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# Adaptive observer-based $H_\infty$ FTC for T-S fuzzy systems. Application to cart motion model

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## Abstract

In this paper, an adaptive observer-based **fault-tolerant control (FTC)** strategy is proposed for a class of Takagi-Sugeno (T-S) fuzzy systems with both actuator and sensor faults under external disturbances. FTC approach is developed to compensate the actuator faults and to stabilize the faulty system. Furthermore, using  $H_\infty$  optimization technique, an adaptive fuzzy observer is developed, not only to achieve a simultaneous estimation of system states, sensor and actuator faults, but also to attenuate the influence of disturbances. In terms of **linear matrix inequalities (LMIs)**, sufficient conditions of the existence of observer and controller are derived. **We overcome the drawback of two-step algorithm by proposing a single-step one which allows to solve only the strict LMIs. Therefore, the obtained results present an acceptable compromise between conservatism reduction and computational complexity.** Finally, two numerical examples which one of them is an application to a cart motion model are presented to demonstrate the usefulness of the proposed method.

*Keywords:* T-S fuzzy systems, Adaptive fuzzy observer, Actuator/Sensor fault estimation, Fault Tolerant Control, Linear Matrix Inequality

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## 1. Introduction

With the development of practical engineering, industrial systems are more and more dependent on actuators and sensors which could represent faulty behavior. Sensor and/or actuator faults may not only affect system performances and system stability but may even cause catastrophic accidents. Many researchers have been already interested to fault detection and isolation (FDI), fault estimation (FE) and fault-tolerant control (FTC) problems for nonlinear systems. However, most of them have focused separately on either actuator faults Gassara et al. (2014b); Kharrat et al. (2017); Gao and Ding (2007a); Jia et al. (2016b) or sensor faults Bouattour et al. (2011); Gao and Ding (2007b); Estrada et al. (2015); Qiao and Yang (2018).

It is well known that FE is more general than FDI because it can not only detect and isolate the faults but also provide more detailed information such as location, magnitude and shape. Several works have used different kinds of observers to achieve FE. For instance, sliding mode observers have been used in Gao and Ding (2007a); Liu et al. (2013); Yin et al. (2017); Liu et al. (2011), unknown input proportional multiple-integral observers have been proposed in Hamdi et al. (2013); Koenig (2005) and both proportional multiple-integral observer and proportional derivative observer have been designed in Challoo and Dubey (2011), to achieve a simultaneous estimation of system states and actuator faults. In Jia et al. (2014, 2015, 2016a); Kharrat et al. (2018c), the reconstruction of actuator faults and system states has been carried out by developing some approaches based on learning observer. Nevertheless, the main disadvantage of the latter observers is that they are not effective when dealing with simultaneous sensor and actuator faults. Recently, several researchers have been paid much attention to adaptive observers (such in Gassara et al. (2014b); Kharrat et al. (2017); Zhang et al. (2008)) to achieve a simultaneous estimation of system states and actuator faults. Noting that adaptive observers are able to estimate actuator faults as well as sensor faults Zhang et al. (2016a). However, most of papers that have dealt with the estimation of both sensor and actuator faults, have not considered the control problem such as in Zhang et al. (2014); Li and Zhu (2016); Youssef et al. (2017); Kharrat et al. (2018b). However, in the presence of a potential actuator or sensor faults, it is impossible to maintain the damaged system at an acceptable level of performance. Thus, it becomes of prime importance to apply the FTC design to guarantee the stability of the closed-loop faulty system. The basic idea of the FTC approach in addition to the maintain

of system stability, is to take into account the fault occurrence into a new control which will become tolerant to this fault by canceling its undesirable effects. It is worth noting that the design of FTC requires obviously FE schema to make adequate decision. In fact, the FE step can provide information to achieve the reliability of controlled systems. Generally speaking, FTC is an effective way to improve system performances and to preserve the safety and productivity of the manufacturing processes especially for systems affected by sensor and/or actuator faults (see for example Han et al. (2016); Li and Zhu (2015); Shen et al. (2017); Liu and Gao (2017) and references therein).

Since the development of technology and the complexity of modern systems, it is quite common that nonlinear behaviors and faults in actuators and sensors are inevitable to exist simultaneously in normal system operations. Takagi-Sugeno (T-S) fuzzy modeling technique is an effective approach used to approximate nonlinear models by mixing some local linear representation models with the help of IF-THEN fuzzy rules and fuzzy membership functions (see for example Feng and Shi (2016); Zhang et al. (2018, 2019); Dong et al. (2019); Wu et al. (2019)). Considerable efforts have been devoted to the issue of FTC and FE for T-S fuzzy systems Liu et al. (2013); Han et al. (2016); Chen et al. (2018); Kharrat et al. (2018a); Li et al. (2017), for Markovian jump systems Liu et al. (2011); Li and Zhu (2017); Chen et al. (2016) and Itô stochastic systems Liu and Shi (2013), subject to sensor and/or actuators faults. However in all the papers mentioned above, the observer and FTC design have been given separately and solved using a two-step algorithm. Actually, the present paper improves the previous works in terms of conservatism reduction since the design conditions of the observer and FTC are formulated in a set of Linear matrix Inequalities (LMIs) that can be solved on a single step. To the best of our knowledge, for T-S nonlinear systems subject to simultaneously actuator, sensor faults and external disturbances, there is no work in the literature that gives both the gains of the robust  $H_\infty$  observer and those of the FTC by solving all LMIs in only one step.

In this paper, we contribute to the further development of the stability and stabilization analysis for T-S fuzzy systems with establishing new results for this problem. Indeed, we were inspired by the observer proposed in Zhang et al. (2016a) and Gao and Wang (2006) to establish an adaptive fuzzy observer allowing a simultaneous estimation of system states, sensor and actuator faults. Therefore, an observer-based FTC is developed for T-S fuzzy

systems to guarantee the stability of overall system. The observer and controller design is formulated in a set of LMIs which can be solved in only one step using YALMIP toolbox of MATLAB software.

The proposed findings ameliorate those existing in the literature on the following points:

1. First, we have proposed an adaptive observer-based FTC strategy in the presence of time-varying sensor/actuator faults and disturbances at the same time outperforming some previous works, which have dealt with FE and FTC by considering either sensor faults Qiao and Yang (2018); Ichalal et al. (2016) or actuator faults Kharrat et al. (2017); Gao and Ding (2007a); Jia et al. (2016b).
2. Second, in contrast to Jia et al. (2015); Shen et al. (2017) which didn't take into account the disturbances when analyzing control systems, the proposed FTC is robust to external disturbances. Thus,  $H_\infty$  optimization technique is developed to minimize the effect of disturbances not only on the controlled output but also on state estimation error  $e_{\bar{x}}(t)$  and fault estimation error  $e_{f_a}(t)$ .
3. Third, advanced techniques, aimed at reducing the conservatism, have been proposed. The design conditions of the observer and the FTC for T-S fuzzy systems are formulated under LMI constraints that can be solved in only one step. As it is known, the single step algorithm requires to solve only the strict LMIs which reduce the complexity and computational analyses.

To show the effectiveness of one step algorithm, a comparison with the paper of Han et al. (2016) in terms of the minimization of  $H_\infty$  performance level has been given.

The remainder of this paper is organized as follows. An overview of the T-S fuzzy systems is presented in the second section. Section 3 contains the main results. Besides the augmented system, an adaptive fuzzy observer and observer-based FTC are proposed. Then sufficient design conditions are expressed in the form of LMIs. Thereafter, two examples are proposed in section 4 to validate our theoretical results. A comparison study has been considered to show the performance of the proposed FTC strategy. Finally, a conclusion is given in section 5.

**Notations.** The notations throughout the paper are standard. For a matrix  $A \in \mathbb{R}^{n \times n}$ ,  $A > 0$  (respectively,  $A < 0$ ) represents a real symmetric positive definite matrix (respectively, negative definite matrix). Superscripts "T" and

" - 1" stand for matrix or vector transposition and matrix inverse, respectively.  $sym(A)$  signifies  $A + A^T$ . In LMIs, symbol  $(*)$  is used to represent the transposed element in a symmetric position.  $\lambda_{max}(\cdot)$  and  $\lambda_{min}(\cdot)$  represent the maximum and minimum eigenvalues of  $A$ , respectively.

## 2. Problem formulation

In this section we introduce the T-S fuzzy systems described by a set of If-Then rules where each rule is a local linear representation of the nonlinear system. The  $i^{th}$  rule of the system is of the following form.

Plant Rule  $i(i = 1, 2, \dots, r)$ : If  $\nu_1$  is  $\mu_{i1}$  and,  $\dots$ , and  $\nu_p$  is  $\mu_{ip}$ , Then

$$\dot{x}(t) = A_i x(t) + B_i(u(t) + f_a(t)) + D_i d(t) \quad (1)$$

$$y(t) = Cx(t) + Ff_s(t) \quad (2)$$

$$z(t) = C_{L_i}x(t) \quad (3)$$

$x(t) \in \mathbb{R}^{n_x}$  is the state vector,  $u(t) \in \mathbb{R}^{n_u}$  is the control input,  $y(t) \in \mathbb{R}^{n_y}$  is the measured output,  $d(t) \in \mathbb{R}^{n_d}$  is the external disturbance,  $f_a(t) \in \mathbb{R}^{n_u}$  and  $f_s(t) \in \mathbb{R}^{n_f}$  represent actuator and sensor fault, respectively. They can be constant or time-varying function.  $z(t) \in \mathbb{R}^{n_z}$  is the controlled output.  $A_i, B_i, D_i, C, F$  and  $C_{L_i}$  are known real constant matrices of appropriate dimensions.

We assume that  $n_y \geq n_u + n_f$ , pairs  $(A_i, B_i)$  are controllable,  $rank(B_i) = n_u$ ,  $rank(F) = n_f$ , and  $rank([C, F]) = n_y$ .

$\nu_j(x(t))(j = 1, \dots, p)$  are the premise variables which are supposed to be measurable,  $\mu_{ij}(i = 1, \dots, r, j = 1, \dots, p)$  are the fuzzy sets which are characterized by the membership functions.  $r$  and  $p$  are the total numbers of If-Then rules and the premise variables, respectively.

The overall fuzzy system inferred by fuzzy blending of each individual plant rule is given by

$$\dot{x}(t) = \sum_{i=1}^r h_i(\nu(x(t))) [A_i x(t) + B_i(u(t) + f_a(t)) + D_i d(t)] \quad (4)$$

$$y(t) = Cx(t) + Ff_s(t) \quad (5)$$

$$z(t) = \sum_{i=1}^r h_i(\nu(x(t))) [C_{L_i} x(t)] \quad (6)$$

in which

$$\nu(x(t)) = [\nu_1(x(t)), \dots, \nu_p(x(t))]$$

$$h_i(\nu(x(t))) = \frac{\sigma_i(\nu(x(t)))}{\sum_{i=1}^r \sigma_i(\nu(x(t)))}; \sigma_i(\nu(x(t))) = \prod_{j=1}^p \mu_{ij}(\nu_i(x(t)))$$

and  $\mu_{ij}(\nu_i(x(t)))$  is the grade of the membership of  $\nu_i(x(t))$  in  $\mu_{ij}$ .  $h_i(\nu(x(t)))$  is the weighting function which is in general nonlinear and satisfies  $0 \leq h_i(\nu(x(t))) \leq 1$  and  $\sum_{i=1}^r h_i(\nu(x(t))) = 1$ .

For simplicity, we introduce  $h_i$  to denote  $h_i(\nu(x(t)))$ .

Then the T-S fuzzy model can be rewritten as follows

$$\dot{x}(t) = \sum_{i=1}^r h_i[A_i x(t) + B_i(u(t) + f_a(t)) + D_i d(t)] \quad (7)$$

$$y(t) = Cx(t) + Ff_s(t) \quad (8)$$

$$z(t) = \sum_{i=1}^r h_i[C_{L_i} x(t)] \quad (9)$$

Prior to presenting the main results, the following two assumptions are assumed.

**Assumption 1.** *Koenig (2006) System  $(\bar{E}, \bar{A}_i, \bar{C})$  is observable,*

$$\text{rank} \begin{bmatrix} \bar{E} \\ \bar{C} \end{bmatrix} = n_x + n_f \quad (10)$$

and

$$\text{rank} \begin{bmatrix} s\bar{E} - \bar{A}_i \\ \bar{C} \end{bmatrix} = n_x + n_f, \forall s \in \mathbb{C}, \text{Re}(s) \geq 0, \forall i = [1, \dots, r] \quad (11)$$

**Assumption 2.** *Zhang et al. (2016b) Both actuator fault  $f_a(t)$  and sensor fault  $f_s(t)$  are differentiable after their occurrence (note that the faults don't need to be differentiable at the time of their occurrence).*

*Both  $\dot{f}_a(t)$  and  $\dot{f}_s(t)$  satisfy the following norm bounded constraints:*

$$\|f_a(t)\| \leq \alpha_a, \quad \|\dot{f}_a(t)\| \leq f_{a_{max}} \quad \text{with } 0 \leq \alpha_a, f_{a_{max}} < \infty$$

$$\|f_s(t)\| \leq \alpha_s, \quad \|\dot{f}_s(t)\| \leq f_{s_{max}} \quad \text{with } 0 \leq \alpha_s, f_{s_{max}} < \infty$$

### 3. Main results

#### 3.1. Augmented system

Inspired by the descriptor approach used in Gao and Ding (2007b) and Du et al. (2012), we take the sensor fault  $f_s(t)$  as an auxiliary state, so we obtain a descriptor system. The faulty system given by (7- 9) can be rewritten as follows:

$$\bar{E}\dot{\bar{x}}(t) = \sum_{i=1}^r h_i[\bar{A}_i\bar{x}(t) + \bar{B}_i(u(t) + f_a(t)) + \bar{D}_id(t)] \quad (12)$$

$$y(t) = \bar{C}\bar{x}(t) \quad (13)$$

$$z(t) = \sum_{i=1}^r h_i[C_{Li}x(t)] \quad (14)$$

where

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ f_s(t) \end{bmatrix}, \bar{E} = \begin{bmatrix} I_{n_x \times n_x} & 0_{n_x \times n_f} \\ 0_{n_f \times n_x} & 0_{n_f \times n_f} \end{bmatrix}, \bar{A}_i = \begin{bmatrix} A_i & 0_{n_x \times n_f} \\ 0_{n_f \times n_x} & 0_{n_f \times n_f} \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0_{n_f \times n_x} \end{bmatrix},$$

$$\bar{D}_i = \begin{bmatrix} D_i \\ 0_{n_f \times n_x} \end{bmatrix} \text{ and } \bar{C} = [C \quad F]$$

#### 3.2. Design of Adaptive Fuzzy Observer-based Fault Tolerant Control

In this section, an adaptive fuzzy observer is designed to simultaneously estimate system states, actuator and sensor faults for system (12-14).

$$\begin{cases} \dot{\xi}(t) = \sum_{i=1}^r h_i[(\bar{A}_i - L_{P_i}\bar{C})\hat{x}(t) + \bar{B}_i(u(t) + \hat{f}_a(t)) + L_{P_i}y(t)] \\ \hat{x}(t) = (\bar{E} + L_D\bar{C})^{-1}(\xi(t) + L_Dy(t)) \\ e_y(t) = y(t) - \hat{y}(t) \\ \hat{y}(t) = \bar{C}\hat{x}(t) \\ \dot{\hat{f}}_a(t) = \Gamma \sum_{i=1}^r h_i N_i(\dot{e}_y(t) + \sigma e_y(t)) \end{cases} \quad (15)$$

and the active **FTC** is:

$$u(t) = - \sum_{i=1}^r h_i K_i \hat{x}(t) - \hat{f}_a(t) \quad (16)$$



where  $\xi(t) \in \mathbb{R}^{n_x+n_f}$  is the middle variable,  $\hat{x}(t) \in \mathbb{R}^{n_x+n_f}$  is the estimation of augmented descriptor state vector  $\bar{x}(t) \in \mathbb{R}^{n_x+n_f}$ .  $\hat{y}(t) \in \mathbb{R}^{n_y}$  is the estimation of the output vector.  $e_y(t) \in \mathbb{R}^{n_y}$  is the output estimation error and  $\hat{f}_a(t) \in \mathbb{R}^{n_u}$  is the estimated of actuator fault  $f_a(t)$ .  $L_{P_i} \in \mathbb{R}^{(n_x+n_f) \times n_y}$  and  $L_D \in \mathbb{R}^{(n_x+n_f) \times n_y}$  are, respectively the proportional gains and derivative gain to be designed.  $\sigma \in \mathbb{R}$  is a positive scalar.  $\Gamma \in \mathbb{R}^{n_u \times n_u}$  is a symmetric positive definite matrix representing the learning rate which is set to produce a faster convergence speed of the states and actuator faults.  $N_i \in \mathbb{R}^{n_u \times n_y}$  are matrices to be determined.

By some manipulations and by posing  $\bar{S} = \bar{E} + L_D \bar{C}$ , the proposed observer (15) can be transformed into the following form

$$\bar{S} \dot{\hat{x}}(t) = \sum_{i=1}^r h_i [(\bar{A}_i - L_{P_i} \bar{C}) \hat{x}(t) + \bar{B}_i(u(t) + \hat{f}_a(t)) + L_{P_i} y(t)] + L_D \dot{y}(t) \quad (17)$$

By adding  $L_D \dot{y}(t)$  on both sides of equation (12) we obtain

$$\bar{S} \dot{\bar{x}}(t) = \sum_{i=1}^r h_i [\bar{A}_i \bar{x}(t) + \bar{B}_i(u(t) + f_a(t)) + D_i d(t)] + L_D \dot{y}(t) \quad (18)$$

$$\begin{aligned} \bar{S} \dot{\hat{x}}(t) &= \sum_{i=1}^r h_i [(\bar{A}_i - L_{P_i} \bar{C}) \bar{x}(t) + \bar{B}_i(u(t) + f_a(t)) + D_i d(t) + L_{P_i} y(t)] \\ &+ L_D \dot{y}(t) \end{aligned} \quad (19)$$

Define the state estimation errors as follows :

$$e_x(t) = x(t) - \hat{x}(t) \quad , \quad \bar{e}_x(t) = \bar{x}(t) - \hat{x}(t)$$

By taking into account (17), (19) and (15) the error dynamics and the output estimation error are obtained as

$$\dot{\bar{e}}_x(t) = \sum_{i=1}^r h_i [\bar{S}^{-1}(\bar{A}_i - L_{P_i} \bar{C}) \bar{e}_x(t) + \bar{S}^{-1} \bar{B}_i e_{f_a}(t) + \bar{S}^{-1} \bar{D}_i d(t)] \quad (20)$$

$$e_y(t) = \bar{C} \bar{e}_x(t) \quad (21)$$

where  $e_{f_a}(t) = f_a(t) - \hat{f}_a(t)$  depicts for the actuator fault estimation error. Note that in this framework, both time-varying actuator and sensor faults are considered, then it follows that  $\dot{f}(t) \neq 0$ .

Consequently, the dynamic of actuator fault estimation error is given by the following expression:

$$\dot{e}_{f_a}(t) = \dot{f}_a(t) - \dot{\hat{f}}_a(t) \quad (22)$$

Then

$$\dot{e}_{f_a}(t) = \dot{f}_a(t) - \Gamma \sum_{i=1}^r h_i N_i (\dot{e}_y(t) + \sigma e_y(t)) \quad (23)$$

By taking into consideration (20) and (21), one can obtain

$$\begin{aligned} \dot{e}_{f_a}(t) &= \dot{f}_a(t) - \sigma \Gamma \sum_{i=1}^r h_i N_i \bar{C} \bar{e}_x(t) - \Gamma \sum_{i=1}^r \sum_{j=1}^r h_i h_j N_i \bar{C} \bar{S}^{-1} ((\bar{A}_j - L_{p_j} \bar{C}) \\ &\quad \times \bar{e}_x(t) + \bar{B}_j e_{f_a}(t) + \bar{D}_j d(t)) \end{aligned} \quad (24)$$

Considering equation (16), the closed-loop of the T-S fuzzy system becomes

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j [(A_i - B_i K_j) x(t) + B_i K_j e_x(t) + B_i e_{f_a}(t) + D_i d(t)] \quad (25)$$

$$z(t) = \sum_{i=1}^r h_i [C_{L_i} x(t)] \quad (26)$$

### 3.3. Analysis and design conditions

**Theorem 1.** Considering system (7-9) under assumptions 1 and 2. Given a real positive scalar  $\gamma > 0$ , two scalar tuning parameters  $\sigma, \mu > 0$  and definite positive matrices  $M_1, M_2, M_3$  and  $M_4$ , the adaptive fuzzy observer proposed in (15) and the FTC designed in (16) ensure, under  $H_\infty$  performance level  $\gamma$ , the stability of closed-loop system (25-26) and the convergence of state estimation error  $\bar{e}_x(t)$  and fault estimation error  $e_{f_a}(t)$  in a uniformly bounded compact, which means

$$\int_0^\infty (z^T(t) M_1 z(t) + \bar{e}_x^T(t) M_2 \bar{e}_x(t) + e_{f_a}^T(t) M_3 e_{f_a}(t)) dt \leq \gamma^2 \int_0^\infty d^T(t) M_4 d(t) dt \quad (27)$$

if there exist symmetric positive definite matrices  $P_1$  and  $P_2$  and positive definite matrix  $G$  as well as matrices  $N_i, L_{P_i}$  and  $K_i$  such that  $\forall i \in [1, \dots, r]$  the following conditions hold:

$$(P_2 \bar{S}^{-1} \bar{B}_i)^T = N_i \bar{C}, \quad i = 1, 2, \dots, r \quad (28)$$

$$\Phi_{ij} + \Phi_{ji} < 0, \quad i, j = 1, 2, \dots, r, i \leq j \quad (29)$$

where

$$\Phi_{ij} = \begin{bmatrix} \varphi_{ij}^{11} & \varphi_{ij}^{12} & P_1 B_i & P_1 D_i \\ * & \varphi_i^{22} & \varphi_{ij}^{23} & P_2 \bar{S}^{-1} \bar{D}_i \\ * & * & \varphi_{ij}^{33} & \varphi_{ij}^{34} \\ * & * & * & -\gamma^2 M_4 \end{bmatrix} \quad (30)$$

in which

$$\begin{aligned} \varphi_{ij}^{11} &= \text{sym}(P_1(A_i - B_i K_j)) + C_{Li}^T M_1 C_{Li} \\ \varphi_{ij}^{12} &= P_1 B_i K_j^* \\ \varphi_i^{22} &= \text{sym}(P_2 \bar{S}^{-1}(\bar{A}_i - L_{P_i} \bar{C})) + M_2 \\ \varphi_{ij}^{23} &= -\frac{1}{\sigma}(\bar{A}_j - L_{P_i} \bar{C})^T \bar{S}^{-T} P_2 \bar{S}^{-1} \bar{B}_i \\ \varphi_{ij}^{33} &= \frac{G}{\sigma \mu} + M_3 - \frac{1}{\sigma} \text{sym}(\bar{B}_i^T \bar{S}^{-T} P_2^T \bar{S}^{-1} \bar{B}_j) \\ \varphi_{ij}^{34} &= -\frac{1}{\sigma}(\bar{B}_i^T \bar{S}^{-T} P_2^T \bar{S}^{-1} \bar{D}_j) \end{aligned}$$

*Proof.* Choose the following Lyapunov function

$$V(t) = x^T(t) P_1 x(t) + \bar{e}_x^T(t) P_2 \bar{e}_x(t) + \frac{1}{\sigma} e_{f_a}^T(t) \Gamma^{-1} e_{f_a}(t) \quad (31)$$

The time derivative of  $V(t)$  can be shown to be

$$\begin{aligned} \dot{V}(t) &= \dot{x}^T(t) P_1 x(t) + x^T(t) P_1 \dot{x}(t) + \dot{\bar{e}}_x^T(t) P_2 \bar{e}_x(t) + \bar{e}_x^T(t) P_2 \dot{\bar{e}}_x(t) \\ &+ \frac{2}{\sigma} e_{f_a}^T(t) \Gamma^{-1} \dot{e}_{f_a}(t) \end{aligned} \quad (32)$$

Since for any given vectors  $x, y \in \mathbb{R}^n$ , a scalar  $\mu \in \mathbb{R}^+$  and a symmetric positive definite matrix  $Q$ , the following inequality  $2x^T y \leq \frac{1}{\mu} x^T Q x + \mu y^T Q^{-1} y$  holds Gassara et al. (2014b) then we have, if there exist  $G = G^T > 0$  and  $\mu > 0$ , the following inequality:

$$\begin{aligned} \frac{2}{\sigma} e_{f_a}^T(t) \Gamma^{-1} \dot{f}_a(t) &\leq \frac{1}{\sigma \mu} e_{f_a}^T(t) G e_{f_a}(t) + \frac{\mu}{\sigma} \dot{f}_a^T(t) \Gamma^{-1} G^{-1} \Gamma^{-1} \dot{f}_a(t) \\ \frac{2}{\sigma} e_{f_a}^T(t) \Gamma^{-1} \dot{f}_a(t) &\leq \frac{1}{\sigma \mu} e_{f_a}^T(t) G e_{f_a}(t) + \delta \end{aligned} \quad (33)$$

where  $\delta = \frac{\mu}{\sigma} f_{1max}^2 \lambda_{max}(\Gamma^{-1}G^{-1}\Gamma^{-1})$

By substituting (20), (24) and (25) into equation (32) and by considering (33), one can obtain:

$$\begin{aligned}
\dot{V}(t) \leq & \sum_{i=1}^r \sum_{j=1}^r h_i h_j \{x(t)^T [sym(P_1^T(A_i - B_i K_j))]x(t) + 2x(t)^T P_1 B_i K_j e_x(t) \\
& + 2x(t)^T P_1 B_i e_{f_a}(t) + 2x(t)^T P_1 D_i d(t) + \bar{e}_x^T(t) [sym(P_2 \bar{S}^{-1}(\bar{A}_i - L_{P_i} \bar{C}))] \\
& \times \bar{e}_x(t) + 2\bar{e}_x^T(t) P_2 \bar{S}^{-1} \bar{B}_i e_{f_a}(t) + 2\bar{e}_x^T(t) P_2 \bar{S}^{-1} \bar{D}_i d(t) - 2e_{f_a}^T(t) N_i \bar{C} \bar{e}_x(t) \\
& - \frac{2}{\sigma} e_{f_a}^T(t) N_i \bar{C} \bar{S}^{-1} (\bar{A}_j - L_{P_i} \bar{C}) \bar{e}_x(t) - \frac{1}{\sigma} e_{f_a}^T(t) sym(N_i \bar{C} \bar{S}^{-1} \bar{B}_j) e_{f_a}(t) \\
& + \frac{1}{\sigma \mu} e_{f_a}^T(t) G e_{f_a}(t) - \frac{2}{\sigma} e_{f_a}^T(t) N_i \bar{C} \bar{S}^{-1} \bar{D}_j d(t) + \delta\} \tag{34}
\end{aligned}$$

Taking into account equation (28), one can obtain

$$\begin{aligned}
\dot{V}(t) \leq & \sum_{i=1}^r \sum_{j=1}^r h_i h_j \{x(t)^T [sym(P_1^T(A_i - B_i K_j))]x(t) + 2x(t)^T P_1 B_i K_j e_x(t) \\
& + 2x(t)^T P_1 B_i e_{f_a}(t) + 2x(t)^T P_1 D_i d(t) + \bar{e}_x^T(t) [sym(P_2 \bar{S}^{-1}(\bar{A}_i - L_{P_i} \bar{C}))] \\
& \times \bar{e}_x(t) + 2\bar{e}_x^T(t) P_2 \bar{S}^{-1} \bar{D}_i d(t) - \frac{2}{\sigma} e_{f_a}^T(t) (P_2 \bar{S}^{-1} \bar{B}_i)^T \bar{S}^{-1} (\bar{A}_j - L_{P_i} \bar{C}) \bar{e}_x(t) \\
& - \frac{1}{\sigma} e_{f_a}^T(t) sym((P_2 \bar{S}^{-1} \bar{B}_i)^T \bar{S}^{-1} \bar{B}_j) e_{f_a}(t) \frac{1}{\sigma \mu} e_{f_a}^T(t) G e_{f_a}(t) \\
& - \frac{2}{\sigma} e_{f_a}^T(t) (P_2 \bar{S}^{-1} \bar{B}_i)^T \bar{S}^{-1} \bar{D}_j d(t) + \delta\} \tag{35}
\end{aligned}$$

Besides above analyses, to satisfy the attenuation level in (27) under  $H_\infty$  sense, we consider the following performance index:

$$J(t) = \dot{V}(t) + z^T(t) M_1 z(t) + \bar{e}_x^T(t) M_2 \bar{e}_x(t) + e_{f_a}^T(t) M_3 e_{f_a} - \gamma^2 d^T(t) M_4 d(t) \tag{36}$$

$$\begin{aligned}
J(t) \leq & \sum_{i=1}^r \sum_{j=1}^r h_i h_j \{ x(t)^T [\text{sym}(P_1^T(A_i - B_i K_j)) + C_{L_i}^T M_1 C_{L_i}] x(t) \\
& + 2x(t)^T P_1 B_i K_j e_x(t) + 2x(t)^T P_1 B_i e_{f_a}(t) + 2x(t)^T P_1 D_i d(t) \\
& + \bar{e}_x^T(t) [\text{sym}(P_2 \bar{S}^{-1}(\bar{A}_i - L_{P_i} \bar{C})) + M_2] \bar{e}_x(t) + 2\bar{e}_x^T(t) P_2 \bar{S}^{-1} \bar{D}_i d(t) \\
& - \frac{2}{\sigma} e_{f_a}^T(t) (P_2 \bar{S}^{-1} \bar{B}_i)^T \bar{S}^{-1} (\bar{A}_j - L_{P_i} \bar{C}) \bar{e}_x(t) + \frac{1}{\sigma \mu} e_{f_a}^T(t) G e_{f_a}(t) \\
& - \frac{1}{\sigma} e_{f_a}^T(t) [\text{sym}((P_2 \bar{S}^{-1} \bar{B}_i)^T \bar{S}^{-1} \bar{B}_j) + M_3] e_{f_a}(t) \\
& - \frac{2}{\sigma} e_{f_a}^T(t) (P_2 \bar{S}^{-1} \bar{B}_i)^T \bar{S}^{-1} \bar{D}_j d(t) - \gamma^2 d^T(t) M_4 d(t) + \delta \} \quad (37)
\end{aligned}$$

Noting extended state vector  $\xi(t) = [x^T(t) \ \bar{e}_x^T(t) \ e_{f_a}^T(t) \ d^T(t)]^T$  and defining  $K_j^* = K_j [I_{n_x} \ 0_{n_x \times n_f}]$ , inequality (37) can be reformulated as follows

$$J(t) \leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \{ \xi^T(t) \Phi_{ij} \xi(t) + \delta \} \quad (38)$$

If condition (29) holds, then  $\frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r h_i h_j (\Phi_{ij} + \Phi_{ji}) < 0$  and we can note that

$$\sum_{i=1}^r \sum_{j=1}^r h_i h_j \Phi_{ij} < 0.$$

Pre- and post-multiplying the previous inequality by  $\xi^T(t)$  and  $\xi(t)$  we get

$$\sum_{i=1}^r \sum_{j=1}^r h_i h_j \xi^T(t) \Phi_{ij} \xi(t) \leq 0$$

For  $\vartheta = \lambda_{\min}(-\Phi_{ij})$ , we can obtain:

$$J(t) \leq -\vartheta \|\xi(t)\|^2 + \delta \quad (39)$$

It follows that

$$\begin{aligned}
\dot{V}(t) + z^T(t) M_1 z(t) + \bar{e}_x^T(t) M_2 \bar{e}_x(t) + e_{f_a}^T(t) M_3 e_{f_a} - \gamma^2 d^T(t) M_4 d(t) \leq 0 \\
\text{for } \vartheta \|\xi(t)\|^2 > \delta \quad (40)
\end{aligned}$$

- when  $d(t) = 0$ , (40) means  $\dot{V}(t) \leq 0$  for  $\vartheta \|\xi(t)\|^2 > \delta$  and under the Lyapunov stability theory,  $\xi(t)$  will converge to a small set  $\Psi = \{\xi(t) / \|\xi(t)\|^2 \leq \frac{\delta}{\vartheta}\}$ ; thus  $\xi(t)$  is uniformly bounded in the case of  $d(t) = 0$ .

- when  $d(t) \neq 0$ , integrating both sides of (40) with respect to  $t$  over time period  $[0 \infty]$  yields

$$\begin{aligned} & \int_0^\infty \dot{V}_1(s) ds + \int_0^\infty z^T(s)M_1z(s) ds + \int_0^\infty \bar{e}_x^T(s)M_2\bar{e}_x(s) ds \\ & + \int_0^\infty e_{f_a}^T(s)M_3e_{f_a}(s) ds - \gamma^2 \int_0^\infty d^T(s)M_4d(s) ds \leq 0 \\ & \text{for } \vartheta \|\xi(t)\|^2 > \delta \end{aligned} \quad (41)$$

As  $V_1(\infty) \geq 0$ , and with zero initial condition  $V_1(0) = 0$ , one obtains

$$\begin{aligned} & \int_0^\infty z^T(s)M_1z(s) ds + \int_0^\infty \bar{e}_x^T(s)M_2\bar{e}_x(s) ds + \int_0^\infty e_{f_a}^T(s)M_3e_{f_a}(s) ds \\ & \leq \gamma^2 \int_0^\infty d^T(s)M_4d(s) ds \quad \text{for } \vartheta \|\xi(t)\|^2 > \delta \end{aligned} \quad (42)$$

therefore,  $J < 0$  for  $\vartheta \|\xi(t)\|^2 > \delta$

□

**Remark 1.** *We should point out that the main advantage of the proposed Lyapunov function is that observer and controller gains are computed in only one step. Also notice that other Lyapunov functions, such as parameter-dependent Lyapunov function, may be used to further reduce the conservatism Rodrigues et al. (2014). However, this can significantly increase the computational load. Furthermore, adopting such a technique to establish LMI conditions based on parameter-dependent Lyapunov function, remains unexplored subject.*

**Remark 2.** *Noting that conditions in theorem 1 are given in bilinear matrix inequality (BMI) form, which can not be solved using existing solvers such as LMI Toolbox or Yalmip in the MATLAB software.*

*Accordingly we need to make further development to convert the bilinear conditions in theorem 1 in LMIs.*

**Theorem 2.** Consider system (7-9) under assumptions 1 and 2. Given a real positive scalar  $\rho = \gamma^2$ , two scalar tuning parameters  $\sigma, \mu > 0$  and definite positive matrices  $M_{11}$ ,  $M_2$ ,  $M_3$  and  $M_4$ , if there exist symmetric definite positive matrices  $X_1$  and  $P_2$  and positive definite matrix  $G$  as well as matrices  $N_i$ ,  $Y_i$  and  $W_i$  such that  $\forall i \in [1, \dots, r]$  the following conditions hold:

$$(P_2 \bar{S}^{-1} \bar{B}_i)^T = N_i \bar{C}, \quad i = 1, 2, \dots, r \quad (43)$$

$$\Psi_{ij} + \Psi_{ji} < 0, \quad i, j = 1, 2, \dots, r, i \leq j \quad (44)$$

where

$$\Psi_{ij} = \begin{bmatrix} \Omega_{ij}^{11} & \Omega_{ij}^{12} & 0 \\ * & \Omega^{22} & \lambda I \\ * & * & \Omega_{ij}^{33} \end{bmatrix} \quad (45)$$

in which

$$\Omega_{ij}^{11} = \text{sym}(A_i X_1 - B_i W_j), \quad \Omega_{ij}^{12} = [ B_i W_j \quad 0 \quad B_i \quad D_i \quad X_1 C_{Li}^T ]$$

$$\Omega^{22} = \begin{bmatrix} -2\lambda X_1 & 0 \\ * & -2\lambda I \end{bmatrix}, \quad \Omega_{ij}^{33} = \begin{bmatrix} \omega_i^{11} & \omega_{ij}^{12} & P_2 \bar{S}^{-1} \bar{D}_i & 0 \\ * & \omega_{ij}^{22} & \omega_{ij}^{23} & 0 \\ * & * & -\rho M_4 & 0 \\ * & * & * & -M_{11} \end{bmatrix}$$

in which

$$\begin{aligned} \omega_i^{11} &= \text{sym}(P_2 \bar{S}^{-1} \bar{A}_i - Y_i \bar{C}) + M_2 \\ \omega_{ij}^{12} &= -\frac{1}{\sigma} \bar{A}_j^T \bar{S}^{-T} P_2 \bar{S}^{-1} \bar{B}_i + \frac{1}{\sigma} \bar{C}^T Y_i^T \bar{S}^{-1} \bar{B}_i \\ \omega_{ij}^{22} &= \frac{G}{\sigma \mu} + M_3 - \frac{1}{\sigma} \text{sym}(\bar{B}_i^T \bar{S}^{-T} P_2^T \bar{S}^{-1} \bar{B}_j) \\ \omega_{ij}^{23} &= -\frac{1}{\sigma} (\bar{B}_i^T \bar{S}^{-T} P_2^T \bar{S}^{-1} \bar{D}_j) \end{aligned}$$

then the adaptive fuzzy observer proposed in (15) and the FTC designed in (16) ensure, under  $H_\infty$  performance level  $\gamma$ , the stability of closed-loop system (25-26) and the convergence of state estimation error  $\bar{e}_x(t)$  and fault estimation error  $e_{fa}(t)$  in a uniformly bounded compact.

In this case, the gains of the adaptive fuzzy observer and controller are respectively given by  $L_{P_i} = \bar{S} P_2^{-1} Y_i$  and  $K_i = W_i X^{-1}$ .

*Proof.* Using Schur complement, (30) can further be written as

$$\Upsilon_{ij} = \begin{bmatrix} \text{sym}(P_1(A_i - B_i K_j)) & P_1 B_i K_j^* & P_1 B_i & P_1 D_i & C_{Li}^T \\ * & \varphi_i^{22} & \varphi_i^{23} & P_2 \bar{S}^{-1} \bar{D}_i & 0 \\ * & * & \varphi_{ij}^{33} & \varphi_{ij}^{34} & 0 \\ * & * & * & -\gamma^2 M_4 & 0 \\ * & * & * & * & -M_1^{-1} \end{bmatrix} \quad (46)$$

We can write the previous equality in this form

$$\Upsilon_{ij} = \begin{bmatrix} \Upsilon_{ij}^{11} & \Upsilon_{ij}^{12} \\ * & \Upsilon_{ij}^{22} \end{bmatrix} < 0 \quad (47)$$

where

$$\Upsilon_{ij}^{11} = [ \text{sym}(P_1(A_i - B_i K_j)) ], \quad \Upsilon_{ij}^{12} = [ P_1 B_i K_j^* \quad P_1 B_i \quad P_1 D_i \quad C_{Li}^T ] \quad (48)$$

$$\Upsilon_{ij}^{22} = \begin{bmatrix} \varphi_i^{22} & \varphi_i^{23} & P_2 \bar{S}^{-1} \bar{D}_i & 0 \\ * & \varphi_{ij}^{33} & \varphi_{ij}^{34} & 0 \\ * & * & -\gamma^2 M_4 & 0 \\ * & * & * & -M_1^{-1} \end{bmatrix}$$

Letting the following symmetric matrix

$$\Upsilon = \begin{bmatrix} \Upsilon_{11} & 0 \\ 0 & \Upsilon_{22} \end{bmatrix}$$

where  $\Upsilon_{11} = P_1^{-1}$ ,  $\Upsilon_{22} = \text{diag}(P_1^{-1}, I, I, I)$

Pre- and post-multiplying inequality (47) by  $\Upsilon$ , one can obtain

$$\begin{bmatrix} \Upsilon_{11} \Upsilon_{ij}^{11} \Upsilon_{11}^T & \Upsilon_{11} \Upsilon_{ij}^{12} \Upsilon_{22}^T \\ * & \Upsilon_{22} \Upsilon_{ij}^{22} \Upsilon_{22}^T \end{bmatrix} < 0 \quad (49)$$

Since for a given negative definite matrix  $\Pi < 0$  and a matrix  $X$  of appropriate dimension such that  $X^T \Pi X < 0$ , then  $\exists \lambda > 0$  such that  $X^T \Pi X \leq -2\lambda X - \lambda^2 \Pi^{-1}$  holds Gassara et al. (2014a), it follows

$$\Upsilon_{22} \Upsilon_{ij}^{22} \Upsilon_{22}^T \leq -2\lambda \Upsilon_{22} - \lambda^2 (\Upsilon_{ij}^{22})^{-1} \quad (50)$$

Using Schur complement, the previous inequality can further be written as

$$\begin{bmatrix} \Upsilon_{11} \Upsilon_{ij}^{11} \Upsilon_{11}^T & \Upsilon_{11} \Upsilon_{ij}^{12} \Upsilon_{22}^T & 0 \\ * & -2\lambda \Upsilon_{22} & \lambda I \\ * & * & \Upsilon_{ij}^{22} \end{bmatrix} < 0 \quad (51)$$

By posing  $X_1 = P_1^{-1}$ ,  $Y_i = P_2 \bar{S}^{-1} L_{P_i}$ ,  $W_i = K_i P_1^{-1}$ ,  $M_{11} = M_1^{-1}$  and  $\rho = \gamma^2$  we obtain inequality (45).  $\square$



**Remark 3.** *It is noticed that the equality constraint (43) can be easily solved by using a transformation into the following optimization problem Zhang et al. (2009):*

*Minimize  $\eta > 0$  subject to:*

$$\begin{bmatrix} \eta I & (P_2 \bar{S}^{-1} \bar{B}_i)^T - N_i \bar{C} \\ * & \eta I \end{bmatrix} < 0, i = 1, 2, \dots, r \quad (52)$$

**Remark 4.** *Derivative gain  $L_D$  can be chosen so that new defined matrix  $\bar{S} = \bar{E} + L_D \bar{C}$  be nonsingular and the conditions given in (44) be linear. Thus, for an arbitrary  $L_D$ , by solving conditions in theorem 2, if we find feasible solutions of  $X_1$ ,  $P_2$ ,  $N_i$  and  $G$  and then by computing the proportional gains  $L_{P_i}$  of the observer and the controller gains  $K_i$  we can estimate the system states, sensor and actuator faults, else we select another  $L_D$  until a feasible solution be found.*

**Remark 5.**  *$\hat{x}(t)$  is the estimation of the augmented descriptor state vector which can be estimated reliably. So it is easy to have the estimation of states and sensor faults, they can be absolutely formulated as follows :  $\hat{x}(t) = \begin{bmatrix} I_{n_x} & 0_{n_f} \end{bmatrix} \hat{\hat{x}}(t)$  and  $\hat{f}_s(t) = \begin{bmatrix} 0_{n_x} & I_{n_f} \end{bmatrix} \hat{\hat{x}}(t)$ .*

*The estimation of the actuator faults can be provided by the adaptive observer proposed in (15). More precisely, based on the proposed adaptive fault estimation scheme, the learning rate  $\Gamma$  should be adjusted such that the **FTC** can rapidly recover the performance of the system even in the presence of sensor/actuator faults and external disturbances simultaneously. Increasing or decreasing this parameter may lead to an unsatisfactory convergence speed of system states and actuator faults estimation.*

#### 4. NUMERICAL EXAMPLE

In this section, to validate the proposed analytical results in the previous section, two examples will be given.

##### **Example 1:**

Consider fuzzy system of form (7-9) with

$$A_1 = \begin{bmatrix} 0.5 & 1 & 0 \\ 0.2 & -1.5 & 0 \\ 2 & 0.1 & 0.3 \end{bmatrix}; A_2 = \begin{bmatrix} 0.5 & 1 & 0.1 \\ 0 & -1 & 0 \\ 0.5 & 0 & 0.1 \end{bmatrix}; B_1 = \begin{bmatrix} 0 \\ 1.5 \\ 0 \end{bmatrix}; B_2 = \begin{bmatrix} 0 \\ 0.9 \\ 0 \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0.5 \end{bmatrix}; D_1 = \begin{bmatrix} 0 \\ 0.1 \\ 0 \end{bmatrix}; D_2 = \begin{bmatrix} 0 \\ 0.2 \\ 0 \end{bmatrix}; F = \begin{bmatrix} -1 \\ 1 \end{bmatrix};$$

$$C_{L1} = [ 0.1 \ 0 \ 0 ], C_{L2} = [ 0 \ 0.2 \ 0 ]$$

The membership functions for rules 1 and 2 are choosing as follows :

$$h_1(y_1(t)) = e^{-y_1^2(t)}, h_2(y_1(t)) = 1 - h_1(y_1(t)) \quad (53)$$

By solving LMI conditions in theorem 2, by selecting

$$L_D = \begin{bmatrix} 0 & 0.6 & 8 & 4 \\ 1 & 0 & 5 & 6 \end{bmatrix}^T$$

and by choosing the tuning parameters as follows :  $\lambda = 2$ ,  $\sigma = 0.2$ ,  $\mu = 0.1$  and the index performance  $\rho = 0.4$  ( $\gamma = 0.6325$ ), we can get a set of feasible solutions.

Equality (43) and inequality (44) were implemented with Yalmip toolbox and SDPT3-4 solver to obtain the corresponding observer gains and FTC gains respectively:

$$L_{P_1} = \begin{bmatrix} 1,966 & -0,244 \\ -1,397 & 0,851 \\ 2,734 & 3,862 \\ 1,865 & 2,081 \end{bmatrix}, L_{P_2} = \begin{bmatrix} 1,332 & 0,169 \\ -0,281 & 0,304 \\ 2,207 & 3,161 \\ 1,698 & 2,145 \end{bmatrix}$$

and

$$K_1 = [ 4,775 \ 1,655 \ 1,453 ], K_2 = [ 5,532 \ 2,391 \ 1,790 ]$$

The external disturbance is given by  $d(t) = 0.01(0.015\pi t) - 0.01x$ .

The system mentioned in this example is affected by an actuator and sensor faults which satisfy assumption A2 and are described by  $f_s(t)$  as a square-wave signal between 10s and 30s:

$$f_s(t) = \begin{cases} 0 & t \leq 10 \\ -0.2x & 10 < t \leq 14 \\ 0.01x & 14 < t \leq 17 \\ 0.2 + 0.05\sin(3\pi t + \frac{\pi}{2})\cos(0.2\pi t) + 0.02x & 17 < t \leq 23 \\ 0.02x & 23 < t \leq 26 \\ -0.3 + 0.01x & 26 < t \leq 29 \\ 0 & t > 29 \end{cases} \quad (54)$$

$x$ : is a random value between 0 and 1. and the actuator fault as

$$f_a(t) = \begin{cases} 0 & t \leq 6 \\ 0.55(1 - e^{-3(t-6)}) + 0.01x & 6 < t \leq 30 \end{cases} \quad (55)$$

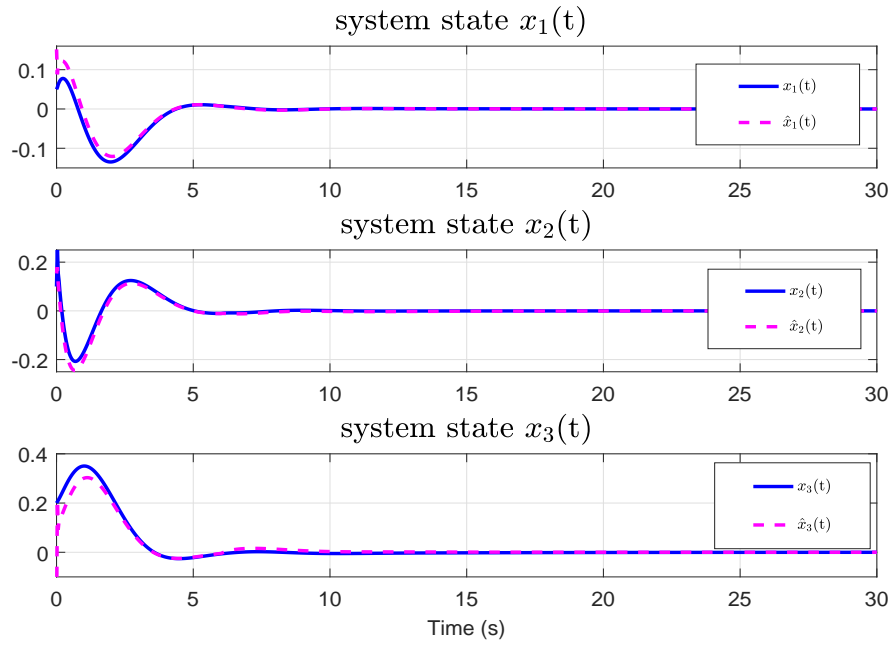


Figure 1: System states and their estimations under FTC law for example 1.

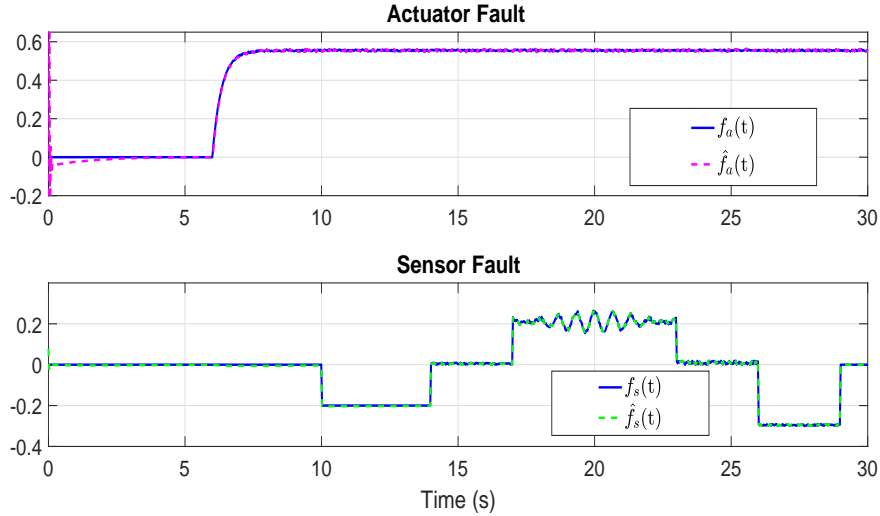


Figure 2: Actuator and sensor fault and their estimations.

The simulation initial conditions are  $x_0 = [0.05 \ 0.1 \ 0.2]^T$  and  $\xi_0 = [0.1 \ 0.2 \ 0.1 \ 0]^T$ , respectively. The learning rate is set to  $\Gamma = 0.9$ , so it can provide a convergence speed of system states and actuator faults estimation.

**Example 2:**

In this example, as shown in Figure 6, we consider a cart motion model from Han et al. (2016); Qiu et al. (2012) which is simulated to illustrate the effectiveness of the proposed observer for estimating system states, sensor and actuator faults and the performance of the FTC to force a cart on the  $x - y$  plane to follow the straight line  $y = 0$  with a constant velocity  $v_0 = 1m/s$ . It is supposed that a controller is designed previously to maintain a constant forward velocity. The cart's path is then controlled by the torque  $u$  about the  $z$ -axis according to the following dynamic model:

$$\begin{aligned}
 \begin{bmatrix} \dot{\theta}(t) \\ \dot{\psi}(t) \\ \dot{y}(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & -k/I & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta(t) \\ \psi(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ v_0 \sin(\theta(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/I \\ 0 \end{bmatrix} u(t) \\
 &+ \begin{bmatrix} 0 \\ 0.1 \\ 0 \end{bmatrix} d(t) \tag{56}
 \end{aligned}$$

$\theta(t)$  is the heading angle with time derivative  $\psi(t)$ ,  $I = 1kgm^2$  is the moment of inertia of the cart with respect to the center of mass,  $k = 0.01$  is the damping coefficient, the initial angle is in the range  $(-\frac{3\pi}{5}, \frac{3\pi}{5})$ ,  $u(t)$  is the control torque and  $d(t)$  is the external disturbance. Define the states of the

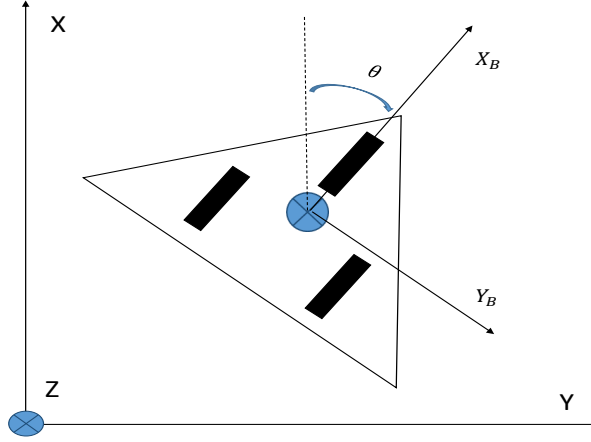


Figure 3: Autonomous land vehicle.

system as  $x = [x_1^T(t), x_2^T(t), x_3^T(t)] = [\theta^T(t), \psi^T(t), y^T(t)]$ .

The measured output and controlled output are given, respectively, by

$$y(t) = Cx(t) + Ff_s(t) = [y_1^T(t), y_2^T(t), y_3^T(t)]$$

$$z(t) = \sum_{i=1}^r h_i [C_{L_i} x(t)]$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.5 \end{bmatrix}, F = \begin{bmatrix} 2 \\ -2 \end{bmatrix}, C_{L1} = [0.3 \ 0 \ 0], C_{L2} = [0 \ 0 \ 0.2]$$

Referring to the local sector non linearity method Tanaka and Wang (2004), the system can be approximated by the following T-S and the following fuzzy rules can be employed:

Plant Rule i: If  $x_1(t)$  is about  $h_i$ , Then

$$\dot{x}(t) = A_i x(t) + B_i u(t) + D_i d(t)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -k/I & 0 \\ 1 & 0 & 0 \end{bmatrix}; A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -k/I & 0 \\ \frac{\sin(3\pi/5)}{3\pi/5} & 0 & 0 \end{bmatrix}; B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix};$$

$$D_1 = D_2 = \begin{bmatrix} 0 \\ 0.1 \\ 0 \end{bmatrix}$$

Define  $x_1(t) \in (-\frac{3\pi}{5}, \frac{3\pi}{5})$  and  $z(t) = \sin(x_1(t))$

The membership functions  $h_1(z(t))$  and  $h_2(z(t))$  are obtained from the property  $h_1(z(t)) + h_2(z(t)) = 1$

Following the membership functions used in Han et al. (2016), their expressions are given as follows:

$$h_1(z(t)) = \frac{\sin(x_1(t)) - \frac{5}{3\pi}x_1(t)}{x_1(t)(1 - \frac{5}{3\pi})}, h_2(z(t)) = 1 - h_1(z(t)) \quad (57)$$

Considering the overall fuzzy model given in (7-9).

**Comparison:** To show the effectiveness of our results, we compare the approach proposed in this paper to the one proposed in the paper of Han et al. (2016). Theorems 1 and 2 in Han et al. (2016) work well with  $\gamma_1 = 1, \gamma_2 = 2$  as performances indexes. However, from  $\gamma_1 = 0.0995$  and  $\gamma_2 = 0.8367$  downwards theorems 1 and 2 does not lead to a conclusion since the two-step LMI conditions cannot produce a set of feasible solutions.

Now, by applying theorem 2 to solve the corresponding LMIs, by selecting

$$L_D = \begin{bmatrix} 0 & 0.6 & 8 & 4 \\ 1 & 0 & 5 & 6 \end{bmatrix}^T$$

and by choosing the tuning parameters as follows :  $\lambda = 3, \sigma = 3, \mu = 0.5$  and the index performance  $\rho = 0.0099$  (and even for lower values) we can obtain a set of feasible solutions.

Equality (43) and inequality (44) were implemented with Yalmip toolbox and SDPT3-4 solver to obtain:

The gains of the adaptive fuzzy observer

$$L_{P_1} = \begin{bmatrix} 3.2593 & 1.6628 \\ 3.5586 & 0.9411 \\ -2.5826 & 0.6070 \\ 1.2006 & 1.9057 \end{bmatrix}, L_{P_2} = \begin{bmatrix} 0.6578 & 1.0046 \\ 5.2083 & 1.3574 \\ -1.4641 & 0.7598 \\ 1.2021 & 1.9057 \end{bmatrix}$$

and controller gains

$$\begin{aligned} K_1 &= [ 2.8912 \quad 2.5303 \quad 1.2165 ] \\ K_2 &= [ 2.8127 \quad 2.5081 \quad 1.2128 ] \end{aligned}$$

The external disturbance is given by  $d(t) = 0.1\sin(y_1(t)) - y_1(t)$ . The system mentioned in this example is affected by an actuator and sensor faults which satisfy assumption *A2* and are described by  $f_s(t)$  as a square-wave signal between 15s and 30s and the actuator fault as

$$f_a(t) = \begin{cases} 0 & t \leq 6 \\ 0.2(t - 6) & 6 < t \leq 11 \\ 0.1 + 0.2\sin(0.2\pi(t - 11)) & 11 < t \leq 30 \end{cases} \quad (58)$$

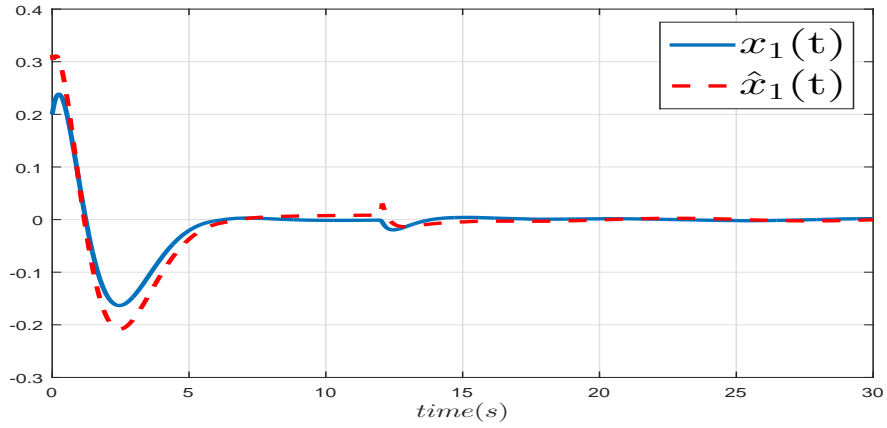


Figure 4: System state  $x_1(t)$  and its estimation  $\hat{x}_1(t)$  under FTC law with a time-varying fault  $f_a(t)$

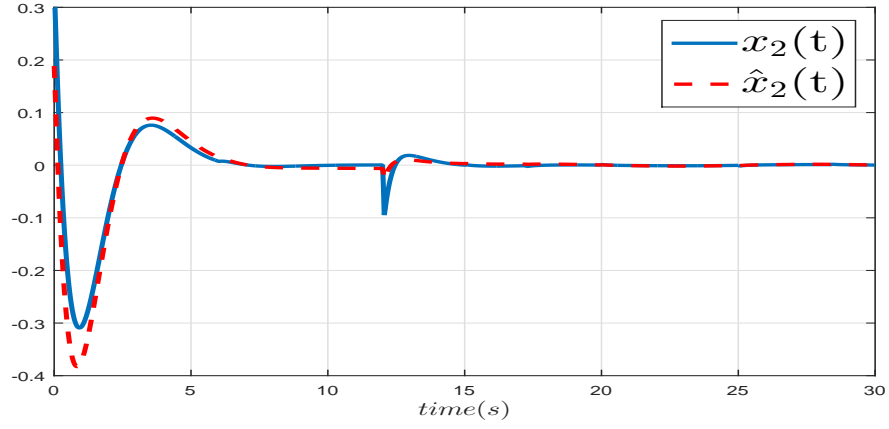


Figure 5: System state  $x_2(t)$  and its estimation  $\hat{x}_2(t)$  under FTC law with a time-varying fault  $f_a(t)$ .

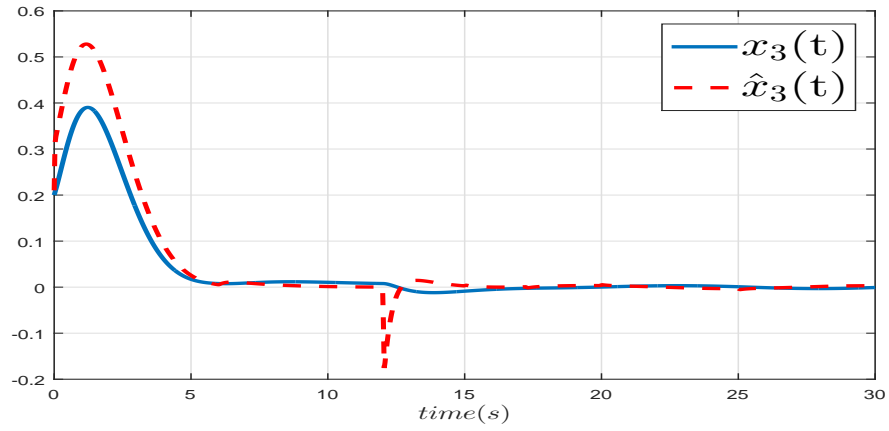


Figure 6: System state  $x_3(t)$  and its estimated  $\hat{x}_3(t)$  under FTC law with a time-varying fault  $f_a(t)$ .



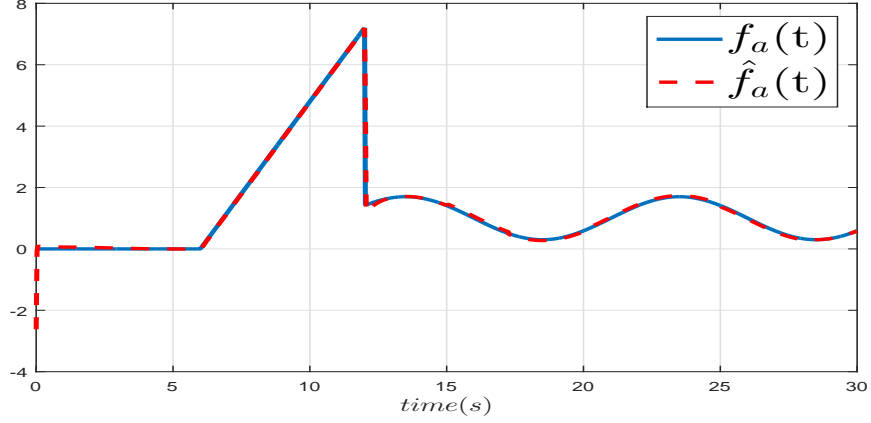


Figure 7: Actuator fault  $f_a(t)$  and its estimation  $\hat{f}_a(t)$  under FTC law.

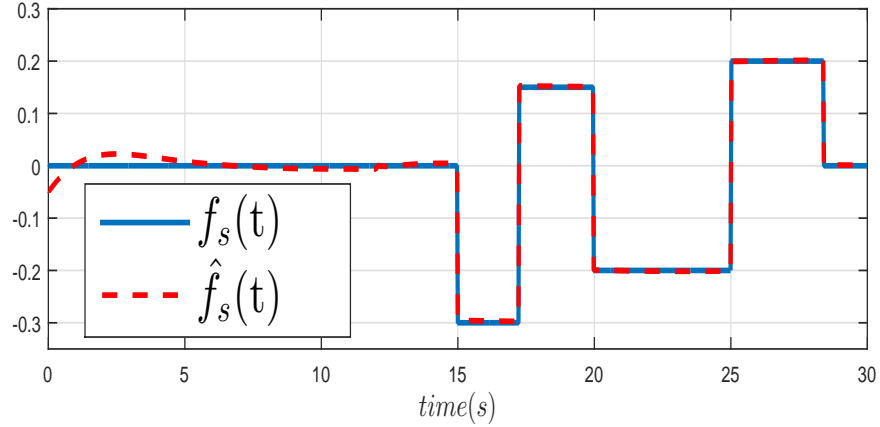


Figure 8: Sensor fault  $f_s(t)$  and its estimation  $\hat{f}_s(t)$  under FTC law.

The simulation initial conditions are  $x_0 = [0.2 \ 0.3 \ 0.3]^T$  and  $\xi_0 = [0.3 \ 0.2 \ 0.3 \ 0]^T$ , respectively.

By choosing  $\Gamma = 0.7$ , the derivative of  $f_a(t)$  over time is norm bounded by  $f_{a_{max}} = 0.1257$ . The term  $\delta = \frac{\mu}{\sigma} f_{a_{max}}^2 \lambda_{max}(\Gamma^{-1}G^{-1}\Gamma^{-1}) = 1.5653 \cdot 10^{-4}$ , which reduces the radius of the ball in which the estimation errors converge.

The simulation results of both examples are given in Figures 1 – 2 and 4 – 8. On the basis of these figures, it can be seen that the adaptive fuzzy observer

proposed in this work can estimate system states, actuator and sensor faults under the FTC law. The simulation results verify that the adaptive fuzzy observer-based FTC used in this paper can guarantee the performance and the stability of the fuzzy closed-loop system despite the presence of external disturbances, sensor and actuator faults.

**Results with FTC strategy versus results with nominal control:**

In order to illustrate the effectiveness of the FTC strategy, we proceed to compare the evolution of system states in presence of actuator faults under FTC law and nominal control which is given in (59).

*Nominal control:*

$$u(t) = - \sum_{i=1}^r h_i K_i \hat{x}(t) \tag{59}$$

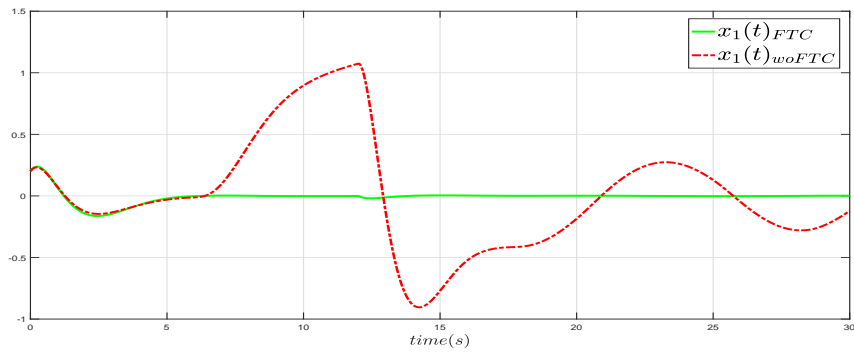


Figure 9: System state  $x_1(t)$  with FTC and nominal control law.

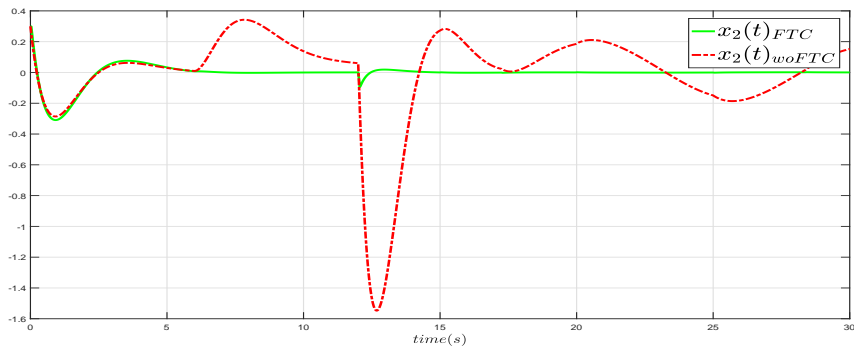


Figure 10: System state  $x_2(t)$  with FTC and nominal control law.

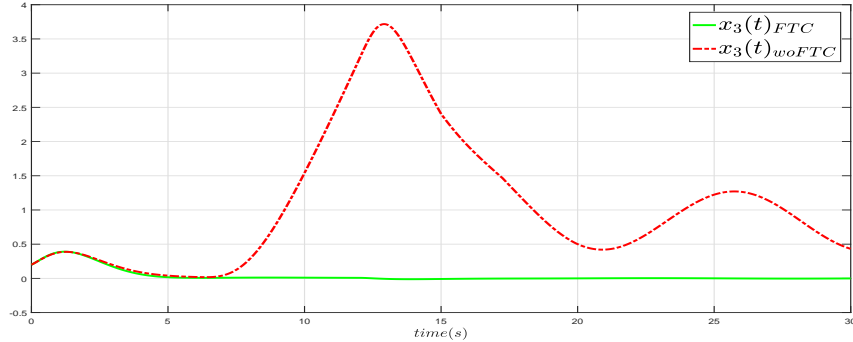


Figure 11: System state  $x_3(t)$  with FTC and nominal control law.

We notice from Figures 9 – 11 that when the actuator faults occur, the system states under FTC law converge to zero, which isn't the case under the nominal control. That means that the FTC strategy is essential in a faulty system to maintain the stability of the closed-loop system.

## 5. Conclusion

In this paper, an adaptive fuzzy observer-based FTC design has been developed to establish FE and to maintain the robust stability of the closed-loop T-S fuzzy systems in presence of sensor and actuator faults. Using the  $H_\infty$  optimization technique, an augmented adaptive fuzzy observer has been developed to estimate both sensor/actuator faults and the system states. Sufficient conditions of stability and stabilization have been expressed in LMI form which can be easily solved in one step with existing solvers. The effectiveness and the advantages of the proposed approach have been illustrated by two numerical examples which one of them concerns the cart motion non-linear model. Moreover, the efficiency of the FTC law compared with the nominal one is illustrated by simulating system states with both controllers.

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