A Generic Method to Model CO 2 Emissions of Combined Cycle Power Plants for Environmental Power Dispatch

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***Abstract*—This paper proposes a generic methodology for combined cycle gas turbines (CCGT) modeling. The main objectives are the estimation of the CO_2 emissions for specific units and their integration in an environmental power dispatch that considers several plants. At first a design procedure aims at calibrating the model using the sparse information advised by the manufactures. Off-design points are also investigated in order to estimate the CO_2 emissions on the whole operating range of the units. The obtained results show a good consistency with the emission coefficients found in the literature for that type of units. Then those carbon costs are used as input parameters for a unit commitment problem (UC). The Mixed Integer Linear Programming (MILP) formulation minimizes the global emissions for a set of different units on Jurong Island in Singapore. The grid emission factor obtained for the simulated network displays values close to the registered field data which validates the developed model. Finally, a tightened formulation of the power dispatch problem is introduced. The objective is to reduce the computational time while guaranteeing good performance of the return solutions.**

Keywords—combined cycle gas turbine, design, CO_2 emissions, environmental unit commitment, power dispatch, optimization

1. Introduction

The work presented in this paper has been led in the framework of a wider study whose objective is to simulate the operation of the Jurong Island power grid in Singapore. The island has four thermal power plants with thirteen identified combined cycle gas turbines (CCGT) [1]. The first concern is to model the power generation on site to perform a power dispatch that minimizes the amount of CO_2 generated. Such unit commitment (UC) problems have been widely studied in the literature. The traditional objective is to minimize the generation cost while meeting the load and fulfilling system operating constraints [2]. Here the objective only refers to the minimization of the CO_2 emissions. In the literature those emissions are commonly added to the objective function as a linear cost with the generated power as in [3-4]. Another alternative consists in considering a quadratic curve for the CO_2 emissions – the same way it is commonly done for the generation cost – as well as CO_2 start-up costs [5]. Then either a bi-objective (cost/CO2) optimization is performed or the CO_2 is simply added to the conventional objective function while introducing carbon penalties [2]. Cubic functions can also be found in the literature [6] as well as exponential formula when other types of pollutant considered [7]. In addition to that variety of models, the attached coefficients are rarely justified. They come from pre-established test cases and can display wide ranges of values even for the same type of unit – from 336 kg/MWh to 616 kg/MWh for CCGT emissions in [3]. Indeed, a great part of such power systems studies mainly focus on validating UC problems formulation and the algorithms used to solve them – which is still a hot topic in research on power systems. In order to accurately model those emissions for existing thermal units one must pay attention to energy conversion aspects and model calibration. CCGT has received extensive attention, since it offers enhanced performances compared to conventional gas turbines (GT). An increasing number of heavy duty units has been installed in electrical systems across the world in recent decades to replace conventional coal/oil power plants [8]. Most CCGT studies are concerned with optimal design that minimizes the cost for the plant owner as in [9-10]. However, their performance dramatically decreases when deviating from their nominal operating point. Thus off-design operations have to be considered carefully, especially when a power dispatch will require the system to work on its whole operating range [11]. A particular attention also has to be paid to the computational time once the power dispatch problem is written. Some more complex studies such as grid expansion planning with grid constraints, and sizing for renewable or storage units can be considered [12]. In such cases prohibitive computational times should be avoided and tight formulations can prove to be of interest [2]. The main motivation of the present study is then to use a CCGT model in order to get realistic coefficients for carbon emissions in the environmental UC problem. With only on type of unit considered (CCGT here) it is important to discriminate the costs attached to the different technologies. The biggest challenge lies in obtaining consistent values from the sparse information given by the manufacturers for the identified units - usually only the rated power and efficiency are advised. Then major contributions of this work are:

- The cross disciplinary bridge between CCGT modeling and power dispatch strategies.
- The development of a systematic comprehensive CCGT model whose granularity is adapted to with the MILP formulation for UC problems - the scalability does not allow too much detailed representation.
- The CO_2 start-up costs are not explicitly entered as parameters in the UC. Instead the off design

performance is estimated below the minimum operating point and start-up phases are modeled.

- A tight formulation of the environmental UC is proposed. Especially the units are gathered with distinct power plants. It allows the reduction of the number of variables and equations compared to a more complete representation.

The rest of the paper is organized as follows. Section 2 describes the model used to represent the considered units and the obtained results. The general equations are given and a particular attention is attached to the optimal procedures that allow estimating both nominal and off design performance. Section 3 present the obtained results and compare the performance for the different units. The environmental unit commitment is introduced in Section 4 with thirteen independent units distributed among four power plants. Section 5 describes the tight formulation of the power dispatch problem while the conclusions of the work are presented in Section 6.

2. CCGT Modeling

2.1. Fundamentals and Identified Generation on Site

Fig. 1 shows a simplified diagram of a CCGT. The operation of such a unit lies on the combination of two thermodynamic cycles. In the top Brayton cycle the input air mass low m_a (in kg/s) at ambient conditions is compressed with a ratio r_c before entering the combustion chamber of the gas turbine. The combustion with a fuel mass flow m_f (in kg/s) allows the flue gases to reach the turbine inlet temperature TIT (in ^{O}C) before their expansion that produces a work W_{GT} (in MW). A nomenclature of all the considered parameters is given on Table 1. Among those both r_c and TIT have the greatest influence on the GT performances with highest values around 40 and 1600 ^{O}C respectively for the most efficient systems [8]. The main limitations come from the maximum stress that the different materials can endure under operating constraints. In the bottom Rankine cycle, the hot gases at the turbine exhaust temperature TET (in ^{O}C) are used in a heat recovery steam generator (HSRG) to produced superheated steam at high temperature for a steam turbine. Note that typical architectures

Fig. 1. Simplified diagram of a typical CCGT unit

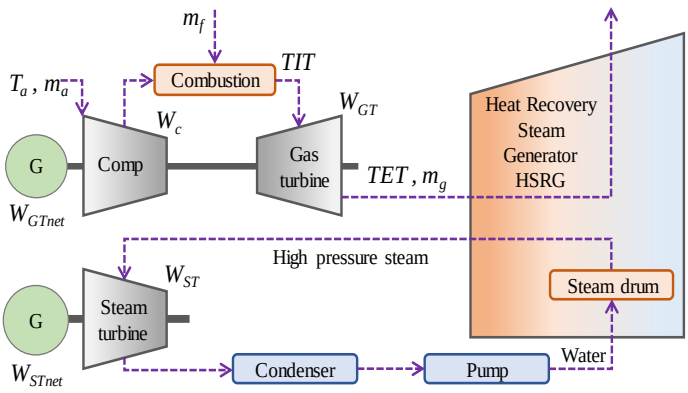


Table 1 Nomenclature for the CCGT model

T_a	ambient temperature	^{O}C
r_c	compression ratio	-
n_c	compressor isentropic efficiency	-
n_m	mechanical efficiency	-
m_a	air mass flow	kg/s
γ_c	air specific heat ratio in compression	-
W_c	compressor consumed power	MW
C_{pa}	specific heat of air	kJ/kg/K
n_{cc}	combustion chamber efficiency	-
λ	air/fuel ratio in combustion chamber	-
LHV	fuel lower heat value	kJ/kg
TIT	turbine inlet temperature	^{O}C
m_f	fuel mass flow	kg/s
m_g	flue gas mass flow	kg/s
C_{pg}	specific heat of flue gas	kJ/kg/K
γ_e	air specific heat ratio gas turbine expansion	-
n_{gt}	gas turbine isentropic efficiency	-
TET	turbine exhaust temperature	^{O}C
W_{GT}	shaft work of the gas turbine	MW
W_{GTnet}	gas turbine net output power	MW
Q_{GT}	heat supplied to the gas turbine	kJ/s
n_{gen}	electrical generator efficiency	-
η_{GT}	gas turbine efficiency	-
Q_{AV}	available heat in the flue gas	kJ/s
W_{STnet}	steam net output power	MW
W_{CC}	combined cycle net output power	MW
η_{CC}	combined cycle efficiency	-
SCE	specific carbon emission	kg/MWh
$\cdot_{,des}$	subscript for design values	-

for HSRG display up to three different pressure systems with corresponding low, intermediate and high pressure steam turbines. Thus the combine cycle (CC) units allow exploiting the available heat in the exhausts gas of heavy duty and enhancing the power generation efficiency from 40 % (η_{GT}) to 60 % (η_{CC}) [8]. Such units represent more than half the installed power generation capacity in Singapore and up to 95 % of mix in term of electricity generation [1]. Field data [13] and manufacturers websites identify five types of units on Jurong Island with the characteristics advised in Table 2 and distributed among four power plant as follows:

- Keppel Merlimau (KPL): Units A_1 and A_2, Units B_1 and B_2.
- Pacific Light Power (PLP): Units C_1 and C_2.
- SembCorp Cogen (SMB): Unit A_1, Units D_1 and D_2.
- YLT Seraya (SRY): Units C_1 and C_2, Units E_1 and E_2.

Table 2 Identified units on site

	Unit A	Unit B	Unit C	Unit D	Unit E
W_{GTnet} (MW)	240.0	165.0	260.0	256.0	292.0
W_{STnet} (MW)	160.0	85.0	140.0	152.0	115.0
W_{CC} (MW)	400.0	250.0	400.0	408.0	407.0
TET (^{O}C)	630.0	505.0	580.0	596.0	580.0
η_{GT} (%)	37.0	36.4,	38.5	37.8	40.7
η_{CC} (%)	32.0	53.1	58.8	56.7	59.0

2.2. Model Equations

This subsection describes the steady state equations commonly used in the literature for the modeling of CCGT units. Firstly the temperature at the output of the compressor $T_{c,out}$ (in ^{O}C) is computed as well as the corresponding work W_c (in MW) as in (1) and (2) [14]. Note that C_{pa} (in kJ/kg/K) is estimated at the average temperature between Ta and $T_{c,out}$ and referring to the tables for air properties.

$$T_{c,out} = T_a \times \left(1 + \frac{r_c^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{n_c}\right) \quad (1)$$

$$W_c = \frac{m_a \times C_{pa} \times \left(T_{c,out} - T_a\right)}{1000 \times n_m} \quad (2)$$

A complete combustion of pure methane is considered (3) [15]. Similarly to [10] the combustion equations has to be solved in order to estimate the fuel mass flow m_f (in kg/s) required to reach the turbine inlet temperature. The heat capacity of the flue gas C_{pg} (in kJ/kg/K) is also obtained after solving a second order polynomial equation with the molar air fuel ratio λ (4). C_{pg} is computed with the molar quantities and specific heat of each component in the flues gas and using the corresponding property tables. M_a, M_f, n_a, and nf denote the molar masses and quantities of fuel and air.

$$\begin{aligned} &CH_4 + 2.\lambda.\left(O_2 + 3{,}76.N_2\right) \rightarrow .... \\ &....CO_2 + 2H_2O + 2.\lambda.3{,}76.N_2 + \left(2.\lambda - 2\right).O_2 \end{aligned} \quad (3)$$

$$\begin{cases} m_a \times \left(C_{pg} \times TIT - C_{pa} \times T_{c,out}\right) = m_f \times \left(LHV \times n_{cc} - C_{pg} \times TIT\right) \\ \dfrac{m_f}{m_a} = \dfrac{M_f \times n_f}{M_a \times n_a} = \dfrac{M_f}{M_a \times \lambda} \\ C_{pg} = \dfrac{n_{CO_2}.C_{pCO_2} + n_{H_2O}.C_{pH_2O} + n_{N_2}.C_{pN_2} + n_{O_2}.C_{pO_2}}{n_{CO_2} + n_{H_2O} + n_{N_2} + n_{O_2}} = \ldots \\ \ldots\dfrac{C_{pCO_2} + 2.C_{pH_2O} - 2.C_{pO_2}.C_{pN_2} + \left(2 \times 3.76.C_{pN_2} + 2.C_{pCO_2}\right)\lambda}{1 + \left(2 \times 3.76 + 2\right).\lambda} \end{cases} \quad (4)$$

The computation of the GT output work W_{GT} (in MW) requires the estimation of the turbine exhaust temperature TET (in ^{O}C) as shown in (5) and (6). The different net output powers are then obtained following (7) and (8). Note that the HSRG is not fully modeled with the superheater, the economizer, the deaerator and condenser equations. In this work the steam turbine work output W_{STnet}, once the heat is extracted from the exhaust gas, is given as an input fixed parameter at design conditions. The specific CO_2 emissions SCE (in kg/MWh) consider the CO_2 molar mass M_{CO2} and the fuel molar quantity n_f (9).

$$TET = TIT \times \left(1 - n_{gt} \times \left(1 - \frac{1}{r_c^{\frac{\gamma_e - 1}{\gamma_e}}}\right)\right) \quad (5)$$

$$W_{GT} = \frac{m_g \times C_{pg} \times \left(TIT - TET\right)}{1000} \quad (6)$$

$$W_{GTnet} = W_{GT} \times n_m \times n_{gen} - W_c \quad (7)$$

$$W_{CC} = W_{GTnet} + W_{STnet} \quad (8)$$

$$SCE = \frac{n_f \times M_{CO_2} \times 3600}{1000 \times W_{CC}} \quad (9)$$

The heat supplied to the gas turbine Q_{GT} (in MW) [13] is used to compute the thermal efficiencies for the GT and the overall CC unit as follows:

$$Q_{GT} = \frac{m_g \times C_{pg} \times \left(TIT - T_{c,out}\right)}{1000} \quad (9)$$

$$\eta_{GT} = \frac{W_{GT\,net}}{Q_{GT}} \text{ and } \eta_{CC} = \frac{W_{CC}}{Q_{GT}} \quad (10)$$

2.3. Off-Design Operation

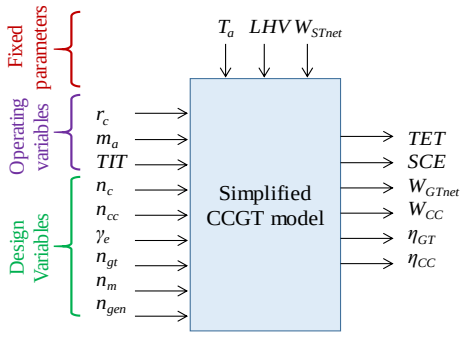
The previous equations describe the behavior of a typical CCGT at the design operating point. Such a representation with a more detailed model of the HSRG is used in [9] to perform the optimal sizing of a CCGT unit with regard to capital and operating costs. In power system studies the units never operate at the nominal point all along the considered time horizon [11]. It becomes more obvious with the totality of a variable load fed by a finite numbers of units as it is the case in the system modeled here. Thus a particular attention has to be paid to the off-design operating points and constraints. Especially the objective is to estimate the carbon emissions on the whole operating range of the units. The first off-design constraint refers to the fulfillment of the schoked conditions of the gas turbine with a constant swallowing capacity (11). Here the compression ratio is used instead of the compressor output pressure [11].

$$\frac{m_a \times \sqrt{TIT}}{r_c} = \frac{m_{a,des} \times \sqrt{TIT_{des}}}{r_{c,des}} \leftrightarrow G_{sh} = G_{sh}^{D} \quad (11)$$

Reference [16] establishes semi empirical equations for the behavior of the steam turbine under off-design conditions. The output power has to be corrected regarding the deviations of the flue gas mass flow m_g (in kg/s) and TET from their design values while considering η_c, the Carnot cycle efficiency (12). W_{STnet} then tends to decrease in the same time than Q_{AV} the heat available in the GT exhaust gas ((13) and (14)). Note that a more accurate representation of the off-design performances should take the deviations of the isometric efficiencies for the compressor and the gas turbine into account. It is not considered here and constant values are used.

$$\eta_c = 1 - \frac{T_a}{TET} \quad (12)$$

Fig. 2. Inputs/Outputs of the CCGT model



$$\frac{Q_{AV}}{Q_{AV,des}} = \left(\frac{TET}{TET_{des}}\right)^2 \left(\frac{m_g}{m_{g,des}}\right) \tag{13}$$

$$W_{STnet} = \frac{W_{STnet,des}}{n_m \times n_{gen}} \times \left(\frac{\eta_c}{\eta_{c,des}}\right) \times \ldots \\ \ldots \left(-0.32063 + 1.64575 \times \frac{Q_{AV}}{Q_{AV,des}} - \times 0.32872 \frac{{Q_{AV}}^2}{{Q_{AV,des}}^2}\right) \tag{14}$$

2.4. *Optimization for Design and Part-Load Operating Points*

The model developed for CCGT units displays the architecture shown on Fig. 2 with fixed parameters LHV = 48685 kJ/kg and T_a = 30 °C. The equations are validated with the injection of the parameters found in the literature for the PG9351FA unit [17]. Acceptable values of the error for the output variables are observed (Table 3). The next step consists in calibrating the model. The objective is to optimize all the inputs (i.e. both design and operating variables) in order to minimize the error between the outputs and the values advised by the manufacturers (noted with the superscript $.^D$). The design objective odj^D is then expressed as the sum of the errors regarding the values for TET, W_{GTnet} and η_{GT}. (15). The CC output power and efficiency are implicitly fit as the ST net power is entered as a fixed parameter here. For the off-design operations only the three operating variables are optimized. The unit operation consists in the control of the inlet guide vane of the compressor. The operation is based on IVG control up to 30 % air flow reduction with constant TET followed by a maximum air flow control [11]. The operating controls are then optimized considering their design values as upper bounds ($m_a > 0.7 \times m_{a.des}$). The off-design objective odj^{OD} aims at ensuring the shocked conditions, maintaining TET at the highest possible value while minimizing the error with the part load CC output power W_{CC}^{OD} .

$$obj^D = \frac{\left(W_{GTnet} - W_{GTnet}^D\right)^2}{{W_{GTnet}^D}^2} + \frac{\left(TET - TET^D\right)^2}{{TET^D}^2} + \frac{\left(\eta_{GT} - \eta_{GT}^D\right)^2}{{\eta_{GT}^D}^2} \tag{15}$$

$$obj^{OD} = \frac{\left(W_{CC} - W_{CC}^{OD}\right)^2}{W_{CC}^{OD^2}} + \frac{\left(TET - TET^D\right)^2}{{TET^D}^2} + \frac{\left(G_{sh} - G_{sh}^D\right)^2}{G_{sh}^{D^2}} \tag{16}$$

Table 3 Model validation

	Test Unit	Model	Error
W_{GTnet} (MW)	258.7	261.2	2.8 %
W_{STnet} (MW)	404.2	406.2	0.9 %
TET (°C)	617.5	408.0	0.4 %
η_{GT} (%)	37.0	38.0	2.7 %
η_{CC} (%)	57.8	56.8	1.7 %

The two optimization problems described above are solved using the Sequential Quadratic Programming embedded in the Matlab Optimization Toolbox [18]. For every design and every part load point of each unit the problems are run with a hundred starting points randomly taken within the bounds for the variables. Note that the off-design operation points are considered between 30 % and 100 % off the nominal load with a step of 5 %.

3. Units Compared Performances

3.1. *Design Results*

Table 4 summarizes the result obtained while running the model calibration in order to meet the performances advised by the manufacturer for all the five units. The variables in bold refer to the performances points used to calibrate the model while running the design procedures. The observed values are very close to the targeted data on Table I. On the whole the observed errors for the GT/CC works and TET are below 0.1 % while the maximum registered deviation for the efficiencies is only 0.6 %. The results also display the specific CO_2 emission estimated for the different units. The observed values are close as it is expected for units with similar performances in terms of efficiency. Those SCE values are consistent with the ones found in the literature and field data in Singapore. [19] refers to a combined power plant with a SCE of 437 kg/MWh while the official energy statistics estimates the total emissions at 431 kg/MWh [4]. They also comply the references concerning power systems studies [3] but remain higher than the best performances advised by the manufacturers – down to 325 kg/MWh [20].

Table 4 Design results

	Unit A	Unit B	Unit C	Unit D	Unit E
r_c	30.85	15.07	15.18	16.05	18.54
m_a (ks/s)	639.02	595.29	712.02	735.24	700.47
TIT (°C)	1390.00	1083	1261.2	1274.2	1296
n_c	0.95	0.91	0.8	0.83	0.9
n_{cc}	0.98	0.99	0.98	0.98	0.98
γ_e	1.29	1.31	1.3	1.32	1.29
n_{gt}	0.83	0.89	0.94	0.9	0.91
n_m	0.99	0.97	0.99	0.96	0.99
n_{gen}	0.99	0.99	0.99	0.97	0.99
W_{GTnet} (MW)	**239.99**	**165.00**	**259.99**	**256.00**	**291.98**
W_{STnet} (MW)	**399.99**	**250.00**	**399.99**	**408.00**	**406.98**
TET (°C)	**630.00**	**504.99**	**580.00**	**596.00**	**579.99**
η_{GT} (%)	**37.47**	**36.4**	**38.74**	**37.8**	**41.87**
η_{CC} (%)	**61.20**	**53.10**	**58.42**	**56.70**	**57.20**
SEC (kg/MWh)	392.21	457.33	411.78	424.53	421.19

Fig. 3. Performances of Unit A – *a)* Operating veraibles – *b)* efficiencisies

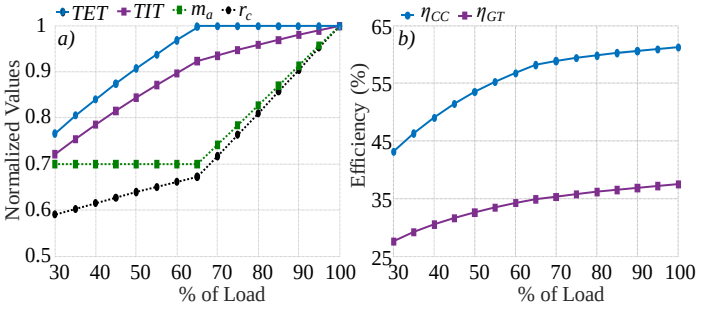
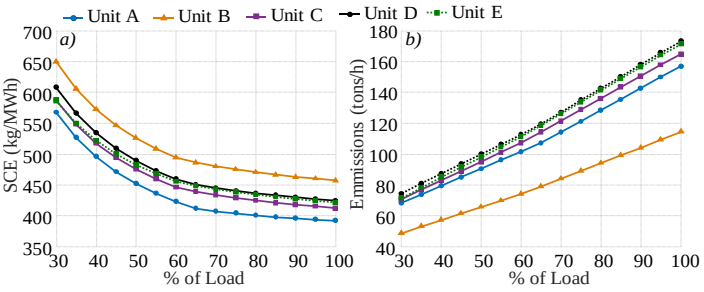


Fig. 4. Units performances – *a) SCE* – *b)* CO_2 emissions



3.2. *Off-Design Performance*

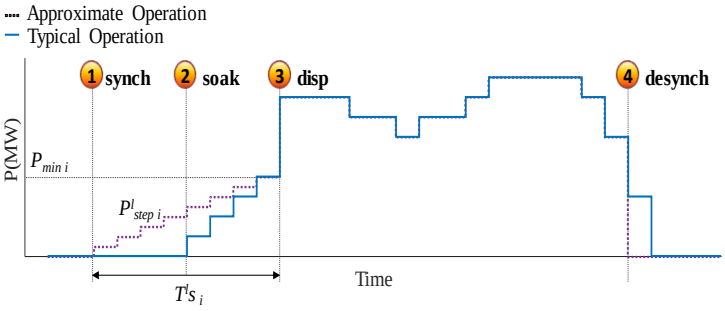
Fig. 3 shows the results obtained while running the off-design model for Unit A. Especially the operating variables variations are consistent with the considered control strategy [11]. *TET* remain constant while the air mass flow and the compressions ratio decrease to reach load points below 100 % of the nominal load (Fig. 3a). Once the lower bound for m_a is met (0.7), *TET* starts decreasing allowing the unit to work a lower output powers. Consequently, the CC efficiency drops around 20% for the unit works at 30 % of the nominal load. That strong decrement then justifies the consideration of part load performances when estimating the CO_2 emissions. The same observation can be made concerning the *SCE* which increase significantly when the output power decreases (Fig. 4a). Plotting the *SCE* allows ranking the units from the dirtiest (Unit B) to the cleanest (Unit A). Multiplying the specific emissions by the corresponding output power for the units allows estimating the net emissions in tons. Note that the emissions for Unit B are significantly lower as the rated power is less important than for the other technologies (250 MW Vs 400 MW). Fig. 4b highlights the fact that the emissions E_i for for the i^{th} unit can be expressed with a linear formulation (17) that will be used as an input parameter of the UC problem in the next section. Table V gives the values for the different a_i, b_i obtained with regression coefficients greater than 0.99.

$$E_i(W_{CC}) = a_i \times \frac{W_{CC}}{W_{CC,des}} + b_i \qquad (17)$$

Table 5 Linear coefficients for CO_2 emissions

	Unit A	Unit B	Unit C	Unit D	Unit E
a_i (in tons/h)	126.56	94.00	135.16	141.51	143.34
b_i (in tons/h)	27.67	19.13	28.00	29.59	26.94

Fig. 5. CCGT unit operation



4. Environmental UC

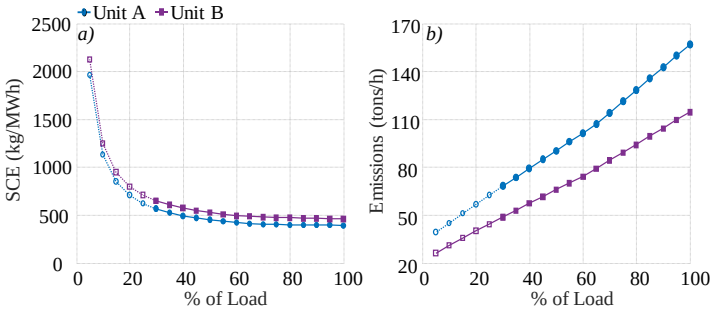
4.1. *CCGT units Operation*

Classical unit commitment aims at minimizing the cost of power generation while meeting the load and fulfilling the operating constraints [21]. The outputs are the schedules for a set of units U all along a time horizon T. Typical constraints for thermal units refer to the ramping limitation and minimum up/down times TU_i, TD_i defining the minimum duration for operating or non-operating phases [2]. More accurate approaches for thermal units consist in introducing the different phase for the operations as well as discriminating the start-up types [22]. When a CCGT unit is committed it first enters in a synchronization phase where the turbine shaft reaches the synchronization speed with the grid frequency (Fig. 5). Then a soak phase occurs with the output power that increases until the minimum operating point. The dispatchable phase then starts with the i^{th} unit power flowing within the range $P_{min,i}$ and $P_{max,i}$. When the unit is decommitted the power then returns to zero during the desynchronization phase. In this work both synchronization and soak phase are combined as the only available duration is the total start time (dotted line model in Fig. 5). The start-up time tends to increase with the prior reservation time during which the unit is turned off. Three start-up types l = hot, warm, cold are then commonly considered with associated duration Ts^l_i. Table 6 gives the operating variables for the considered units. Note that with a daily UC cold start (after more than 120 h off) is not considered. The load is considered with a 30 min discretization (Δt = 30 min) based on the data for the whole year 2016 [1]. With such a time step the ramp constraints do not need to be considered. It appears that the studied units can fully operates on their whole range during only one increment of time. Also the duration of the desynchronization phase is neglected as they are usually lower than the start-up phase time [23].

Table 6 Operating parameters

	Unit A	Unit B	Unit C	Unit D	Unit E
Ramp (MW/min)	50	30	10	22	10
TU_i , TD_i (min)	60	120	120	90	120
Ts^{hot}_i (min)	30	60	60	30	60
Ts^{warm}_i (min)	90	120	120	90	120
$P_{min,i}$ (MW)	80	75	120	142	122
$P_{max,i}$ (MW)	400	250	400	408	407

Fig. 6. Extended Performances – *a) SCE* – *b)* CO_2 emissions



4.2. *Problem Formulation*

A great number of methodologies are commonly considered to solve UC problems from priority listing to heuristics such as interior points or genetic algorithms [24]. In the present study the MILP is considered. That method gained great important in the past decades with the solvers improvement. The problem is formulated in MATLAB using YALMIP [25] and is solved using CPLEX 12.7.1. As already mentioned the objective of the UC here is to meet the load with the minimum amount of CO_2. The objective function is expressed as in (18) with the output power of the i^{th} unit at time t $P_{i,t}$ and the on/off status $u_{i,t}$. An exhaustive nomenclature of the symbols used in the UC is given in Table 7.

$$obj: \text{Min} \sum_{t\in T}\sum_{i\in U}\left(a_i \times \frac{P_{i,t}}{P_{max,i}} + b_i \times u_{i,t}\right) \times \Delta t \tag{18}$$

Note that the start-up costs (in tons of CO_2) don't appear in the function as it is commonly done in such studies [5]. The main reason is that the information is not available for the considered units. The models previously developed are used to estimate the start-up emissions while extending the off-design operations a lower values of the delivered power (Fig. 6). Especially the *SCE* drastically increase to reach values more than five time greater than the ones observed at nominal operating points (Fig. 6a). With a non-null duration for the start-up phase the corresponding CO_2 cost is computed when considering the output power trajectories from 0 to $P_{min,i}$ (Fig. 5). Consequently, and as expected, warm start-ups will be costlier with a longer duration. Then the first set of constraints attached to the UC problem describes the start up type selection based on the reservation times bounds T^d_l and T^u_l prior the start up. T^u_l = 8 h for hot start and T^u_l = 24 h for warm start. T^d_l = 0 h for hot start and T^d_l = 8 h for warm start. The constraints in (19) allows selecting the start-up type depending on the prior reservation time and (20) ensure that only one type is selected among S [21]. The unit immediately enters in start-up phase when $v_{i,t}$ is set to 1 and (21) and (22) ensure that only the appropriate start phase is considered. The power delivered by the unit i in start-up phase l at time t is computed by multiplying the step power $P^l_{step,i}$ by the duration of the phase before t ((23) and (24)).

Table 7 Nomenclature for the UC problem

$u_{i,t}$	unit i committed at time t
$us_{i,t}$	unit i in start phase at time t
$us^l_{i,t}$	unit i in start phase type l at time t
$ud_{i,t}$	unit i in dispatchable phase at time t
$v_{i,t}$	unit i started at time t
$v^l_{i,t}$	unit i started type l at time t
$w_{i,t}$	unit i shut down at time t
$P_{i,t}$	power generated by unit i at time t
$Ps_{i,t}$	power generated by unit i in start phase at time t
a_i , b_i	coefficients for the CO_2 emissions of unit i
$P_{min\ i}$	minimum power of unit i in dispatchable phase
$P_{max\ i}$	maximum power in dispatchable phase of unit i
TU_i, TD_i	minimum up/down time of unit i
Ts_l	start up type l duration
D_t	demand load at time t
$P^l_{step\ i}$	power increment of unit of during start up type l
T^d_l, T^u_l	bounds for start up type l intervals
T, U, S	set of time steps, units and start-up types

$$v^l_{i,t} \le \sum_{t-T^u_l+1}^{t-T^d_l} w_{i,t} \tag{19}$$

$$v_{i,t} = \sum_{l\in S} v^l_{i,t} \tag{20}$$

$$us^l_{i,t} = \sum_{t-Ts_l+1}^{t} v^l_{i,t} \tag{21}$$

$$us_{i,t} = \sum_{l\in S} us^l_{i,t} \tag{22}$$

$$Ps_{i,t} = \sum_{t-Ts_l+1}^{t}\sum_{l\in S} us^l_{i,t} \times P^l_{step\,i} \tag{23}$$

$$Ps_{i,t}, \le P_{max,i} \times us_{i,t} \tag{24}$$

Constraints in (25) and (26) introduce the bounds for the units power considering both dispatchable and start-up phases while (27), (28) and (29) represent the typical tight formulation for the minimum up/down time and power balance [2]. The last constraints refer to logical operations of the units that cannot start-up or shut down at the same time or have to operates either in patchable or start-up phase. ((30), (31) and (32)) [2].

$$ud_{i,t} \times P_{min,i} + Ps_{i,t} \le P_{i,t} \tag{25}$$

$$P_{i,t} \le ud_{i,t} \times P_{max,i} + Ps_{i,t} \tag{26}$$

$$u_{i,t} \ge \sum_{t-TU_i+1}^{t} v_{i,t} \tag{27}$$

$$1 - u_{i,t} \ge \sum_{t-TD_i+1}^{t} w_{i,t} \tag{28}$$

$$\sum_{i \in U} P_{i,t} = D_t \tag{29}$$

$$u_{i,t-1} - u_{i,t} + v_{i,t} - w_{i,t} = 0 \tag{30}$$

$$u_{i,t} = us_{i,t} + ud_{i,t} \tag{31}$$

$$v_{i,t} + w_{i,t} \leq 1 \tag{32}$$

4.3. *Obtained Results*

At first the simulation is performed without considering the constraints for the start-up phases. Before commenting the results, it is necessary to recall the units ranking from the cleanest to the dirtiest. That ranking is establishes by plotting the specific carbon emissions for the different technologies in Fig. 4 – 1) units A, 2) units C, 3) units E, 4) units D and 5) units E. The obtained results confirm that the cleanest units (A and C) are run in priority all along the day while the dirtiest (D and B) are much less solicited (Fig. 7). Without considering the start-up phases some units displays multiple starts and stops during the simulated day which is unrealistic and would dramatically increases the costs of generation (Fig. 7a and Fig. 7c). Introducing the start-up constrains solves the problem by avoiding multiple commitments within the same day for the dirtiest units as shown on Fig. 8. It can be noticed that the base load is still fed by the cleanest units while the peak load is meet with the commitment of units D in PLP and E_1 in SRY (Fig. 8b and Fig. 8d). Note that the dirtiest units B in KPL are not used at all. The simulation allows computing the grid emission factor (*GEF*) which represent the amount of CO_2 per kWh generated. The obtained values is 0.406 kg/kWh. That is consistent with the recorded *GEF* of 0.431 kg/kWh for the year 2016 in Singapore which also considers grid losses as well as 5 % of the electricity mixed coming from oil steam plants [1].

Fig. 7. UC without start-up phases – *a)* KPL – *b)* PLP – *c)* SMB – *d)* SRY

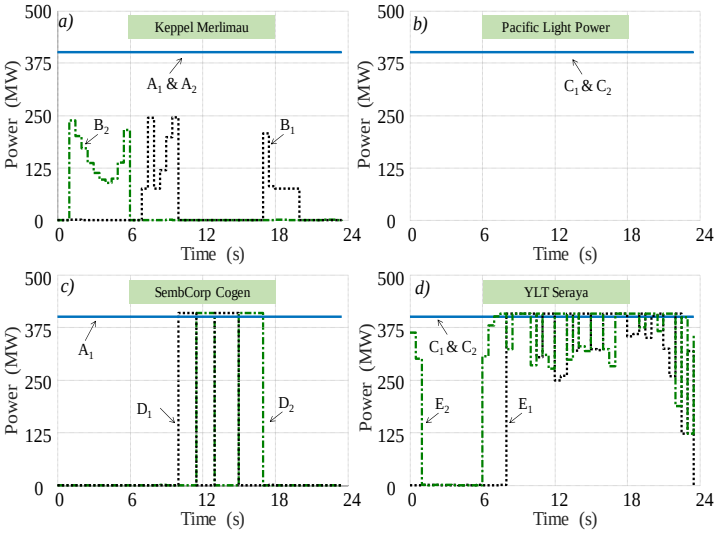
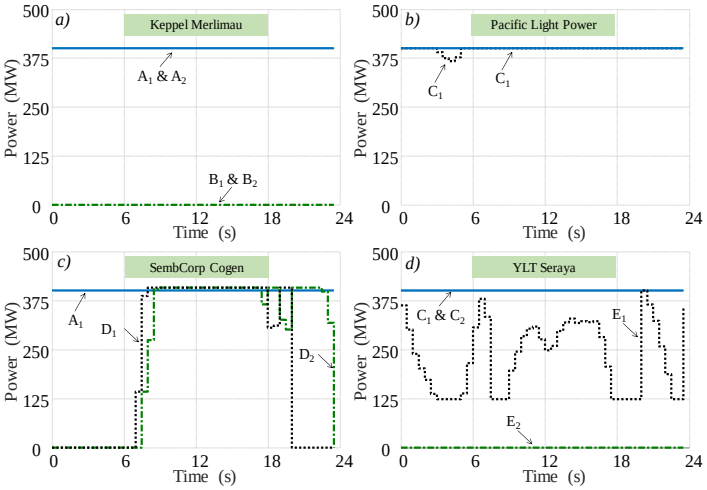


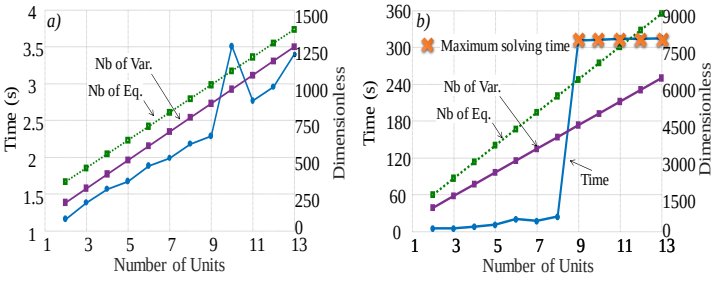
Fig. 8. UC without start-up phases – *a)* KPL – *b)* PLP – *c)* SMB – *d)* SRY



4.4. *Computational Performance*

This subsection brings some comments about the computational times for the simulations. As already mentioned the problem model is formulated in MATLAB using YALMIP [24] and is solved using CPLEX 12.7.1 with 16 threads in parallel. The computer has a RAM of 16 GB with a 3.2 GHz processor. Fig. 9 plots the computational performances (time, numbers of variables and equations) with an increasing number of units considered in the power dispatch while adapting the level of load demand to meet. Without considering any start-up phase the computational time remains very low (Fig. 9a). Introducing start-up constraints significantly increases the number of variables leading to more complex problems to solve. An arbitrary limit of solving time is set to 5 min and is reached for 9 units or more. The attention is then paid to the gap value to estimate the optimality of the return solutions as the error between the solution of the relaxed linear problem and the MILP solution (9 units, 1.3 % for 10 units, 0.7 % for 12 units, 0.4 % for 12 units, 0.6 % for 13 units). The observed times are much greater than the one advised in for the same type of studies and up to 180 units [21]. Here the complexity comes from the multiple equivalent optimal solutions. The non-unicity of the solution is obvious with identical units in game. Also, as observed in [26], the load level here does not require the commitment of all the units at any moment which hardens the solving by increasing the number of equivalent nodes in the branch and bounds MILP algorithm.

Fig. 9. Computation info – *a)* no start-up phase – *b)* with start-up phases



5. THIGHTENED FORMLATION

5.1. Motivations

The environmental power dispatch described in the previous sections aims at being integrated in wider power systems studies. Grid constraints might be considered as well as expansion planning problems involving energy storage systems (ESS), renewables or controllable loads [12]. Thus the number of equations and variables would tend to increase and might lead to prohibitive computational times. With the relatively simple previous problem important solving times are observed with gap values around 1 % after 5 min of resolution (arbitrary limit). Thus simplifications should be made in order allow the computation of more complex studies. Especially the attention is paid to the reduction of variables and constraints to described the afore mentioned power dispatch.

5.2. From Units Models to Plants Representation

The main simplification lies on the aggregation of the different units within the same power plant. Then the power dispatch will consider four power plants instead of thirteen independent units. The cost function (i.e. the CO2 emissions) of each plant i takes account of the a_k, b_k coefficients for the corresponding individual units. A traditional priority list (PL) algorithm is used to determine the commitment of every unit depending on the total power delivered by the plant - with the units ranked from the cleanest to the dirtiest (Fig. 4). The main limitation of such PL approaches is that they do not consider any runtime and start up phases constraints [27]. Here those constraints are considered only for the first unit switched on when the plant is committed. Eventually the emissions cost function for the plant displays the shape on Fig. 10. The steps occurring when additional units are committed implicitly correspond to the attached start-up costs.

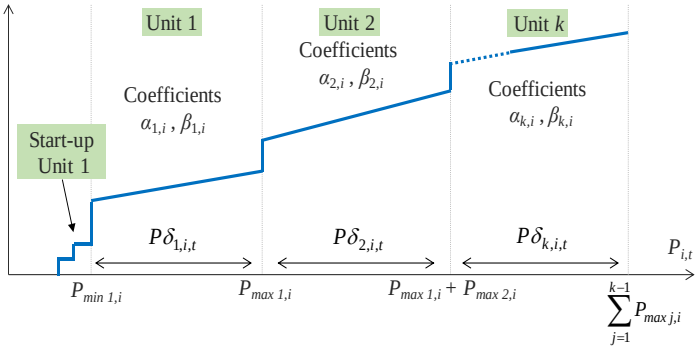
5.3. Formulation

The MILP formulation has to be modified in order to take the afore mentioned simplifications into account. The considered emissions function for each plant is similar to the traditional piecewise linear cost curve for generator in UC [28]. However, the discontinuities occurring when additional units are committed require the introduction of additional binary variables and specific constraints. A list of the added symbols (variables and parameters) is given in Table 8. The first added set of constraints ensure that the plant only works in one specific interval. Eq (33) and (34) allows applying the appropriate bounds while (35) allows identifying the operating interval if the corresponding plant i is committed ($u_{i,t} = 1$). The power delivered in the specific operating interval equals to the power generated by the power plant $P_{i,t}$ (36). Note that in the present modified formulation i refers to the different plants and not the units as before.

$$u\delta_{k,i,t} \times P\delta^{d}_{k,i} \le P\delta_{k,i,t} \tag{33}$$

$$P\delta_{k,i,t} \le u\delta_{k,i,t} \times P\delta^{u}_{k,i} \tag{34}$$

Fig. 10. Units aggregation in power plant i



$$\sum_{k=1}^{K_i} u\delta_{k,i,t} \le u_{i,t} \tag{35}$$

$$\sum_{k=1}^{K_i} P\delta_{k,i,t} = P_{i,t} \tag{36}$$

The objective function in (18) is modified and consists in summing the emissions for all the intervals of every plants ($U = 4$ plants here) (37). The objective function in (18) is modified and consists in summing the emissions for all the intervals of every plants ($U = 4$ plants here) (37). The slope coefficients $\alpha_{k,i}$ are equivalent to the ones associated to the operating units in interval k (a_k). The coefficients $\beta_{k,i}$ need to be computed by iteratively including the b_k coefficient of the committed units with $P_{max\ k,i}$ the maximum power of unit k in plant i (38). Note that the problem simplification is enhanced by considering only one type of start-up when plants are switched on.

$$obj: \sum_{t\in T}\sum_{i\in U}\sum_{k\in K_i}\left(\frac{\alpha_{k,i}}{P_{maxk,i}} \times \left(P\delta_{k,i,t} - P\delta^{d}_{k,i}\right) + \beta_{k,i} \times u\delta_{k,i,t}\right) \times \Delta t \tag{37}$$

$$\beta_{k,i} = b_k + \sum_{j=1}^{k-1}\left(a_j + b_j\right) \tag{38}$$

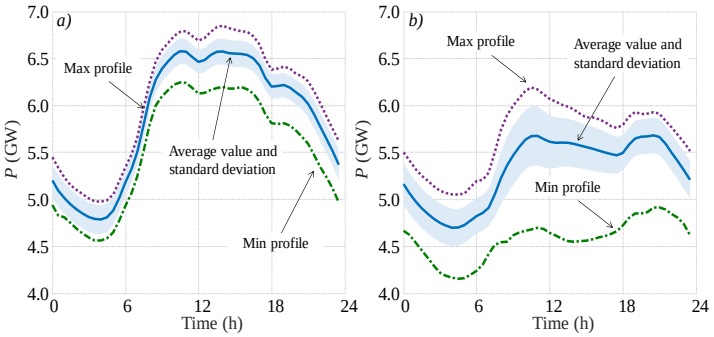
5.4. Simulations

Once more the tightened problem is implemented using YALMIP. The formulation displays a reduced number of variables and constraints compared to the previous problem with multiple start up types and thirteen independent units (Table 9). Especially the number of constraints is divided by four. The problem is run for a hundred of random days taken from the Singapore load profile for the year 2016 whose

Table 8 Added symbols in the tightened problem

$P\delta_{k,i,t}$	power in interval k of plant i at time t
$u\delta_{k,i,t}$	plant i operating in interval k at time t
$\alpha_{k,i}, \beta_{k,i}$	Emissions coefficients of interval k of plant i
$P\delta^{d}_{k,i,}, P\delta^{u}_{k,i}$	lower/upper limits of interval k of plant i
$us^{l}_{i,t}$	unit i in start phase type l at time t
K_i	number of units in plant i

Fig. 11. Singapore Electrical Demand – *a)* Week days – *b)* Week ends



characteristics are presented on Fig. 11. The daily profiles are consistent all along the year with very low standard deviations observed. Also note that the base load is important representing more than 70 % of the peak values. To enhance the diversity of test case the load demand D_t is adapted for every run as in (39). A uniform distribution **U** randomly adjusts the load level between 30 % and 100% of the maximum value for the considered set of units (maximum of 4.9 GW here).

$$D_t \leftarrow \sum_{i \in U} \sum_{k \in K_i} P_{max\,k,i} \times \frac{D_t}{\max(D_t)} \times \mathbf{U}(0.3,1) \tag{39}$$

The tight problem performance is compared to the one for the simple model where no start-up phase is considered with thirteen independent units. The multiple-start up types problem is used as a reference case to compute the average relative error over the runs. The average computational time to reach the convergence (gap below 10^{-6}) with the tight model is significantly reduced to 30 s even if it remains longer than the simple model computation (4s previously observed). The relative error (average after 100 runs) is very low below at 0.69 %. That value can be explained by the relatively low contribution of start-up emissions compared to the CO2 generated in dispatchable phase. For instance, for Unit A a hot start-up would generate 16 tons of carbon dioxide which represent 20 % of the hourly emissions at nominal point (154 tons/h). As expected the deviations with the reference problem tend to increase with higher numbers of start-ups in the considered commitment profiles but remains below 1 % (Table 10).

Table 9 Tightened model performance

	Multiple Start-up Model			Tight Problem	
Continuous variables	1248			384	
Binary variables	5616			960	
Constraints	8867			2060	
CPU Time	> 300s			30s	
Error (%)	-			**0.69**	
Number of starts	1	2	3	4	≥ 5
Error (%)	**0.28**	**0.62**	**0.73**	**0.81**	**0.84**

Fig. 12. Tight model performance – *a)* KPL – *b)* PLP – *c)* SMB – *d)* SRY

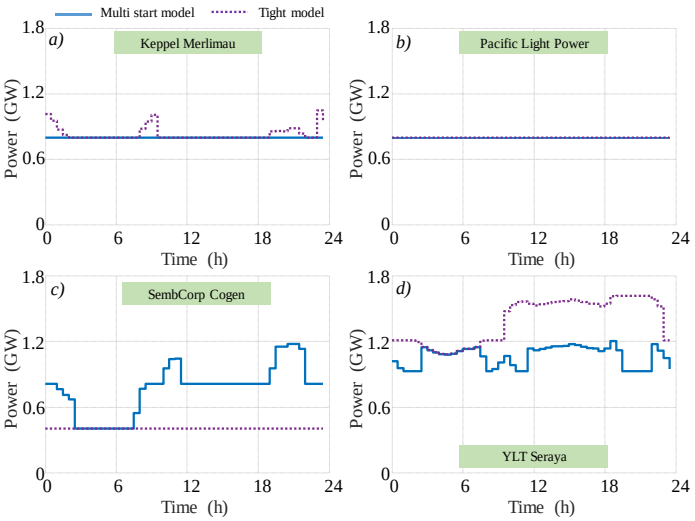


Fig. 12 shows the results obtained while running the complete multi-starts and the tight models. For the tested day the computed error is only 0.3 % even if the plant generation profiles can display strong deviations. For both Keppel and Pacific Light Power plants the two solutions are equivalent with the cleanest units working at full load all along the day (units A and C at 2×400 MW). For the tight formulation Sembcorp generation is limited to the power produced by the cleanest units on site (units A at 2×400 MW). The surplus of energy is provided by the units E in YLT power where it is provided by units D in Sembcorp plant while running the multiple start-up types model. At the end the result in term of CO2 emissions are close as the performance of units D and E are very close (Fig. 4).

CONCLUSIONS

The CCGT model described in this paper aims at estimating the CO_2 emission coefficients that are used in an environmental UC. Compared to conventional UC studies with CO_2 costs taken from the literature, the methodology developed here allows the representation of specific units. Especially the start-up emission doesn't have to be guessed and is estimated by introducing the start-up phase with a specific duration. Despite the sparsity of the information available from unit manufacturers, the calibrated models display very good agreement with real data in terms of GEF. Finally, a tightened formulation of the power dispatch allows reduced computational times while maintaining the same performance compared to a multiple start-up types model. With the individual power plants considered as aggregations of different units that approach is appropriate if grid constraints would have to be considered as well as loss computation. Indeed, the global objective of the study is to model the power system management on Jurong Island in Singapore. Further work should then focus on the grid modeling that would lead to additional operating constraints to be fulfilled in UC.

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