emphasize once more that the model is fully unitary and doesn't require a mind/brain dualism (i.e., unlike some readings of the original MMI [4]). Here, the minds are physically linked to the brain and define some quantum excitations of a mechanical structure. Of course, we agree that the model of 'quantum' minds and brain advocated in the present work is very sketchy and speculative at this level and we yet will not attempt to develop it further here.

7 Conclusion and comments

To conclude this work several remarks are necessary. First, observe that the present derivation is strongly related to Zurek's work about the Laplace indifference principle. Going back to Eq. 36 and Eq. 37 we see that the main difference concern the permutation between two observer mind states $\mathcal{O}^{(1)}$ and $\mathcal{O}^{(2)}$ in the two branches corresponding to the observable $|\uparrow\rangle$ or $|\downarrow\rangle$. Removing the irrelevant degrees of freedom we see that at the end of the unitary evolution we get either

$$\sqrt{\frac{1}{2}}\left(|\uparrow\rangle\otimes|E_\uparrow,\mathcal{O}^{(1)}_\uparrow,\emptyset^{(2)}\rangle + |\downarrow\rangle\otimes|E_\downarrow,\emptyset^{(1)},\mathcal{O}^{(2)}_\downarrow\rangle\right), \tag{48}$$

or

$$\sqrt{\frac{1}{2}}\left(|\uparrow\rangle\otimes|E_{\uparrow},\emptyset^{(1)},\mathcal{O}_{\uparrow}^{(2)}\rangle+|\downarrow\rangle\otimes|E_{\downarrow},\mathcal{O}_{\downarrow}^{(1)},\emptyset^{(2)}\rangle\right), \tag{49}$$

depending which state (i.e., $|\spadesuit^{(1)}\rangle\otimes|\heartsuit^{(2)}\rangle$ or $|\heartsuit^{(1)}\rangle\otimes|\spadesuit^{(2)}\rangle$) the Nature 'randomly' assigned to the full system. Now, the objective probability (i.e., relative frequency) of each evolution is $\frac{1}{2}$ for symmetry reasons and in agreement with our previous discussion of the Stosszahlansatz. Therefore, we get

$$\mathcal{P}(\mathcal{O}_{\uparrow}^{(i)})=\mathcal{P}(\mathcal{O}_{\downarrow}^{(i)})=\frac{1}{2} \tag{50}$$

for $i=1,2$. There is complete indifference for the observer mind state i where he will finish his journey. This conveys the spirit of Laplace indifference principle which is here driven by symmetry like for a classical die tossing. On the one side, this clean self-locating uncertainty for each mind state is completely classical since it is driven by statistical distributions associated with a Stosszahlansatz for the heart and spade permutations. On the other side, this result is fully quantum and unitary. Unlike in the original MMI of Albert and Loewer no genuine stochastic process breaking the unitarity of the quantum evolution has to to be invoked. Compared to the self-locating discussion given by Zurek, Deutsch and Wallace and Carroll and Sebens our procedure doesn't suffer from the incoherence problem associated with the standard MWI. The price to be paid is of course heavy since we have to introduce several mind states in a kind of schizophrenic quantum superposition.

The previous analysis prompts at least two fundamental criticisms which I think are very well founded. The first criticism concerns the causal structure of the model used in that work which is very conspiratorial. Indeed, the model used by Zurek is already conspiratorial since in order to recover Born's rule for a general quantum state we must include a fine graining procedure (which I named a trick) which looks mathematically fine but which is physically superdeterministic. In this approach the fine graining procedure can be experimentally implemented by using logical gates and beam splitters (see for instance the discussions and proposals made by Vaidman). The idea is to introduce states like $|a^{(S)}\rangle=\frac{1}{\sqrt{N_a}}\sum_{\alpha\in\Delta_a}|\alpha^{(S)}\rangle$ to transform a general wave function given by Eq. 25 into a symmetric Schmidt state as given by Eq. 27. However, this fine graining is necessarily wave function dependent and by changing the probability coeeficents in the initial we would have to modify completely the fine grains for the later experiments. Therefore, from the point of view of causality where the observer is selecting the beam splitters and other apparatus this looks like the fine grains was decided in advance in a conspiratorial way for reproducing Born's rule for a very specific problem. This is of course not necessarily a fatal objection if we are ready to accept such features. Many other interpretations of quantum mechanics involve superdeterministic or even retrocausal properties but we have at least to be aware of the problem.

The second criticism concerns of course the notion of minds introduced in the present work. We fully agree that this notion is highly speculative. In fact,

we agree with Albert recent negative comments concerning the value of this whole business about many minds and about his own work done with Loewer. They wrote:

> *The many-minds interpretation of quantum mechanics that Barry Loewer and I discussed twenty-five or so years ago, and which is rehearsed in chapter of [5], was a (bad, silly, tasteless, hopless, explicitely dualist) attempts of coming to terms with that realization.* [30], p. 163.

The 'tasteless' model discussed here is presented as a kind of disparate remedy for making sense of the MMI and MWI or may be should we say for saving the whole MWI enterprise. Yet, several authors got interested in the past years about the possible role of minds and consciousness in the interpretations of quantum mechanics. So, may be this is a good strategy to try. The 'toy' model considered here is far from being perfect and while it allows us to recover Born's rule in the MWI (without mind/brain dualism) it opens many fundamental and unsettled questions which could constitute fatal objections to the proposal if we can not answer them. Therefore, I only hope that the model suggested here could motivate further work in that fascinating area.

8 Appendix: The Wallace proof in the semantics of Zurek

This leads us to Wallace scenario that we analyze using Zurek semantics. Wallace like Deutsch was interested into the observation of the system S by an observer Alex that we can directly identify with the state of the environment E. Wallace thus considers the following state

$$|\Psi^{(SE)}\rangle = \sqrt{\frac{1}{N}} \sum_{\alpha \in \Delta} |\alpha^{(S)}\rangle \otimes |\mathrm{Alex}_\alpha^{(E)}\rangle \tag{51}$$

as well as the swapped state $\hat{U}^{(S)}(\alpha \leftrightarrow \beta)|\Psi^{(SE)}\rangle$. If the original state contains the terms

$$|\Psi^{(SE)}\rangle = |\alpha^{(S)}\rangle \otimes |\mathrm{Alex}_\alpha^{(E)}\rangle + |\beta^{(S)}\rangle \otimes |\mathrm{Alex}_\beta^{(E)}\rangle + |R\rangle \tag{52}$$

the new swapped state contains instead the terms

$$\hat{U}^{(S)}(\alpha \leftrightarrow \beta)|\Psi^{(SE)}\rangle = |\beta^{(S)}\rangle \otimes |\mathrm{Alex}_\alpha^{(E)}\rangle + |\alpha^{(S)}\rangle \otimes |\mathrm{Alex}_\beta^{(E)}\rangle + |R\rangle \tag{53}$$

where $|R\rangle$ is the 'rest' which is irrelevant. Now, by direct application of Laplace indifference principle we obtain (see Eq.19)

$$\begin{aligned} \mathcal{P}_\Psi(\alpha^{(S)}, \mathrm{Alex}_\alpha^{(E)}) &= \mathcal{P}_{\hat{U}^{(S)}\Psi}(\beta^{(S)}, \mathrm{Alex}_\alpha^{(E)}) \\ \mathcal{P}_\Psi(\beta^{(S)}, \mathrm{Alex}_\beta^{(E)}) &= \mathcal{P}_{\hat{U}^{(E)}\Psi}(\alpha^{(S)}, \mathrm{Alex}_\beta^{(E)}). \end{aligned} \tag{54}$$

Unlike Zurek Wallace didn't used a counterswap on the E subsystem. Instead, he used a trick by supposing that (i) the subsystem S has also a ground

state $|\emptyset^{(S)}\rangle$ and that (ii) we apply the erasing operation $\hat{U_e}^{(S)}|\alpha^{(S)}\rangle = |\emptyset^{(S)}\rangle$, $\hat{U_e}^{(S)}|\beta^{(S)}\rangle = |\emptyset^{(S)}\rangle$. After application of the erasing process on the states given by Eqs. 52, 53 we obtain

$$\begin{aligned}\hat{U_e}^{(S)}|\Psi^{(SE)}\rangle &= |\emptyset^{(S)}\rangle \otimes (|\mathrm{Alex}_\alpha^{(E)}\rangle + |\mathrm{Alex}_\beta^{(E)}\rangle + |R\rangle \\ \hat{U_e}^{(S)}\hat{U}^{(E)}(\alpha \leftrightarrow \beta)|\Psi^{(SE)}\rangle &= |\emptyset^{(S)}\rangle \otimes (|\mathrm{Alex}_\alpha^{(E)}\rangle + |\mathrm{Alex}_\beta^{(E)}\rangle + |R\rangle \end{aligned} \quad (55)$$

This motivates the set of equations:

$$\begin{aligned}\mathcal{P}_\Psi(\alpha^{(S)}, \mathrm{Alex}_\alpha^{(E)}) &= \mathcal{P}_{\hat{U_e}^{(S)}\Psi}(\emptyset^{(S)}, \mathrm{Alex}_\alpha^{(E)}) \\ \mathcal{P}_\Psi(\beta^{(S)}, \mathrm{Alex}_\beta^{(E)}) &= \mathcal{P}_{\hat{U_e}^{(S)}\Psi}(\emptyset^{(S)}, \mathrm{Alex}_\beta^{(E)}) \\ \mathcal{P}_{\hat{U}^{(S)}\Psi}(\alpha^{(S)}, \mathrm{Alex}_\beta^{(E)}) &= \mathcal{P}_{\hat{U_e}^{(S)}\hat{U}^{(S)}\Psi}(\emptyset^{(S)}, \mathrm{Alex}_\beta^{(E)}) \\ \mathcal{P}_{\hat{U}^{(S)}\Psi}(\beta^{(S)}, \mathrm{Alex}_\alpha^{(E)}) &= \mathcal{P}_{\hat{U_e}^{(S)}\hat{U}^{(S)}\Psi}(\emptyset^{(S)}, \mathrm{Alex}_\alpha^{(E)}). \end{aligned} \quad (56)$$

Finally we use the fact that we have

$$\begin{aligned}\mathcal{P}_{\hat{U_e}^{(S)}\Psi}(\emptyset^{(S)}, \mathrm{Alex}_\alpha^{(E)}) &= \mathcal{P}_{\hat{U_e}^{(S)}\hat{U}^{(S)}\Psi}(\emptyset^{(S)}, \mathrm{Alex}_\alpha^{(E)}) \\ \mathcal{P}_{\hat{U_e}^{(S)}\Psi}(\emptyset^{(S)}, \mathrm{Alex}_\alpha^{(E)}) &= \mathcal{P}_{\hat{U_e}^{(S)}\hat{U}^{(S)}\Psi}(\emptyset^{(S)}, \mathrm{Alex}_\alpha^{(E)}) \end{aligned} \quad (57)$$

to get after combining with Eqs. 54, 56 the result

$$\mathcal{P}_\Psi(\alpha^{(S)}, \mathrm{Alex}_\alpha^{(E)}) = \mathcal{P}_\Psi(\beta^{(S)}, \mathrm{Alex}_\beta^{(E)}) \quad (58)$$

which is Zurek Eq.22. By proceeding as in Zurek case we again obtain equiprobability and thus Born's rule Eq. 29.

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