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Integrating Facility Layout Design and Aisle Structure in Manufacturing Systems: Formulation and Exact Solution

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Abstract

In manufacturing systems, aisles are paths which are used for the movement of workers, transportation devices, and materials. The aisle structure contributes to layout efficiency by reducing material handling costs, mean flow time and the amount of space needed, and providing smooth transportation. Therefore, to achieve a good layout, it is essential to determine the position of facilities such as machines and workstations, but also the corresponding aisle structure. In this article, we analyze the requirements for the design of an efficient aisle structure and propose a formulation of the corresponding layout problem as a mixed-integer linear programming model. This formulation allows the layout of unequal-area facilities and the aisle structure to be simultaneously optimized. In optimizing the aisle structure, issues such as optimizing the number, position, and width of the aisles, the position of the entrance and exit doors, and how to connect them to the aisles are studied. By optimizing the number and width of the aisles, the proposed approach contributes towards optimizing transportation traffic. A branch-and-cut algorithm, improved by adding optimality cuts and efficient branching and node strategies, is used to solve the problem. Finally, a set of computational experiments is performed to show the effectiveness of the proposed approach.

Keywords: *Facilities planning and design; Manufacturing systems; Aisle structure; Mixed-integer linear programming*.

1. Introduction

Facility layout problems (FLPs) are concerned with determining the position of a set of facilities within a designated section of a building while satisfying placement restrictions and optimizing desired objective functions (Tompkins, White, Bozer, & Tanchoco, 2010). FLPs play an important role in enhancing all major aspects of manufacturing systems (Sule, 2008). A good layout in manufacturing systems can lead to improved machine utilization and can contribute to reducing work-in-process inventory and material handling costs (MHC). In this regard, Tompkins et al. (2010) claimed that effective facility layout in manufacturing systems can reduce the total operating costs by approximately 20%. Thus, designing a proper facility layout is essential for manufacturing systems.

Most previous studies of FLP in manufacturing systems have focused on finding the best position of facilities such as machines and workstations on the shop floor (Friedrich, Klausnitzer, & Lasch, 2018). However, the facilities require paths to connect with each other. These paths, called aisles, are used for the transportation of materials and humans between the facilities. The aisles occupy part of the floor and can require a lot of plant space (Stephens & Meyers, 2013). Furthermore, since transporting materials

between facilities is performed through the aisle network, designing a good aisle structure can contribute to reducing the transportation distance between facilities and to cutting MHC as well as transportation times. Therefore, designing a good aisle structure is very important for the efficiency of manufacturing systems, and can be included in the FLP (Friedrich et al., 2018). Despite the importance of this matter, only a few articles in the literature have addressed how to deal with aisles in the layout of manufacturing systems. Thus, this article analyses aisle requirements in the layout of manufacturing systems and proposes an approach that allows the position of facilities and the structure of aisles to be determined. The proposed approach satisfies the principles of aisles in manufacturing systems, such as the accessibility of all facilities via aisles, avoiding redundant design and irregularly-shaped aisles, and having no overlap between aisles and other facilities.

Furthermore, the traffic of transportation devices in the aisles is considered in the proposed approach. In this regard, the width of each aisle is set according to aisle traffic. The position of entrance/exit doors and how to connect them to the aisles is another issue that is studied in the proposed approach. Considering the aforementioned issues, the problem is formulated as a mixed-integer linear programming (MILP) model. The model can simultaneously find the optimum structure of the aisles, the position of the facilities, the width of the aisles, and the position of the entrance/exit doors. To solve the model, an improved branch-and-cut (B&C) algorithm is applied, in which the quality of branching is enhanced by adding optimality cuts and selecting an efficient branching strategy. The rest of this article is organized as follows:

Section 2 provides the literature review; in Section 3 the general principles, the problem description and the mathematical model of the problem are explained; the B&C algorithm is described in Section 4; and Section 5 discusses the experimental results. Finally, a summary and conclusions of the study are presented in Section 6.

2. Literature review

2. 1. Literature addressing FLP with aisles

Many articles have been published in the area of facility layout design (Drira, Pierreval, & Hajri-Gabouj, 2007; Hosseini-Nasab, Fereidouni, Ghomi, & Fakhrzad, 2018). There are few articles that have included aisles in the final layout. Among these articles, many have used a predefined aisle structure. In the present study, the emphasis is put on those articles which deal with optimizing aisle structure. We have classified these articles into three groups, based on whether the space required for the aisles is taken into account or not, and whether the problem is solved in an integrated way or not.

2.2. Non-integrated approaches without aisle space requirements

The articles in this group consider that the borders of facilities are used as aisles, and so assume that aisles occupy no space or a negligible space on the shop floor. In these articles, the position of the facilities and aisle structure were determined in two separate steps and were not optimized simultaneously. In all of these articles, except Tretheway and Foote (1994), the position of facilities was first optimized and then some parts of the borders of the facilities were selected as aisles. In Tretheway and Foote (1994), the aisle structure was determined first and then the departments were located around the aisle structure. The aisle structure consisted of the main aisles in conjunction with sub-aisles. The main aisles were single-direction parallel aisles running the full length of the plant. Sub-aisles were perpendicular to the main aisles and did not cross any main aisles. Norman, Arapoglu, and Smith (2001) presented a method for concurrently optimizing the shapes and locations of the departments and the number and the position of input/output (I/O) points for each department. In their model, they identified an aisle structure that contained un-capacitated, bidirectional aisles. A genetic algorithm (GA) was applied to select the best departmental boundaries to be used as aisles. Wu and Appleton (2002) proposed a two-stage method for designing a facility layout and an aisle structure. In the first stage, the location of the facilities was determined so that MHC were minimized, and all the borders around the facilities were considered as the aisle structure. In the second stage, the aisle structure was modified to minimize the number of aisles. In this way, by taking into account the pick-up/drop-off (P/D) points of facilities, the redundant aisles were removed. Minimizing the MHC was considered as an objective function (OF) in which the shortest path between the P/D points of two departments via the aisle structure was used to calculate the distance between facilities. GA and random search methods were applied and compared to optimize both stages. Alagoz, Norman, and Smith (2008) presented a hierarchy of algorithms to find the aisle structure for a given flexible bay block layout. The borders around the departments were considered as candidate aisles. In their approach, a layout based on a flexible bay structure was first obtained, then a heuristic procedure was applied to construct candidate aisles without significantly changing the areas of the departments. Lastly, the final aisle structure was obtained through an enumeration algorithm. Xiao, Xie, Kulturel-Konak, and Konak (2017) presented a MILP model for the dynamic facility layout problem. The objective of the model was to minimize the total MHC and re-layout costs over multiple planning periods. The authors considered a rectangular area for each facility and mentioned that the zone boundaries of the facilities were possible locations to place aisles and material-handling equipment. Friedrich et al. (2018) presented an approach for the efficient arrangement of facilities and their I/O locations within a floor area

to minimize the total transportation distance. In their research, the boundaries of the departments were considered as the aisles and each facility had one I/O point next to an aisle.

The approaches reviewed in this subsection do not take into account the spaces occupied by the aisles. As Friedrich et al. (2018) mentioned, the space used by the aisles is an important consideration in designing the aisle structure. Aisles spaces are passages between workstations or machines to allow the free movement of workers and transportation devices on the floor. In a real layout, aisles require significant floor space, and ignoring this issue may lead to a layout with low efficiency in terms of worker and transportation device movements.

2.3. Non-integrated approaches with aisle space requirements

The second group's articles take into account the space required for aisles. Benson and Foote (1997) considered locating the I/O points for the departments and selected the best aisle structure from a set of possible aisle structures, with both vertical and horizontal aisles. A genetic algorithm was applied to find the best location of I/O points on several pre-determined aisle structures. Zhou, Ye, Cao, and Ye (2006) discussed a particular multi-objective facility layout problem with aisles. The first objective was to minimize the total material handling costs. The second objective was to maximize the adjacency requirements between the facilities, which means, for example, placing some facilities as far apart as possible (due to factors such as noise, dust, or safety reasons). The problem was formulated as a bi-criteria nonlinear mixed-integer programming model. The authors fixed some spaces and considered them as aisles for the transportation devices. A multi-objective genetic algorithm with a local search method was developed to obtain Pareto solutions. Li (2011) studied the problem of the optimization of aisle structure and department allocation for the layout of a retail area. First, a set of potential aisle structures were defined. These structures were evaluated by exposure metric. The exposure metric is the probability that a point in the retail area is noticed by customers. Then the department allocation problem was solved. They considered central aisles and branch aisles, which divided the entire retail area into a number of sub-areas. Wang and Chang (2015) developed a two-stage model called a spine bay layout to allocate the workstations for several inter-bay systems to minimize the MHC. In their approach, a central vertical aisle was used for inter-bay material handling. This vertical aisle was connected to a set of horizontal aisles that were used for intra-bay material handling. To optimize the spine bay layout, a two-stage mixed-integer programming (MIP) model was developed. In the first stage, each workstation was allocated to a bay, and in the second stage, the exact position of the workstations inside their relevant bay was determined. Allahyari and Azab (2018) proposed a mathematical model for FLP which considered operations sequence, parts demand, and aisle structure. They used one central horizontal aisle or one

central vertical aisle in the middle of the shop floor. The position and the width of the aisles were predefined. By considering a horizontal aisle in the center of the floor, the floor was divided into two horizontal levels. Minimizing the total distance traveled by materials between facilities was considered as the objective function. A multi-start search simulated annealing algorithm was developed to solve large instances of the problem, in which a unique heuristic algorithm was used for initialization. Even if the second group's articles were concerned with the aisle structure, where aisles occupied spaces on the floor, they did not explicitly address how to define the problem of designing the aisle structure and the position of facilities simultaneously. Solving these problems sequentially can result in solutions that are far from the global optimum solution (Hu, Chen, Zhou, & Fang, 2007).

2.4. Integrated approaches with aisle space requirements

The previously-discussed articles tried to find an aisle structure before or after determining the position of facilities; the aisle structure and the facility layout were determined in two separate steps. The articles reviewed in this subsection study the problem of optimizing the aisle structure and facility position in an integrated manner. Peters and Yang (1997) investigated the position of facilities and aisles for a material handling system design integration problem in the semiconductor manufacturing industry. They proposed a methodology to solve this integrated design problem. In their approach, the aisle structure was limited to having aisles around the floor or one central aisle. Gomez, Fernandez, De la Fuente García, and Garcıa (2003) designed a GA which incorporates vertical aisles in the plant layout problem. Their aisle structure was limited to a set of vertical aisles. Lee, Roh, and Jeong (2005) proposed an improved GA for multi-floor FLP considering vertical and horizontal aisles. The best position of facilities and aisles on each floor was determined to satisfy the two objectives of minimizing total transportation costs and maximizing adjacency requirements between facilities. They assumed that the number and width of each vertical and horizontal aisle on each floor were given by the designer. Chang, Lin, and Lin (2006) addressed an optimal multi-floor layout with aisles. In their study, the aisle structure was determined simultaneously during the layout construction stage. A K-means clustering algorithm was applied in order to group the departments which were allocated to the same floor. Then a hybrid genetic algorithm was used to improve the layout on each floor. Klausnitzer and Lasch (2016) proposed an MIP approach to optimize simultaneously the aisle structure and the position of facilities. They assumed that the aisles have a predefined width and can be placed around the facilities and next to the floor space borders. By this assumption, they used the rectilinear distance metric to calculate the distances between facilities.

2.5. Summary

We have seen in this section that only a few articles deal with aisles in FLP. Most of them do not optimize the aisle structure and often use very simple structures, such as one central aisle or aisles located only on the border of the shop floor. As shown in Table 1, only a very few of these articles have addressed more complex aisle structures that combine vertical and horizontal aisles. Unfortunately, these articles either use a predefined number of aisles, do not provide an exact approach, or do not optimize the position of facilities and the design of the aisle structure simultaneously. In the reviewed research works, the studied aisle structures do not consider certain important aspects, such as aisle width and the positions of the entrance/exit doors. As a consequence, because of these limitations, in the next section we analyze the requirements related to aisle structure design and we derive a new integrated mathematical formulation to simultaneously determine the position and orientation of facilities as well as the aisle structure.

Table 1

Summary of the related work.

Reference	**Integrated facility layout and aisle structure [A]**	**Aisle considerations**				**Global optimization [F]**
		Type of aisle [B]	**Aisle area [C]**	**Aisle width [D]**	**Entrance/ exit doors [E]**	
Tretheway and Foote (1994)	No	Horizontal or vertical	No	No	No	No
Benson and Foote (1997)	No	Horizontal or vertical	Yes	No	No	No
Peters and Yang (1997)	Yes	One central aisle	Yes	No	No	No
Wu and Appleton (2002)	No	Vertical	No	No	No	No
Norman et al (2001)	No	Horizontal	No	No	No	No
Gomez et al (2003)	Yes	Vertical	Yes	No	No	No
Lee et al (2005)	Yes	Horizontal and vertical	Yes	No	No	No
Chang et al (2006)	Yes	Horizontal and vertical	Yes	No	No	No
Zhou et al (2006)	No	Horizontal and vertical	Yes	No	No	No
Alagoz et al (2008)	No	Horizontal and vertical	No	No	No	No
Li (2011)	No	Horizontal and vertical	Yes	No	No	Yes
Wang and Chang (2015)	No	Horizontal	Yes	No	No	No
Klausnitzer and Lasch (2016)	Yes	Aisles around floor border	Yes	No	No	Yes
Xiao et al (2017)	No	Horizontal and vertical	No	No	No	No
Friedrich et al (2018)	No	Horizontal and vertical	No	No	No	No
Allahyari and Azab (2018)	No	One central aisle	Yes	No	No	No

Our work	Yes	Horizontal and vertical	Yes	Yes	Yes	Yes

[A] Whether the problem was solved in an integrated manner or in two steps; [B] The type of aisle that was considered in the problem; [C] Whether the area occupied by the aisle was considered or not; [D] Whether the aisle width was optimized or not; [E] Whether the issue of entrance/exit doors was studied or not; [F] Whether the global optimum solution was obtained or not.

3. Problem description and mathematical model

In this section, several principles concerning the construction of aisles in manufacturing systems are first presented. These principles are based on the essential requirements of the aisle structure in designing the layout of manufacturing systems. Then they are taken into consideration in the problem formulation that is proposed in the next subsections.

3.1. Principles of aisles

Aisles are the areas of the shop floor that are used by workers and transportation devices. The space of the shop floor that is dedicated to aisles cannot be occupied by a facility. An appropriate aisle structure can contribute to reducing the MHC (by reducing the distances between facilities) and the mean flow time (by reducing the transportation distances and the waiting times by providing smooth material transportation). Several considerations should be taken into account when configuring the aisles. The main one is to configure the aisles in such a way that the P/D points of all the facilities have access to the aisles and there is a path through the aisles between each pair of facilities. The aisles should also be connected to the entrance/exit doors.

Entrance and exit doors are locations where the parts, the raw materials, and the transportation devices enter/exit the manufacturing system. It is not usually possible to build entrance and exit doors everywhere around the building because of technical or environmental considerations (e.g., if there is a street next to one side of the building or some parts of the building are in the vicinity of another building so that there is not enough space for loading/unloading operations). In this situation, within the allowable area for constructing entrance/exit doors, the points that are closest to the aisles are considered as the potential points for constructing entrance/exit doors to reduce transportation costs. Thus determining the location of entrance/exit doors within the allowed area and connecting them to the aisle structure should be addressed when designing the layout.

Aisle width is another important consideration in designing the aisle structure. The width of an aisle refers to the lateral distance between two opposing sides of an aisle. The width of the aisles should be large enough so that when a transportation device stops to complete a pick-up/drop-off process, the path is not blocked. Furthermore, in case of a machine breakdown and the need to move a machine out of the manufacturing system, the aisle width should be wide enough for the largest disassembled part of the

machine to be transported. However, allowing too much space for aisles would be detrimental to the system. Determining the suitable width for each aisle is therefore an important issue. The aisles in which more materials transit should be wider. Hence, the quantity of materials that pass through an aisle should be taken into account when determining the width of the aisle. It should be noted that the space needed for walkways should be added in the calculations related to aisle width.

3.2. Problem analysis

Let us consider that each machine, along with its required surrounding space used for operations such as machine maintenance and tool changing, is represented as a rectangular-shaped facility. There is a fixed point at each facility that is used to pick-up/drop-off the products from/to that facility, named P/D point. Facilities can be rotated clockwise with one of four angles: 0°, 90°, 180°, and 270°. Hence, they can be placed either in a horizontal orientation, where the longer side of the facility is parallel to the *x*-axis, or in a vertical orientation, where the longer side of the facility is parallel to the *y*-axis (Fig. 1).

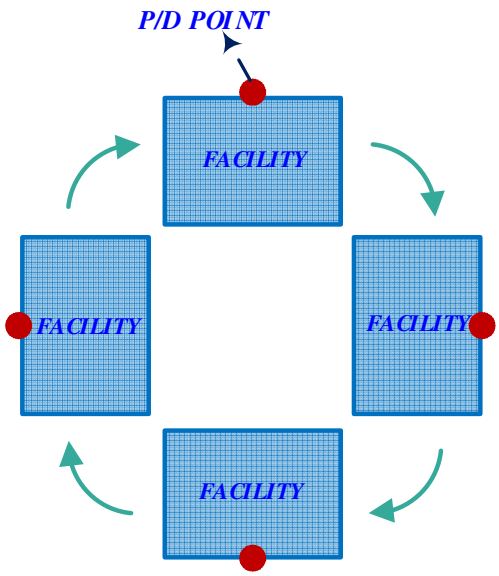

Fig. 1. Location of P/D point according to facility rotation

The products move from the P/D point of one facility to the next, based on their operational sequence, until their process is completed. Transportation of products is performed by industrial transportation devices that can only move through the aisles.

The goal is to find an optimal position for the facilities and aisle structure to minimize total transportation distances. Finding the best structure for the aisles consists of determining the optimum number, position and width of the aisles so that the following conditions are satisfied:

- There should be no overlap between facilities and aisles.
- All the facilities should have access to the aisle network.
- Each pair of facilities should be connected to each other through the aisle network.
- Aisles should be connected to the entrance and exit doors.
- Entrance and exit doors can be located only in certain limited areas around the shop floor.

Moreover, the location of the entrance and exit doors, through which the materials enter/exit the system, must be determined. Thus, there are three groups of decision variables: 1) the position and orientation of the facilities, 2) the number, position, and width of the aisles and 3) the position of the entrance/exit doors.

The aisle structure consists of vertical and horizontal aisles; the horizontal aisles are extended to the boundaries of the shop floor and a single vertical aisle connects two consecutive horizontal aisles. Extending horizontal aisles to the boundaries of the shop floor creates long and straight aisles that are desirable in manufacturing systems (Kim, 1992; Stephens & Meyers, 2013). Furthermore, this structure helps to reduce the number of turns and provides smoother and faster transportation (Friedrich et al., 2018; Leno, Saravanasankar, & Ponnambalam, 2012). The horizontal aisles divide the floor into several levels. The area between the upper side of the shop floor and the first horizontal aisle is considered as level one; the area between the first horizontal aisle and the second horizontal aisle is considered as level two, and so on. Fig. 2 illustrates a typical layout according to the problem constraints, assumptions and decision variables. In this figure, the green lines on the edges of the shop floor are possible areas to construct the doors. In this layout, for example, facilities 7, 10, 12, 19 and 20 are supplied by a horizontal aisle and facilities 9, 5 and 25 are supplied by a vertical aisle. Some facilities such as 2, 4 and 9 are placed in their horizontal orientation and others, such as 1, 3 and 20, are rotated and placed in their vertical orientation. Since facilities can be located in any position in the continuous space of the shop floor, placing the facilities with a suitable orientation to minimize transportation distances is merely a complex problem. Integrating the aisles and entrance/exit doors into the problem increases this complexity. In the next section, a mathematical model is proposed to address this problem.

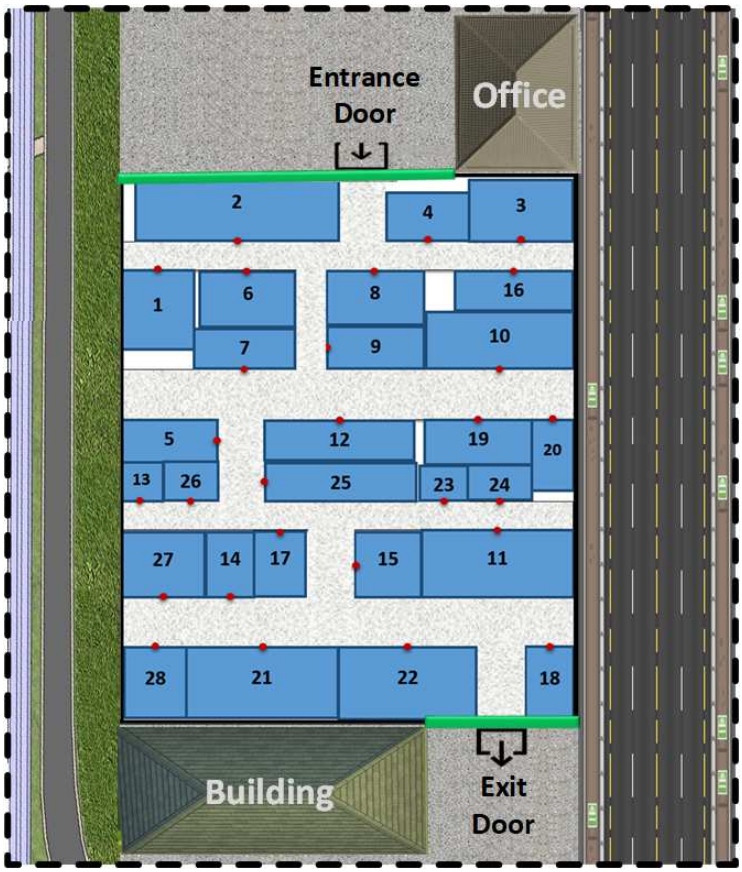

Fig. 2. An example of a manufacturing layout design.

3.3. Mathematical model

In this subsection, a mathematical model that copes with the considerations discussed in subsection 3.2 is proposed. It is shown that the problem can be formulated using a mixed-integer linear mathematical model.

3.3.1. Notations

The notations used in the model are presented as follows. Superscripts provide information about the variables/parameters and subscripts are indexes.

Indexes	
i, j	Index set of facilities $i, j = 1, 2, \dots, N$
k, l	Index set of horizontal aisles $k, l = 1, 2, \dots, K$
m, n	Index set of vertical aisles $m, n = 1, 2, \dots, K - 1$

p	Index set of levels $p = 1, 2, \ldots, K + 1$

Parameters

$length$:	Length of the plant floor along the *x*-axis
$width$:	Width of the plant floor along the *y*-axis
$f_{i,j}$:	Total material flow between facilities *i* and *j*
sh_i^M:	Length of the shorter side of facility *i*
lng_i^M:	Length of the longer side of facility *i*
(l^{ENT}, u^{ENT}):	Start and ending point of allowable area for constructing entrance door
(l^{EXT}, u^{EXT}):	Start and ending point of allowable area for constructing exit door
m_w^H:	Minimum width of horizontal aisles
m_w^V:	Minimum width of vertical aisles
α:	Coefficient of traffic impact for determining the width of horizontal aisles
β:	Coefficient of traffic impact for determining the width of vertical aisles
$PD_i =$	$\begin{cases} 1 & \text{If the P/D point of facility } i \text{ is on its longer side} \\ 0 & \text{Otherwise} \end{cases}$
M:	A large positive number

Decision variables

(x_i^C, y_i^C):	The centroid (or position) of facility *i*
x_m^V:	The centroid of vertical aisle *m* in the *x*-axis
y_k^H:	The centroid of horizontal aisle *k* in the *y*-axis
$d_{i,j}^{MM}$:	The distance between the P/D point of facility *i* and *j* through aisles
$(d_x_{i,j}^{MM}, d_y_{i,j}^{MM})$:	The distances between the P/D point of facilities *i* and *j* in the *x* and *y*-axis directions
(x_i^{PD}, y_i^{PD}):	The *x* and *y*-axis coordinate of the P/D point of facility *i*
$d_x_{k,l}^{HH}$:	The horizontal distance required to travel from horizontal aisle *k* to horizontal aisle *l*
$d_x_{m,n}^{VV}$:	The horizontal distance required to travel from vertical aisle *m* to vertical aisle *n*
$d_x_{i,m}^{MV}$:	The horizontal distance between the P/D point of facility *i* and the centroid of vertical aisle *m*
$(l_x_i^M, l_y_i^M)$:	The length of facility *i* in the *x* and *y*-axis directions
w_k^H:	The width of horizontal aisle *k*
w_m^V:	The width of vertical aisle *m*
$traffic_k^H$:	The amount of traffic in the horizontal aisle *k*
$traffic_m^V$:	The amount of traffic in the vertical aisle *m*

$$h_i = \begin{cases} 1 & \text{If facility } i \text{ has a horizontal orientation} \\ 0 & \text{Otherwise} \end{cases}$$

$$left_{i,j}^{MM} = \begin{cases} 1 & \text{if facility } i \text{ is to the left of facility } j \\ 0 & \text{Otherwise} \end{cases}$$

$$below_{i,j}^{MM} = \begin{cases} 1 & \text{if facility } i \text{ is below facility } j \\ 0 & \text{Otherwise} \end{cases}$$

$$above_{i,k}^{MH} = \begin{cases} 1 & \text{if facility } i \text{ is above the horizontal aisle } k \\ 0 & \text{Otherwise} \end{cases}$$

$$below_{i,k}^{MH} = \begin{cases} 1 & \text{if facility } i \text{ is below the horizontal aisle } k \\ 0 & \text{Otherwise} \end{cases}$$

$$left_{i,m}^{MV} = \begin{cases} 1 & \text{if facility } i \text{ is to the left of vertical aisle } m \\ 0 & \text{Otherwise} \end{cases}$$

$$right_{i,m}^{MV} = \begin{cases} 1 & \text{if facility } i \text{ is to the right of vertical aisle } m \\ 0 & \text{Otherwise} \end{cases}$$

$$u_{i,p}^{MP} = \begin{cases} 1 & \text{if facility } i \text{ is placed in level } p \\ 0 & \text{Otherwise} \end{cases}$$

$$z_{i,k}^{MH} = \begin{cases} 1 & \text{if facility } i \text{ is supplied via horizontal aisle } k \\ 0 & \text{Otherwise} \end{cases}$$

$$z_a_{i,k}^{MH} = \begin{cases} 1 & \text{if facility } i \text{ is supplied via horizontal aisle } k \text{ which is located above it} \\ 0 & \text{Otherwise} \end{cases}$$

$$z_b_{i,k}^{MH} = \begin{cases} 1 & \text{if facility } i \text{ is supplied via horizontal aisle } k \text{ which is located below it} \\ 0 & \text{Otherwise} \end{cases}$$

$$z_{i,m}^{MV} = \begin{cases} 1 & \text{if facility } i \text{ is supplied via vertical aisle } m \\ 0 & \text{Otherwise} \end{cases}$$

$$z_l_{i,m}^{MV} = \begin{cases} 1 & \text{if facility } i \text{ is supplied via vertical aisle } m \text{ which is located left of it} \\ 0 & \text{Otherwise} \end{cases}$$

$$z_r_{i,m}^{MV} = \begin{cases} 1 & \text{if facility } i \text{ is supplied via vertical aisle } m \text{ which is located right of it} \\ 0 & \text{Otherwise} \end{cases}$$

$$c_{k,i,j}^{H} = \begin{cases} 1 & \text{if horizontal aisle } k \text{ is in the path of materials transported between facilities } i \text{ and } j \\ 0 & \text{Otherwise} \end{cases}$$

$$c_{m,i,j}^{V} = \begin{cases} 1 & \text{if vertical aisle } m \text{ is in the path of materials transported between facilities } i \text{ and } j \\ 0 & \text{Otherwise} \end{cases}$$

3.3.2. Objective function

Total Transportation Distance (TTD), defined as the sum of the material flow between each pair of facilities, f_{ij}, multiplied by the corresponding transportation distances, $d_{i,j}^{MM}$, is considered as the objective function.

$$Min\ TTD = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} f_{i,j} d_{i,j}^{MM} \tag{1}$$

Since the product route and demand is known, f_{ij} can be calculated easily. To calculate $d_{i,j}^{MM}$, the total vertical path plus the total horizontal path that should be traveled through the aisles to connect the P/D points of facilities i and j must be calculated. Fig. 3 illustrates the aisle-based distances for facility pairs (3,6), (10,5) and (16,14). As shown in Fig. 3, the total vertical distances between P/D points of a facility pair is not dependent on the aisle structure and can be obtained by calculating the difference between the y-coordinates of their P/D points.

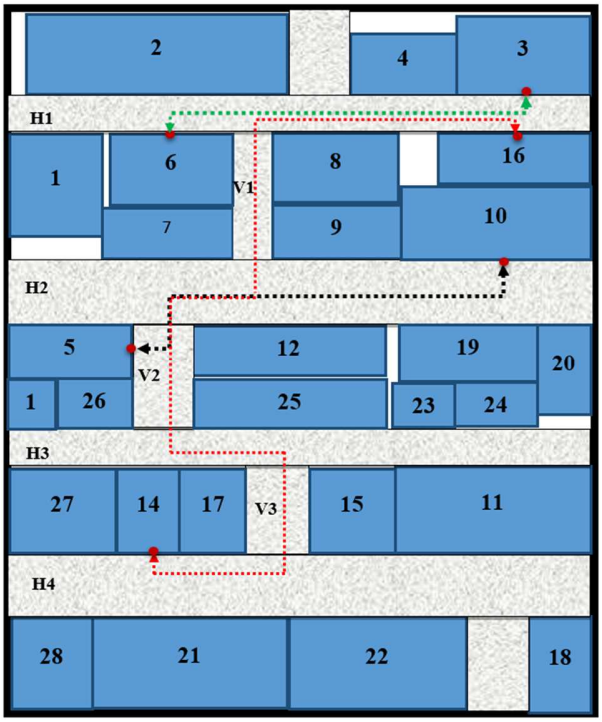

Fig. 3. An example of the aisle-based distances between the P/D points of three pairs of facilities.

To calculate the horizontal distances, each horizontal aisle along with its connected vertical aisle, which is below it, is considered as a class of aisle. For example, in Fig. 3, horizontal aisle 1 (H1) and vertical aisle 1 (V1) are in one class, horizontal aisle 2 (H2) and vertical aisle 2 (V2) are in one class and so on.

If two facilities are supplied by an identical class of aisle (e.g., facility pairs (3,6) and (10,5) in Fig. 3), their horizontal distance is equal to the difference between the x-coordinates of their P/D points. It can be stated that in this situation, the aisle-based distance between each pair of facilities is equal to the rectilinear distance between their P/D points. This distance is calculated using constraints (2)-(6). The first two parts of the right-hand side of constraint (2) calculate the rectilinear distance between the P/D

points of facilities i and j and the last part is to control if both facilities i and j are supplied through an identical class of aisle.

$$d_{i,j}^{MM} \geq d_x_{i,j}^{MM} + d_y_{i,j}^{MM} - M(2 - z_{i,k}^{MH} - z_{j,l}^{MH} - z_{i,m}^{MV} - z_{j,n}^{MV}) \qquad \forall i < j, \forall\, k = l = m = n \qquad (2)$$

$$d_x_{i,j}^{MM} \geq x_i^{PD} - x_j^{PD} \qquad \forall i, j \qquad (3)$$

$$d_x_{i,j}^{MM} \geq x_j^{PD} - x_i^{PD} \qquad \forall i, j \qquad (4)$$

$$d_y_{i,j}^{MM} \geq y_i^{PD} - y_j^{PD} \qquad \forall i, j \qquad (5)$$

$$d_y_{i,j}^{MM} \geq y_j^{PD} - y_i^{PD} \qquad \forall i, j \qquad (6)$$

Calculating the horizontal distance between facilities i and j that are not supplied by the same class of aisle is more complicated. Let us consider the path that should be traveled between two facilities 16 and 14 in Fig. 4. The transportation device has to move through arcs a-b, c-d, e-f and g-h in the horizontal direction. The sum of these arcs determines the horizontal traveling distance between facilities 16 and 14. The lengths of these arcs are different and depend on the position of the aisles and facilities. To deal with this difficulty, the distance between facilities i and j, which are not supplied through an identical class of aisles, is split into two distance types.

1. The first type comprises the horizontal distances that should be traveled inside the aisles that supply facilities i and j. Arcs a-b and g-h in Fig. 4 are examples of this type of distance and are calculated using Eqs. (11) and (12).
2. The second type comprises the horizontal distances that should be traveled through the aisles other than the ones that supply facilities i and j. These distances, shown by arcs c-d and e-f in Fig. 4, are calculated by Eq. 7. In fact, these distances are equal to the horizontal distances required to travel from horizontal aisle k to horizontal aisle l.

$$d_x_{k,l}^{HH} = \sum_{m=k}^{l-2} d_x_{m,m+1}^{VV} \qquad \forall k < l - 1 \qquad (7)$$

$$d_x_{m,n}^{VV} \geq x_m^V - x_n^V \qquad \forall m, n \qquad (8)$$

$$d_x_{m,n}^{VV} \geq x_n^V - x_m^V \qquad \forall m, n \qquad (9)$$

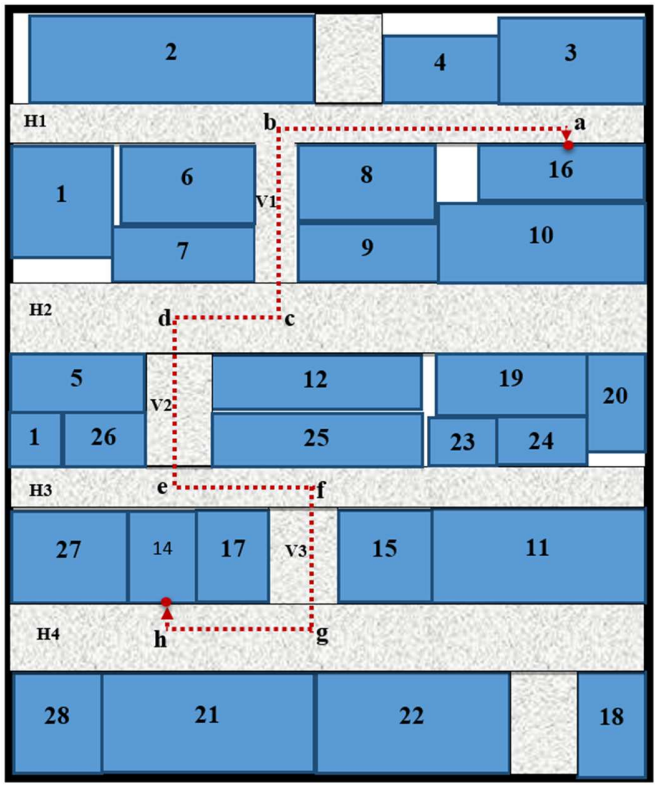

Fig. 4. Aisle-based distance between facilities 16 and 14.

The total distances between facilities through aisles are calculated using constraints (10). In constraints (10) $d_x_{i,m}^{MV}$ and $d_x_{j,n}^{MV}$ are type 1 horizontal distances, $d_x_{k,l}^{HH}$ are type 2 horizontal distances, and $d_y_{i,j}^{MM}$ is a vertical distance.

$$d_{i,j}^{MM} \geq d_x_{i,m}^{MV} + d_x_{j,n}^{MV} + d_x_{k,l}^{HH} + d_y_{i,j}^{MM} - M\left(2 - z_{i,k}^{MH} - z_{j,l}^{MH} - z_{i,m}^{MV} - z_{j,n}^{MV}\right) \quad \forall i \neq j, k < l, \; m = k, n = l - 1 \tag{10}$$

$$d_x_{i,m}^{MV} \geq x_i^{PD} - x_m^{V} \quad \forall i, m \tag{11}$$

$$d_x_{i,m}^{MV} \geq x_m^{V} - x_i^{PD} \quad \forall i, m \tag{12}$$

It should be noticed that in constraints (10) k is less than l. This means that it calculates the distance between a facility and its downstream facilities. In this way, only half of the distance matrix is calculated, which is not necessarily a lower/upper triangular matrix. Since the distance between facilities i and j is exactly equal to the distance between facilities j and i, the distance matrix is symmetrical and the unfilled parts can easily be obtained using constraints (13). In this way, repetitive computations are avoided and computational complexity is reduced.

$$d_{i,j}^{MM} \geq d_{j,i}^{MM} \quad \forall i, j \tag{13}$$

3.3.3. Within-site boundary constraints

These groups of constraints, similar to those used in Dunker, Radons, and Westkämper (2005), guarantee that all the facilities and aisles are placed inside the boundaries of the shop floor. First, it is necessary to determine the length of each facility in the *x* and *y*-axis directions. Facilities are allowed to take either a horizontal or vertical orientation. If a facility is placed in the horizontal direction, its length along the *x*-axis is equal to its longer side and its length along the *y*-axis is equal to its shorter side (Fig. 5). Eqs. (14) and (15) are used to control the length of the facilities considering their orientations.

$$l_x_i^M = lng_i^M h_i + sh_i^M (1 - h_i) \qquad \forall i \qquad (14)$$

$$l_y_i^M = lng_i^M (1 - h_i) + sh_i^M h_i \qquad \forall i \qquad (15)$$

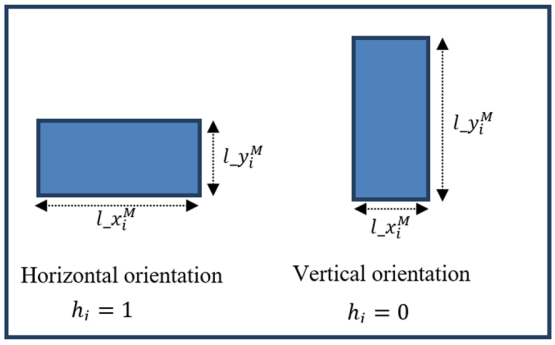

Fig. 5. The orientation of facilities.

Constraints (16)-(19) guarantee that facilities are located inside the boundaries of the shop floor.

$$x_i^C + 0.5 l_x_i^M \leq length \qquad \forall i \qquad (16)$$

$$x_i^C - 0.5 l_x_i^M \geq 0 \qquad \forall i \qquad (17)$$

$$y_i^C + 0.5 l_y_i^M \leq width \qquad \forall i \qquad (18)$$

$$y_i^C - 0.5 l_y_i^M \geq 0 \qquad \forall i \qquad (19)$$

Constraints (20)-(23) guarantee that aisles are placed within the boundaries of the shop floor. This group of constraints is the same as the ones used for the facilities. In line with our assumptions, the horizontal aisles are extended to the *x*-axis boundaries of the floor and thus they should only be verified as being located inside the *y*-axis boundaries of the floor. In the same way, the vertical aisles are verified as being placed inside the *x*-axis boundaries of the floor.

$$y_k^H + 0.5 w_k^H \leq width \qquad \forall k \qquad (20)$$

$$y_k^H - 0.5 w_k^H \geq 0 \qquad \forall k \qquad (21)$$

$$x_m^V + 0.5w_m^V \leq length \qquad \forall m \qquad (22)$$

$$x_m^V - 0.5w_m^V \geq 0 \qquad \forall m \qquad (23)$$

3.3.4. Non-overlapping constraints

Constraints (24)-(26), similar to those presented in Meller, Narayanan, and Vance, (1998), are used to guarantee non-overlapping between facilities. If two facilities are located at the same level on the shop floor, the right-hand side of constraints (24) is equal to 1, and consequently one of the left-side variables has to take the value 1. As an example, if facility *i* is to the left of facility *j*, $left_{i,j}^{MM} = 1$, constraint (25) is imposed and constraint (26) is relaxed. Therefore, the *x*-coordinate of the centroid of facility *i* plus half of its length in the *x*-axis direction ($x_i^C + 0.5l_x_i^M$) should be less than the *x*-coordinate of the centroid of facility *j* minus half of its length in the *x*-axis direction ($x_j^C - 0.5l_x_j^M$).

$$below_{i,j}^{MM} + below_{j,i}^{MM} + left_{i,j}^{MM} + left_{j,i}^{MM} \geq u_{i,p}^{MP} + u_{j,p}^{MP} - 1 \qquad \forall i \neq j, p \qquad (24)$$

$$x_i^C + 0.5l_x_i^M \leq x_j^C - 0.5l_x_j^M + M(1 - left_{i,j}^{MM}) \qquad \forall i \neq j \qquad (25)$$

$$y_i^C + 0.5l_y_i^M \leq y_j^C - 0.5l_y_j^M + M(1 - below_{i,j}^{MM}) \qquad \forall i \neq j \qquad (26)$$

The following constraints guarantee non-overlapping between facilities and horizontal aisles. Each facility has two states relative to the horizontal aisles, above or below. If facility *i* is above horizontal aisle *k*, $above_{i,k}^{MH} = 1$, constraints (28) are relaxed and constraints (29) are imposed (Fig. 6).

$$below_{i,k}^{MH} + above_{i,k}^{MH} = 1 \qquad \forall i, k \qquad (27)$$

$$y_k^H - 0.5w_k^H \geq y_i^C + 0.5l_y_i^M - M(1 - below_{i,k}^{MH}) \qquad \forall i, k \qquad (28)$$

$$y_k^H + 0.5w_k^H \leq y_i^C - 0.5l_y_i^M + M(1 - above_{i,k}^{MH}) \qquad \forall i, k \qquad (29)$$

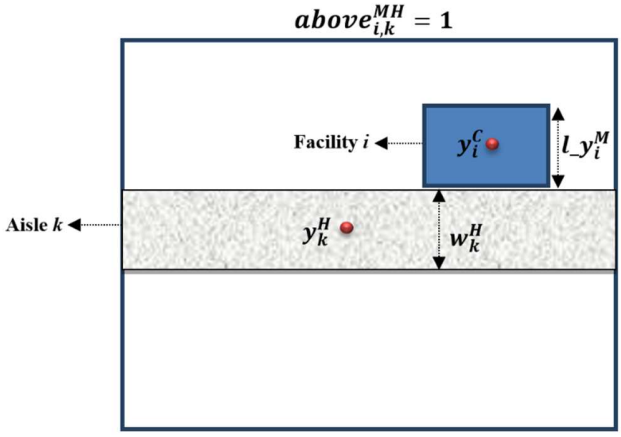

Fig. 6. Non-overlapping between facilities and horizontal aisles.

The following constraints guarantee that at the same level the facilities and the vertical aisles do not overlap. Each facility has two states relative to vertical aisles, left or right. If facility i is located on the right side of vertical aisle n, $right_{i,m}^{MV} = 1$, then constraints (31) are relaxed and constraints (32) are imposed (Fig. 7).

$$left_{i,m}^{MV} + right_{i,m}^{MV} \geq u_{i,p}^{MP} \qquad \forall i, m = p - 1, \; 1 < p < K + 1 \qquad (30)$$

$$x_m^V - 0.5w_m^V \geq x_i^C + 0.5l_x_i^M - M(1 - left_{i,m}^{MV}) \qquad \forall i, m \qquad (31)$$

$$x_m^V + 0.5w_m^V \leq x_i^C - 0.5l_x_i^M + M(1 - right_{i,m}^{MV}) \qquad \forall i, m \qquad (32)$$

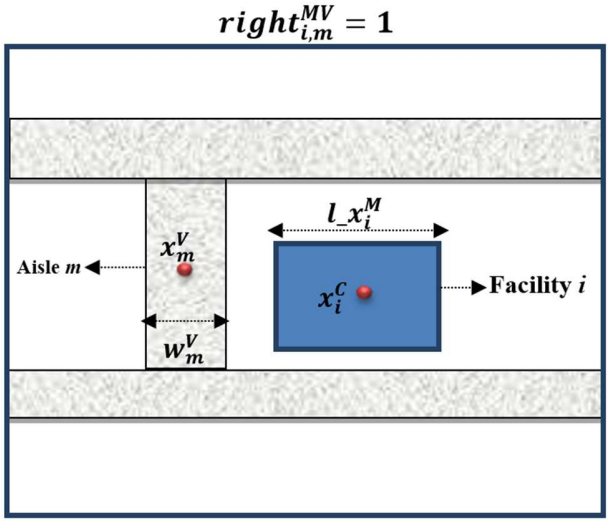

Fig. 7. Non-overlapping between facilities and vertical aisles.

Constraints (33) guarantee non-overlapping between horizontal aisles.

$$y_k^H - 0.5w_k^H \geq y_l^H + 0.5w_l^H \qquad \forall k > l \qquad (33)$$

3.3.5. P/D points and aisle accessibility constraints

Each facility is assumed to have one fixed P/D point in the center of one of its sides. To connect a facility to the aisle structure, the facility's P/D point should be at the edge of a horizontal or vertical aisle. Depending on a facility's orientation and the location of its P/D point, one of the following situations can occur, as illustrated in Fig. 8:

1. The facility is placed horizontally and its P/D point is on its longer side.
2. The facility is placed vertically and its P/D point is on its shorter side.
3. The facility is placed horizontally and its P/D point is on its shorter side.
4. The facility is placed vertically and its P/D point is on its longer side.

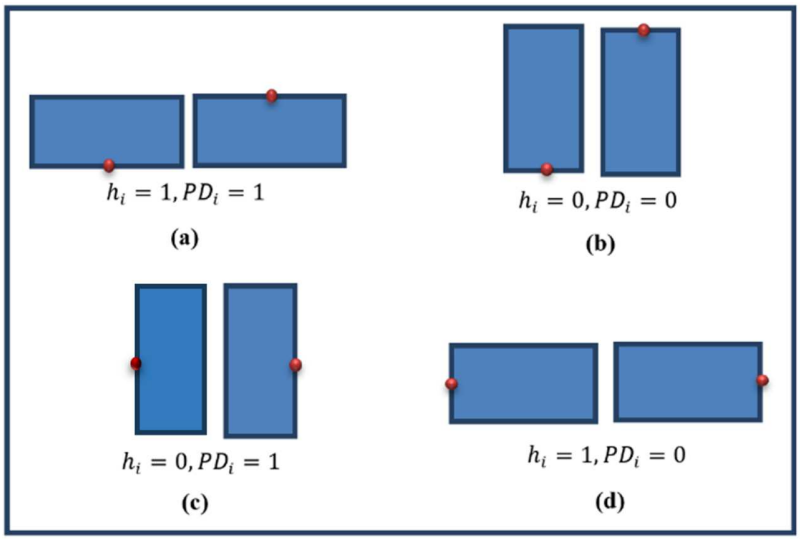

Fig. 8. Facility orientation and position of P/D point

A facility should be supplied by a horizontal aisle if its situation matches one of the conditions shown in Fig. 8 (a) and (b), and it should be supplied by a vertical aisle if its situation matches one of the conditions shown in Fig. 8 (c) and (d). Taking into account facility orientations and the positions of their P/D points, constraints (34)-(36) guarantee that each facility is supplied by only one aisle.

$$\sum_{k=1}^{K} z_{i,k}^{MH} + \sum_{m=1}^{K-1} z_{i,m}^{MV} = 1 \qquad \forall i \qquad (34)$$

$$z_{i,k}^{MH} \geq h_i PD_i + (1 - h_i)(1 - PD_i) \qquad \forall i, k \qquad (35)$$

$$z_{i,m}^{MV} \geq 1 - h_i PD_i - (1 - h_i)(1 - PD_i) \qquad \forall i, m \qquad (36)$$

As shown in Fig. 9, if a facility is supplied by a horizontal (vertical) aisle, it can be placed above/below (to the left/right of) the corresponding aisle. Eqs. (37) and (38) determine the position of a facility relative to the aisle that supplies it.

$$z_a_{i,k}^{MH} + z_b_{i,k}^{MH} = z_{i,k}^{MH} \qquad \forall i, k \qquad (37)$$

$$z_l_{i,m}^{MV} + z_r_{i,m}^{MV} = z_{i,m}^{MV} \qquad \forall i, m \qquad (38)$$

By knowing four variables $z_a_{i,k}^{MH}$, $z_b_{i,k}^{MH}$, $z_l_{i,m}^{MV}$, and $z_r_{i,m}^{MV}$, the coordinates of a facility's P/D point can be calculated. As shown in Fig. 9, if a facility is supplied by a horizontal aisle, the *x*-coordinate of its P/D point is equal to the *x*-coordinate of the facility's centroid. This is guaranteed by constraints (39) and (40).

$$x_i^{PD} \leq x_i^{C} + M(1 - z_{i,k}^{MH}) \qquad \forall i, k \qquad (39)$$

$$x_i^{PD} \geq x_i^C - M(1 - z_{i,k}^{MH}) \qquad \forall i,k \qquad (40)$$

Determining the *y*-coordinate of the P/D point of a facility that is supplied by a horizontal aisle depends on the position of the facility relative to that aisle. If a facility is supplied by a horizontal aisle that is above it, $z_a_{i,k}^{MH} = 1$, the *y*-coordinate of its P/D point should be equal to the *y*-coordinate of its centroid plus one half of its length in the *y*-axis direction (see Fig. 9). Constraints (41) and (42) are responsible for controlling this issue. In the same way, the *y*-coordinate of the P/D point of a facility that is supplied by a horizontal aisle situated below it is calculated through constraints (43) and (44).

$$y_i^{PD} \leq y_i^C + 0.5l_y_i^M + M(1 - z_a_{i,k}^{MH}) \qquad \forall i,k \qquad (41)$$

$$y_i^{PD} \geq y_i^C + 0.5l_y_i^M - M(1 - z_a_{i,k}^{MH}) \qquad \forall i,k \qquad (42)$$

$$y_i^{PD} \leq y_i^C - 0.5l_y_i^M + M(1 - z_b_{i,k}^{MH}) \qquad \forall i,k \qquad (43)$$

$$y_i^{PD} \geq y_i^C - 0.5l_y_i^M - M(1 - z_b_{i,k}^{MH}) \qquad \forall i,k \qquad (44)$$

Constraints (45)-(50) are similar to constraints (39)-(44), yet are used to determine the P/D point coordinates of the facilities that are supplied through vertical aisles. As shown in Fig. 9, the *y*-coordinate of the P/D points of these facilities is equal to the *y*-coordinate of their centroid. Determining the *x*-coordinate of the P/D points of these facilities depends on the position of the facilities relative to their supplying vertical aisles. According to constraints (47) and (48), if a facility is supplied by a vertical aisle that is located on its left side, $z_l_{i,m}^{MV} = 1$, the *x*-coordinate of its P/D point should be equal to the *x*-coordinate of the facility's centroid minus one half of its length in the *x*-axis direction. In the same way, the *x*-coordinate of the P/D point of a facility supplied by a vertical aisle on its right side is calculated through constraints (49) and (50).

$$y_i^{PD} \leq y_i^C + M(1 - z_{i,m}^{MV}) \qquad \forall i,m \qquad (45)$$

$$y_i^{PD} \geq y_i^C - M(1 - z_{i,m}^{MV}) \qquad \forall i,m \qquad (46)$$

$$x_i^{PD} \leq x_i^C + 0.5l_x_i^M + M(1 - z_l_{i,m}^{MV}) \qquad \forall i,m \qquad (47)$$

$$x_i^{PD} \geq x_i^C + 0.5l_x_i^M - M(1 - z_l_{i,m}^{MV}) \qquad \forall i,m \qquad (48)$$

$$x_i^{PD} \leq x_i^C - 0.5l_x_i^M + M(1 - z_r_{i,m}^{MV}) \qquad \forall i,m \qquad (49)$$

$$x_i^{PD} \geq x_i^C - 0.5l_x_i^M - M(1 - z_r_{i,m}^{MV}) \qquad \forall i,m \qquad (50)$$

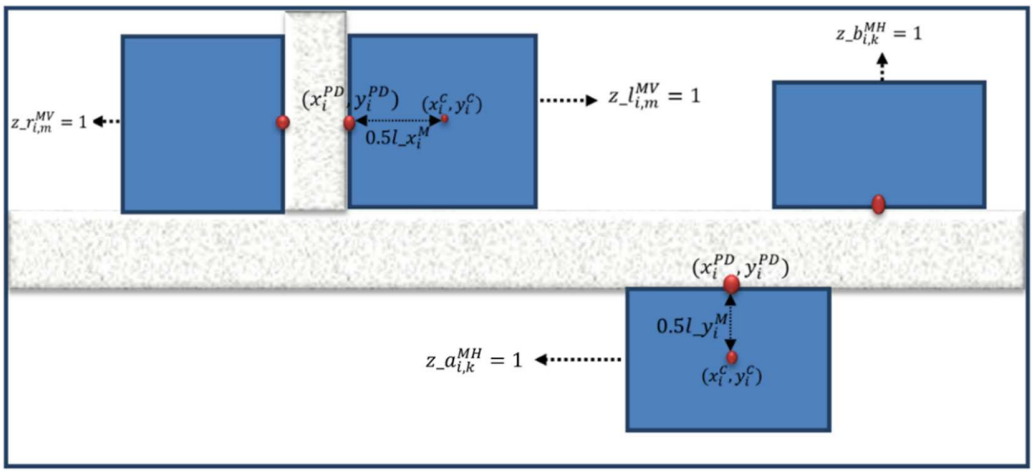

Fig. 9. Coordinates of P/D points of facilities relative to the aisles that supply them

Constraints (51)-(54) guarantee that if a facility is supplied by a horizontal aisle, its P/D point should be located at the common border with that aisle. Two decision variables, $z_a_{i,k}^{MH}$ and $z_b_{i,k}^{MH}$, determine whether facility *i* is supplied by horizontal aisle *k*. For example, if facility *i* is supplied by horizontal aisle *k* which is above it, variable $z_a_{i,k}^{MH}$ will be equal to 1 and variable $z_b_{i,k}^{MH}$ will be equal to 0. This means that constraints (51) and (52) are imposed and constraints (53) and (54) are relaxed. Thus the *y*-coordinate of the P/D point of facility *i*, y_i^{PD} should be equal to the *y*-coordinate of the lower side of aisle *k*, $y_k^H - 0.5w_k^H$.

$$y_i^{PD} \leq y_k^H - 0.5w_k^H + M(1 - z_a_{i,k}^{MH}) \qquad \forall i,k \qquad (51)$$

$$y_i^{PD} \geq y_k^H - 0.5w_k^H - M(1 - z_a_{i,k}^{MH}) \qquad \forall i,k \qquad (52)$$

$$y_i^{PD} \leq y_k^H + 0.5w_k^H + M(1 - z_b_{i,k}^{MH}) \qquad \forall i,k \qquad (53)$$

$$y_i^{PD} \geq y_k^H + 0.5w_k^H - M(1 - z_b_{i,k}^{MH}) \qquad \forall i,k \qquad (54)$$

Similar to constraints (51)-(54), constraints (55)-(58) are used to ensure that if a facility is supplied by a vertical aisle, its P/D point should be located at the edge of that aisle.

$$x_i^{PD} \leq x_m^V - 0.5w_m^V + M(1 - z_r_{i,m}^{MV}) \qquad \forall i,k \qquad (55)$$

$$x_i^{PD} \geq x_m^V - 0.5w_m^V - M(1 - z_r_{i,m}^{MV}) \qquad \forall i,k \qquad (56)$$

$$x_i^{PD} \leq x_m^V + 0.5w_m^V + M(1 - z_l_{i,m}^{MV}) \qquad \forall i,k \qquad (57)$$

$$x_i^{PD} \geq x_m^V + 0.5w_m^V - M(1 - z_l_{i,m}^{MV}) \qquad \forall i,k \qquad (58)$$

3.3.6. Facility level constraints

Constraints (59) ensure that each facility is allocated to only one level, and constraints (60) and (61) control the level of each facility.

$$\sum_{\forall p} u_{i,p}^{MP} = 1 \qquad \forall i \qquad (59)$$

$$y_k^H - 0.5w_k^H \geq y_i^C + 0.5l_y_i^M - M(1 - u_{i,p}^{MP}) \qquad \forall i, k = p - 1, \; 1 < p \leq K + 1 \qquad (60)$$

$$y_k^H + 0.5w_k^H \leq y_i^C - 0.5l_y_i^M + M(1 - u_{i,p}^{MP}) \qquad \forall i, k = p, \; 1 \leq p < K + 1 \qquad (61)$$

3.3.7. Aisle width constraints

Aisle width refers to the lateral distance between two opposing sides of an aisle. Generally, aisles are not necessarily supposed to have the same width, and aisles with more traffic should be wider. In the proposed formulation, the traffic in an aisle is considered as the sum of the material flow which uses that aisle to reach its destination. Constraints (62)-(65) calculate the amount of traffic in each horizontal and vertical aisle.

$$traffic_k^H = \sum_i \sum_j f_{ij} c_{k,i,j}^H \qquad \forall k \qquad (62)$$

$$c_{k,i,j}^H \geq z_{i,l}^{MH} + z_{j,l'}^{MH} + z_{i,m}^{MV} + z_{j,n}^{MV} - 1 \qquad \forall i \neq j, l, m \leq k \leq l', n \qquad (63)$$

$$traffic_m^V = \sum_i \sum_j f_{ij} c_{m,i,j}^V \qquad \forall m \qquad (64)$$

$$c_{m,i,j}^V \geq z_{i,k}^{MH} + z_{j,l}^{MH} + z_{i,n}^{MV} + z_{j,n'}^{MV} - 1 \qquad \forall i \neq j, k, n \leq m \leq l, n' \qquad (65)$$

It is assumed that if an aisle is created, its width should be at least equal to a minimum value. This value is determined such that the aisles are wide enough to allow the largest disassembled part of a facility to be transported in case of breakdown. In addition, the space needed for walkways is considered to determine the minimum aisle width. An aisle with more traffic needs to be wider. Therefore, the width of the aisles should be equal to a minimum value plus a traffic-based value. Constraints (66) and (67) are defined to determine the width of each aisle based on the traffic passing through it. Coefficients α and β are determined by the opinions of expert designers. It can be noted that any other strategies can be defined, based on expert designers' ideas to determine the width of the aisles.

$$w_k^H \geq m_w^H + \alpha\ traffic_k^H - M(1 - c_{k,i,j}^H) \qquad \forall\ k \qquad (66)$$

$$w_m^V \geq m_w^V + \beta\ traffic_m^V - M(1 - c_{m,i,j}^V) \qquad \forall m \qquad (67)$$

According to constraints (66) and (67), if no facility uses a horizontal aisle k in its travel path ($c_{k,i,j}^H = 0, \forall i, j$), then its width is set to be equal to zero. By determining the width of the horizontal aisles, the number of aisles can be obtained. In this way, horizontal aisles with a width >0 are included in the layout and those with a width of 0 are not taken into account. The number of vertical aisles is equal to the number of horizontal aisles minus one.

3.3.8. The entrance and exit door constraints

Another important consideration in a manufacturing system is the entrance/exit doors through which the materials and transportation devices enter or exit the system. Usually, there are technical and environmental restrictions that limit the possible locations around the building for the construction of entrance/exit doors. In this research, the environmental restrictions are assumed to be such that the entrance/exit doors can only be constructed in some parts of the top/bottom side of the building (see green line in Fig. 10).

In the mathematical model, the entrance/exit doors are considered as two virtual facilities indexed by *0* and *N+1*. Thus the P/D point of the entrance door, x_0^{PD}, and the P/D point of the exit door, x_{N+1}^{PD}, have to satisfy the following constraints.

$$l^{ENT} \leq x_0^{PD} \leq u^{ENT} \qquad (68)$$

$$l^{EXT} \leq x_{N+1}^{PD} \leq u^{EXT} \qquad (69)$$

The *y*-coordinate of the entrance/exit doors is equal to the *y*-coordinate of the top/bottom side of the shop floor (see Fig. 10). After locating the entrance and exit doors on the top and bottom sides of the building, two new vertical aisles are required to connect the aisle network to the entrance and exit doors. Since there are *K-1* normal vertical aisles in the system, the two new vertical aisles, called entrance and exit aisles, are indexed by 0 and *K*. As shown in, Fig. 10, the *x*-coordinate of the centroid of the entrance (exit) aisle is equal to the *x*-coordinate of the entrance (exit) door.

$$x_0^{PD} = x_0^V \qquad (70)$$

$$x_{N+1}^{PD} = x_K^V \qquad (71)$$

The two new vertical aisles are treated in the same way as other vertical aisles. The facilities should have no overlap with these new vertical aisles. This is guaranteed by constraint (30)-(32) for $m = 0$ and $m = K$.

By determining the position of the entrance and the exit doors, the distances between these doors and the facilities can be calculated and considered when calculating the TTD. Constraints (72)-(75) are used to calculate the distances between the facilities and the entrance/exit doors. These constraints are similar to constraints (2) and (10).

$$d_{0,i}^{MM} \geq d_x_{0,i}^{MM} + d_y_{0,i}^{MM} - M\left(1 - z_{i,1}^{MH} - z_{i,1}^{MV}\right) \qquad \forall i \qquad (72)$$

$$d_{0,i}^{MM} \geq d_x_{0,1}^{MV} + d_x_{i,m}^{MV} + d_x_{1,k}^{HH} + d_y_{0,i}^{MM} - M\left(1 - z_{i,k}^{MH} - z_{i,m}^{MV}\right) \qquad \forall i, 1 < m = k - 1 \qquad (73)$$

$$d_{i,N+1}^{MM} \geq d_x_{i,N+1}^{MM} + d_y_{i,N+1}^{MM} - M\left(1 - z_{i,K}^{MH} - z_{i,K-1}^{MV}\right) \qquad \forall i \qquad (74)$$

$$d_{i,N+1}^{MM} \geq d_x_{N+1,K-1}^{MV} + d_x_{i,m}^{MV} + d_x_{k,K}^{HH} + d_y_{i,N+1}^{MM} - M\left(1 - z_{i,k}^{MH} - z_{i,m}^{MV}\right) \qquad \forall i, m = k < K \qquad (75)$$

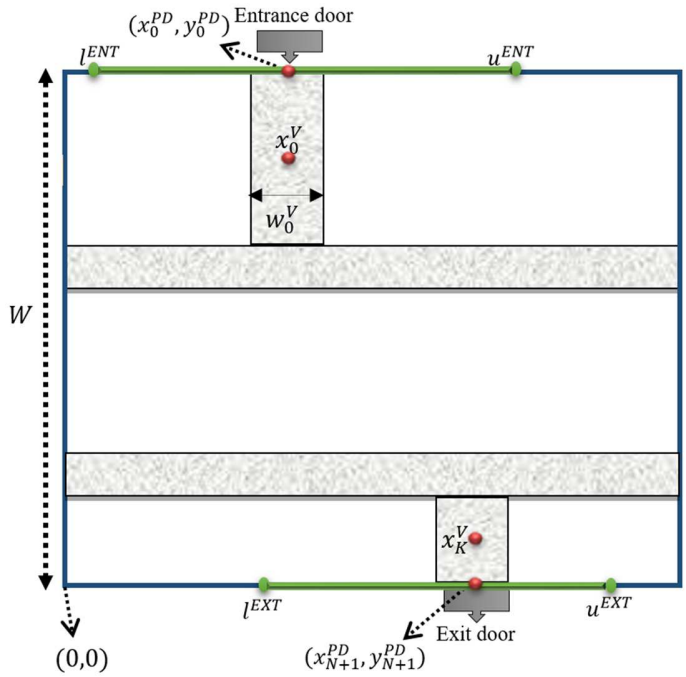

Fig. 10. Possible areas for placing the entrance and exit doors and their corresponding vertical aisles.

3.3.9. Domain constraints

The decision variables are defined by constraints (76) and (77).

$$x_i^C, y_i^C, x_m^V, y_k^H \geq 0 \qquad \forall i, j, k, m \qquad (76)$$

$h_i, left_{i,j}^{MM}, below_{i,j}^{MM}, above_{i,k}^{MH}, below_{i,k}^{MH}, left_{i,m}^{MV}, right_{i,m}^{MV}, u_{i,p}^{MP}, z_{i,k}^{MH}, z_a_{i,k}^{MH}, z_b_{i,k}^{MH}, z_{i,m}^{MV}, z_l_{i,m}^{MV},$
$z_r_{i,m}^{MV}, c_{k,i,j}^{H}, c_{m,i,j}^{V} \in \{0,1\} \qquad \forall i,j,k,m,p$
(77)

3.3.10. Updated objective function

The objective function, shown in Eq. (1), calculates the transportation distance between each pair of facilities and does not consider the transportation distance between the facilities and the entrance and exit doors. The updated objective function calculates the total transportation distance between each pair of facilities and between the facilities and the entrance and exit doors.

$$Min\ TTD = \sum_{i=0}^{N} \sum_{j=i+1}^{N+1} f_{ij} d_{i,j}^{MM} \tag{78}$$

In Eq. (78), $f_{0,j}$ shows the quantity of materials that are moved from the entrance door to facility *j*, and $f_{i,N+1}$ shows the quantity of materials that are moved from facility *i* to the exit door. This new objective function, along with constraints (2)-(77), shapes the model. Since the objective function and the constraints are represented entirely by linear equations, this model is linear and hence it is possible to achieve a global optimum solution. However, the model is computationally complex, so the global optimum solution can be achieved only for small-sized problems. Without considering the aisle problem, complexity is reduced to an unequal area facility layout problem, which is NP-Hard (Anjos & Vieira, 2017), with *2N* continuous. In our case, *2* continuous variables are needed to determine the position of the entrance/exit doors, *K* for the center of the horizontal aisles and *K-1* for the vertical aisles. In total, there are *2N+2K+1* continuous decision variables for the positioning of facilities and aisles. In addition to these continuous decision variables, there are continuous variables for the width of the aisles, and many binary variables in the model to satisfy the non-overlapping and aisle accessibility constraints. In the next section, a branch-and-cut algorithm, along with improvement mechanisms, is proposed to solve the model.

4. Proposed branch-and-cut

The branch-and-cut (B&C) method has successfully been used to solve a variety of MILP problems (Karaoğlan, Erdoğan, & Koç, 2018) and it can also guarantee global optimality. The B&C algorithm follows the general scheme of the branch-and-bound (B&B) algorithm. In the B&C algorithm, some cuts are added to the problem to limit the possible value of the continuous relaxed variables. Cuts are additional constraints that may remove the feasible points of the continuous relaxed problem while

leaving the feasible set of MIP unchanged, which is very efficient for solving MILP. Interested readers can consult Caccetta (2000), Mitchell (2002) and Conforti, Cornuéjols, and Zambelli (2014) for more details about the B&C algorithm. Even though the B&C algorithm is an effective method to solve MILP, some mechanisms are applied to improve the B&C algorithm in solving the proposed model. These mechanisms consist of 1) adding optimality cuts, 2) defining the branching order of variables and 3) choosing the best node selection strategies.

4.1. Optimality cuts

Four families of valid inequalities are introduced to strengthen the model through the implementation of the B&C algorithm. These inequalities are called optimality cuts. The optimality cuts applied here are quite simple yet very efficient. The first group of optimality cuts are used as follows:

$$\sum_{\forall i} z_a_{i,k}^{MH}\, l_x_i^M \leq length \qquad \forall k \qquad (79)$$

$$\sum_{\forall i} z_b_{i,k}^{MH}\, l_x_i^M \leq length \qquad \forall k \qquad (80)$$

Inequalities (79) and (80) prevent variables $z_a_{i,k}^{MH}$ and $z_b_{i,k}^{MH}$ from taking the value one if this results in generating an unfeasible layout that violates the limitation of shop floor length. The following two inequalities are used as the second group of cuts:

$$z_a_{i,k}^{MH} \leq u_{i,p}^{MP} \qquad \forall i, \forall p = k+1 \qquad (81)$$

$$z_b_{i,k}^{MH} \leq u_{i,p}^{MP} \qquad \forall i, \forall p = k \qquad (82)$$

Constraints (81) imply that if facility *i* is supplied by aisle *k,* which is above it, facility *i* is certainly placed in level *k+1*. Constraints (82) imply that if facility *i* is supplied by aisle *k,* which is below it, facility *i* is certainly placed in level *k*.

4.2. Branching strategy

Selecting a branching variable is an essential task in any B&C algorithm, since it can significantly affect its computational time. Ideally, it is preferable to select a branching variable that helps to explore the minimum number of nodes. In this study, the objective function is strongly dependent on the variables related to the position of the facilities. At the same time, the decision variable of the position of each facility is directly influenced by the decision variable that is concerned with the aisle that supplies it. Hence, it is reasonable to focus on the variables that determine which aisle is supplying each facility.

Consequently, two variables, $z_a_{i,k}^{MH}$ and $z_b_{i,k}^{MH}$, are the first priorities for branching. Once variables $z_a_{i,k}^{MH}$ and $z_b_{i,k}^{MH}$ are set, variables $u_{i,p}^{MP}$, $above_{i,k}^{MH}$ and $below_{i,k}^{MH}$ will be fixed.

Through this strategy, when the problem is solved as a relaxed linear programming problem at each node and the integer variables are relaxed, some relaxed integer variables are derived to take integer value. Furthermore, by first branching on $z_a_{i,k}^{MH}$ certain other variables will be relatively determined. For example, if we know the value of $z_a_{i,k}^{MH}$ for two facilities then this implies that the variable $left_{i,j}^{MM}$ should take value 1.

4.3. Node selection strategies

Another important task in any B&C algorithm is to select the node that should be branched at each iteration. Node selection strategies aim to prune open nodes and end the queue as quickly as possible.

Depth-first, best-bound, breadth-first, or best-first search strategies are the most commonly used strategies. The depth-first strategy is suitable for problems where a feasible solution is difficult to find. Since finding a feasible solution in which facilities have no overlap is difficult, the depth-first search strategy is implemented. In this strategy, the most recent node added to the tree is chosen for branching. The advantage of this strategy is that a feasible solution, and consequently a lower bound, can be found quickly (Belov & Scheithauer, 2006). The obtained lower bound is used to keep the list of open nodes minimal and thus memory usage is low.

5. Computational experiments

In this section, a set of test instances is used to verify and evaluate the proposed approach. The B&C described in section 4 is compared with general B&C. In the general version of the B&C algorithm, the default settings of CPLEX were used, in which CPLEX chooses a variable to branch on. The node selection strategy is the best-bound search in which the node with the best objective function for the associated LP relaxation is selected. The five following versions of the B&C algorithm are used for this comparison:

- The B&C1, which is the general version of the B&C algorithm.
- The B&C2, which is the B&C1 algorithm plus the proposed branching strategy.
- The B&C3, which is the B&C2 algorithm plus the proposed node selection strategy.
- The B&C4, which is the B&C3 algorithm plus the proposed first group of optimality cuts (Eqs. 79, 80).
- The B&C5, which is the B&C4 algorithm plus the proposed second group of optimality cuts (Eqs. 81, 82).

All these versions of B&C algorithms are implemented using GAMS modeling language (version 28.2.0), and CPLEX (version 12.9) is used as the solver. The numerical tests are conducted on a laptop computer with an Intel Core i7 CPU (2.3 GH) and 8 GB of memory under the Windows 10 operating system.

5.1. Test instances

Since none of the reviewed studies in Section 2 address our problem, using their test instances would not provide an appropriate comparison for the proposed approach. Therefore, new test instances inspired by a real manufacturing system are designed, and the facility dimensions are taken from a real case study (Zhou et al., 2006). Table 2 shows 12 facilities and their sizes.

Table 2

Facilities and their dimensions.

No.	Facility name	Area (m×m)
1	Raw material warehouse	30×10
2	Casting	18×15
3	Heat-treat	12×10
4	Mechanical	20×15
5	Precision	14×10
6	In-process product	24×20
7	Assembly	14×8
8	Performance testing	18×12
9	Final product warehouse	20×16
10	Maintenance	12×10
11	Service houses	40×30
12	Depot	30×30

In the test instances, the width of the largest disassembled part of the facilities is 3.5 meter (m). In addition, 0.5 m is added for the space needed for walkways. The minimum width for the aisles is therefore considered to be 4 m. The aisles can become wider, based on their traffic. The values of α and β are set to 0.0033. Based on the number of facilities, the test instances are classified into six groups of 7, 8, 9, 10, 11 and 12 facilities. For each group, four instances are generated. These instances have different sizes for the plant floor and different possible positions for the entrance/exit doors that are presented in Table 3. For example, test instance 1 corresponds to a problem with 7 facilities (facilities 1 to 7 in Table 2) where the plant length is 45m, the plant width is 55m, the width of the entrance door is 4m and it can be placed on the upper side of the plant in the range 20m to 45m, and the width of the exit door is 4m and

can be located on the bottom side of the plant in the range 10m to 40m. These instances are designed so that the total area of all the facilities is around 70% of the total plant area. The material flow between facilities is determined using a uniform distribution on the interval [30, 50].

Table 3
General data of the test instances.

Problem No.	No. of facilities	Plant area (m^2)	Entrance door		Exit door	
			Possible position	width(m)	Possible position	width(m)
1	7	45×55	Up. 20m-45m	4	Down. 10m-40m	4
2		40×52	Down. 20m-40m	4	Up. 20m-40m	4
3		43×51	Up. 10m-43m	4	Bottom. 0m-20m	4
4		41×58	Down. 20m-41m	4	Up. 30m-41m	4
5	8	50×58	Up. 10m-40m	4	Down. 30m-40m	4
6		52×55	Down. 30m-52m	4	Up. 20m-40m	4
7		48×60	Up. 10m-40m	4	Down. 0m-48m	4
8		50×60	Down. 10m-50m	4	Up. 0m-30m	4
9	9	50×65	Up. 10m-40m	5	Down. 30m-50m	5
10		52×63	Down. 10m-50m	5	Up. 0m-40m	5
11		48×68	Up. 10m-48m	5	Down. 10m-40m	5
12		50×68	Down. 30m-50m	5	Up. 0m-50m	5
13	10	55×65	Up. 30m-40m	6	Down. 10m-40m	6
14		52×70	Down. 20m-52m	6	Up. 20m-40m	6
15		50×70	Up. 10m-40m	6	Down. 20m-50m	6
16		53×68	Down. 20m-53m	6	Up. 10m-40m	6
17	11	70×70	Up. 30m-40m	6	Down. 0m-40m	6
18		68×72	Down. 50m-60m	6	Up. 50m-68m	6
19		65×75	Up. 0m-50m	6	Down. 10m-65m	6
20		67×74	Down. 0m-50m	6	Up. 10m-67m	6
21	12	75×90	Up. 0m-50m	7	Down. 0m-40m	7
22		67×93	Down. 20m-67m	7	Up. 40m-67m	7
23		65×95	Up. 40m-65m	7	Down. 0m-50m	7
24		68×90	Down. 50m-68m	7	Up. 0m-50m	7

5.2. Branch-and-cut experiments

In this subsection, the performance of five different B&C algorithms (B&C1- B&C5) is evaluated. For all versions of the B&C algorithms, a maximum time of 36000 seconds is allowed for CPLEX to find the

global optimum solution. If no global optimum solution is found, the algorithm stops and the best solution found is recorded, along with the optimality gap (the gap between the best possible solution with relaxed integrality constraints and the best feasible solution found). Table 4 presents the computational results of the algorithms and Fig. 11 illustrates the mean objective function value (OFV) of the algorithms for each problem size. As can be seen in Table 4, for problems with seven facilities ($N = 7$), the performance of all the algorithms with respect to OFV is similar and they obtain the global optimum solution. If the number of facilities increases to eight or more, B&C1 cannot find the global optimum solution in the allowed computation time. The average OFV obtained by B&C1 for problems with eight facilities ($N = 8$) is 24485 with an average optimality gap of 26.32%. However, for this problem size, all the other algorithms reach the global optimum solution. For problems with nine facilities ($N = 9$), B&C3, B&C4 and B&C5 can provide optimal solutions with zero optimality gap, whereas B&C1 and B&C2 find solutions with an average optimality gap of 88.61% and 30.67%, respectively. As the number of facilities increases, the difference between the algorithms becomes more and more apparent. The average optimality gaps of B&C1 and B&C2 for problems with more than nine facilities are more than 75.00% and therefore these algorithms cannot provide an acceptable solution from a practical perspective. This makes these two algorithms out of competition for problems with 10,11, and 12 facilities.

For problems with 10 facilities ($N = 10$) and 11 facilities ($N = 11$), only B&C5 can reach the global optimum solution. For problems with 10 facilities, the average OFV of B&C3 and B&C4 is respectively 77846 and 74987 with an average optimality gap of 31.80% and 13.39%. This shows that the performance of B&C4 is slightly better than B&C3 for finding the global optimum solution. The superiority of B&C4 over B&C3 in terms of OFV can also be seen in larger-sized problems with 11 facilities and 12 facilities ($N = 12$) which is due to the use of the first group of optimality cuts. However, this group of cuts does not lead to a noticeable improvement in OFV and so the average OFV of B&C3 and B&C4 are very similar. Even though the average OFV of B&C3 and B&C4 are similar, there is a significant difference between their average optimality gaps. Since the gap is calculated by comparing the current possible solution with a bound, obtained by relaxing integrality constraints, it can be concluded that the bound obtained by B&C3 is weaker than the bound of B&C4.

For problems with 12 facilities ($N = 12$), B&C5 can reach solutions with an average optimality gap of 12.05%. There are many real manufacturing systems with less than 13 facilities (Tompkins et al. 2010) and hence, B&C5 determines acceptable solutions for these with low optimality gaps. B&C3 and B&C4 are not suitable algorithms for problems with 12 facilities because they find solutions with low-quality OFV and high average optimality gaps. Consequently, analysis proves that B&C5 provides the best performance in terms of OFV.

Table 4

Comparison between the algorithms.

Problem No.	No. of facilities	B&C1		B&C2		B&C3		B&C4		B&C5	
		OFV	Opt Gap	OFV	Opt Gap	OFV	Opt Gap	OFV	Opt Gap	OFV	Opt Gap
1	7	23887	0.00%	23887	0.00%	23887	0.00%	23887	0.00%	23887	0.00%
2		20753	0.00%	20753	0.00%	20753	0.00%	20753	0.00%	20753	0.00%
3		21652	0.00%	21652	0.00%	21652	0.00%	21652	0.00%	21652	0.00%
4		20789	0.00%	20789	0.00%	20789	0.00%	20789	0.00%	20789	0.00%
Average		21770	0.00%	21770	0.00%	21770	0.00%	21770	0.00%	21770	0.00%
5	8	24647	25.47%	20038	0.00%	20038	0.00%	20038	0.00%	20038	0.00%
6		24537	22.44%	20276	0.00%	20276	0.00%	20276	0.00%	20276	0.00%
7		24214	28.38%	19387	0.00%	19387	0.00%	19387	0.00%	19387	0.00%
8		24540	29.00%	19411	0.00%	19411	0.00%	19411	0.00%	19411	0.00%
Average		24485	26.32%	19778	0.00%	19778	0.00%	19778	0.00%	19778	0.00%
9	9	51921	89.00%	37226	35.00%	27899	0.00%	27899	0.00%	27899	0.00%
10		49084	81.00%	33051	28.27%	27407	0.00%	27407	0.00%	27407	0.00%
11		45116	97.28%	29833	29.39%	23608	0.00%	23608	0.00%	23608	0.00%
12		46464	87.17%	32613	30.00%	25348	0.00%	25348	0.00%	25348	0.00%
Average		48146	88.61%	33181	30.67%	26066	0.00%	26066	0.00%	26066	0.00%
13	10	129697	100.00%	92915	78.48%	82690	25.90%	80142	12.36%	73960	0.00%
14		122859	100.00%	84191	81.00%	72507	30.72%	69710	14.16%	63858	0.00%
15		126679	100.00%	91560	78.47%	80555	34.57%	77456	15.50%	72055	0.00%
16		123603	100.00%	87502	72.85%	75632	36.00%	72639	11.56%	66912	0.00%
Average		125710	100.00%	89042	77.70%	77846	31.80%	74987	13.39%	69196	0.00%
17	11	210796	100.00%	163078	100.00%	95262	100.00%	90896	27.58%	74761	0.00%
18		219616	100.00%	179359	100.00%	110341	100.00%	105965	24.04%	89016	0.00%
19		217338	100.00%	167402	100.00%	101935	100.00%	97742	29.23%	78050	0.00%
20		224677	100.00%	175981	100.00%	108303	100.00%	103827	22.97%	88011	0.00%
Average		218107	100.00%	171455	100.00%	103960	100.00%	99608	25.96%	82460	0.00%
21	12	247113	100.00%	209478	100.00%	159673	100.00%	154272	32.73%	123682	7.14%
22		250830	100.00%	219967	100.00%	175377	100.00%	169590	48.29%	134697	14.54%
23		248214	100.00%	213115	100.00%	169856	100.00%	163736	44.48%	129631	12.29%
24		249256	100.00%	211079	100.00%	170700	100.00%	164707	43.08%	131693	14.24%
Average		248853	100.00%	213410	100.00%	168902	100.00%	163076	42.15%	129926	12.05%

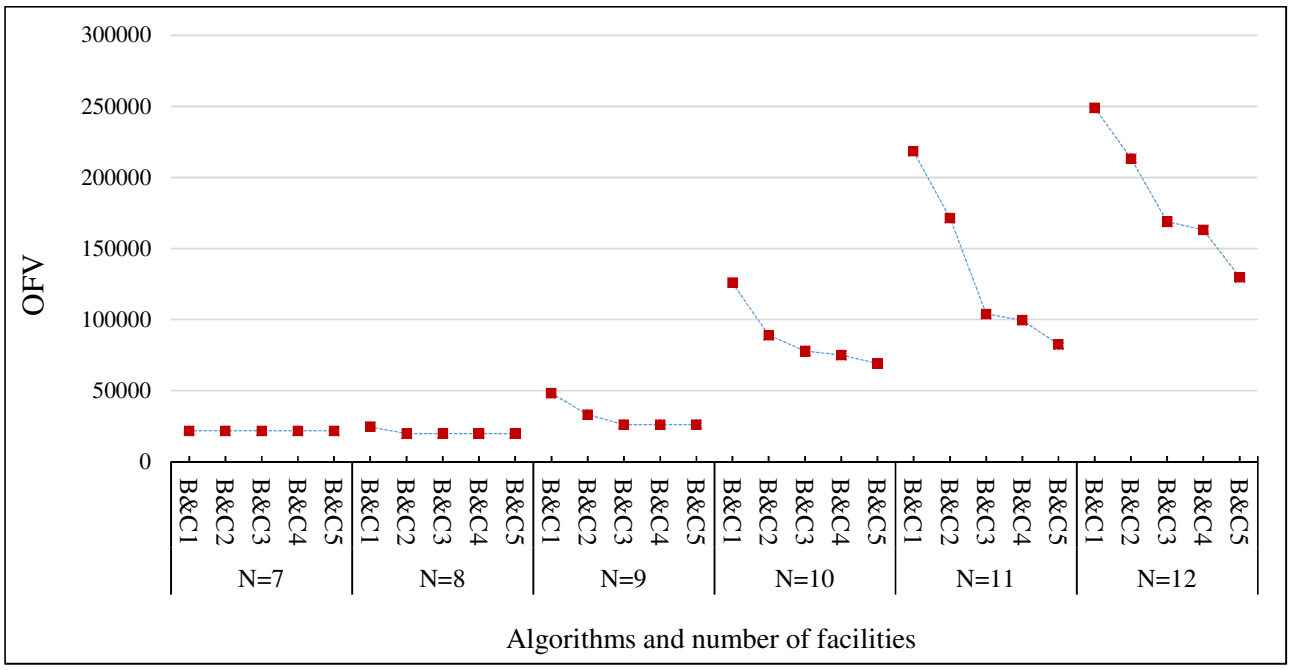

Fig. 11. Mean OFV of the algorithms for each problem size.

In addition to the OFV, the performance of the algorithms is evaluated in terms of computation time. This comparison is made for problems that the algorithms can reach the zero optimality gap. Fig. 12 shows the average computation time for each problem size. For problems with seven facilities, to which all the algorithms find the global optimum solution, B&C1, with a computation time of around 7085s, has the worst performance. For this problem size, the computation time of B&C2 is 2024s and that of the other algorithms is less than 2000s. By increasing the problem size to eight facilities, the computation time of B&C2 increases significantly and it reaches the optimum solution in 7677s, whereas the B&C3, B&C4 and B&C5 find the global optimum solution in 2503s, 956s and 807s respectively. For problems with nine facilities, the performance of B&C3 in terms of computation time decreases, and the algorithm requires 13877s to find the global optimum solution. However, B&C4 and B&C5 find the global optimum solution in 1435s and 1379s, respectively, which are much better performances. Therefore, even though B&C3 and B&C4 achieve similar results in terms of OFV, B&C4 performs much better than B&C3 in terms of computation time. This superiority of B&C4 is due to the use of optimality cuts that help the algorithm to avoid searching the unfeasible nodes. It can be said that even though the cuts in B&C4 do not increase the performance of the algorithm to solve larger-sized problems, they can reduce computation time. For problems with 10 and 11 facilities, B&C5 can reach the global optimum solution in 3011s and 10261s respectively.

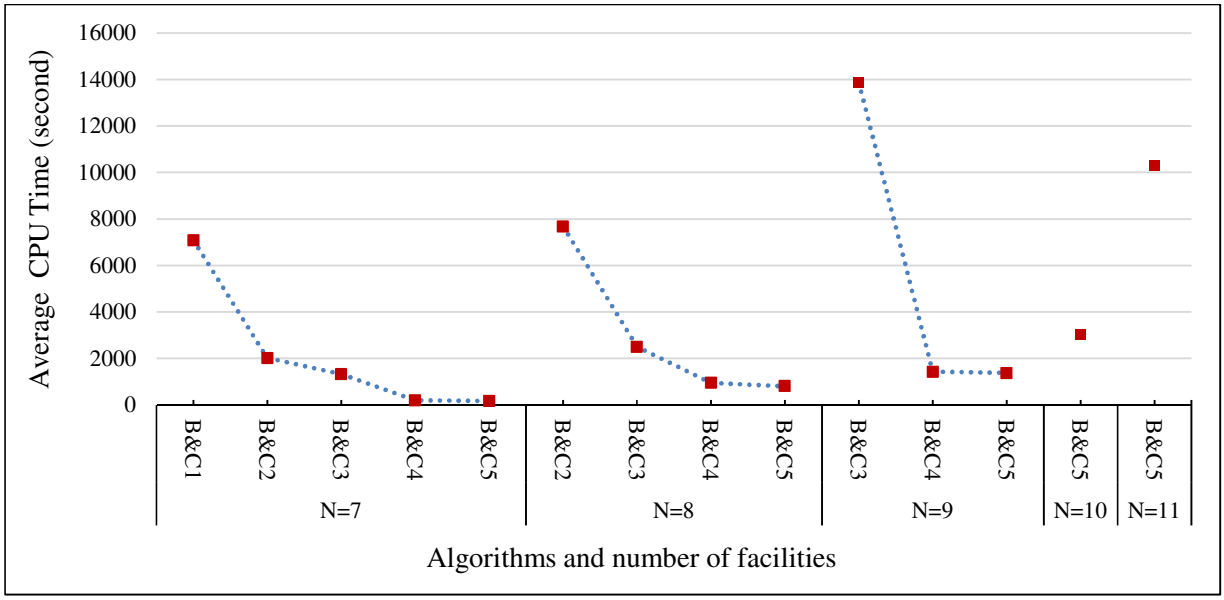

Fig. 12. Computation times of the algorithms to find global optimum solution.

The best layout obtained via the B&C5 algorithm for problem 21 is illustrated in Fig. 13. In this layout, 12 facilities are placed on three levels. The aisle structure consists of two horizontal aisles, one vertical aisle between the horizontal aisles, and two vertical aisles connected to the entrance/exit doors. Via this aisle structure, all the facilities have access to each other and to the entrance/exit doors. The upper horizontal aisle is busier than the lower horizontal aisle because it supplies six facilities; therefore it is wider. The central vertical aisle is the busiest aisle because it directly supplies facilities 2 and 5 and is also the communication aisle between two other horizontal aisles. Thus, it is the widest aisle. Since the facilities are allowed to rotate and be placed in their optimal orientation, the layout is coherent with a minimum of unused space. The area of the shop floor is 6300m^2 and the total space occupied by the facilities and aisles is 4487m^2 and 1573m^2, respectively. Thus, around 3.8% of the area is unused. A large portion of this unused space is related to the zone in the top-left corner. From a practical point of view, since this space has a regular rectangular shape, it could be used efficiently according to the needs of the manufacturing system (e.g., to install new facilities or an office).

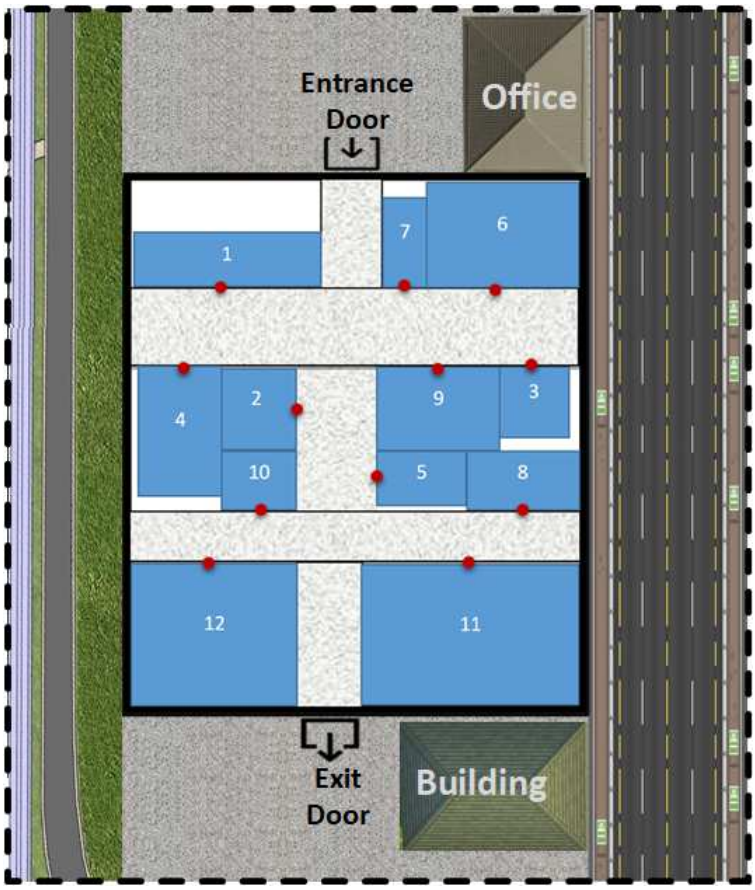

Fig. 13. The best layout obtained for test instance 21.

6. Conclusions

In many practical applications, designing a layout necessitates studying how parts, transportation devices, materials, and operators circulate in the system through aisles. The system's performance depends not only on the position of facilities but also on the aisle structure. The approach presented in this article addresses these two aspects in an integrated manner. In this respect, both practical considerations (e.g., to be able to access every facility, to be able to move them for maintenance and to avoid turns) and theoretical requirements (e.g., where to place the entrance and exit doors, what the width of the aisles should be and what the best aisle structure is) are pointed out. A MILP formulation of this problem is proposed, making it possible to solve the problem optimally using an enhanced B&C algorithm. This approach is successfully tested for a set of instances with up to 12 facilities. Problems with 11 facilities are solved optimally and problems with 12 facilities are solved with a 7%-15% optimality gap. The proposed approach can therefore be applied to real manufacturing systems comprising up to 11 facilities.

This work opens up several interesting research directions. Increasing the size of the problems that can be solved is an important aspect. Future research could explore this by applying other possible cuts and designing metaheuristic procedures. Moreover, in this model, minimizing the TTD was considered as the performance criterion. However, the problem can be studied as a multi-objective FLP, and other objectives such as minimizing the number of turns in the aisles, minimizing the mean flow time, and maximizing space utilization could also be considered. Furthermore, allowing non-rectangular-shaped facilities and having several entrance/exit doors are promising directions for future research. Studying different types of aisle structure, where the length of horizontal aisles can be shorter than the length of the shop floor and where there are multiple vertical aisles between each pair of horizontal aisles, could be another promising topic for further research.

References

Alagoz, O., Norman, B. A., & Smith, A. E. (2008). Determining aisle structures for facility designs using a hierarchy of algorithms. *IIE Transactions*, *40*(11), 1019–1031.

Allahyari, M. Z., & Azab, A. (2018). Mathematical modeling and multi-start search simulated annealing for unequal-area facility layout problem. *Expert Systems with Applications*, *91*, 46–62.

Anjos, M. F., & Vieira, M. V. (2017). Mathematical optimization approaches for facility layout problems: The state-of-the-art and future research directions. *European Journal of Operational Research*, *261*(1), 1–16.

Belov, G., & Scheithauer, G. (2006). A branch-and-cut-and-price algorithm for one-dimensional stock cutting and two-dimensional two-stage cutting. *European Journal of Operational Research*, *171*(1), 85–106.

Benson, B., & Foote, B. L. (1997). DoorFAST: A constructive procedure to optimally layout a facility including aisles and door locations basedon anaisle flow distance metric. *International Journal of Production Research*, *35*(7), 1825–1842.

Caccetta, L. (2000). Branch and cut methods for mixed integer linear programming problems. In *Progress in Optimization* (pp. 21–44). Springer.

Chang, C.-H., Lin, J.-L., & Lin, H.-J. (2006). Multiple-floor facility layout design with aisle construction. *Industrial Engineering & Management Systems*, *5*(1), 1–10.

Conforti, M., Cornuéjols, G., & Zambelli, G. (2014). *Integer programming* (Vol. 271). Springer.

Drira, A., Pierreval, H., & Hajri-Gabouj, S. (2007). Facility layout problems: A survey. *Annual Reviews in Control*, *31*(2), 255–267.

Dunker, T., Radons, G., & Westkämper, E. (2005). Combining evolutionary computation and dynamic programming for solving a dynamic facility layout problem. *European Journal of Operational Research*, *165*(1), 55–69.

Friedrich, C., Klausnitzer, A., & Lasch, R. (2018). Integrated slicing tree approach for solving the facility layout problem with input and output locations based on contour distance. *European Journal of Operational Research*, *270*(3), 837–851.

Gomez, A., Fernandez, Q. I., De la Fuente García, D., & Garcıa, P. J. (2003). Using genetic algorithms to resolve layout problems in facilities where there are aisles. *International Journal of Production Economics*, *84*(3), 271–282.

Hosseini-Nasab, H., Fereidouni, S., Ghomi, S. M. T. F., & Fakhrzad, M. B. (2018). Classification of facility layout problems: a review study. *The International Journal of Advanced Manufacturing Technology*, *94*(1–4), 957–977.

Hu, G. H., Chen, Y. P., Zhou, Z. D., & Fang, H. C. (2007). A genetic algorithm for the inter-cell layout and material handling system design. *The International Journal of Advanced Manufacturing Technology*, *34*(11–12), 1153–1163.

Karaoğlan, İ., Erdoğan, G., & Koç, Ç. (2018). The multi-vehicle probabilistic covering tour problem. *European Journal of Operational Research*, *271*(1), 278–287.

Kim, C.-B. (1992). *Algorithms for automated three dimensional facility layout problem* [PhD Thesis]. University of Oklahoma.

Klausnitzer, A., & Lasch, R. (2016). Extended model formulation of the facility layout problem with aisle structure. In *Logistics Management* (pp. 89–101). Springer.

Lee, K.-Y., Roh, M.-I., & Jeong, H.-S. (2005). An improved genetic algorithm for multi-floor facility layout problems having inner structure walls and passages. *Computers & Operations Research*, *32*(4), 879–899.

Leno, I. J., Saravanasankar, S., & Ponnambalam, S. G. (2012). Layout design for efficient material flow path. *Procedia Engineering*, *38*, 872–879.

Li, C. (2011). *A facility layout design methodology for retail environments* [PhD Thesis]. University of Pittsburgh.

Meller, R. D., Narayanan, V., & Vance, P. H. (1998). Optimal facility layout design. *Operations Research Letters*, *23*(3–5), 117–127.

Mitchell, J. E. (2002). Branch-and-cut algorithms for combinatorial optimization problems. *Handbook of Applied Optimization*, *1*, 65–77.

Norman, B. A., Arapoglu, R. A., & Smith, A. E. (2001). Integrated facilities design using a contour distance metric. *IIE Transactions*, *33*(4), 337–344.

Peters, B. A., & Yang, T. (1997). Integrated facility layout and material handling system design in semiconductor fabrication facilities. *IEEE Transactions on Semiconductor Manufacturing*, *10*(3), 360–369.

Stephens, M. P., & Meyers, F. E. (2013). *Manufacturing facilities design and material handling*. Purdue University Press.

Sule, D. R. (2008). *Manufacturing facilities: location, planning, and design*. CRC press.

Tompkins, J. A., White, J. A., Bozer, Y. A., & Tanchoco, J. M. A. (2010). *Facilities planning*. John Wiley & Sons.

Tretheway, S. J., & Foote, B. L. (1994). Automatic computation and drawing of facility layouts with logical aisle structures. *The International Journal of Production Research*, *32*(7), 1545–1555.

Wang, H.-F., & Chang, C.-M. (2015). Facility layout for an automated guided vehicle system. *Procedia Computer Science*, *55*, 52–61.

Wu, Y., & Appleton, E. (2002). The optimisation of block layout and aisle structure by a genetic algorithm. *Computers & Industrial Engineering*, *41*(4), 371–387.

Xiao, Y., Xie, Y., Kulturel-Konak, S., & Konak, A. (2017). A problem evolution algorithm with linear programming for the dynamic facility layout problem—A general layout formulation. *Computers & Operations Research*, *88*, 187–207.

Zhou, G., Ye, M., Cao, Z., & Ye, F. (2006). A genetic algorithm approach on a facility layout design problem with aisles. *International Conference on Intelligent Computing*, 1008–1013.