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NUMBERING THE DIVINE PERSONS: MENTAL EXISTENCE OF NUMBERS IN JOHN DUNS SCOTUS, HENRY OF HARCLAY AND PETER AURIOL

Maria SOROKINA*

Abstract

At the beginning of the fourteenth century, three authors are particularly dissatisfied with the realist theory of number: John Duns Scotus, Henry of Harclay, and Peter Auriol assert that it leads to several paradoxes and therefore postulate the purely mental existence of numbers. However, although all three expound the conceptualist theory of number, they do not define the form of number in the same way. For this reason, while Scotus and Harclay admit the existence of number in divine persons, that is totally impossible for Auriol, even if a number is an *ens rationis* and not a real accidental form. According to Auriol, the concept of number is a sum of units that is produced by the intellect; since the human mind cannot conceive of the divine persons separately from each other, it cannot produce this sum.

Among the various texts dealing with the nature of numbers (commentaries on Aristotle's *Metaphysics* and *De anima, quodlibeta*, etc.), commentaries on the Peter Lombard's *Sentences* hold a special status. In these works, the question of numerical terms is a part of the discussion on the Trinity: in *distinctio 24* of book I of his *Sentences*, Peter Lombard asked if there was a number in the divine persons. On the one hand, the affirmative answer seems to be self-evident: for example, according to the First Canonical Letter of John, "there are *three* that give testimony in heaven, the Father, the Word, and the Holy Spirit."¹ However, on the other hand, according to Aristotle's *Categories*, a number is a kind of quantity (a discrete quantity) and an accidental form, whereas there is no quantity and no accident in God.² In this case, what is the Trinity if not the number of the divine persons? In their search for a solution, the commentators on the *Sentences* had to define number as clearly as possible.

Confronted with this difficult question, medieval authors were bound to distinguish several types of numbers. According to Peter Lombard himself, while number usually means a quantity or a diversity of things, it is only a negation in the particular case of the divine persons: God is nothing but three persons, i.e., the Father, the Son, and the Holy Spirit.³ Thomas Aquinas presents a more detailed classification: he distinguishes between a *numerus simpliciter* and a *numerus secundum quid*. The *numerus simpliciter* is of two types: a number of material things (for instance, a number of stones), which is a quantity, and a "transcendental number" of immaterial things each with different essences (angels, for instance), which is not a quantity, since the latter is proper only to the matter. Unlike the *numerus simpliciter*, which really exists,

* This article has benefitted from the advice of Russell Friedman, Lukáš Lička, Azélina Jaboulet-Vercherre, Guy Guldentops, and an anonymous reviewer for *RTPM*. I thank all of them.

¹ For example, THOMAS DE AQUINO, *In Sent.*, I, d. 24, q. 1, a. 2, ed. P. MANDONNET, Paris 1929, p. 578: "Contra est quod habetur I Joan., V, 7: *Tres sunt qui testimonium dant in caelo: Pater, Verbum et Spiritus sanctus [...]. Sed 'tres' dicit aliquem numerum. Ergo videtur quod ibi sit numerus.*"

² For example, IOANNES DUNS SCOTUS, *Reportatio Parisiensis*, I-A, d. 24, q. u., § 5, ed. A. WOLTER – O. BYCHKOV, vol. 2, St. Bonaventure/New York 2008, p. 23: "Item, omnis numerus est quantitas essentialiter, quia superius est de intellectu inferioris essentialiter. In Deo non est quantitas, ergo nec numerus"; PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., ms. Città del Vaticano, Biblioteca Apostolica Vaticana, Borgh. lat. 329 [= Vb], f. 265va, ed. 1596, p. 543a: "Praeterea, nullum accidens est in Deo. Sed numerus est accidens [...]. Ergo non potest in Deo poni."

³ PETRUS LOMBARDUS, *Sententiae in quattuor libris distinctae*, I, d. 24, ed. I. BRADY, vol. 1, Grottaferrata 1971, pp. 187-188: "Cum enim dicitur unus Deus, multitudo deorum excluditur, nec numeri quantitas in divinitate ponitur. [...] Ita etiam cum dicimus 'tres personas', nomine ternarii non quantitatem numeri in Deo ponimus vel aliquam diversitatem, sed intelligentiam non ad alium nisi ad Patrem et Filium et Spiritum Sanctum dirigendam significamus." Cf. THOMAS DE AQUINO, *Summa theologiae*, I, q. 30, a. 3, ed. Leonina, vol. 4, Rome 1888, p. 340: "Termini ergo numerales significant in divinis illa de quibus dicuntur, et super hoc nihil addunt nisi negationem, ut dictum est: et quantum ad hoc, veritatem dixit Magister in *Sententiis*."

the number *secundum quid* often has only a mental existence: for instance, a point at the origin of, for instance, three lines is triple, not in an extramental reality, but *secundum rationem*.⁴ Regarding the divine persons, in the *Summa theologiae*, Aquinas defines the Trinity as “a transcendental number,” while in his *Sentences* commentary, he speaks of an intermediate number between the *numerus simpliciter* and the *numerus secundum quid*: there is neither matter nor diversity of essences in God, which are necessary to constitute a number in a proper sense; nevertheless, there is a “triplicity of relations” between the persons, i.e., the real number and not only the number *secundum rationem*.⁵

These typologies of numbers are rather complex. This made the conceptualist theory of number, at the beginning of the fourteenth century, appear as a simplification: in its framework, *any* number is an *ens rationis*. At that time, at least three philosophers support this opinion: John Duns Scotus, Henry of Harclay, and Peter Auriol assert that numbers are mental beings, dependent on a cognitive operation. But how do they resolve the trinitarian problem, the actual starting point of their reasoning? If a number is an *ens rationis*, does it exist in the divine persons? Before examining this question, we should first explain the reasons why the realist theory of number came to be rejected and then present different versions of the conceptualist theory they formulate in its stead. This will finally help us understand whether or not, according to Scotus, Harclay, and Auriol, the divine Trinity could be regarded as a number.

1. *The realist theory of number: reasons to reject it*

The medieval explanation of number – not just the realist views, but as we will see, the conceptualist ones as well – relied on a type of hylomorphic explanation of number on which the matter of number was fairly standardly taken to be the enumerated things, while the form of number, i.e., the numeral that could be applied to the collective enumerated things, could be

⁴ THOMAS DE AQUINO, *In Sent.*, I, d. 24, q. 1, a. 2, ed. MANDONNET, p. 578: “Respondeo dicendum quod sicut ratio unitatis consistit in indivisione, ita et ratio numeri vel multitudinis consistit in divisione vel distinctione aliqua. Unde ea quae invenimus divisa simpliciter, dicimus esse multa simpliciter; et quae invenimus divisa secundum quid, dicimus esse multa secundum quid. Divisio autem simpliciter attenditur vel secundum essentiam, sive formam; vel secundum quantitatem, seu materiam; unde ea quae differunt secundum essentiam, dicimus esse multa, ut hominem et lapidem; et similiter duas partes lineae jam divisae dicimus duas lineas. Divisio autem secundum quid est quae attenditur secundum proprietates rei, sicut dicimus hominem album esse alium et distinctum a se nigro, et adhuc magis secundum quid in illis in quibus attenditur diversitas relationum secundum rationem tantum; sicut punctus si diceretur multiplex, secundum quod est principium plurium linearum”; ID., *Summa theologiae*, I, q. 30, a. 3, ed. Leonina, p. 339: “Ad evidentiam igitur huius, considerandum est quod omnis pluralitas consequitur aliquam divisionem. Est autem duplex divisio. Una materialis, quae fit secundum divisionem continui: et hanc consequitur numerus qui est species quantitatis. Unde talis numerus non est nisi in rebus materialibus habentibus quantitatem. Alia est divisio formalis, quae fit per oppositas vel diversas formas: et hanc divisionem sequitur multitudo quae non est in aliquo genere, sed est de transcendentibus, secundum quod ens dividitur per unum et multa. Et talem multitudinem solam contingit esse in rebus immaterialibus.” For number theory in Aquinas, see: A. MAURER, “Thomists and Thomas Aquinas on the Foundation of Mathematics,” in: *Review of Metaphysics* 47 (1993), pp. 43-61; P. SOUSEDÍK, D. SVOBODA, “The Ontology of Number: Is Number an Accident?,” in: L. NOVÁK – D.D. NOVOTNÝ – P. SOUSEDÍK – D. SVOBODA (eds.), *Metaphysics: Aristotelian, Scholastic, Analytic*, Frankfurt 2012, pp. 123-140, esp. pp. 124-127; D. SVOBODA, P. SOUSEDÍK, “Mathematical One and Many: Aquinas on Number,” in: *The Thomist* 78/3 (2014), pp. 401-418 (see p. 402, n. 3 for further literature).

⁵ THOMAS DE AQUINO, *In Sent.*, I, d. 24, q. 1, a. 2, ed. MANDONNET, p. 578: “Sciendum est igitur quod in divinis non est numerus simplex, qui est per divisionem essentiae vel quantitatis; sed est numerus quidam, scilicet numerus relationum, non tamen relationum existentium in Deo secundum rationem tantum, sed realiter in ipso subsistentium. Unde numerus divinarum personarum est medius inter numerum qui est numerus simpliciter, et numerum qui est in ratione tantum”; ID., *Summa theologiae*, I, q. 30, a. 3, ed. Leonina, p. 339: “Nos autem dicimus quod termini numerales, secundum quod veniunt in praedicationem divinam, non sumuntur a numero qui est species quantitatis; quia sic de Deo non dicerentur nisi metaphorice, sicut et aliae proprietates corporalium, sicut latitudo, longitudo, et similia: sed sumuntur a multitudine secundum quod est transcendens.” It is still an open research question as to whether Aquinas’ opinion on number in the divine persons is representative of a larger trend in the thirteenth century.

defined in various ways. Thus, if we count some stones, the matter of number is the stones themselves, for the realists as well as for conceptualists. The real argument is over *the form of number*. For the conceptualists, this form does not exist outside the mind of the one who enumerates the stones. In contrast, for the realists, this form is an extramental accidental form. Realists, however, found it difficult to describe its existence. This form is neither entirely in each of these stones, nor is it divided between these stones (i.e., one part in one stone, one part in another, etc.). In fact, the form is entirely in all the numbered stones taken together; thus, it provides their unity.⁶ It remains to be defined what this form is. Theories holding the extramental being of numbers seem to be widespread in the thirteenth and at the beginning of the fourteenth century, and these realist views of number are many and varied. For example, Henry of Ghent defined the form of number as a continuity (*continuitas*). The common point among the realists was considering number to be a discrete quantity owing its origin to a division of continuous quantity.⁷ Henry of Ghent therefore concludes that the form of every number should be continuity or an initial unity. When a stone is divided into two, there is no other unity between these two pieces than the initial continuity of the stone. In other cases, the units of number do not originate from the division of continuum, but have an innate disposition (*apta nata*) to originate from it; that is why in the case of, say, five human beings, the form of the number five is also the continuity. Bernard of Auvergne expounded another version of realist theory: he took the form of number to be the last unity (*ultima unitas*), not as that unit itself, but as that unit in accordance with a distance from the first unit. In other words, when we have four stones, the form is the fourth stone, not as a material unit, but as the foundation of the distance from the first unit, the first stone. Still another view – mentioned, for instance, by Scotus, but not attributed to any particular author – was that the form of number is a discreteness (*discretio*) which is a discontinuity of numbered things.⁸

Duns Scotus, Henry of Harclay, and Peter Auriol each formulate a conceptualist theory of number through the same process of elimination: a number has mental existence, because the only alternative, extramental existence, is excluded. Scotus and Harclay attack the three theories mentioned above; Auriol presents and criticizes six different realist views.⁹ However, it is not very important how many opinions they refute, since they emphasize the absurdity of *any* realist theory of number. One of Scotus' arguments to prove that *no* realist number theory can work runs as follows: if numbers are real, we should recognize the real existence of an infinity of numbers. If we take a pair of stones and a pair of pieces of wood, it seems at first that there are only two numbers, two binaries. But then a third number appears: we also have the third pair, the pair of pairs, so there are three binaries. Then, that third binary and, for example, the binary of stones form another number, the fourth binary; that fourth binary and the binary of pieces of

⁶ See, for example, THOMAS DE ARGENTINA, *In Sent.*, I, d. 24, q. u., a. 3, ed. Genuae 1585, f. 86ra: “[...] forma quae est numerus nec est tota in qualibet unitate secundum se sumpta, nec est tota in <una> unitate tantum, sed est tota in omnibus suis unitatibus simul sumptis”.

⁷ Some authors confirm the view by referring to Aristotle's *Physics*. See *Les Auctoritates Aristotelis. Un florilège médiéval*, ed. J. HAMESSE, Paris/Leuven 1974, p. 148, 78 (*Physica*, III, 111); THOMAS DE AQUINO, *In Aristotelis libros Physicorum*, III, lect. 12, n. 5, ed. Leonina, vol. 2, Rome 1884, p. 140: “[...] divisio continuae quantitatis causat numerum”; for Aristotle's mathematical theories, see H. MENDELL, “Aristotle and Mathematics”, in: E. N. ZALTA (ed.), *The Stanford Encyclopedia of Philosophy* (Fall 2019 Edition), <https://plato.stanford.edu/archives/fall2019/entries/aristotle-mathematics/> (last accessed 18 January 2020).

⁸ See HENRICUS DE GANDAVO, *Quodlibet IV*, q. 6, ed. J. ETZKORN – G. WILSON, Leuven 1987, pp. 24-27; BERNARDUS DE ALVERNIA, *Reprobationes contra Henricum*, IV, 6, ms. Paris, BnF, n.a.l. 1465, f. 54va; IOANNES DUNS SCOTUS, *Rep.*, I-A, d. 24, q. u., § 11, 15-16, 23-32, ed. WOLTER – BYCHKOV, pp. 25, 27-28, 30-33.

⁹ These six theories attacked by Auriol are: (1) Hervaeus Natalis's position (number is “a whole really different from its parts,” *quodlibet totum aliud realiter a partibus*); (2) a definition of number as a “oneness” (*semelitas*); (3) a definition of number as an aggregation (*unitas aggregationis*); (4-6) the three opinions mentioned above and also criticized by Scotus and Harclay (cf. *supra*, note 7). See PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 1, Vb, f. 266ra-267rb; ed. 1596, p. 543b-547a (I have made a working edition of this text, which will appear on the *Electronic Scriptum* of the Peter Auriol Homepage: <http://www.peterauriol.net/editions/electronicscriptum/>). Given this spectrum of views, the realist theory of number deserves a dedicated study.

wood constitute the fifth binary, and so forth *ad infinitum*.¹⁰ Peter Auriol puts forward a similar argument: a pair of stones and a pair of pieces of wood give the third number, a pair of pairs; these three numbers (three binaries) form a triplicity; that triplicity and three binaries constitute the number four; four, three and three binaries give five, and so forth.¹¹ Henry of Harclay also knows Scotus' demonstration. When he paraphrases it, he adds a judicious remark based on the characteristics of *entia rationis*: while an infinity of real things is impossible, *entia rationis* could be infinite.¹² Apparently, for Harclay as well as for Scotus and Auriol, that is the case for numbers. More research would be needed to determine whether other medieval authors found this argument of Scotus's convincing or not, but for some of them, it was quite weak. While the Franciscan friar Landulph Caracciolo attacks Peter Auriol's similar demonstration, he makes an important remark: the real existence of infinity is excluded, since only the initial numbers of real things (the binary of pieces of wood and the binary of stones) are real; these two numbers do not give the third real number.¹³

While reproducing Scotus' arguments, Henry of Harclay and Peter Auriol add to them some paradoxes of any realist theory of number. According to Harclay, if a number is a real being, we should recognize that a greater number is equal to a smaller number. Arithmetically speaking, a duality is equal to any other duality. Thus, a duality of stones is equal to a duality of pieces of wood. It is obvious that these two dualities together form the third one, i.e., a duality of pairs. It is obvious too that, when two quantities are equal to each other, their halves are also equal to each other. But in Harclay's example, the half of the duality of pairs includes two units (for instance, two stones), while the half of the duality of stones includes only one unit. Therefore, the number two is paradoxically equal to the number one.¹⁴ Peter Auriol borrows this

¹⁰ IOANNES DUNS SCOTUS, *Rep.*, I-A, d. 24, q. u., § 21, ed. WOLTER – BYCHKOV, p. 29: “Si forma numeri sit aliquid in ipsis unitatibus discretis extra animam, sequitur quod infiniti erunt numeri in actu, positis tantum duobus binariis in actu. – Probatio consequentiae: accipiatur hic binarius lapidum et ille lignorum, qui sunt duo binarii singulares, quorum neuter est alius. Illorum duorum binariorum est unus numerus binarius tertius, qui habet unum binarium pro una unitate et alium binarium pro alia unitate [...]. Illi ergo duo binarii tamquam duae unitates faciunt unum binarium tertium alium ab utroque istorum; et iste tertius binarius cum alio binario, tamquam duae unitates, constituent quartum binarium, et ille et alius quintum, et sic in infinitum; ergo, etc.”

¹¹ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 2, Vb, f. 268va; ed. 1596, p. 549b: “Praeterea, si numerus dicit formam realem extra animam existentem, datis duobus numeris, darentur necessario infinitae realitates. Posita namque dualitate lapidum et dualitate lignorum, quaeritur utrum illa dualitas duarum dualitatum sit aliquid reale aut non, et si non, pari ratione nec dualitates lapidum vel lignorum fuerint aliquid reale. Si vero haec tertia dualitas aliquid sit reale, iam occurunt tres realitates: prima quidem dualitatis lapidum, secunda vero dualitatis lignorum, tertia quoque dualitatis dualitatum. Et tunc quaeritur de trinitate istarum trium rerum, utrum sit aliquid reale, et si sic, iam habentur quattuor res, et quaeretur de quaternitate illarum, utrum sit quinta res, et sic in infinitum.”

¹² HENRICUS DE HARCLAY, *In Sent.*, I, d. 24, q. u., mss. Casale Monferrato, Biblioteca del Seminario Vescovile, B2 [= C], f. 57rb; München, Bayerische Staatsbibliothek, Clm 8717 [= M], f. 93ra: “Praeterea, si numerus esset aliquid reale extra animam, datis duobus numeris, darentur infiniti. [...] Dicis: eodem modo dicam tibi, si ponantur esse tantum in intellectu istae dualitates, quod scilicet erit processus in infinitum. Sed respondeo quod non est inconveniens in entibus rationis procedere in infinitum.” On Henry of Harclay's *Sentences* commentary and the list of its manuscripts, see W. O. DUBA – R. L. FRIEDMAN – C. D. SCHABEL, “Henry of Harclay and Aufredo Gonteri Brito,” in: Ph. ROSEmann (ed.), *Mediaeval Commentaries on the Sentences of Peter Lombard*, vol. 2, Leiden/Boston 2009, pp. 263–368. My edition of this question of Harclay's commentary is forthcoming.

¹³ LANDULPHUS CARACCIOLI, *I Sent.*, d. 24, q. u., Bologna, Collegio di Spagna, 46 [= B], f. 114va; Dole, Bibliothèque Municipale, 80 [= D], f. 99rb; Firenze, Biblioteca Nazionale Centrale, Conv. Soppr. B. 5. 640 [= F], f. 78rb; Padova, Biblioteca Antoniana, 166 Scatt. IX [= P], f. 106rb; Wien, Österreichische Nationalbibliothek, Palatinus lat. 1496 [= W], f. 91va: “Item, dato ternario et denario, sequuntur numeri infiniti in actu”; *ibid.*, B, f. 115va-vb; D, f. 100ra-rb; F, f. 79ra-rb; P, f. 107rb; W, f. 92va: “Ad secundum consequens dico quod inter ternarium et binarium non exurgit alia multitudo ab ipsismet numeris duabus”.

¹⁴ HENRICUS DE HARCLAY, *In Sent.*, I, d. 24, q. u., C, f. 57ra; M, f. 93ra: “Praeterea, si numerus est aliquid existens in rerum natura praeter res continuas et res numeratas, maior numerus esset aequalis minori. Probatio: omnis dualitas est aequalis omni dualitati in numero. Posito hoc quod hic sint duo lapides et ibi duo ligna, hic sunt duo binarii. [...] ergo hic est una dualitas duarum rerum, scilicet duorum numerorum, scilicet binariorum vera dualitas realis, sicut est dualitas lignorum; ergo per primam propositionem haec dualitas binariorum est aequalis numeri cum dualitate lignorum. Sed quorum quantitates sunt aequales, eorum medietates sunt aequales; ergo medietas huiusmodi

argument from Harclay with minor changes. A duality of pieces of wood is equal to duality of two *quinarii*, i. e., two groups each of which consists of five ‘units’. The half of the first duality should be equal to the half of the second duality, but, if we compare them, the number one (one piece of wood) will equal the number five.¹⁵

As for Peter Auriol, he also invented a paradox concerning numbers. If the form of number is real, in ten stones which are the matter of the number ten, there are *several* forms of this same number ten and not only one form. The form is constituted according to an order which exists among numbers: 2 precedes 3, 3 precedes 4, and so on. But the form of number could be constituted in different ways. Let us suppose that, when we describe the constitution of this form, we were to start from one stone (for example, the nearest one), then move to the second (less close), then to the third, and so on, up to the tenth; finally, we have the form of ten. But we can also start from another stone (for example, the farthest), then move to the second (which previously was the ninth), then to the third (formerly the eighth), and so on, up to the tenth. This new form of ten is not the same as the first one. In brief, in ten stones there are as many forms of number ten as there are ways to combine these stones. None of these combinations is better than another, because they are all natural to the same extent: every stone could be the first part of the form, or the second, or the last, etc. That is why, Peter Auriol concludes, considering this plurality of forms, someone who borrowed 100 deniers could give back only 10 deniers (since in these 10 deniers the form of ten is present 100 times or even more). A good business idea... The only way to avoid this trap is to admit that a number is an *ens rationis*.¹⁶

2. The conceptualist number theory: different versions

Since the realist theory of number is untenable, Duns Scotus, Henry of Harclay, and Peter Auriol agree in claiming that a number is a mental being. However, even if the arguments of the three philosophers are similar, their conclusions are not identical: in particular, the definition Auriol gives of the form of number is not the same as those of Scotus and Harclay.

Scotus presents his number theory as follows: the only type of quantity that exists outside the mind is not a discrete quantity, i.e., a number, but rather a continuous quantity, “divided as this, this, this.”¹⁷ According to this definition, we can say that ten human beings are the parts of a continuum, but each of them is a *separate* part, and its unity with other parts is only in the mind. This unity is a discrete quantity, namely the number ten. Scotus explains how the intellect produces it: by grasping some parts of a continuum as *one* object, the intellect then chooses one unit, one part of this conceived plurality, and applies it as a measure to this plurality, as many times as required. The result of this measuring is a number, and undoubtedly it has only a mental

binarii lapidum est aequalis medietati binarii dualitatum. Igitur haec unitas est aequalis huic binario, quod est impossibile.”

¹⁵ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 1, Vb, f. 268va; ed. 1596, p. 549b: “Praeterea, impossibile est minorem numerum aequalem esse numero maiori, sic quod realitas minor aequalis sit maiori. Sed si dualitas sit aliquid in rerum natura, realitas dualitatis duorum quinariorum est maior realitas quam <dualitas> duorum lignorum, quoniam illa realitas dividi potest in duas partes maiores, videlicet in realitatem unius quinarii et realitatem alterius quinarii, dualitas vero duorum lignorum dividitur in duas unitates, de quibus constat quod sunt minores quam realitates quinariorum. [...] Ergo si forma numeri est in rerum natura, maior realitas est aequalis minori, quod est impossibile, et per consequens impossibile est quod numerus sit res extra.”

¹⁶ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 2, Vb, f. 269va-vb; ed. 1596, p. 552a: “Tertio quoque patet idem ex ordine quem habent numeri ad invicem secundum Philosophum in *Praedicamentis*. Binarius enim praecedet ternarium, et ternarius quaternarium. Ex quo potest sic argui: si forma denarii esset in decem lapidibus actualiter existens, sequitur quod in illis decem lapidibus sunt in actu duo denarii, immo tot quot modis possunt lapides ad invicem combinari. Incipiens namque decem illos lapides computare ab una parte formabit suum binarium, et suum ternarium, et sic deinceps usque ad denarium; incipiens vero a parte altera formabit aliud binarium, et aliud ternarium, quia connectet alias et alias unitates, et per consequens deveniet ad aliud denarium [...]. Secundum hoc ergo [...] universaliter tot denarii possunt hic apprehendi, quot modis potest unitatum ordo et connexio variari. [...] Sed hoc est impossibile et absolum manifeste: sequeretur enim quod debens centum denarios non haberet solvere nisi decem. Igitur impossibile est quod denarius decem lapidum sit in actu nisi ab anima computante.”

¹⁷ Here and in what follow, I quote the English translation of Scotus’ *Reportatio* by A. Wolter and O. Bychkov.

existence. This creation of number by the mind is similar to the work of a geometrician. He cannot establish the exact length of some distance right away, so he chooses a measure (for example, a foot or a cubit) and applies it to the distance as many times as required.¹⁸ Henry of Harclay defines number and its form in a similar way to Scotus, but more clearly: for him, a number is a “repetition of units by the intellect.”¹⁹ Either it could be several units which are distinct from each other, or it could be the same unit repeated several times. In this last assertion, Harclay refers to Boethius: according to the latter’s *De Trinitate*, when we say “sun, sun, sun,” we form in our intellect the number three.²⁰

At first glance, the number theory of Peter Auriol seems close to those of Scotus and Harclay, and Auriol is just clearer in his explanations than his predecessors. Yet there is a major difference between them, and Auriol himself stresses this difference. While he makes no comment on Scotus’ explanations, he explicitly questions Harclay’s the number theory. For Auriol, the form of number is not a simple repetition of units (one, one, one...), as Harclay maintains, but a *sum* of units (*summatio* or *summa*). The very process of calculation, the addition of one thing to another, defines number: in other words, for Auriol, a number is a numbering. But to repeat the same does not mean to number: we add one thing to another, not one thing to itself. Auriol insists time and again that the addition is the *sine qua non* condition: without addition, while just conceiving the units one after another (a stone, another stone, another one...), the intellect obtains no certitude about the quantity of enumerated things.²¹ Similarly, according to Auriol, a number is not an *aggregatio* of things: we can conceive several objects together without numbering them.²²

¹⁸ IOANNES DUNS SCOTUS, *Rep.*, I-A, d. 24, q. u., § 42-43, ed. WOLTER – BYCHKOV, pp. 38-39: “Aliter dicitur quod de quantitate nihil est extra animam realiter nisi quantitas continua [...], cuius partes divisae extra animam non constituunt numerum extra animam [...], sed tantum sunt illae *partes continui divisae hoc, hoc, hoc*, quarum unitas formalis tantum est ab anima, et sic numerus est quantitas secundum rationem. Nam anima potest accipere aliquam multitudinem partium extra ut unum intelligibile, et negotiando circa illam per actum suum, potest per unam partem illius multitudinis mensurare totam multitudinem intellectam. Exemplum istius patet de geometra imperfecto qui non mensurat primo per visum aliquam certam quantitatem, quam habeat de distantia, sed mensurando primam quantitatem per pedes, vel cubitos, vel perticam, vel virgam, applicat illam ut mensuram ad aliam, ut sciat eius certam quantitatem. – Ita in proposito: intellectus primo mensurat aliquam multitudinem intellectam, quam postea applicat ad mensurandum multitudines extra.”

¹⁹ As Nielsen and Trifogli indicate, when he discusses the question about the ontology of number in his *Quodlibet* (Paris, 1315), Thomas Wylton refers to Harclay’s definition of number and rejects it. L. O. NIELSEN – C. TRIFOGLI, “Thomas Wylton’s Questions on Number, the Instant, and Time,” in: *Documenti e studi sulla tradizione filosofica medievale* 16 (2005), pp. 57-177, esp. pp. 8-59 and 74-76. Wylton also alludes to some arguments invented by Harclay in favour of his opinion (cf. my forthcoming edition of question 24 of Harclay’s commentary).

²⁰ HENRICUS DE HARCLAY, *In Sent.*, I, d. 24, q. u., C, f. 57rb; M, f. 93ra; Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 1113 [= A], f. 128vb, Wrocław, Biblioteka uniwersytecka, I.F.184 [= W], f. 213rb: “[...] numerus nihil aliud est nisi repetitio unitatum per intellectum, et hoc potest fieri ubi nulla est diversitas rerum, sicut dicit Boethius, libro *De Trinitate*. [...] non constituitur numerus necessario per distinctionem rerum numeratarum, sed per repetitionem unitatum, sicut cum dicitur “sol, sol, sol”, secundum ipsum.” Cf. *infra*, notes 32-33.

²¹ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 1, Vb, f. 267ra; ed. 1596, p. 546b: “[...] numerans non repetit priorem unitatem, sed addit aliam novam et distinctam. Unde patet quod forma numeri non est repetitio eiusdem unitatis”; *ibid.*, Vb, f. 267rb-va; ed. 1596, p. 547a-b: “Non tamen computare est [...] unum post aliud sumere ordinatim: non enim dicens intra mentem ‘unum, et unum, et unum’, et sic deinceps dicitur computare; in fine enim nullam certitudinem habet aut numerum aliquem illorum distinctorum quae mentaliter unum post aliud ordinavit. [...]. Mensurare autem et metiri sunt idem, metiri autem est mente terminare et concludere [...], quae quidem conclusio non est aliud quam quaedam summa distinctorum. Et secundum hoc numerare est summare.”

²² PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 1, Vb, f. 267rb, ed. 1596, p. 547a: “Et propter hoc dixerunt alii quod forma numeri consistit in unitate aggregationis. [...] Sed nec iste modus dicendi est competens. Manifestum est enim quod aggregare non est aliud quam acervare sive adunare et ad quandam simultatem aliqua adducere quasi in unum gregem et in unum cumulum. Sed constat quod coacervare et adunare unum uni non est numerare seu computare. [...] Praeterea, si numerus esset formaliter aggregatio unitatum, [...] tunc intellectus non posset intelligere aliqua intelligibilia aggregatim quin conciperet numerum, cuius oppositum experientia docet: possumus enim aliqua intelligibiliter aggregare non numerando ea.” We find similar terminology, and especially the notion of *acervus*, in many texts. See, e. g., BOETHIUS, *De institutione arithmeticā*, I, 3, ed. J. SCHILLING – H. OOSTHOUT, Turnhout 1999 (CCSL 94a), pp. 15-16: “Numerus est unitatum *collectio*, uel quantitatis *aceruuſ* ex unitatibus profusus”;

When giving his definition of the form of number, Auriol minutely describes how the intellect progressively makes up this form. First of all, the intellect grasps one thing as a distinct thing, indivisible in itself and separated from the others. In order to do so, the intellect conceives a precise *ratio* of this thing, namely the *ratio* of indivisibility. As is well known, in Auriol's theory of cognition, the *ratio* is a key notion: it is an extramental feature of the object of cognition and the content of a concept that the intellect forms while conceiving this object. Any object has several different *rationes* which direct the intellect to generate different concepts: when we observe a rose, our mind can produce the concept of 'rose' as well as the concept of 'flower' by grasping different *rationes*.²³ Nevertheless, in Auriol's theory of number, the term *ratio* obviously has a different meaning. Auriol says explicitly that this *ratio* is not an extramental aspect of the object, but something mind-generated, a mental form imposed by the mind upon extramental objects (matter of number).²⁴ He gives no comments on this issue; however, his further argumentation allows us to better understand the different uses of the same term. For creating a concept, the mind should always proceed with a *ratio*; the only concept without any explicit *ratio* is that of being, the most general and indeterminate concept.²⁵ But Auriol insists on the difference between the conceiving of one thing "as a being" (*unum quod convertitur cum ente*, or *unum* as a synonym of *ens*) and that of one thing "as the principle of number" (*unum principium numeri*). In the first case, we conceive one thing as a distinct thing without any additional *ratio*, but with two negations, namely its indivisibility and its difference from other things, not *in recto*, but *in obliquo*, i.e., secondarily. In the second case, our intellect adds the *ratio* of indivisibility (i.e., the form of the number) to a real thing (i.e., the matter of the number) and obtains the concept of number one, "one as the principle of number."²⁶ Thus, like all *rationes*, the *ratio* of indivisibility is the core of the concept which is the number one. However, in contrast to other *rationes*, this *ratio* only has objective existence in the intellect. Auriol demonstrates this in the following way. Let us suppose that being one ("one as the principle of number") is an accident of a thing, as Averroes claims it to be. To this accident corresponds a *ratio*, namely, the *ratio* of indivisibility: 'one' is something which cannot be divided. If this *ratio* is extramental, it is negative (*ratio negativa*), because it is inherent to a thing taken as "a denied subject" (*in quantum negatum*), i.e., denied to be divisible. In this case,

I. CAIAZZO, *Lectures médiévaies de Macrobe. Les Glosae Colonienses super Macrobius*, Paris 2002, p. 191, n. 4; AVERROES, *De physico auditu libri octo*, III, 68, Venetiis 1562 (Iuntina, IV), f. 117vbK: "[...] quia numerus est coacervus unitatum, numerus est compositus ex indivisibilibus"; THOMAS AQUINAS, *In duodecim libros Metaphysicorum Aristotelis expositio*, VIII, 3, § 1725, ed. M.-R. CATHALA – R. SPIAZZI, Taurini 1964, p. 412b: "[...] unitates aggregantur in eo per modum coacervationis."

²³ For the notion of *ratio* in Auriol, see: R. FRIEDMAN, "Peter Auriol on Intentions and Essential Predication," in: S. EBBESEN – R. FRIEDMAN (eds.), *Medieval Analyses in Language and Cognition*, Copenhagen 1999, pp. 415-430, esp. pp. 420-42; F. WÖLLER, *Theologie und Wissenschaft bei Petrus Aureoli. Ein scholastischer Entwurf aus dem frühen 14. Jahrhundert*, Leiden 2015, pp. 62-72.

²⁴ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 3, Vb, f. 272rb; ed. 1596, pp. 557b-558a: "Haec autem ratio non est nisi clausio distincti in quantum distinctum, vel indivisi in quantum indivisum, quam facit mens, distinctum in quantum huiusmodi apprehendendo [...]. Praeterea, ille conceptus quem format computans de primo distincto, in quantum quoddam indivisum est, est formalis ratio unius quod principiat numerum."

²⁵ WÖLLER, *Theologie und Wissenschaft bei Petrus Aureoli*, pp. 72-73; M. PICKAVÉ, "Metaphysics as First Science: The Case of Peter Auriol," in: *Documenti e studi sulla tradizione filosofica medievale* 15 (2004), pp. 487-516; J. AERTSEN, *Medieval Philosophy as Transcendental Thought. From Philip the Chancellor (ca. 1225) to Francisco Suárez*, Leiden 2012, pp. 433-456.

²⁶ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 3, Vb, f. 272rb-va; ed. 1596, p. 558a-b: "Praeterea, aliud est rem aliquam in sua realitate concludere per duplarem negationem, scilicet per indistinctionem in se et divisionem ab alio quocumque, et aliud rem, ut indivisa est et per negationes conclusa, in illa indistinctione mentaliter concludere. Prima enim conclusio quae debetur rei, in quantum res est, non est nisi per negationes, et est concipere unum quod convertitur cum ente, de quo dictum est supra quod non addit ad rem aliquam rationem in recto, sed in obliquo connotat huiusmodi negationes. Secunda vero, quae debetur indivisioni rei aut ipsi ut sic negata est, pertinet ad unum quod est principium numeri [...]. Claudio indivisi, in quantum indivisum est, non est aliquid in re extra, sed ratio per intellectum circa indivisum in quantum huiusmodi fabricata"; *ibid.*, Vb, f. 272ra-rb; ed. 1596, pp. 557b-558a: "Tertia quoque propositio est quod unum quod est principium numeri addit ad unum quod cum ente convertitur positivam rationem, quae quidem non est nisi in anima obiective."

the *ratio* is less perfect than the accident, since the latter is positive. That is impossible.²⁷ The only solution is to admit that the *ratio* of indivisibility is produced by the intellect and does not correspond to any extramental reality (i.e., being one is not an extramental accident). Auriol underlines that this mental *ratio* is positive, because “one as the principle of number” means an intermediate state (*medietas* or *aequalitas*) between a greater and a lesser quantity: for instance, we say that there is only one phoenix, neither more nor fewer.²⁸ According to Auriol, the intellect can attribute such a *ratio* to a thing taken as “a subject denied” to be divisible; thus, the mind starts numbering the extramental entities.²⁹

After this first step, the intellect grasps the next thing, with the same *ratio*, as a distinct thing, gets a new concept and *adds* it to the first; so it obtains *the sum* of two. Then the next thing is conceived, with the same appropriate *ratio*, and added to this sum of two; now the intellect has the sum of three, and so on, *ad infinitum*. Thus, these sums are the result of *reflexive* acts of the intellect: after conceiving a thing as distinct, the intellect returns to the things conceived earlier and makes an addition. The human mind cannot generate numbers by a direct acquaintance with the extramental world; it needs to go to a higher level of abstraction for forming this concept. Nevertheless, numbers are not second intentions. In the end, the final sum (the form of number) is applied by the intellect to a real plurality of things (i.e., to the matter of number) that the intellect was measuring. When the extramental plurality is conceived with the *ratio* of a sum, the concept of number is finally forged: for example, the intellect comes to the concept of six stones, when it conceives these stones with the *ratio* of the sum of six; therefore, this concept has only objective being in the intellect, not extramental being.³⁰ Because of this last operation, namely the application of the form to the things outside the mind, a number is neither a *res ficta* nor an *intentio secunda*, i.e., a concept formed by the intellect reflecting on its own concepts and without grasping an extramental reality, but an *intentio prima*: there is a direct extramental ground for the concept of number.³¹

²⁷ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 3, Vb, f. 272va; ed. 1596, pp. 558b: “Praeterea, Commentator dicit, X *Metaphysicae*, commento 8, quod unum principium numeri est accidens additum rei, unum vero quod cum ente convertitur est idipsum quod res. [...] Sed non posset poni accidens, si consisteret in solis negationibus circa continuum, et non diceret rationem aliquam positivam circa huiusmodi indivisum. Ergo necesse est quod talis clausio positiva sit formalis ratio unius quod est de genere quantitatis discretae per modum principiantis. Quod autem huiusmodi clausio non sit in re extra, sed tantum in anima obiective, patet ex hoc quod habet pro proprio subiectivo indivisum in quantum indivisum. Manifestum est autem quod nullum positivum reale potest habere pro subiecto indivisum in quantum huiusmodi, sic quod indivisio sit ratio subiectiva, alias sequeretur quod reale positivum inesset alicui per rationem negativam, et tunc accidens esset perfectius sua ratione subiectiva, quod est impossibile.”

²⁸ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 3, Vb, f. 272ra-rb; ed. 1596, pp. 557b-558a: “Quod ratio unius quod est principium numeri est positiva. [...] Praeterea, unum quod est principium numeri consistit in quadam medietate et aequalitate excludente plus vel minus, unde consuevimus dicere quod tantum est unus phoenix, nec plures nec pauciores. Sed conceptus sic mediens inter plura et pauciora videtur esse positivus.”

²⁹ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 3, Vb, f. 272va; ed. 1596, p. 558b: “In intellectu vero possibile est rationem obiectivam positivam alicui in quantum negatum, sic quod in quantum negatum sit subiectum; et per consequens clausio indivisi, in quantum indivisum est, non est aliquid in re extra, sed ratio per intellectum circa indivisum in quantum huiusmodi fabricata.”

³⁰ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 3, Vb, f. 267va; ed. 1596, p. 547b: “[...] summare est actus intellectus reflexus, praesupponens actum intellectus distinctum aliquod prius accipientis; quo accepto statim, dum aliud distinctum accipitur, intellectus reflectens se super illud et super istud reducit ambo in summam dualitatis; deinde vero assumens distinctum aliud et reflectens se super priora distincta, reducit illa et <istud> in summam trinitatis; deinde vero assumens distinctum aliud et reflectens se super priora distincta, illa et istud reducit in summam quaternitatis; et sic in infinitum. Secundum hoc ergo summa est quaedam ratio formata ab intellectu reflectente se super distincta praeaccepta”; *ibid.*, Vb, f. 269vb; ed. 1596, p. 552b: “Ad cuius evidentiam considerandum est quod forma numeri est quaedam ratio positiva et simplex, claudens atque mensurans formaliter distincta in quantum distincta, existens in intellectu obiective.”

³¹ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 3, Vb, f. 270va; ed. 1596, p. 554b: “Si etiam dicatur quod erunt intentiones secundae, praesertim quia supra dictum est quod fit numerus per actum intellectus super praeaccepta iterum redientis et quasi reflectentis, dicendum quod non, quia huiusmodi reflectio exigitur ad hoc quod deveniatur ad summam sive conclusionem quam numerus importat; nihilominus postquam ad eam ventum est, intellectus ipsam rebus attribuit quae sunt extra.” On Auriol on first and second intentions see, e.g., L. M. DE RIJK, “A Study on the

Globally speaking, Auriol's number theory conforms to his idea of concept formation: concepts have mental existence, but their foundation lies in the extramental reality. Concepts are things themselves, but in a special mode of existence, called "intentional" or "objective" existence, in which the extramental objects are "indistinguishably mixed together (*indistinguibiliter immiscetur*) with their conception." Thus, the concept of animal, for example, is an individual animal as conceived by the intellect, and in the formation of this concept, the intellect is directed by an extramental *ratio* of a particular animal.³² In the same way, the concept of number is several extramental things progressively conceived together; for producing it, the intellect proceeds with a mental *ratio* of sum.

Most of the examples quoted by Auriol concern exclusively the numbering of *extramental* beings, material or immaterial (e.g., stones or angels);³³ nevertheless, some of his statements seem to indicate that the use of number is probably wider than that. First, it seems that, in the framework of Auriol's theory, mental items could also be counted. Auriol himself mentions the number of categories, without explaining whether in this case the number pertains to *intentiones secundae* (i.e., the categories as concepts organizing other concepts).³⁴ Second, Auriol himself adds that the forms of numbers are not always applied to the real things: these forms could also exist in the mind in an abstract way, with no relation to any kind of matter. Some people have in their intellect more of these forms, others have fewer: it depends on their knowledge of mathematics.³⁵ In other words, we can perform mathematical operations without any real, extramental object, simply by adding two and three, and not two and three stones.

3. Number in the divine persons

The conceptualist theories of number elaborated by Scotus, Harclay, and Auriol had some important consequences. One of them is the following: this theory could be applied to a larger group of things and phenomena than the realist theory ever could. In fact, the definition of number as an *ens rationis* includes almost all sorts of things, material or immaterial, substances or accidents.³⁶ Both Henry of Harclay and Peter Auriol attack an opinion like the one we saw above from Thomas Aquinas,³⁷ according to which, since number is in the category of quantity (a discrete quantity), only material things which have a quantity (*quanta*) can be numbered in the strict sense. According to that opinion, we can, of course, speak of a number of immaterial things (angels, demons, categories...), since they are distinct from one another, but their number does not originate from a division of continuous quantity proper to the matter. This explained, for Aquinas, why their number is of a special kind: a transcendental number. In the framework of a conceptualist theory of number, this typology is useless: there is only one kind of number, which

Medieval Intentionality Debate up to *ca.* 1350," in: GERALDUS ODONIS O.F.M., *Opera Philosophica*, vol. 2: *De intentionibus*, ed. L. M. DE RIJK, Leiden 2005, pp. 352-357; D. PERLER, *Theorien der Intentionalität im Mittelalter*, Frankfurt 2002, 2nd ed. 2004, pp. 294-313, esp. pp. 306-313.

³² See FRIEDMAN, "Peter Auriol on Intentions and Essential Predication," p. 425.

³³ See also *infra*, 3.

³⁴ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 3, Vb, f. 271va; ed. 1596, p. 556a: "Praeterea, denarius est quaedam species numeri, et tamen dicitur de praedicamentis quod sunt decem."

³⁵ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 3, Vb, f. 270rb; ed. 1596, p. 553b: "Habet enim quilibet penes se huiusmodi clausiones, ut dualitatem, trinitatem, et quaternitatem, et ille qui plures habet, plura scit computare, quae quidem in mente remanent et abstrahunt ab omni materia et genere terminato, ut a nummo, vel lapide, vel latere, et sic de aliis."

³⁶ For example, PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 3, Vb, f. 272vb; ed. 1596, p. 559a: "Tertium vero quod dicitur de numero et unitate est quod reperiuntur in omni genere entis, quia et in angelis, et in quantis, et in substantia, et in accidentibus."

³⁷ Cf. *supra*, note 4.

exists wherever the human intellect is able to form this concept.³⁸ But is this also the case with the divine persons?

In considering numbers as *entia rationis*, Scotus defines the divine Trinity as the number three and rightly points out that it was impossible in the framework of any realist theory of number. For instance, if the form of number is a distance between the first and the last units (the opinion of Bernard of Auvergne), the Trinity is not a number: there is no distance between the divine persons. Similarly, if the form of number is a discreteness, a discontinuity of numbered units, then number is not proper to God: according to this definition, every divine person should be a material unit which has in potency the form of number; this form becomes actual when one unit is considered with others. But nothing is potential or material in God.³⁹ Thus, the only way to postulate the existence of number in the divine persons is to admit that numbers have only a mental existence. According to Scotus, the intellect could conceive one of the divine persons or all of them; in either case, there is a number, one or three.⁴⁰ However, this does not imply an accidental form in God. In fact, the things which are numbered, whatever they are, do not really constitute some *one* with a common form outside of the intellect. When we speak of three stones, we do not describe any real thing, but we produce a *res in anima* to exclude any other, greater or lesser, aggregation of stones (for instance, two or four). Similarly, when we speak of the divine persons, we do not assert the real existence of something in God; we simply want to say that the divine persons are three, neither fewer nor more. Just as Peter Lombard himself declared in the *Sentences*, in the case of the divine Trinity, the number is used in order to exclude, not in order to posit.⁴¹

Although for Scotus the numbering of the divine persons is entirely comparable to that of real, created things (for example, that of stones), he emphasizes in the *Ordinatio* that the divine Trinity is number *secundum quid* and not number *simpliciter*: if the Trinity were a number *simpliciter*, we would have to recognize there to be a diversity between the divine persons and thus an imperfection. Since all divine persons have the same essence and there is no imperfection, we cannot use the concept of number *simpliciter*; nevertheless, since the divine persons are distinct, we can still use the concept of number, namely, number *secundum quid*.⁴² In

³⁸ For example, HENRICUS DE HARCLAY, *In Sent.*, I, d. 24, q. u., C, f. 56va; Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 13687 [=H], f. 76rb; M, f. 92vb; A, f. 128va; W, f. 212va: “[...] ostendo primo quod numerus non causatur ex divisione continui, nec tantum habet esse in continuis materialibus, sed in aliis”; *ibid.*, C, f. 56vb; H, f. 76va; M, f. 92vb; A, f. 128va; W, f. 212vb: “alias species numeri non assignavit Boethius nec aliquis alias arithmeticus.”

³⁹ IOANNES DUNS SCOTUS, *Rep.*, I-A, d. 24, q. u., § 45-46, ed. WOLTER – BYCHKOV, p. 40: “Secundum istas diversas opiniones diversimode est respondendum ad quaestionem – et quidem secundum primam opinionem, quod in divinis non est numerus, quia ibi non est distantia unitatis ab unitate, quia omnes tres personae sunt indivisibiliter et indistincter et unitissime eadem essentia simplex. Secundum etiam opinionem secundam ponentem unitates in numero quasi partes eius materiales et discretionem quasi formam eius, dicendum esset quod in divinis isto modo non est numerus, quia in divinis nihil est potentiale et materiale respectu ulterioris actus propter compositionem.”

⁴⁰ IOANNES DUNS SCOTUS, *Ord.*, I, d. 24, q. u., in: *Ioannis Duns Scoti Opera omnia* [...], ed. Commissio Scotistica, vol. 5, Vatican 1959, p. 368: “Si tamen numerus habeat esse in intellectu tantum, simul concipiente plura una vel plures unitates, cum intellectus aliquis possit concipere tres personas simul vel unam solam, [...] posset poni quasi unus totalis conceptus Trinitatis in intellectu”; IOANNES DUNS SCOTUS, *Rep.*, I-A, d. 24, q. u., § 52, ed. WOLTER – BYCHKOV, p. 43: “[...] intellectus noster accipit unam personam ut unam unitatem, et aliam ut aliam, et tertiam ut tertiam, et sic numerantur tres personae, non natura sed quantum ad animam numerantem.”

⁴¹ IOANNES DUNS SCOTUS, *Rep.*, I-A, d. 24, q. u., § 49, ed. WOLTER – BYCHKOV, p. 42: “Ad quaestionem igitur, secundum ultimam opinionem, esset dicendum, sicut dicit Magister, quod magis utimur numero in divinis ad removendum quam ad aliquid affirmandum vel ponendum. Nam numerus non dicit in rebus nisi privationem vel praecisionem maioritatis vel minoritatis. [...] multa non faciunt aliquod unum totum extra animam; ut cum dico ‘tres lapides’, non dico aliquam unam formam in ipsis, sed tantum hunc et illum et tertium cum praecisione maioris et minoris aggregationis lapidum. [...] cum dico ‘tres personae sunt Deus’, hic terminus numeralis ‘tres’ non significat nisi Patrem et Filium et Spiritum Sanctum cum praecisione maioris vel minoris pluralitatis, et significat quod in divinis sunt tres personae, non plures nec pauciores.” Cf. *supra*, note 3.

⁴² IOANNES DUNS SCOTUS, *Ord.*, I, d. 24, q. u., ed. Commissio Scotistica, p. 368: “[...] dico quod numerus simpliciter non est in divinis [...]. Conceditur tamen ibi numerus ‘secundum quid’ [...]. Bene tamen conceditur ibi

the *Reportatio*, Scotus does not mention the distinction *simpliciter/secundum quid*, but explains why in the case of the divine persons, we are talking about the Trinity (*Trinitas* or *trinus*) and not about triplicity (*triplicitas* or *triplex*), as we do in the case of other numbered things: the name ‘Trinity’ is less appropriate to the diversity and better suited for the “plurality with the unity of form,” as are the divine persons.⁴³

Henry of Harclay’s opinion is even simpler: number exists in the divine persons, and there is even no need to resort to the concept of number *secundum quid*, because the intellect can produce a number even by the repetition of the same unit. Harclay claims to have borrowed this idea from Boethius, while Peter Auriol, responding to Harclay, claims that the ancient philosopher said exactly the opposite.⁴⁴ Indeed, if we turn to Boethius, we see that he contrasts two notions, that of *numeratio* and that of *iteratio*. Only different things can be numbered; thus, when we repeat “sun, sun, sun,” we do not really number three suns. Peter Auriol therefore concludes that Boethius denies the existence of number in the divine persons. What are we to think of Auriol’s interpretation of Boethius? On the one hand, Auriol is correct to note that, according to Boethius, when we say about the same sword “*ensis, mucro, gladius*,” we still characterize one object, and when we say “the Father, the Son, and the Holy Spirit,” we continue to speak of one God. On the other hand, Boethius’ theory is subtler than Auriol appears to allow for. For Boethius, if we mean by a number the things which are numbered, we should not posit number in God, since God is not triple. Nevertheless, if we mean the number itself (*ipse numerus*), i.e., what Auriol would call the form of number, then, according to Boethius, it can legitimately be stated that it exists in God, because this number is formed by the triple repetition of the same thing; it is God’s triple predicate.⁴⁵ Henry of Harclay refers to this particular passage of Boethius’ *De Trinitate*.⁴⁶

distinctio sine diversitate vel potentialitate (vel etiam aggregatione), quae dicunt imperfectionem aliquam in unitate.”

⁴³ IOANNES DUNS SCOTUS, *Rep.*, I-A, d. 24, q. u., § 49, ed. WOLTER – BYCHKOV, pp. 42-43: “Concedimus tamen magis proprie in divinis nomen Trinitatis vel trini [*coni.*: trium *ed.*] quam nomen triplicis vel triplicitatis [*coni.*: triplicati *ed.*], quia nomen Trinitatis magis abstrahit a diversitate et aggregatione plurium, secundum suum modum significandi, quam nomen triplicitatis vel ternarii. Denotat enim Trinitas pluralitatem cum unitate formae.” For the correction of “triplicati” into “triplicitatis”, cf. the rest of the quotation. “Trinitatis vel trium” should probably be correct into “Trinitatis vel *trini*”. At least, two out of five manuscripts of the *Rep. I-A* give this reading (mss. Città del Vaticano, Biblioteca Apostolica Vaticana, Borgh. lat. 325, f. 60ra; Oxford, Balliol College, 205, f. 105v). For the textual tradition of the text, see IOANNES DUNS SCOTUS, *Reportatio Parisiensis*, I-A, ed. A. WOLTER – O. BYCHKOV, vol. 1, St. Bonaventure/New York 2004, pp. XIX-XXIV. Moreover, according to medieval theologians, when describing Good, one should not use the adjective “triplex,” but “*trinus*” (gen. sing. “*trini*”), since the latter means “three-and-one” or “three-in-one”. See, e. g., PETRUS LOMBARDUS, *Sententiae in IV libris distinctae*, I, d. 19, c. 12, 1, ed. I. BRADY, t. 1, pars 2, Grottaferrata 1971, p. 171: “Propterea, cum Deus dicatur *trinus*, non tamen debet dici *triplex*; ibi enim non est triplicitas, ubi summa est unitas et indifferens aequalitas.”; THOMAS DE AQUINO, *In Sent.*, I, d. 19, *exp. textus*, ed. P. MANDONNET, Paris 1929, p. 499: “Praeterea cum Deus dicatur *trinus*, non tamen debet dici *triplex*. Hujus ratio est, quia triplicitas est quaedam proportio contenta sub multiplicitate, quae est inaequalitatis species; et ideo ubi est triplicitas, oportet inaequalitatem esse.”; *ibid.*, d. 24, q. 2, a. 2, ad 2, p. 591: “Similiter cum dicitur: Deus est *trinus*, significatur quod est habens tres personas in una essentia, unde ly “Deus” supponit tunc vel essentiam vel suppositum indistinctum.” I’m particularly grateful to Guy Guldenops for this suggestion.

⁴⁴ HENRICUS DE HARCLAY, *In Sent.*, I, d. 24, q. u., C, f. 57rb; M, f. 93ra; A, f. 128vb, W, f. 213rb: “Secundum ergo dicendum est ad quaestionem propositam quod numerus vere potest esse in divinis, quia numerus nihil aliud est nisi repetitio unitatum per intellectum, et hoc potest fieri ubi nulla est diversitas rerum, sicut dicit Boethius, libro *De Trinitate*; cf. *supra*, note 15. PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 1, Vb, f. 267ra; ed. 1596, p. 546a: “Quapropter dixerunt alii quod totalitas numeri non est alia quam replicatio unitatis, [...] Et hoc est quod dicit Boethius, I *De Trinitate* [...]. Sed nec iste modus dicendi videtur idoneus, tum quia Boethius expresse dicit oppositum, ait enim quod unitatum iteratio non est numeratio, iteratio enim repetitio est eiusdem, numeratio vero diversorum, et concludit non omnem unitatum repetitionem numerum pluralitatemque perficere.”

⁴⁵ BOETHIUS, *De Trinitate*, c. 3, in: *Boethius. De consolatione philosophiae, opuscula theologica*, ed. C. MORESCHINI, München 2005, p. 172, 142-149: “Potest enim unus tot vocabulis *gladius agnoscit*; haec enim unitatum iteratio potius est, non numeratio, velut si ita dicamus: ‘*ensis mucro gladius*’, repetitio quaedam est eiusdem, non numeratio diversorum, velut si dicam: ‘*sol sol sol*’, non tres soles effecerim, sed de uno totiens praedicaverim. Non igitur si de Patre ac Filio et Spiritu sancto tertio praedicatur Deus, idcirco *trina* praedicatio

While Scotus and Harclay affirm that there is the number three in God, Auriol denies it. His position logically follows from the definition he gives to the form of number: according to Auriol's theory, no *summatio* of divine persons is possible. The human intellect cannot grasp several things with the *ratio* of number immediately, since this *ratio* is a sum which is constituted progressively. To obtain this sum, the intellect must conceive every unit of number with the *ratio* of a distinct thing, indivisible in itself and separate from the others. But, for Auriol, in the case of the divine persons, this is impossible, since none of the divine persons can be conceived separately from the two others. Auriol explicitly claims that it is strictly impossible to conceive the Father without the Son and the Holy Spirit, etc.⁴⁷ For that reason, the intellect cannot produce the concept of one about any of the divine persons, and because the concept of one is the principle of number, this explains why the intellect cannot arrive at the final form of number, i.e., a sum, in this case three or the Trinity.⁴⁸

In other words, Auriol continues, when we are speaking about the Trinity, we do not conceive God with a positive *ratio* of the number three. When we use numerical terms regarding God, we perform an exclusion, similar to that which takes place in the case of numbered things. A number has, in some way, two components, positive and privative. ‘Three stones’ means not only something positive, namely the form (the sum) of three, but also a privation, namely an exclusion of all other stones (that of the fourth, of the fifth, etc.). The ‘divine Trinity’ does not mean the form of number, but signifies the Father, the Son, and the Holy Spirit, all other beings excluded. This exclusion is close to (but not the same as) that which is done when we number the stones.⁴⁹

Auriol’s explanations of the divine Trinity recall those of Scotus. In both cases, the idea that the number three excludes from God anything other than the three divine persons, goes back to Peter Lombard.⁵⁰ However, in spite of the similarity of their reasoning, Auriol openly disagrees with Scotus about his definition of the Trinity as a *numerus secundum quid*. In fact, Auriol adds, a form *simpliciter* does not differ from a form *secundum quid* because of its subject, but because of “the decrease of its own *ratio*.” Auriol does not explain the last expression; he

numerum facit”; *ibid.*, ed. MORESHINI, p. 171, 121-132: “Deus vero a Deo nullo differt [...]. Ubi vero nulla est differentia, nulla est omnino pluralitas, quare nec numerus; igitur unitas tantum. Nam quod tertio repetitur Deus, cum Pater ac Filius et Spiritus sanctus nuncupatur, tres unitates non faciunt pluralitatem numeri in eo quod ipsae sunt, si advertamus ad res numerabiles ac non ad ipsum numerum. Illic enim unitatum repetitio numerum facit. In eo autem numero qui in rebus numerabilibus constat, repetitio unitatum atque pluralitas minime facit numerabilem rerum numerosam diversitatem.”

⁴⁶ On the passage, see, e.g., A. TISSERAND, *Pars theologica. Logique et théologique chez Boèce*, Paris 2008, pp. 377-403, esp. pp. 382-392.

⁴⁷ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 4, Vb, f. 273rb-va; ed. 1596, p. 560b: “Ubi enim est tale indistinctum quod mente claudi non potest in sua indistinctione, ibi non est formalis ratio unitatis quae est principium numeri, cum formalis ratio eius sit praecisa clausio alicuius indivisi in sua indivisione, ut declaratum est supra. Sed nullum quod sit indistinctum in Deo potest a mente praecise concludi in sua indistinctione [...] Idem etiam patet de qualibet persona; non enim mente concludi potest Pater, Spiritum Sanctum vel Filium excludendo, sed per necessitatem, dum aliquis in mente sua numerat Patrem, intelligit Filium et Spiritum Sanctum. [...] Ergo unitas quae est principium numeri non habet locum in divinis.” For the ‘indistinction’ between each of the divine properties (paternity, filiation, passive spiration) and the divine essence according to Auriol (*Scriptum*, I, d. 1), see R. FRIEDMAN, *Intellectual Traditions at the Medieval University. The Use of Philosophical Psychology in Trinitarian Theology Among the Franciscans and Dominicans 1250–1350*, vol. 1, Leiden/Boston 2013, pp. 532-550.

⁴⁸ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 4, Vb, f. 273vb; ed. 1596, p. 561a-b: “[...] ratio trinitatis vel dualitatis, prout sunt quaedam summae numerales, non habent locum in divinis. [...] Nec est possibilis alicubi clausio numeri, ubi non est possibilis clausio unitatis ipsum principiantis [...]. Distincta ergo quae computari non possunt, sumendo praecise quodlibet et claudendo, summam numeralem habere non possunt apud intellectum.”

⁴⁹ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 4, Vb, ff. 273vb-274ra; ed. 1596, p. 561b: “Dicendum tamen ad hoc quod clausio numeralis habet habitudinem ad distincta, quae numerat et claudit, positivam et privativam: positivam quidem in quantum claudit, privativam vero in quantum alia distincta excludit. [...] In divinis itaque admittitur trinitas, non quia sit clausio positiva aspiciens personas per modum distinctorum summabilium, sed quia similis est exclusio in personis illi quam facit trinitas in lapidibus numeratis. Unde, quia Pater, Filius et Spiritus Sanctus sunt res a quibus excluditur alia, propter hoc dicuntur tres.”

⁵⁰ Cf. *supra*, note 3.

only emphasizes that, when we use the concept of *numerus secundum quid*, it is not because of the matter of number, but because of its form. Indeed, three stones (the matter of the number three) are distinct *simpliciter*, while three divine persons are distinct only *secundum quid*. However, in both cases, the form of the number three should be the same, namely the sum of three units made by the intellect (if we admit that this sum could exist in the divine persons), whether or not the units of this sum are really distinct.⁵¹ Thus, for Auriol, the notion of *numerus secundum quid* is meaningless.

In the final analysis, the conceptualist theory of number enabled its adherents to solve several problems, such as a number of immaterial things, the existence of numerical infinity, etc. Nevertheless, when number was defined as an *ens rationis* even when applied to the divine persons (as Scotus and Harclay did), then this attempt soon met with resistance. Already for Peter Auriol, the Trinity is by no means a number, this time not only because of the divine immateriality or the absence in God of some characteristics proper to creatures (quantity, accident, etc.), but because of the cognitive limits of the human mind: our intellect is not able to correctly calculate the divine persons, because it cannot comprehend one person without the other two. However, is there any mind which could conceive the Father without the Son or the Holy Spirit, joined through the essence? Maybe, for Auriol, it is beyond the capability of any intellect, even the divine.

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⁵¹ PETRUS AUREOLI, *Scriptum*, I, d. 24, q. u., a. 4, Vb, f. 273rb; ed. 1596, p. 560a: “Opinio Scotti. Propterea dixerunt alii quod in divinis non est numerus simpliciter, sed secundum quid. [...] Sed nec iste modus videtur competens propter duo. [...] Secundo vero, quia forma non dicitur secundum quid aut simpliciter ratione subiecti, sed per diminutionem suaे propriae rationis. Cum igitur quidditativa ratio numeri consistat in esse summam, sive sit summa distinctorum secundum quid sive distinctorum simpliciter, non propter hoc variabitur summa per simpliciter et secundum quid, licet summatum diversetur.”