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# Analytical modeling and design of high temperature superconducting machine

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# ANALYTICAL MODELING AND DESIGN OF HTS MACHINES

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High Temperature Superconducting Motor School

August 31 – September 4, 2020, Nancy, France

# Presentation Outline

- ❑ **Speaker Biography (short)**
- ❑ **Topology of the studied HTS machine**
- ❑ **2D (Semi)-Analytical Model : assumptions, equations, solution...and discussion**
- ❑ **Back EMF and Torque computation**
- ❑ **Results: Matlab Programm, comparison with FEM**
- ❑ **Multi-Objective Design Optimization with GA:  
P = 3 MW, N = 3000 rpm, PTM > 20 kW/Kg  
(Electrical aircraft application, or other...)**

**Discussions and questions during the presentation**

# Speaker Biography



**1994:** Msc. in electrical engineering, Paris 6, France + “agrégation” in applied physical sciences (to teach in high school),

**2003:** Ph.D. degree in electrical engineering, Nancy, France.

*“Modeling and control of a Synchronous Reluctance Machine with magnetic saturation effects”*

**2016:** HDR (Habilitation) degree in electrical engineering, Nancy, France (confers the right to be sole advisor of a PhD student).

*“Contribution to the analytical modeling of electromagnetic actuators”*

**Since 2007 :** Associate Professor in electrical engineering at Nancy

**Before 2007:** High school teacher

## Teaching topics (Master degree):

- ◆ Electrical machines: modeling, design and control.
- ◆ AC electrical power network: modeling, power quality, protection.
- ◆ High power wind turbine: electrical part (machine, power electronics, control).

## Research topics:

- ◆ 2D and 3D analytical modeling of electrical machines for their design
- ◆ Contactless torque transmissions by magnetic gears and couplers
- ◆ Applied superconductivity in electrical engineering

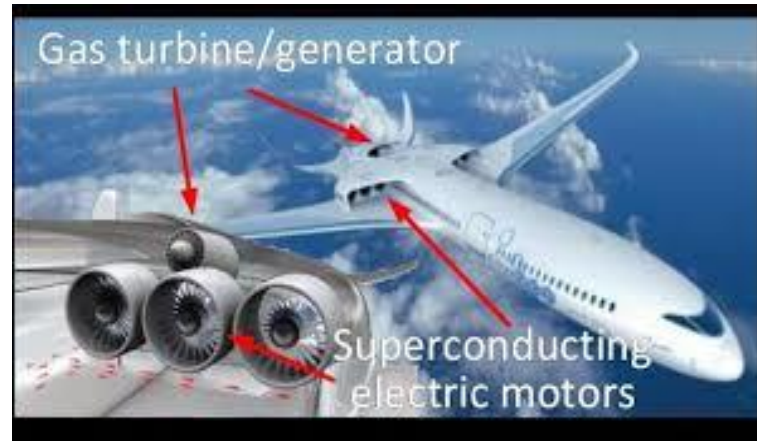
Author and co-author of more than 100 research papers in peer-reviewed journals and international conferences : <https://cv.archives-ouvertes.fr/thierry-lubin>

# Why studied HTS machines?

For high power electrical machines (MW)  
→ Lightweight and better efficiency

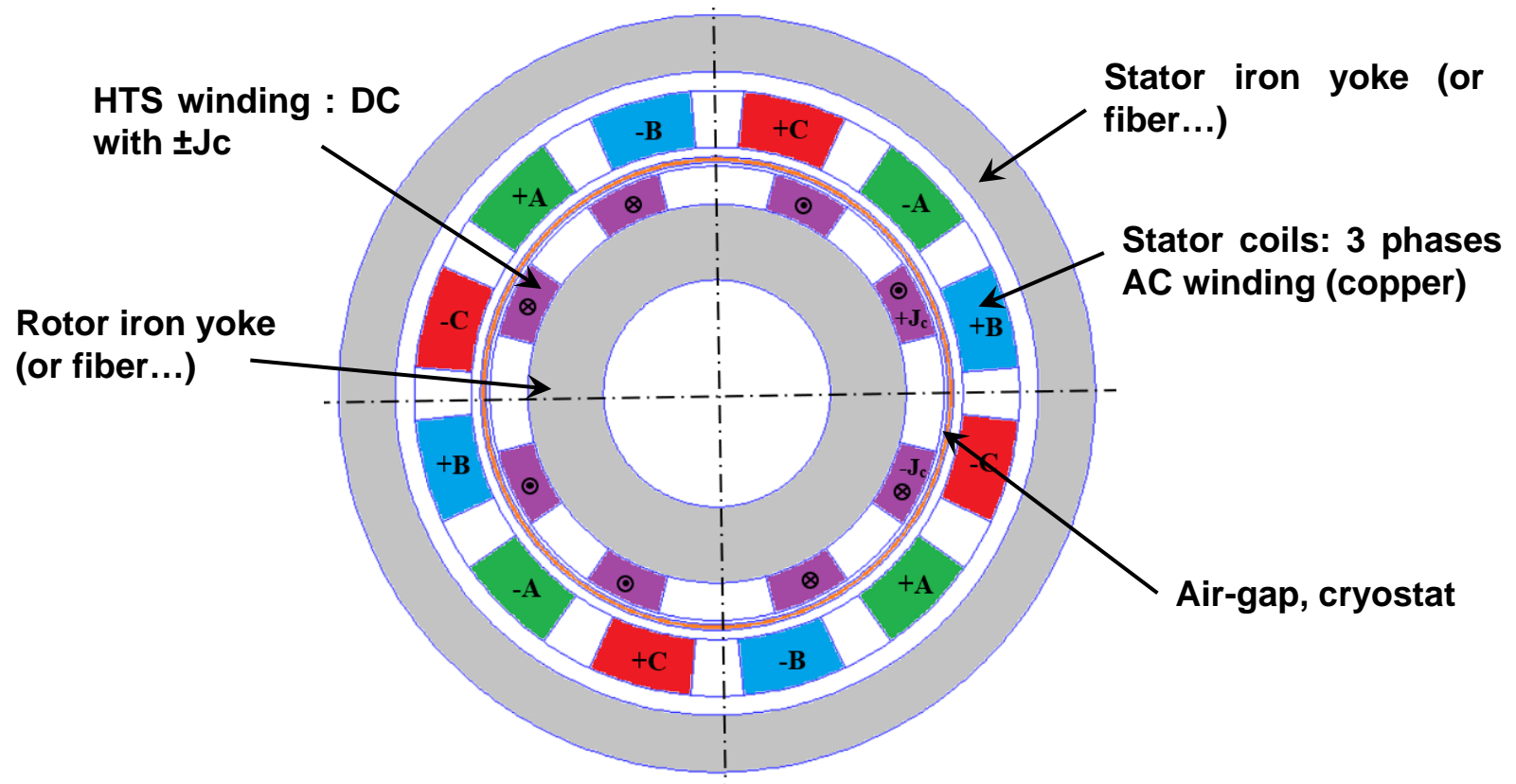


**Off-Shore Wind turbine  
EcoSwing Project (2019)  
3.6MW ; 15 rpm  
(- 40% weight compare to  
conventional system)**



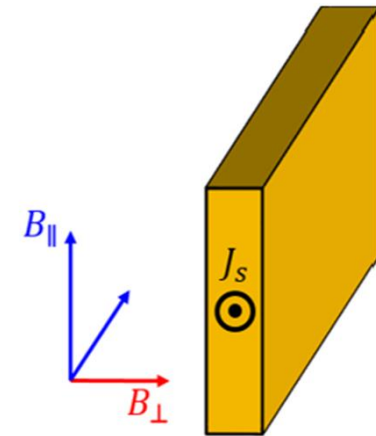
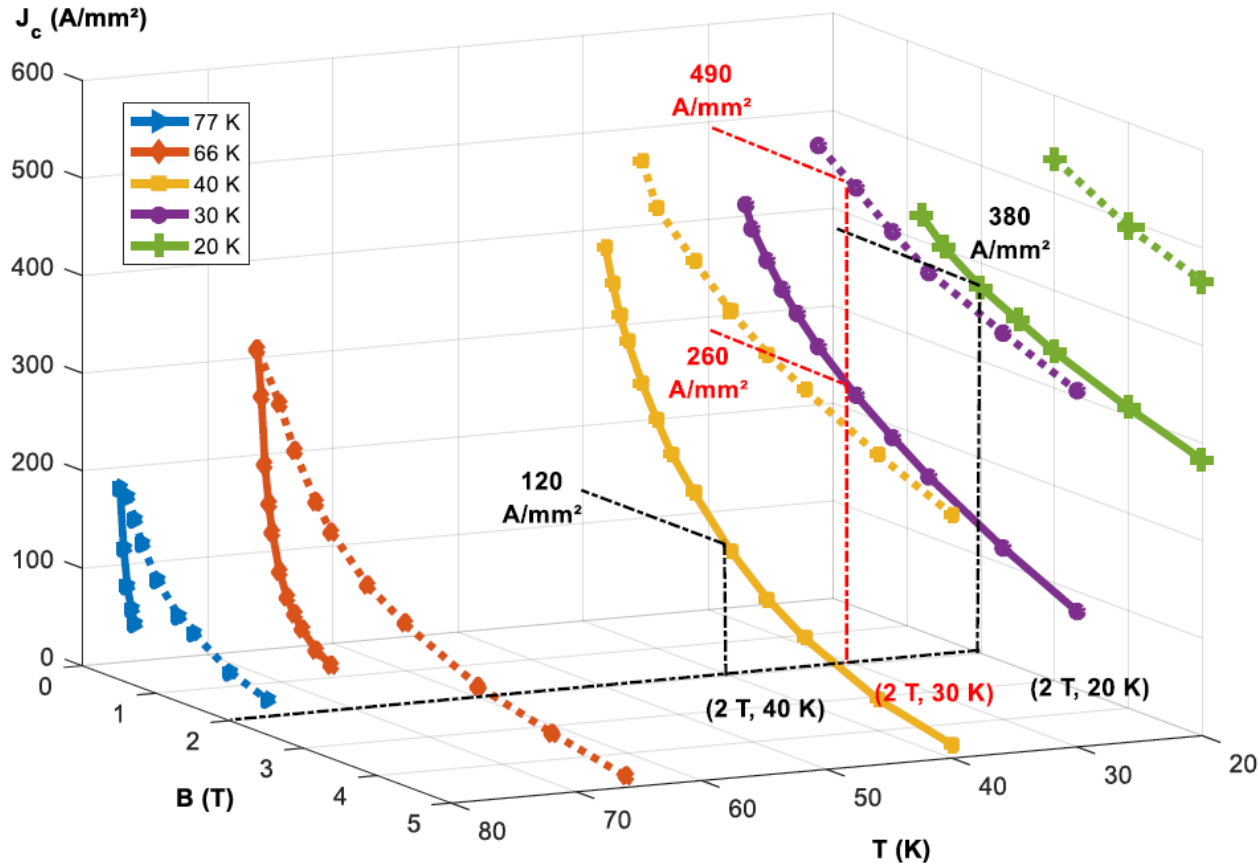
**More Electrical Aircraft  
Motor propulsion  
1-5 MW ; 3000-6000 rpm  
Power to Mass ratio > 20 kW/kg  
Impossible with conventional  
technology (5 kW/Kg, PMs, copper)**

# Topology of the studied HTS machine



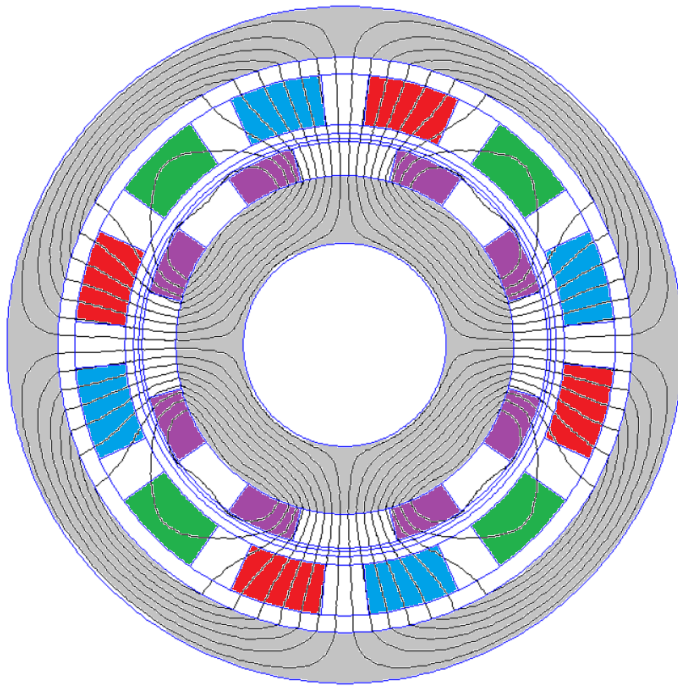
- ◆ Superconducting Synchronous Radial flux machine (axial flux is possible but more complicated to model due to the 3D effect....)
- ◆ Slotless machine: inductor with HTS winding (not possible with copper due to the Joule losses, sometimes with PM...)
- ◆ Rotor HTS winding or bulk:  $\pm J_c$  with  $J_c = 200 \text{ A/mm}^2$  at 20-30K (more complex model next days, the cooling is not studied here)
- ◆ Three phase stator AC winding:  $J_{s_{\text{rms}}} = 20 \text{ A/mm}^2$  (possible with water or oil forced cooling, look at electrical vehicle for example, Litz wire to reduce AC losses)

# HTS wires : $J_c(B,T)$ characteristics

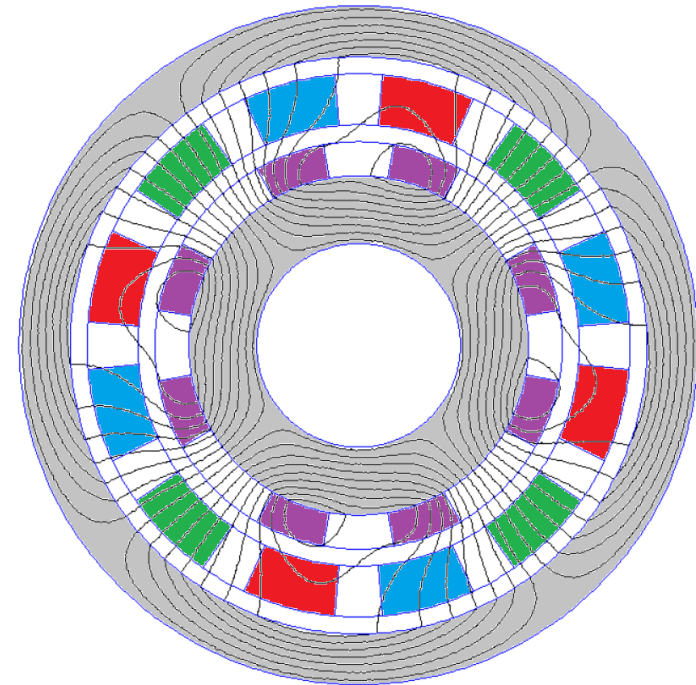


◆  $J_c = 200 \text{ A/mm}^2$  at 30K is correct but this should be include in the design procedure (not done here)... Perhaps the maximal constraint is on the end-winding (3D model.....)

# What we want to obtain « by hand »!



No-load: back EMF computation



Full-load (90°): Torque computation

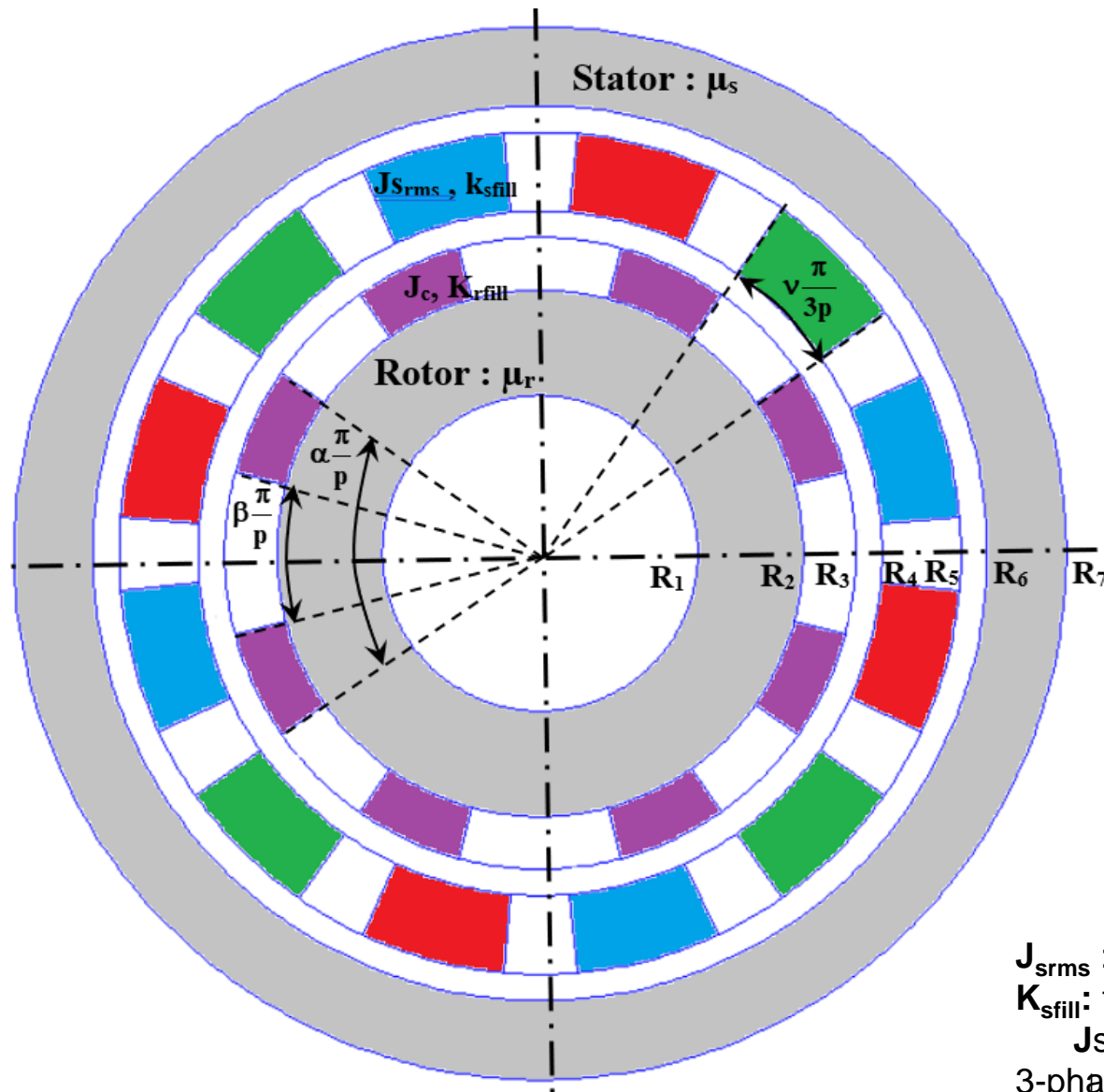
**A very fast and accurate model is needed if we want to use it with an optimization procedure : Analytical model  
First step design (cooling, ac losses....)!**

**We will consider all the source terms (rotor and stator) in the analytical model but this is not mandatory, it depends on what we want to study!**

- The torque can be computed by the Laplace force by only knowing the rotor magnetic field distribution.
- But if we want to know the armature reaction (harmonic terms) on the HTS winding (AC losses computation...), we have to compute it!



# Physical and Geometrical parameters



$p$  : pole-pairs number

$L$  : axial length

$R_1$ : inner radius of the rotor

$a = R_2 - R_1$  : thickness rotor yoke

$b = R_3 - R_2$  : thickness HTS winding

$c = R_4 - R_3$  : air-gap thickness

$d = R_5 - R_4$  : thickness stator winding

$e = R_6 - R_5$  : 2<sup>nd</sup> air-gap thickness

$f = R_7 - R_6$  : thickness stator yoke

$\alpha$  : outer opening of HTS coils

$\beta$  : inner opening of the HTS coils

$v$  : opening of the stator coils

$$0 < \alpha; \beta; v < 1$$

$\mu_s$ : stator yoke relative permeability

$\mu_r$ : rotor yoke relative permeability

$J_c$  : critical current of the HTS coils

$K_{rfill}$ : fill factor HTS coils

$$J_c = 200 \text{ A/mm}^2; K_{rfill} = 0,7$$

$J_{srms}$  : stator RMS current density

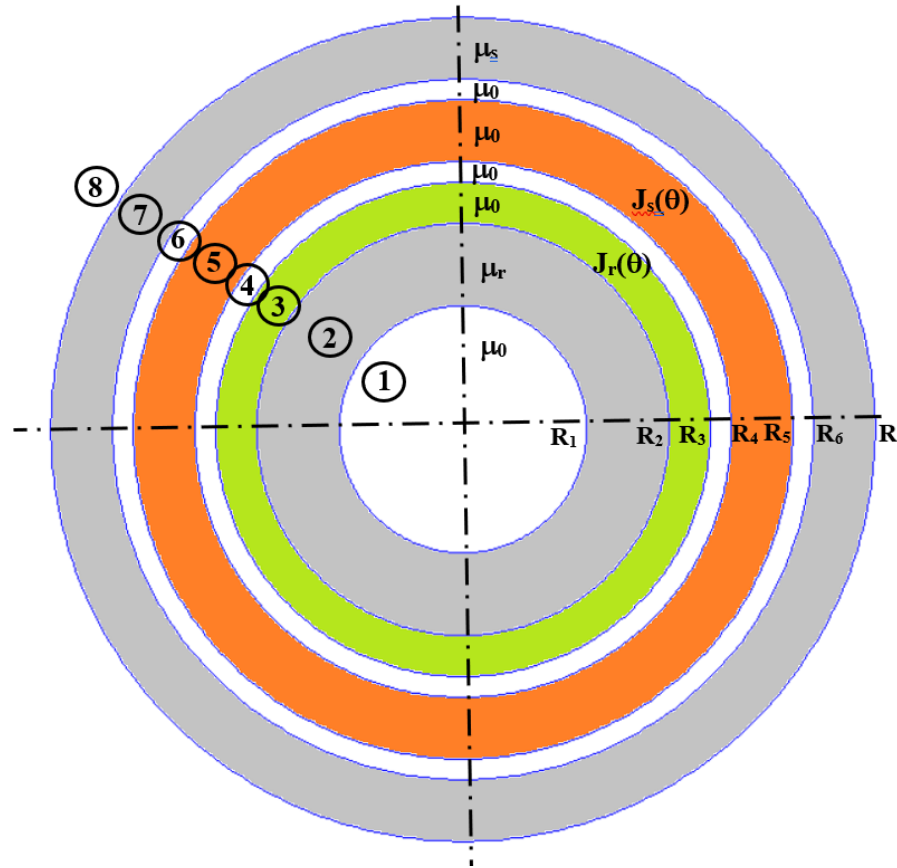
$K_{sfill}$ : fill factor stator coils

$$J_{srms} = 20 \text{ A/mm}^2; K_{sfill} = 0,5$$

3-phase distributed stator winding

# Model and Assumptions

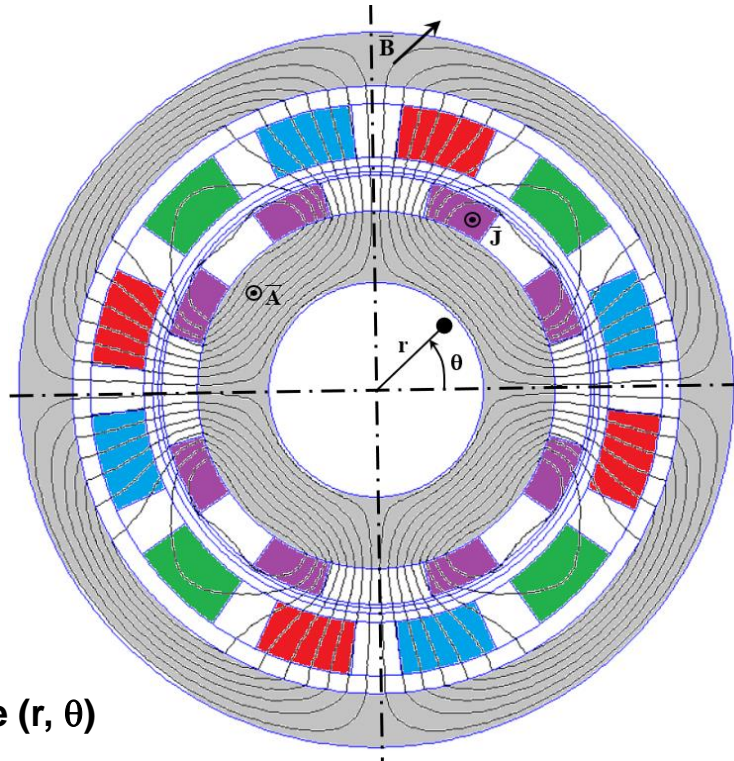
8 homogeneous cylindrical regions with periodicity of  $2\pi/p$



- Perfect cylindrical geometry: cylindrical coordinates  $(r, \theta)$
- End-effects are neglected: 2D analytical model
- Constant permeability for the iron parts: magnetic saturation is not taken into account (be careful that  $B < 2T$  in the iron for the simulation!!)
- Current in the coils is imposed (stator and rotor): equivalent current density distribution  $J_s(\theta)$  and  $J_r(\theta)$

# 2D field problem in cylindrical coordinates

- ◆ Current density  $\mathbf{J} = (0, 0, J_z)$
- ◆ Vector magnetic potential  $\mathbf{A} = (0, 0, A_z)$
- ◆ Magnetic field strength  $\mathbf{H} = (H_r, H_\theta, 0)$
- ◆ Flux density  $\mathbf{B} = (B_r, B_\theta, 0)$



Motor section in the plane  $(r, \theta)$

# Maxwell's equations and Boundary conditions at radius $R_i$

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J} \\ \nabla \cdot \mathbf{B} &= \mathbf{0} \end{aligned} \quad \text{with} \quad \begin{aligned} \mathbf{B} &= \mu_0 \mu_r \mathbf{H} \quad (\text{linear and no magnet}) \\ \mathbf{J} &: \text{current density in the coils} \end{aligned}$$

Magnetic vector potential formulation :  $\mathbf{A} = A_z(r, \theta) \mathbf{u}_z$

$$\nabla \cdot \mathbf{B} = \mathbf{0} \rightarrow \mathbf{B} = \nabla \times \mathbf{A} \quad \text{with} \quad \nabla \cdot \mathbf{A} = \mathbf{0}$$

$$\Delta A = -\mu_0 \mathbf{J} \quad (\text{regions 3 and 5})$$

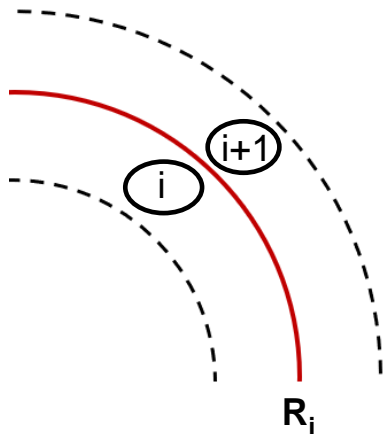
$$\Delta A = \mathbf{0} \quad (\text{for the other regions})$$

- ◆ Continuity of the normal component of the flux density:

$$B_{r_i}(R_i, \theta) = B_{r_{i+1}}(R_i, \theta) \rightarrow A_i(R_i, \theta) = A_{i+1}(R_i, \theta)$$

- ◆ Continuity of the tangential component of the magnetic field:

$$H_{\theta_i}(R_i, \theta) = H_{\theta_{i+1}}(R_i, \theta) \rightarrow \frac{1}{\mu_i} \left. \frac{\partial A_i}{\partial r} \right|_{R_i} = \frac{1}{\mu_{i+1}} \left. \frac{\partial A_{i+1}}{\partial r} \right|_{R_i}$$



# Partial Differential Equations to solve

$$\frac{\partial^2 A_3}{\partial r^2} + \frac{1}{r} \frac{\partial A_3}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A_3}{\partial \theta^2} = -\mu_0 J_r(\theta) \quad \text{for region 3 (HTS windings)}$$

$$\frac{\partial^2 A_5}{\partial r^2} + \frac{1}{r} \frac{\partial A_5}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A_5}{\partial \theta^2} = -\mu_0 J_s(\theta) \quad \text{for region 5 (stator windings)}$$

$$\frac{\partial^2 A_i}{\partial r^2} + \frac{1}{r} \frac{\partial A_i}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A_i}{\partial \theta^2} = 0 \quad \text{for other regions (i = 1, 2, 4, 6, 7 and 8)}$$

## ◆ Source terms: Stator and rotor current density distribution

$$J_r(\theta) = \sum_{n=1,3,5\dots}^N J_m \sin(np(\theta + \delta)) \quad J_s(\theta) = \sum_{n=1,3,5\dots}^N J_{sn} \sin(np\theta)$$

With  $J_m$  and  $J_{sn}$  to be determinate (next slides),  $\delta$  is the load angle attached to the rotor

## ◆ Periodic problem with a period equal to $2\pi/p$ , the solution in each region can be write as

$$A_i(r, \theta) = \sum_{n=1,3,5\dots}^N A_{in}(r) \sin(np(\theta + \alpha))$$

# Complex notation (more compact)

$$A_i(r, \theta) = \sum_{n=1,3,5\dots}^N A_{in}(r) \sin(np(\theta + \alpha)) \quad \rightarrow \quad A_i(r, \theta) = \text{Im} \left( \sum_{n=1,3,5\dots}^N \underline{A}_{in}(r) e^{jnp\theta} \right)$$

The PDE (region 3) becomes an ODE complex equation (Euler):

$$\frac{\partial^2 A_3}{\partial r^2} + \frac{1}{r} \frac{\partial A_3}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A_3}{\partial \theta^2} = -\mu_0 J_r(\theta) \quad \rightarrow \quad r^2 \frac{d^2 \underline{A}_{3n}}{dr^2} + r \frac{d \underline{A}_{3n}}{dr} - (np)^2 \underline{A}_{3n} = -\mu_0 r^2 \underline{J}_m$$

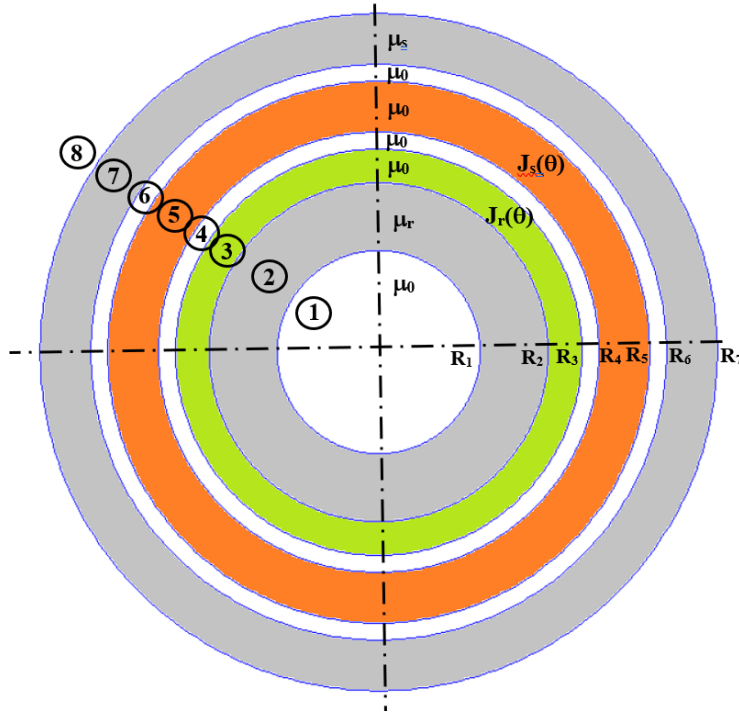
**Solution of the Euler equation:**

$$\underline{A}_{3n}(r) = \underline{K}_1 r^{np} + \underline{K}_2 r^{-np} + \underline{A}_p(r)$$

Where  $\underline{K}_1$  and  $\underline{K}_2$  are the integration constants that will be computed from the interface conditions and  $\underline{A}_p(r)$  is the particular solution :

$$\underline{A}_p(r) = \begin{cases} \mu_0 \frac{r^2}{(np)^2 - 4} \underline{J}_m & \text{for } np \neq 2 \\ \frac{1}{16} \mu_0 r^2 (1 - 4 \ln r) \underline{J}_m & \text{for } np = 2 \end{cases}$$

# Solution in the different regions



$$\underline{A}_{1n}(r) = \underline{K}_1 r^{np}$$

$$\underline{A}_{2n}(r) = \underline{K}_2 r^{np} + \underline{K}_3 r^{-np}$$

$$\underline{A}_{3n}(r) = \underline{K}_4 r^{np} + \underline{K}_5 r^{-np} + \underline{A}_{p3}(r)$$

$$\underline{A}_{4n}(r) = \underline{K}_6 r^{np} + \underline{K}_7 r^{-np}$$

$$\underline{A}_{5n}(r) = \underline{K}_8 r^{np} + \underline{K}_9 r^{-np} + \underline{A}_{p5}(r)$$

$$\underline{A}_{6n}(r) = \underline{K}_{10} r^{np} + \underline{K}_{11} r^{-np}$$

$$\underline{A}_{7n}(r) = \underline{K}_{12} r^{np} + \underline{K}_{13} r^{-np}$$

$$\underline{A}_{8n}(r) = \underline{K}_{14} r^{-np}$$

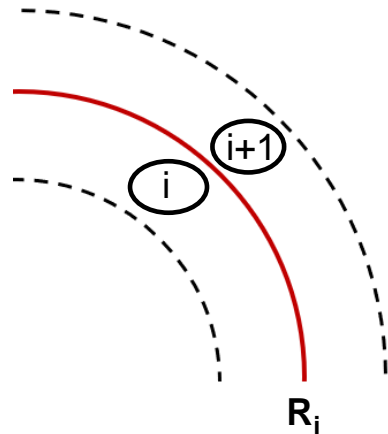
$$\underline{A}_{p3}(r) = \begin{cases} \mu_0 \frac{r^2}{(np)^2 - 4} \underline{J}_{-m} & \text{for } np \neq 2 \\ \frac{1}{16} \mu_0 r^2 (1 - 4 \ln r) \underline{J}_{-m} & \text{for } np = 2 \end{cases}$$

$$\underline{A}_{p5}(r) = \begin{cases} \mu_0 \frac{r^2}{(np)^2 - 4} \underline{J}_{-sn} & \text{for } np \neq 2 \\ \frac{1}{16} \mu_0 r^2 (1 - 4 \ln r) \underline{J}_{-sn} & \text{for } np = 2 \end{cases}$$

**We have 14 coefficients to determine ( $\underline{K}_1$  to  $\underline{K}_{14}$ ) from the interface conditions; Expression of  $\underline{J}_{-m}$  and  $\underline{J}_{-sn}$  to be determinate**

# Relations between the unknown coefficients (1/3)

Relations at the interface between 2 regions:



$$\begin{cases} \underline{A}_{in}(R_i) = \underline{A}_{(i+1)n}(R_i) \\ \frac{1}{\mu_i} \left. \frac{\partial \underline{A}_{in}}{\partial r} \right|_{R_i} = \frac{1}{\mu_{i+1}} \left. \frac{\partial \underline{A}_{(i+1)n}}{\partial r} \right|_{R_i} \end{cases}$$

We have 7 interfaces ( $R_1$  to  $R_7$ ) with 2 relations by interface that corresponds to 14 equations

**It's up to you!**

**Determine the 14 relations between the coefficients and write them in matrix form.**



# Relations between the unknown coefficients (2/3)

$$r = R_1 \rightarrow \begin{cases} \underline{K}_1 R_1^{np} = \underline{K}_2 R_1^{np} + \underline{K}_3 R_1^{-np} \\ \underline{K}_1 R_1^{np-1} = \frac{1}{\mu_r} (\underline{K}_2 R_1^{np-1} - \underline{K}_3 R_1^{-np-1}) \end{cases}$$

$$r = R_2 \rightarrow \begin{cases} \underline{K}_2 R_2^{np} + \underline{K}_3 R_2^{-np} = \underline{K}_4 R_2^{np} + \underline{K}_5 R_2^{-np} + \mu_0 \frac{R_2^2}{(np)^2 - 4} \underline{J}_{-m} \\ \frac{1}{\mu_r} (\underline{K}_2 R_2^{np-1} - \underline{K}_3 R_2^{-np-1}) = \underline{K}_4 R_2^{np-1} - \underline{K}_5 R_2^{-np-1} + 2\mu_0 \frac{R_2}{np((np)^2 - 4)} \underline{J}_{-m} \end{cases}$$

$$r = R_2 \rightarrow \begin{cases} \underline{K}_4 R_3^{np} + \underline{K}_5 R_3^{-np} + \mu_0 \frac{R_3^2}{(np)^2 - 4} \underline{J}_{-m} = \underline{K}_6 R_3^{np} + \underline{K}_7 R_3^{-np} \\ \underline{K}_4 R_3^{np-1} - \underline{K}_5 R_3^{-np-1} + 2\mu_0 \frac{R_3}{np((np)^2 - 4)} \underline{J}_{-m} = \underline{K}_6 R_3^{np-1} - \underline{K}_7 R_3^{-np-1} \end{cases}$$

$$r = R_4 \rightarrow \begin{cases} \underline{K}_6 R_4^{np} + \underline{K}_7 R_4^{-np} = \underline{K}_8 R_4^{np} + \underline{K}_9 R_4^{-np} + \mu_0 \frac{R_4^2}{(np)^2 - 4} \underline{J}_{-sn} \\ \underline{K}_6 R_4^{np-1} - \underline{K}_7 R_4^{-np-1} = \underline{K}_8 R_4^{np-1} - \underline{K}_9 R_4^{-np-1} + 2\mu_0 \frac{R_4}{np((np)^2 - 4)} \underline{J}_{-sn} \end{cases}$$

# Relations between the unknown coefficients (3/3)

$$\begin{aligned}
 r = R_5 &\rightarrow \begin{cases} \underline{K}_8 R_5^{np} + \underline{K}_9 R_5^{-np} + \mu_0 \frac{R_5^2}{(np)^2 - 4} \underline{J}_{sn} = \underline{K}_{10} R_5^{np} + \underline{K}_{11} R_5^{-np} \\ \underline{K}_8 R_5^{np-1} - \underline{K}_9 R_5^{-np-1} + 2\mu_0 \frac{R_5}{np((np)^2 - 4)} \underline{J}_{sn} = \underline{K}_{10} R_5^{np-1} - \underline{K}_{11} R_5^{-np-1} \end{cases} \\
 r = R_6 &\rightarrow \begin{cases} \underline{K}_{10} R_6^{np} + \underline{K}_{11} R_6^{-np} = \underline{K}_{12} R_6^{np} + \underline{K}_{13} R_6^{-np} \\ \underline{K}_{10} R_6^{np-1} - \underline{K}_{11} R_6^{-np-1} = \frac{1}{\mu_s} (\underline{K}_{12} R_6^{np-1} - \underline{K}_{13} R_6^{-np-1}) \end{cases} \\
 r = R_7 &\rightarrow \begin{cases} \underline{K}_{12} R_7^{np} + \underline{K}_{13} R_7^{-np} = \underline{K}_{14} R_6^{-np} \\ \frac{1}{\mu_s} (\underline{K}_{12} R_7^{np-1} - \underline{K}_{13} R_7^{-np-1}) = -\underline{K}_{14} R_7^{-np-1} \end{cases}
 \end{aligned}$$

Valid for  $np \neq 2$ . For  $np = 2$  (that corresponds only to the case  $n = 1$  and  $p = 2$ ), see the previous expression for the particular solution (slide 13).

In the previous relations,  $\mu_s$  and  $\mu_r$  are the relative permeability,  $\mu_s = \mu_r = 1$  if regions 2 and 7 are made with no-magnetic material, that corresponds to a totally ironless machine!

# Writing in Matrix form (Global matrix)

$$\begin{bmatrix}
 M_{11} & M_{11} & M_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 M_{21} & M_{21} & M_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & M_{32} & M_{33} & M_{34} & M_{35} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & M_{42} & M_{43} & M_{44} & M_{45} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & M_{54} & M_{55} & M_{56} & M_{57} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & M_{64} & M_{65} & M_{66} & M_{67} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & M_{76} & M_{77} & M_{78} & M_{79} & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & M_{86} & M_{87} & M_{88} & M_{89} & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{98} & M_{99} & M_{910} & M_{911} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{108} & M_{109} & M_{1010} & M_{1011} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{1110} & M_{1111} & M_{1112} & M_{1113} & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{12} & M_{1211} & M_{1212} & M_{1213} & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{1312} & M_{1313} & M_{1314} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{1412} & M_{1413} & M_{1414}
 \end{bmatrix}
 \times
 \begin{pmatrix}
 \underline{K}_1 \\
 \underline{K}_2 \\
 \underline{K}_3 \\
 \underline{K}_4 \\
 \underline{K}_5 \\
 \underline{K}_6 \\
 \underline{K}_7 \\
 \underline{K}_8 \\
 \underline{K}_9 \\
 \underline{K}_{10} \\
 \underline{K}_{11} \\
 \underline{K}_{12} \\
 \underline{K}_{13} \\
 \underline{K}_{14}
 \end{pmatrix}
 =
 \begin{pmatrix}
 0 \\
 0 \\
 \underline{S}_3 \\
 \underline{S}_4 \\
 \underline{S}_5 \\
 \underline{S}_6 \\
 \underline{S}_7 \\
 \underline{S}_8 \\
 \underline{S}_9 \\
 \underline{S}_{10} \\
 0 \\
 0 \\
 0 \\
 0
 \end{pmatrix}$$

Where  $M_{11}, M_{12}, \dots$  are sub-matrix (n-by-n matrix, diagonal, where n is number of harmonics considered)

$\underline{S}_3, \underline{S}_4, \dots$  are the source terms (vectors of length n)

The length of the global matrix (topological matrix) is 14n-by-14n

# Sub-Matrix form

First line of the global matrix:

For 3 harmonics ( $n = 1, 3, 5$ ) and  $p = 4$

$$\begin{matrix}
 [M_{11}] & [M_{12}] & [M_{13}] \\
 \downarrow & \swarrow & \swarrow \\
 \textcircled{R_1^{np}} K_1 & -\textcircled{R_1^{np}} K_2 & -\textcircled{R_1^{-np}} K_3 = 0
 \end{matrix}$$

$$[M_{11}] = \begin{bmatrix} R_1^4 & 0 & 0 \\ 0 & R_1^{12} & 0 \\ 0 & 0 & R_1^{20} \end{bmatrix} \quad [M_{13}] = - \begin{bmatrix} R_1^{-4} & 0 & 0 \\ 0 & R_1^{-12} & 0 \\ 0 & 0 & R_1^{-20} \end{bmatrix}$$

Third line of the global matrix:

$$\begin{matrix}
 [M_{31}] & [M_{32}] & [M_{33}] & [M_{34}] \\
 \uparrow & \uparrow & \swarrow & \swarrow \\
 \textcircled{R_2^{np}} K_2 & +\textcircled{R_2^{-np}} K_3 & -\textcircled{R_2^{np}} K_4 & -\textcircled{R_2^{-np}} K_5 = \textcircled{\mu_0 \frac{R_2^2}{(np)^2 - 4} J_{-m}}
 \end{matrix} \quad (S_3) = \begin{pmatrix} \frac{\mu_0 R_2^2}{12} J_{-r1} \\ \frac{\mu_0 R_2^2}{140} J_{-r3} \\ \frac{\mu_0 R_2^2}{396} J_{-r5} \end{pmatrix}$$

**Important note : ill-conditioned global matrix and numerical problem**

For example, if we consider  $R_1 = 0,2$  m for 3 harmonics ( $n = 1, 3, 5$ ) and  $p = 4$

$$[M_{11}] = \begin{bmatrix} 1.10^{-3} & 0 & 0 \\ 0 & 4.10^{-9} & 0 \\ 0 & 0 & 1.10^{-14} \end{bmatrix} \quad [M_{13}] = - \begin{bmatrix} 625 & 0 & 0 \\ 0 & 2.10^8 & 0 \\ 0 & 0 & 9.10^{13} \end{bmatrix}$$

Very large numbers and very small numbers in the topological matrix: numerical problem of precision during the inversion: **it is better to write the solution as dimensionless coefficients (radius ratio), but this is not dealt with here.....You can do it!**

# Matrix form : extract of the Matlab programm...

```

% Sub-matrix filling
for n = 1:nn
    k=(2*n-1)*p;

    M11(n,n)=R1^k;
    M12(n,n)=-(R1^k);
    M13(n,n)=-(R1^(-k));

    M21(n,n)=R1^(k-1);
    M22(n,n)=-(1/mur)*(R1^(k-1));
    M23(n,n)=(1/mur)*(R1^(-k-1));

    M32(n,n)=R2^k;
    M33(n,n)=R2^(-k);
    ...

% Global matrix filling

%
%           K1      ,      K2      ,      K3      ,      K4      ,      K5      ,      K6
G = [      M11      ,      M12      ,      M13      ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(
           M21      ,      M22      ,      M23      ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros
           zeros(nn,nn) ,      M32      ,      M33      ,      M34      ,      M35      ,      zeros
           zeros(nn,nn) ,      M42      ,      M43      ,      M44      ,      M45      ,      zeros
           zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      M54      ,      M55      ,      M
           zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      M64      ,      M65      ,      M
           zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      |
           zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      |
           zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros
           zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros
           zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros
           zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros
           zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros
           zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros
           zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros(nn,nn) ,      zeros

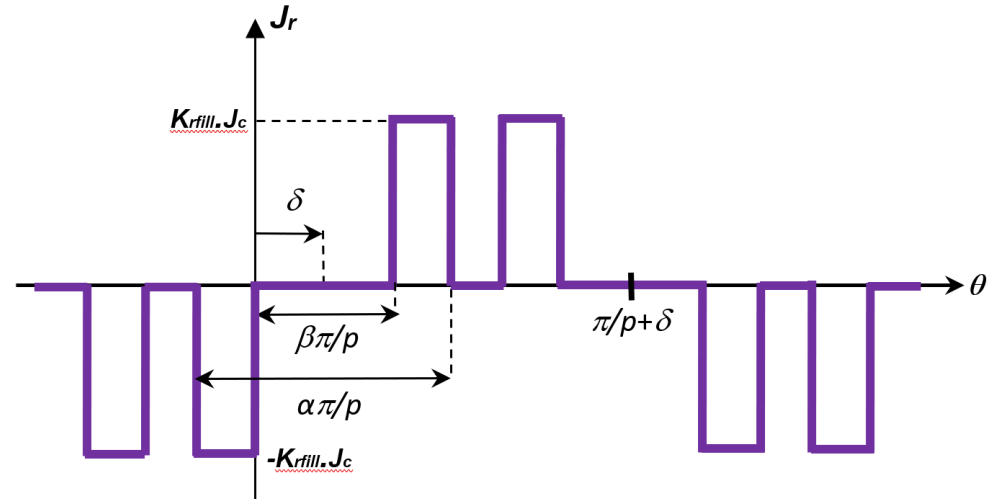
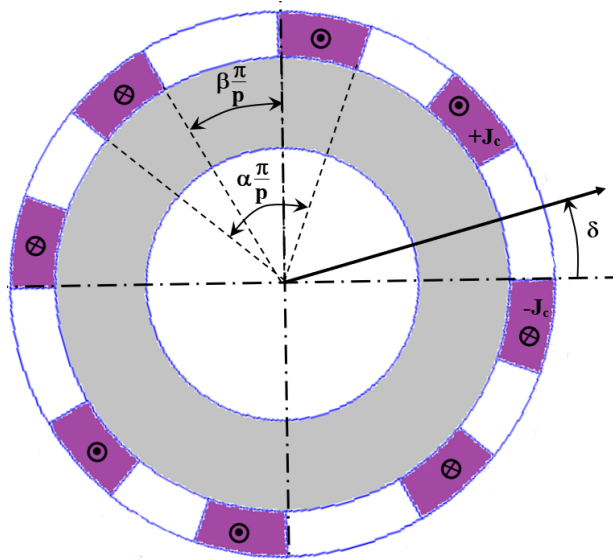
% Source term vector
%
%           K1      ,      K2      ,      K3      ,      K4      ,      K5      ,      K6
S = [ zeros(nn,1) ;      zeros(nn,1) ;      S3      ;      S4      ;      S5      ;      S

% Matrix inversion

X = G\S;

```

# Source terms : HTS current density $J_r(\theta)$



- $p$  : pole-pairs number
- $\alpha$  : outer opening of HTS coils
- $\beta$  : inner opening of the HTS coils
- $\delta$  : rotor position (load torque)
- $J_c$  : critical current of the HTS coils
- $K_{rfill}$  : fill factor for the HTS coils
- $J_c = 300 \text{ A/mm}^2$  ;  $K_{rfill} = 0,7$

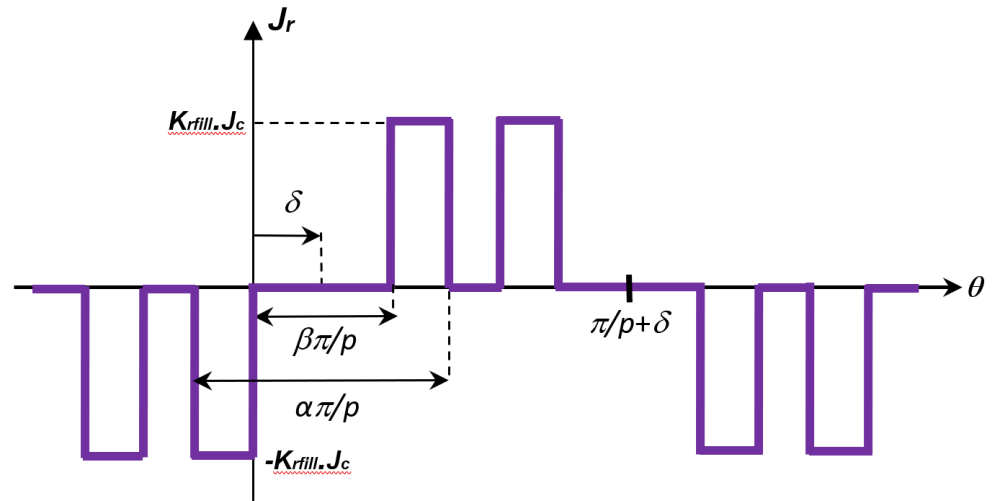
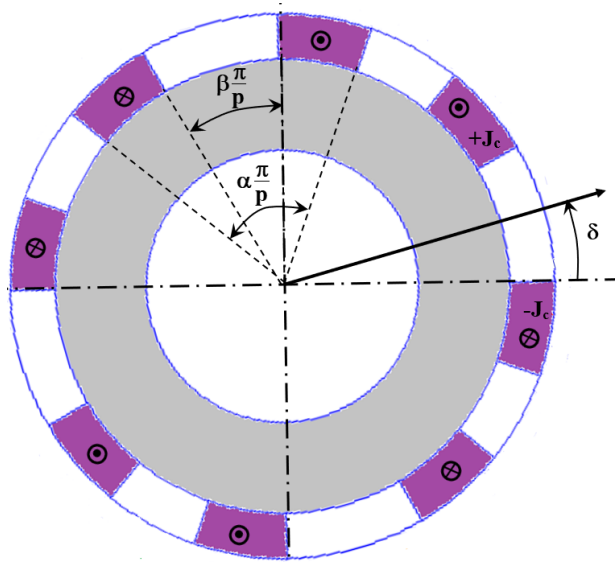
$$J_r(\theta) = \sum_{n=1,3,5,\dots}^N J_m \sin(np(\theta - \delta))$$

$$J_m = \frac{4K_{rfill}J_c}{n\pi} \left( \cos\left(n\beta\frac{\pi}{2}\right) - \cos\left(n\alpha\frac{\pi}{2}\right) \right)$$

In complex notation:

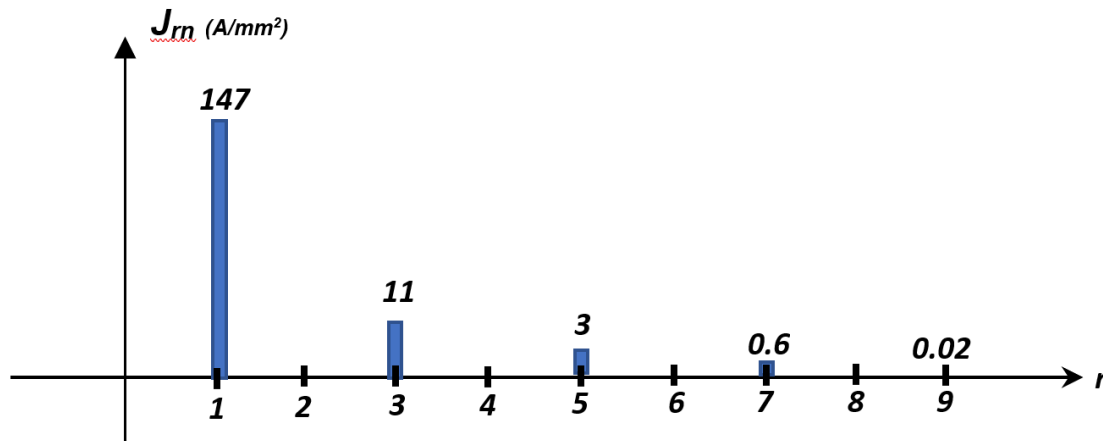
$$\underline{J}_m = J_m e^{-jnp\delta}$$

# Source terms : HTS current density $J_r(\theta)$

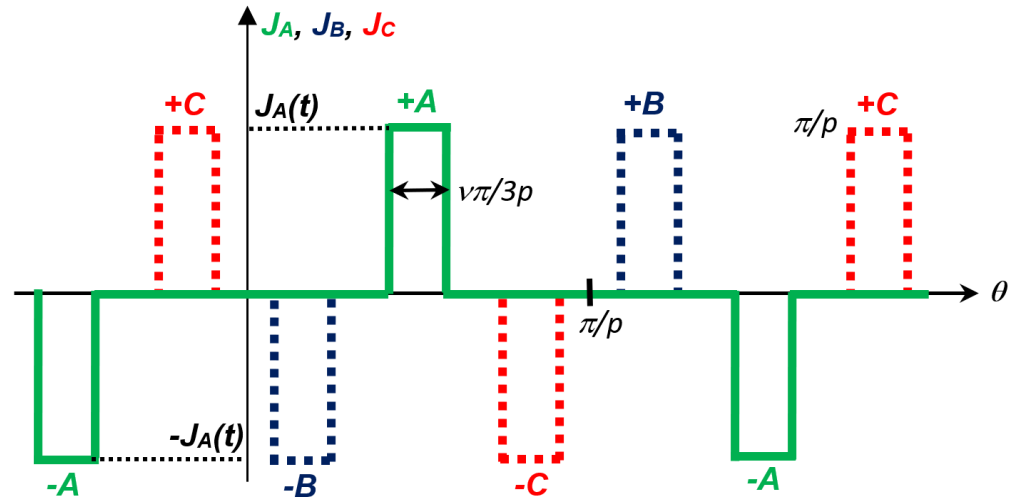
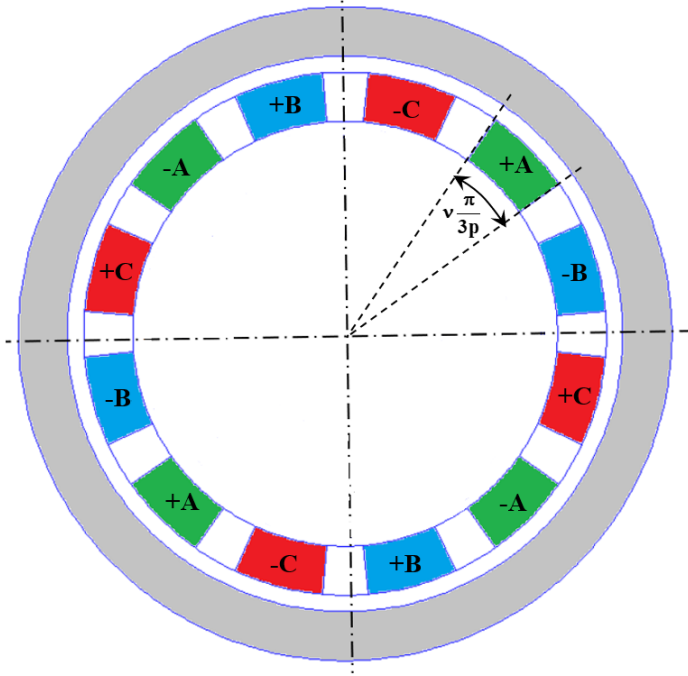


$$J_m = \frac{4K_{rfill}J_c}{n\pi} \left( \cos\left(n\beta\frac{\pi}{2}\right) - \cos\left(n\alpha\frac{\pi}{2}\right) \right)$$

$p = 2; \alpha = 0.9; \beta = 0.5$   
 $J_c = 300 \text{ A/mm}^2; K_{rfill} = 0,7$



# Source terms: three-phase AC winding $J_s(\theta)$



The 3 phases windings are fed by AC  
3 phases current  $\rightarrow$  Rotating field

3-phase distributed stator winding

$p$  : pole-pairs number

$v$  : opening of the stator coils

$J_{srms}$  : stator RMS current density

$K_{sfill}$  : fill factor stator coils

$J_{srms} = 20 \text{ A/mm}^2$  ;  $K_{sfill} = 0,5$

$$J_A(t) = K_{sfill} J_{srms} \sqrt{2} \cos(\omega t)$$

$$J_B(t) = K_{sfill} J_{srms} \sqrt{2} \cos(\omega t - 2\pi/3)$$

$$J_C(t) = K_{sfill} J_{srms} \sqrt{2} \cos(\omega t + 2\pi/3)$$



# Source terms: three-phase AC winding $J_s(\theta)$

$$J_A(t) = K_{sfill} J_{srms} \sqrt{2} \cos(\omega t)$$

$$J_B(t) = K_{sfill} J_{srms} \sqrt{2} \cos(\omega t - 2\pi/3)$$

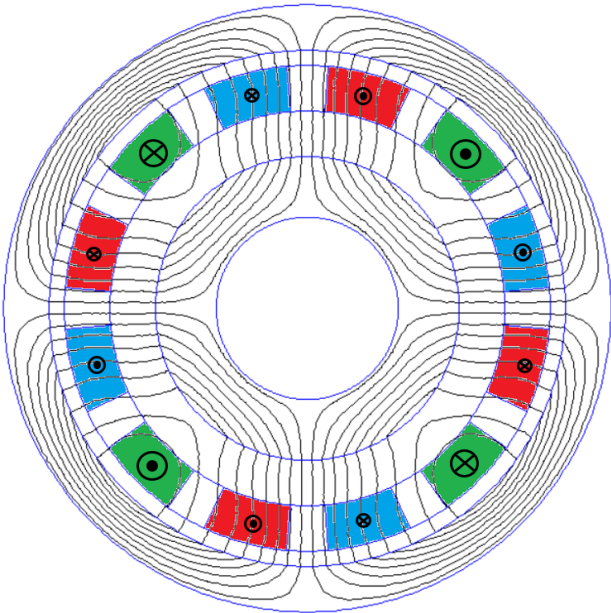
$$J_C(t) = K_{sfill} J_{srms} \sqrt{2} \cos(\omega t + 2\pi/3)$$

At  $t = 0$   
 $\longrightarrow$

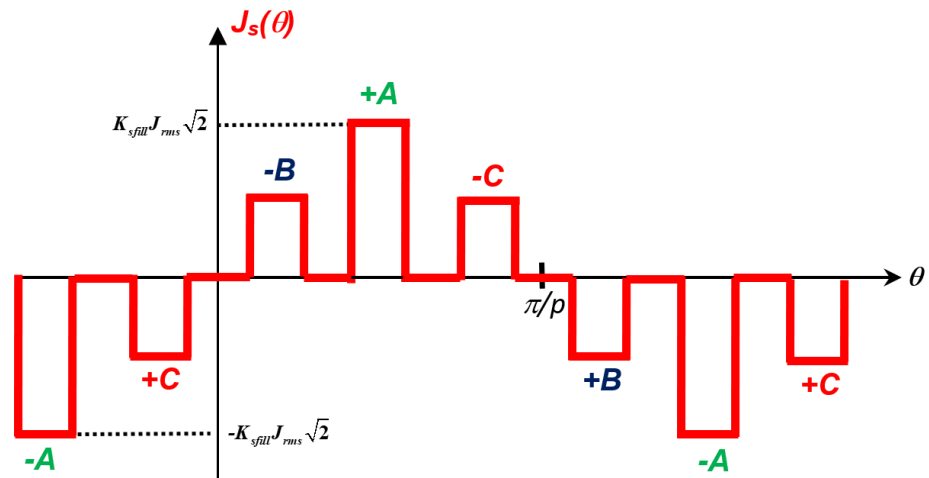
$$J_A(t=0) = K_{sfill} J_{srms} \sqrt{2}$$

$$J_B(t=0) = -\frac{1}{2} K_{sfill} J_{srms} \sqrt{2}$$

$$J_C(t=0) = -\frac{1}{2} K_{sfill} J_{srms} \sqrt{2}$$



Field distribution at  $t = 0$



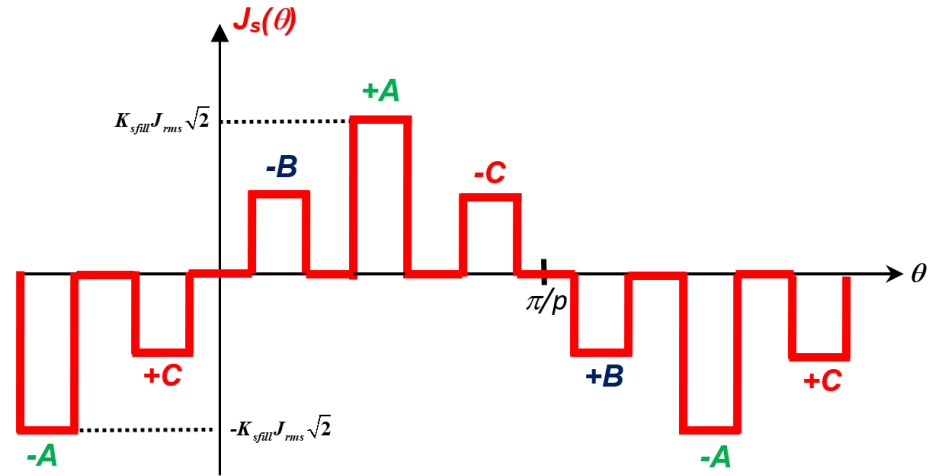
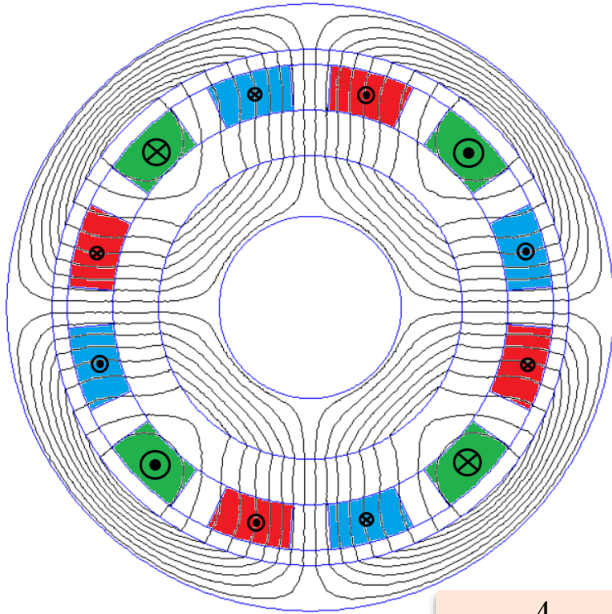
Current density distribution at  $t = 0$

$$J_s(\theta) = \sum_{n=1,3,5\dots}^N J_{sn} \sin(np\theta)$$

with

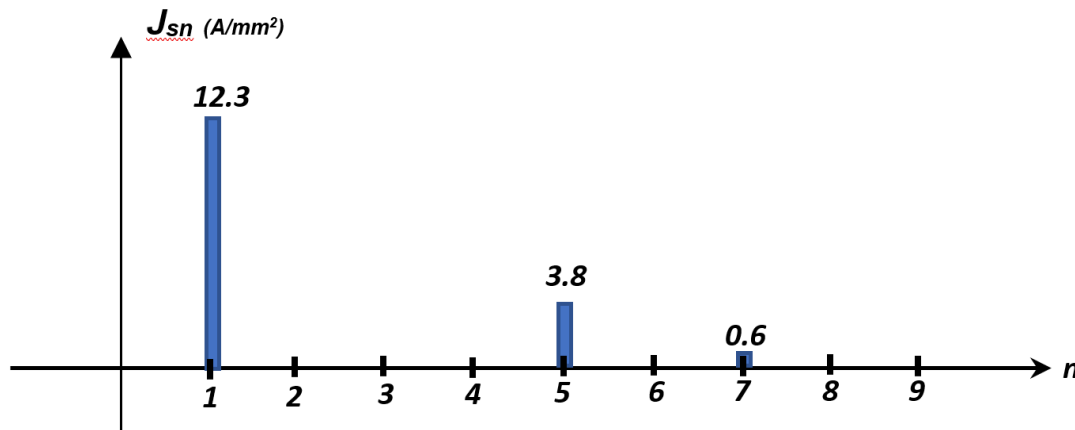
$$J_{sn} = \frac{4}{\pi n} K_{sfill} J_{srms} \sqrt{2} \sin\left(n \frac{\pi}{2}\right) \sin\left(nv \frac{\pi}{6}\right) \left(1 - \cos\left(n \frac{2\pi}{3}\right)\right)$$

# Source terms: three-phase AC winding $J_s(\theta)$



$$J_{sn} = \frac{4}{\pi n} K_{sfill} J_{srms} \sqrt{2} \sin\left(n \frac{\pi}{2}\right) \sin\left(nv \frac{\pi}{6}\right) \left(1 - \cos\left(n \frac{2\pi}{3}\right)\right)$$

$p = 2; v = 0.9$   
 $J_{srms} = 20 \text{ A/mm}^2; K_{rfill} = 0,5$



# More complicated 3-phase AC winding to reduce the harmonic level

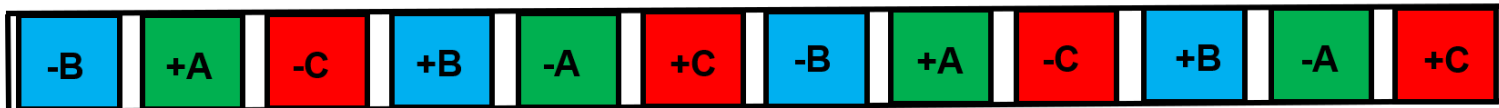
Reduce the harmonic content of the stator winding could be good to avoid AC losses in the HTS coils (due to parasitic rotating field)

## Distributed winding and chording (short pitched coil)

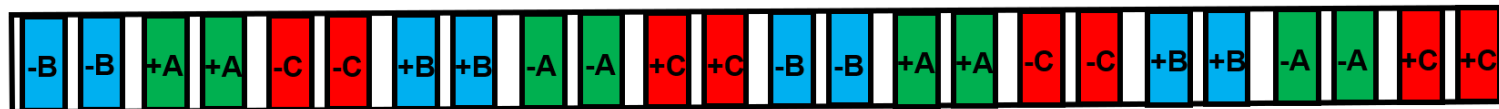
$Q_s$  : number of stator "slots"  
 $p$  : number of pole pairs  
 $q$  : number of slots per pole and per phase

$$q = \frac{Q_s}{6p}$$

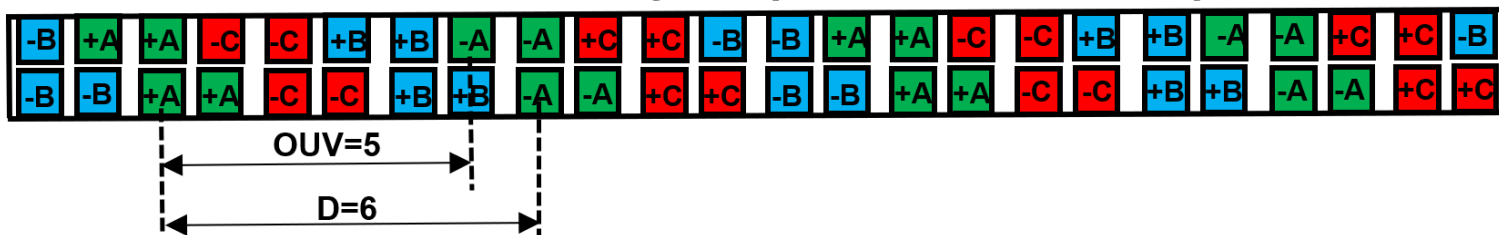
Single layer ;  $q = 1$  ; 6 coils



Single layer ;  $q = 2$  ; 12 coils



Double layer ;  $q = 2$  ; 24 coils, coil span 5/6

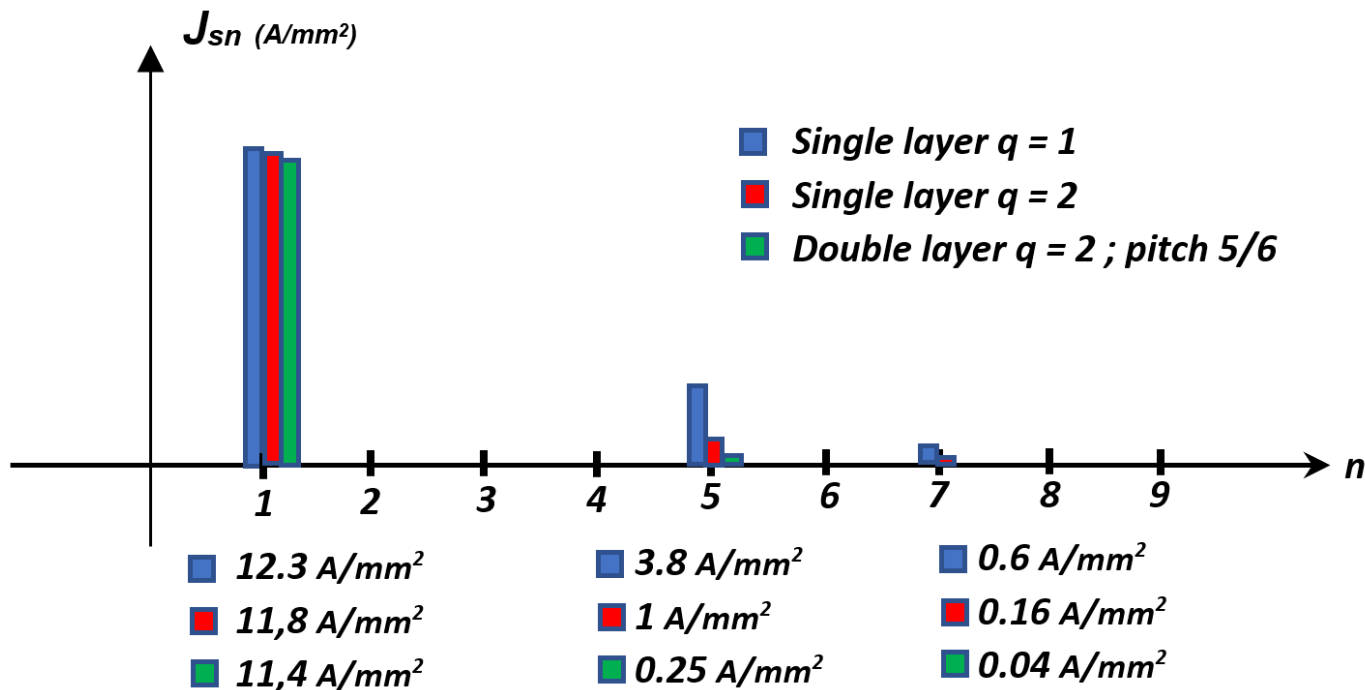


# More complicated 3-phase AC winding to reduce the harmonic level

$$J_{sn} = \frac{4}{\pi n} K_{sfill} J_{srms} \sqrt{2} \sin\left(n \frac{\pi}{2}\right) \sin\left(nv \frac{\pi}{6}\right) \left(1 - \cos\left(n \frac{2\pi}{3}\right)\right) \times \underline{K_d(n) K_p(n)}$$

$$K_d(n) = \frac{\sin\left(npq \frac{\pi}{Q_s}\right)}{q \sin\left(np \frac{\pi}{Q_s}\right)}$$

is the distribution factor ;  $K_p(n) = \sin\left(n \frac{OUV}{D} \frac{\pi}{2}\right)$  is the pitch factor



# Expression of the Flux density

The magnetic vector potential in each region is written as

$$A_i(r, \theta) = \sum_{n=1,3,5\dots}^N A_{in}(r) \sin(np(\theta + \alpha)) \quad \rightarrow \quad A_i(r, \theta) = \text{Im} \left( \sum_{n=1,3,5\dots}^N \underline{A}_{in}(r) e^{jnp\theta} \right)$$

$$\mathbf{B}_i = \nabla \times \mathbf{A}_i \quad \text{with} \quad \mathbf{A}_i = A_i(r, \theta) \mathbf{u}_z$$

In cylindrical coordinates, we have:

$$\mathbf{B}_i = \frac{1}{r} \frac{\partial A_i}{\partial \theta} \mathbf{u}_r - \frac{\partial A_i}{\partial r} \mathbf{u}_\theta$$

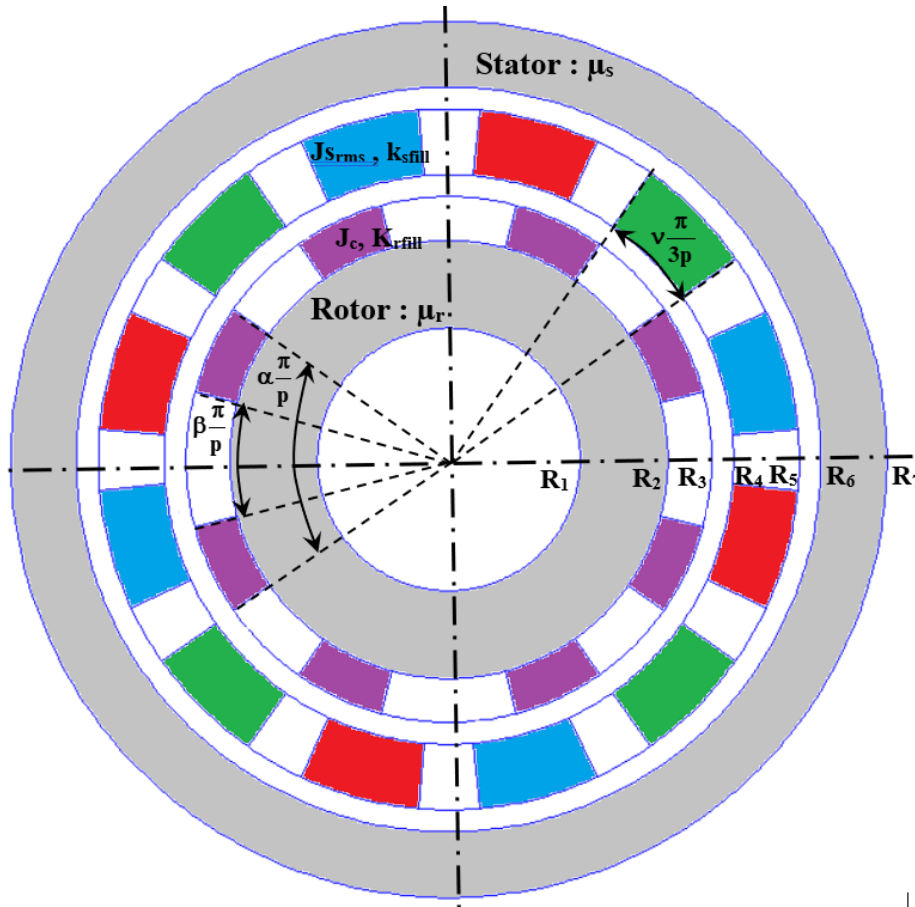
Radial component of  $\mathbf{B}$   $\rightarrow$   $B_{ri}(r, \theta) = \text{Im} \left( \sum_{n=1,3,5\dots}^N \underline{B}_{rin}(r) e^{jnp\theta} \right)$

Tangential component of  $\mathbf{B}$   $\rightarrow$   $B_{\theta i}(r, \theta) = \text{Im} \left( \sum_{n=1,3,5\dots}^N \underline{B}_{\theta in}(r) e^{jnp\theta} \right)$

$$\underline{B}_{rin}(r) = jnp \frac{1}{r} \underline{A}_{in}(r)$$

$$\underline{B}_{\theta in}(r) = -\frac{d\underline{A}_{in}(r)}{dr}$$

# Results and validation with FE simulations



$p = 2$  : pole-pairs number

$L = 30$  cm : axial length

$R_1 = 3$  cm : inner radius of the rotor

$a = 2$  cm : thickness rotor yoke

$b = 1$  cm : thickness HTS winding

$c = 0.5$  cm : air-gap thickness

$d = 1.5$  cm : thickness stator winding

$e = 0.5$  cm : 2<sup>nd</sup> air-gap thickness

$f = 1.5$  cm : thickness stator yoke

$\alpha = 0,777$  : outer opening of HTS coils

$\beta = 0.333$  : inner opening of the HTS coils

$\nu = 0.666$  : opening of the stator coils

$\mu_s = 1000$  : stator relative permeability

$\mu_r = 1000$  : rotor relative permeability

$J_c = 200$  A/mm<sup>2</sup> : critical current of the HTS coils (to avoid magnetic saturation)

$k_{r\text{fill}} = 0.7$  : fill factor for the HTS coils

$J_{s\text{rms}} = 20$  A/mm<sup>2</sup> : stator RMS current density

$k_{s\text{fill}} = 0,5$  : fill factor for the stator coils

# $J_r(\theta)$ and $J_s(\theta)$ computation with Matlab

%Source vector S3 to S6 (HTS rotor current density)

```

for j = 1:nn
    n=2*j-1;
    k=n*p;

    Jrn=(4*k*supra*Jc/(n*pi))*cos(n*beta*pi/2)-cos(n*alpha*pi/2);

    if k==2

        S3(j,1)=(1/16)*muo*Jrn*(R2^2)*(1-4*log(R2))*exp(k*li*delta);
        S4(j,1)=-(1/(16*k))*muo*Jrn*2*R2*(1+4*log(R2))*exp(k*li*delta);
        S5(j,1)=-(1/16)*muo*Jrn*(R3^2)*(1-4*log(R3))*exp(k*li*delta);
        S6(j,1)=(1/(16*k))*muo*Jrn*2*R3*(1+4*log(R3))*exp(k*li*delta);

    else

        S3(j,1)=muo*Jrn*((R2^2)/(k^2-4))*exp(k*li*delta);
        S4(j,1)=muo*Jrn*((2*R2)/(k*(k^2-4)))*exp(k*li*delta);
        S5(j,1)=-muo*Jrn*((R3^2)/(k^2-4))*exp(k*li*delta);
        S6(j,1)=-muo*Jrn*(2*R3/(k*(k^2-4)))*exp(k*li*delta);

    end

    % To plot the rotor current density
    JJR(j,1)=Jrn;
end
    
```

%Source vector S7 to S10 (Stator current density)

```

for j = 1:nn
    n=2*j-1;
    k=n*p;

    Jsn=(4*kcu*Js*sqrt(2)/(pi*n))*sin(n*pi/2)*sin(n*gamma*pi/6)*(1-cos(n*2*pi/3))*(sin(k*q*pi/Ne)/(q*sin(k*pi/Ne)));%

    if k==2

        S7(j,1)= (1/16)*muo*Jsn*(R4^2)*(1-4*log(R4));
        S8(j,1)=-(1/(16*k))*muo*Jsn*2*R4*(1+4*log(R4));
        S9(j,1)= -(1/16)*muo*Jsn*(R5^2)*(1-4*log(R5));
        S10(j,1)=(1/(16*k))*muo*Jsn*2*R5*(1+4*log(R5));

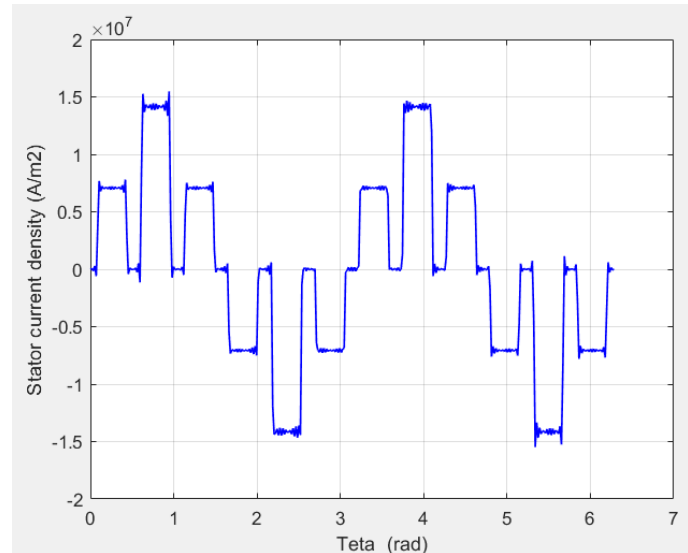
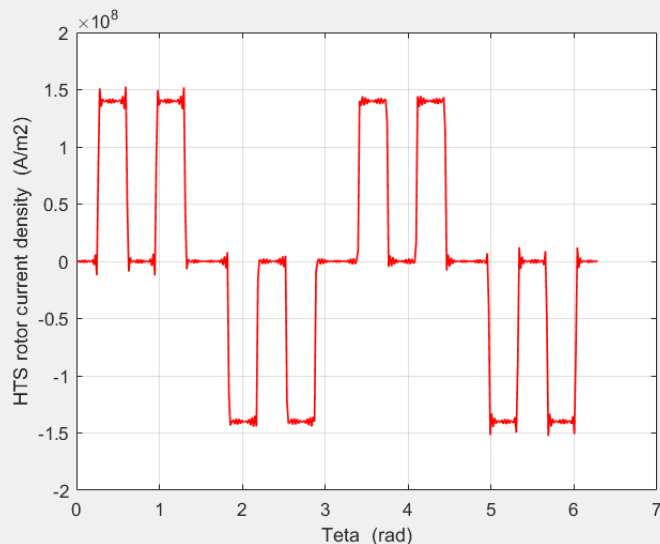
    else

        S7(j,1)= muo*Jsn*(R4^2)/(k^2-4);
        S8(j,1)=muo*Jsn*2*R4/(k*(k^2-4));
        S9(j,1)=-muo*Jsn*(R5^2)/(k^2-4);
        S10(j,1)=-muo*Jsn*2*R5/(k*(k^2-4));

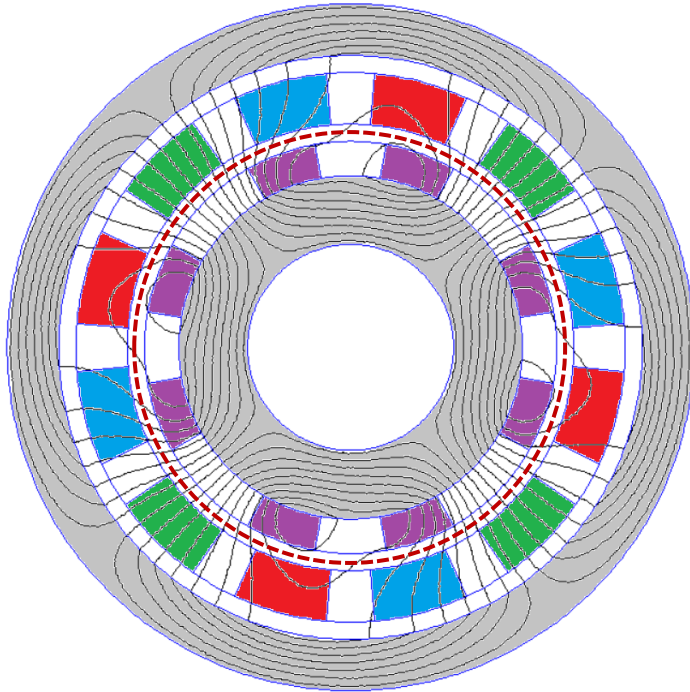
    end

    end

    % To plot the stator current density
    JJ(j,1)=Jsn;
end
    
```



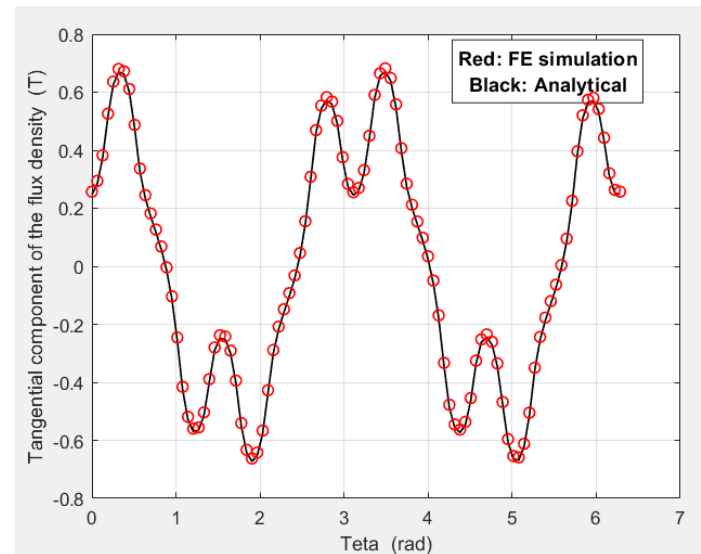
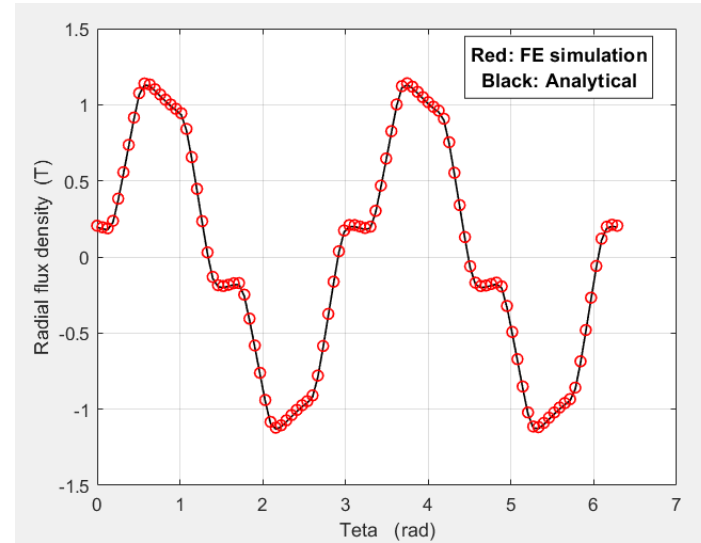
# Flux density in the middle of the air-gap under load condition ( $\delta = 90^\circ$ )



Flux line under Full-load condition  
(FEMM software)

Torque with FEM:  $T = 505 \text{ Nm}$  in 1s  
Torque with Analytical:  $T = 504 \text{ Nm}$  in 20ms

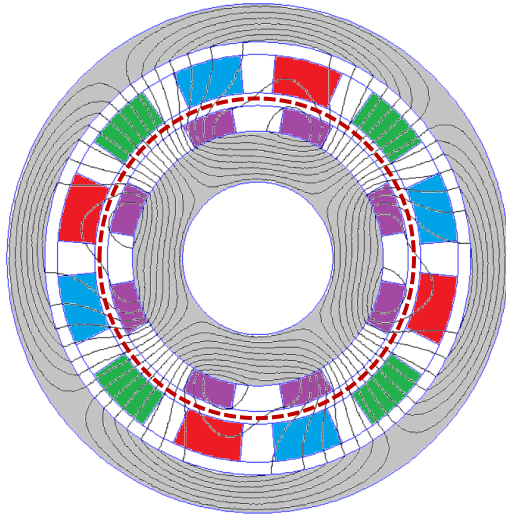
Computational time 50 times better: good for  
design optimization procedure





# Torque computation: Maxwell Stress Tensor

The torque is compute in the middle of the air-gap (region 4) : less harmonic terms are needed



$$T = \frac{LR_m^2}{\mu_0} \int_0^{2\pi} B_{r4}(R_m, \theta) B_{t4}(R_m, \theta) d\theta$$

$$R_m = \frac{R_3 + R_4}{2}$$

With the complex notation, we obtain:

$$\int_0^{2\pi} B_{r4}(R_m, \theta) B_{\theta 4}(R_m, \theta) d\theta = \pi \Re \left\{ \underline{B}_{r4}(R_m) \underline{B}_{\theta 4}^*(R_m) \right\}$$

See slide 27; For harmonic  $np$  we have :

$$\underline{B}_{r4}(R_m) = jnp \frac{1}{R_m} \left( \underline{K}_6 R_m^{np} + \underline{K}_7 R_m^{-np} \right)$$

$$\underline{B}_{\theta 4}^*(R_m) = -np \left( \underline{K}_6^* R_m^{np-1} - \underline{K}_7^* R_m^{-np-1} \right)$$

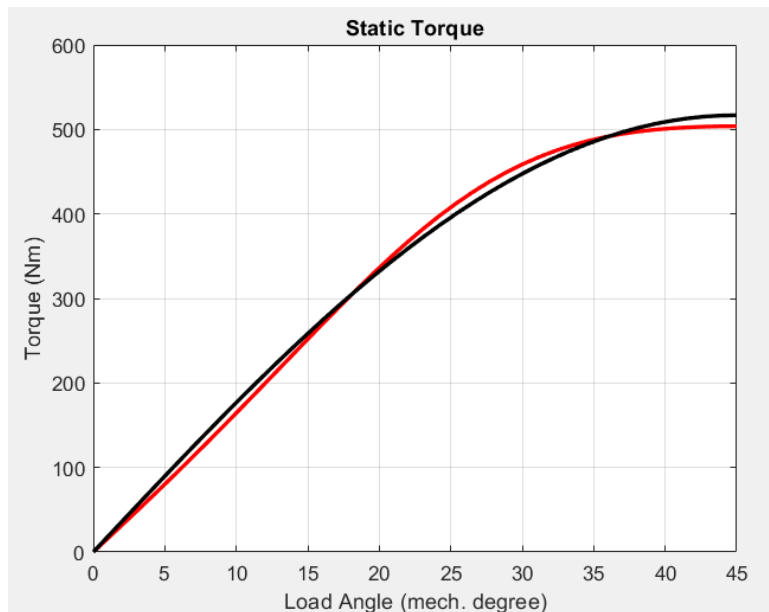
$$T = \frac{\pi L}{\mu_0} \sum_{n=1,3,5\dots} \Re \left\{ j(np)^2 \left( \underline{K}_6^* \underline{K}_7 - \underline{K}_6 \underline{K}_7^* \right) \right\}$$

where  $\Re$  is the real part

# Static and dynamic torque for the studied example

## Static torque

We change the rotor position:  $0 < \delta < 90^\circ$   
 The current in the stator is fixed:  $(I ; -I/2 ; -I/2)$



**Red: 10 harmonics**  
**Black : fundamental only**

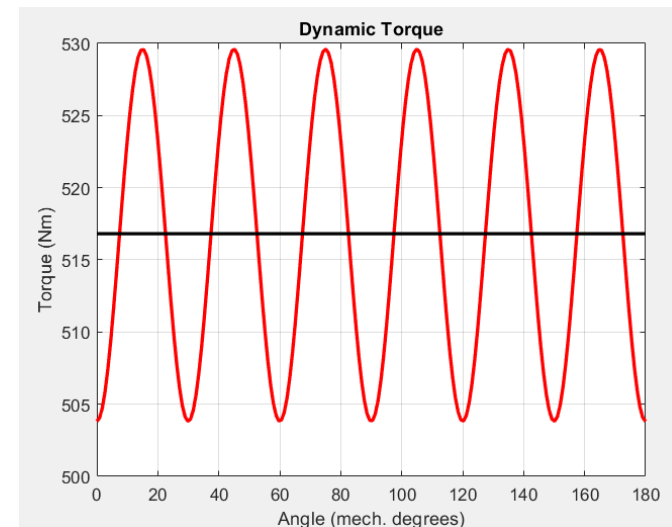
## Dynamic torque

Incremental variation of the rotor position from the maximal value of the torque:  $90^\circ + \Delta\delta$   
 The current in the stator coils change as well (current control with inverter and encoder):

$$J_A(\Delta\delta) = K_{sfill} J_{srms} \sqrt{2} \cos(p\Delta\delta)$$

$$J_B(\Delta\delta) = K_{sfill} J_{srms} \sqrt{2} \cos(p\Delta\delta - 2\pi/3)$$

$$J_C(\Delta\delta) = K_{sfill} J_{srms} \sqrt{2} \cos(p\Delta\delta + 2\pi/3)$$



**Red: 10 harmonics**  
**Black : fundamental only**

# Flux and Back-EMF expression

- ◆ Magnetic flux is obtained from the mean value of magnetic vector potential in a stator coil cross-section (Green-Ostrogradsky theorem)

$$\int_S \mathbf{B} \cdot \mathbf{ndS} = \int_S \text{curl} \mathbf{A} \cdot \mathbf{ndS} = \oint_{l_S} \mathbf{A} \cdot \mathbf{tdl} \quad A_z \rightarrow \frac{1}{S_{slot}} \int_{S_{slot}} A_z dS$$

$$\Phi_{slot} = \frac{L}{S_{slot}} \int_{R_4}^{R_5} \int_{-\frac{v\pi}{6p}}^{\frac{v\pi}{6p}} A_5(r, \theta) r dr d\theta \quad \longrightarrow \quad \Phi_{phase} = \frac{2pL}{S_{slot}} \int_{R_4}^{R_5} \int_{-\frac{v\pi}{6p}}^{\frac{v\pi}{6p}} A_5(r, \theta) r dr d\theta$$

**Regular distributed winding (\*2p)**

$$\Phi_{phase}(\delta) = \frac{4pL}{S_{slot}} \sum_{n=1,3,5\dots} \frac{1}{np} \sin\left(\frac{npv\pi}{6p}\right) \times \text{Im} \left\{ j \left( \underline{K}_8 \frac{R_5^{np+2} - R_4^{np+2}}{np+2} - \underline{K}_9 \frac{R_5^{-np+2} - R_4^{-np+2}}{np-2} \right) \right\} \quad \text{for } np \neq 2$$

$$\Phi_{phase}(\delta) = \frac{2pL}{S_{slot}} \sin\left(\frac{v\pi}{3p}\right) \times \text{Im} \left\{ j \left( \frac{1}{4} \underline{K}_8 (R_5^4 - R_4^4) + \underline{K}_9 \ln\left(\frac{R_5}{R_4}\right) \right) \right\} \quad \text{for } np = 2$$

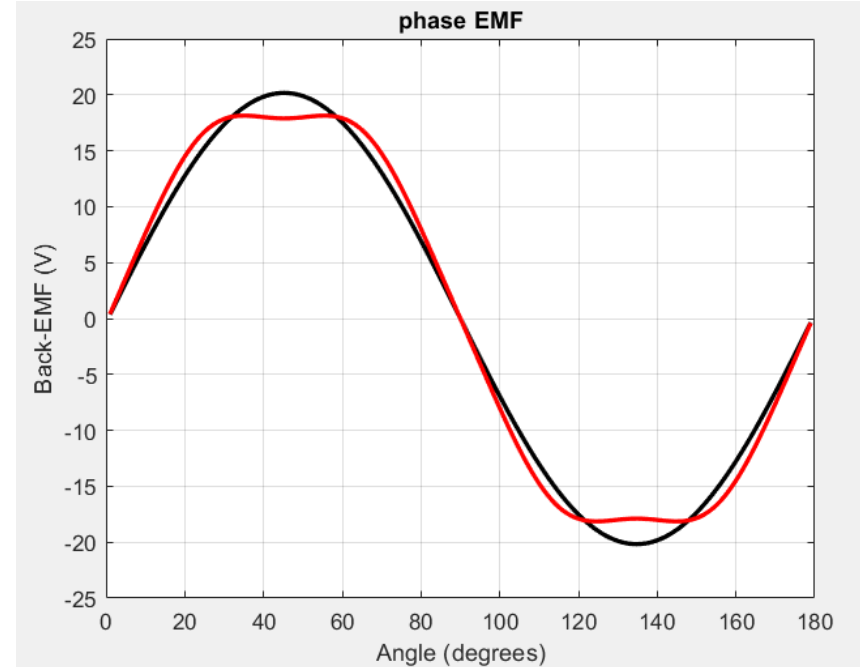
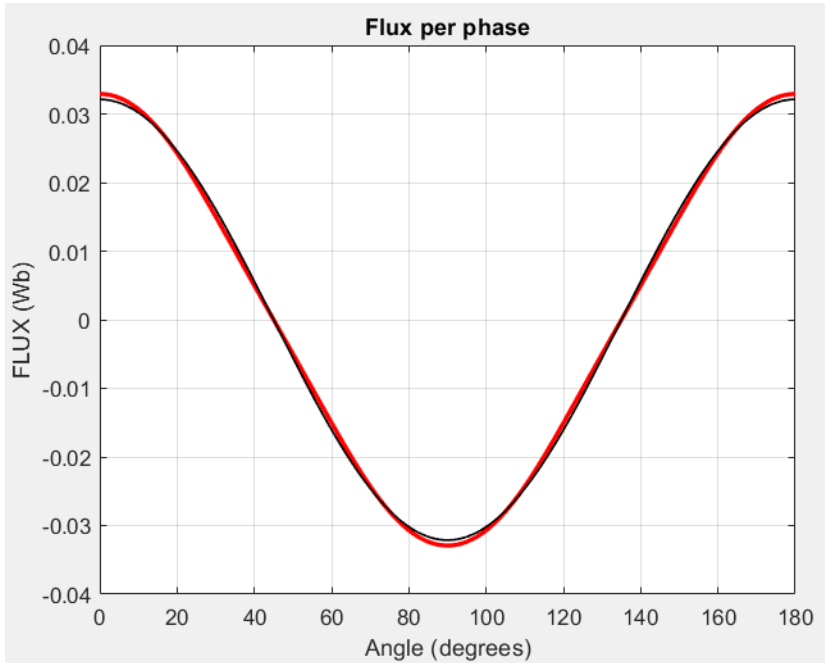
- ◆ The back EMF is obtained from the derivative of the flux

$$E_a(\delta) = -N_c \Omega \frac{d\Phi_{phase}}{d\delta} \quad \text{with } \Omega = \frac{d\delta}{dt} \quad (\delta \text{ is the mechanical position})$$

$N_c$  : number of conductors in the slot

# Flux and Back-EMF computation for the studied example (for 1 conductor/slot)

No load ( $J_{srms} = 0 \text{ A/mm}^2$ ) ;  $\Omega = 314 \text{ rad/s}$   
 Red: 10 harmonics ; Black : fundamental only



## Rapid verification:

$$3EI = C\Omega \quad \text{with} \quad C = 504 \text{ Nm} \quad \text{and} \quad \Omega = 314 \text{ rd/s}$$

$$\text{We have } S_{slot} = 3.7923 \cdot 10^{-4} \text{ m}^2 \quad \text{and} \quad I = k_{sfill} J_{srms} S_{slot} = 0.5 \times 20 \cdot 10^6 \times 3.7923 \cdot 10^{-4} = 3792 \text{ A}$$

$$E = \frac{C\Omega}{3I} = 13.9 \text{ A} \rightarrow E_{\max} = 19.8 \text{ V} \quad (\text{for 1 conductor per slot!})$$

# Design Optimization of an HTS machine



## More Electrical Aircraft Motor propulsion

$$P = 3 \text{ MW} ; N = 3000 \text{ rpm} \rightarrow T = 9500 \text{ Nm}$$

Power to Mass ratio > 20 kW/kg ????

## Genetic Algorithm available in Matlab Optimization Toolbox

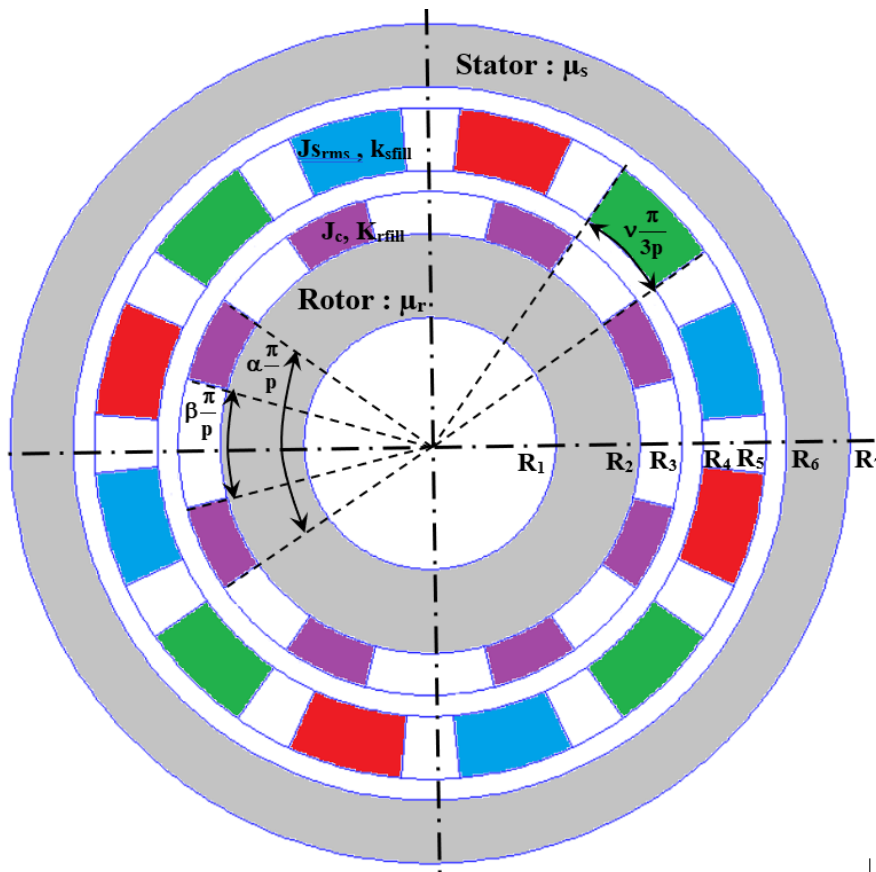
A genetic algorithm (GA) is a method for solving both constrained and unconstrained optimization problems based on a natural selection process that mimics biological evolution. The algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm randomly selects individuals from the current population and uses them as parents to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution.

# Mono-objective Design Optimization

Constraint →  $T = 9500 \text{ Nm}$

1 objective → Minimum active mass (HTS and copper coils)

Totally iron less machine ( $\mu_s = \mu_r = 1$ ) ;  $R_1 = R_2$ ;  $R_5 = R_6 = R_7$



$p = x(1)$  : pole-pairs number

$R_1 = x(2)$  : inner radius of the rotor

$a = x(3) = 0 \text{ cm}$  : thickness rotor yoke

$b = x(4)$  : thickness HTS winding

$c = 1 \text{ cm}$  : air-gap thickness (fixed)

$d = x(5)$  : thickness stator winding

$e = x(6) = 0 \text{ cm}$  : 2<sup>nd</sup> air-gap thickness

$f = x(7) = 0 \text{ cm}$  : thickness stator yoke

$L = x(8)$  : axial length

$\alpha = x(9)$  : outer opening of HTS coils

$\beta = x(10)$  : inner opening of the HTS coils

$v = x(11)$  : opening of the stator coils

$\mu_s = 1$  : stator relative permeability

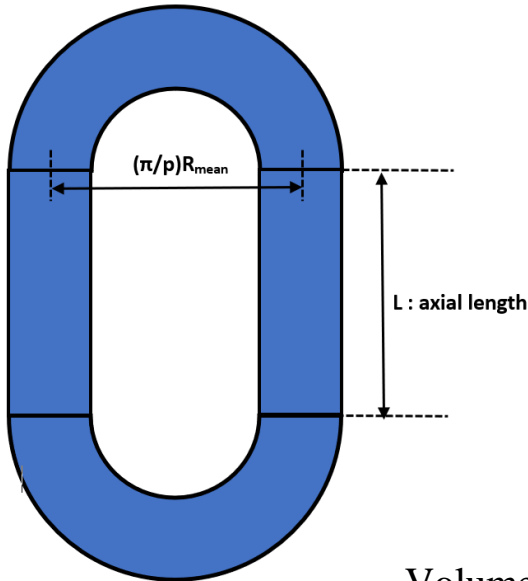
$\mu_r = 1$  : rotor relative permeability

$J_c = 200 \text{ and } 300 \text{ A/mm}^2$  ;  $K_{r\text{fill}} = 0.7$

$J_{s\text{rms}} = 20 \text{ A/mm}^2$  ;  $K_{s\text{fill}} = 0,5$

# Active mass expression

We have to write the active mass (HTS and copper coils) as a function of the geometrical parameters



◆ **Copper coils**  $\rho_{cop} = 8960 \text{kgm}^{-3}$ :

$$\text{Volume active parts : } V_{apcop} = \pi v (R_5^2 - R_4^2) k_{sfill} L$$

$$\text{Volume End winding part : } V_{EWcop} = \frac{\pi^3 v (R_5^2 - R_4^2) (R_5 + R_4) k_{sfill}}{4p}$$

$$\text{Copper mass : } M_{cop} = \rho_{cop} (V_{apcop} + V_{EWcop})$$

◆ **HTS coils**  $\rho_{HTS} = 5500 \text{kgm}^{-3}$ :

$$\text{Volume active parts : } V_{apHTS} = \pi (\alpha - \phi) (R_3^2 - R_2^2) k_{rfill} L$$

$$\text{Volume End winding part : } V_{EWHTS} = \frac{\pi^3 (\alpha + \beta) (\alpha - \beta) (R_3^2 - R_2^2) (R_3 + R_2) k_{rfill}}{8p}$$

$$\text{HTS mass : } M_{HTS} = \rho_{HTS} (V_{apHTS} + V_{EWHTS})$$

$$\text{Total mass : } M_T = M_{apcop} + M_{EWcop}$$

# Design Optimization with Matlab Toolbox

- ◆ We have to define two Matlab function functions: `constraint.m` and `objective.m`  
`constraint.m`: return the value of the constraint (torque)  
`Objective.m`: return the value of the objective (Mass of the windings)
- ◆ We have to choose an optimization method: mono objective Genetic Algorithm (`ga.m`)
- ◆ We have 11 variables with 1 integer (number of pole-pairs). We have to choose lower and upper bounds for this variables (not easy, trial and errors....)

Variable	Number		Lower	Upper
p	Pole-pairs number (integer)	x(1)	1	10
R <sub>1</sub>	Inner radius of the rotor (cm)	x(2)	10	40
a	Thickness rotor yoke (cm)	x(3)	0	0
b	Thickness HTS winding (cm)	x(4)	0.5	2
d	Thickness stator winding (cm)	x(5)	0.5	5
e	2 <sup>nd</sup> air-gap thickness (cm)	x(6)	0	0
f	Thickness stator yoke (cm)	x(7)	0	0
L	Axial length (cm)	x(8)	20	50
α	Outer opening HTS coils	x(9)	0	1
β	Inner opening HTS coils	x(10)	0.1	1
v	Opening of the stator coils	x(11)	0.5	1



# Design Optimization with Matlab Toolbox

```
constraint.m  objective.m  machine_8_domaines.m  machine_8_domaines_couple_statique.m  n
1  function [cin,ceq]=constraint(x)
2
3
4  nn=1;                % Number of harmonic terms
5
6  muo=4*pi*1e-7;      % Air permeability
7  mur=1;              % Relative permeability of the rotor yoke
8  mus=1;              % Relative permeability of the stator yoke
9
10 % Geometrical parameters (cm)
11
12 p=x(1);              % Pole-pairs number
13 R1=x(2)*1e-2;       % Inner radius of the rotor yoke
14 a=x(3)*1e-2;       % Thickness of the rotor yoke
15 b=x(4)*1e-2;       % Thickness of the HTS winding (rotor)
16 c=1e-2;            % Air-gap thickness
17 d=x(5)*1e-2;       % Thickness of the stator winding
18 e=x(6)*1e-2;       % Second air-gap thickness
19 f=x(7)*1e-2;       % Thickness of the stator yoke
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328
329
330
331 % Torque calculation
332 %
333 for n = 1:nn
334     j=2*n-1;
335     Th=Th+(1/muo)*pi*L*((j*p)^2)*real(1i*(conj(K6n(n))*K7n(n)-K6n(n)*conj(K7n(n))));
336
337 end
338
339 C=Th;
340
341 cin(1)=abs(C-9500)-10; % the constaint corresponds to Torque = 9500Nm
342 ceq=[];
343
344 end
```

# Design Optimization with Matlab Toolbox

```
Editor - C:\LUBIN\RECHERCHE\Recherche_2020\Ecole_été_supra\presentation\programme\optimisation_mono_objectif\objective.m
constraint.m  objective.m  machine_8_domaines.m  machine_8_domaines_couple_statique.m  machine_8_domaines_fem.m  mac
1  function vol=objective(x) % active mass
2
3  Rocu=8960; % Density of copper
4  Roacier=8000; % Density of iron
5  Rosupra=5500; % Density of superconducting winding
6  ksupra=0.7; % Filling factor of the HTS coil
7  kcu=0.5; % Filling factor of the stator coil
8  c=1e-2; % Air-gap value
9  VR=1e-6*Roacier*pi*x(3)*(x(3)+2*x(2))*x(8); % Weight of the rotor yoke
10 VS=1e-6*Roacier*pi*x(7)*(x(7)+2*(x(2)+x(3)+x(4)+c+x(5)+x(6)))*x(8); % Weight of the stator yoke
11 Vsupra=1e-6*(Rosupra*ksupra*pi*(x(9)-x(10))*x(4)*(x(4)+2*x(2)+2*x(3))*x(8)+(Rosupra*ksupra*pi^3*(x(
12 Vcu=1e-6*(Rocu*kcu*pi*x(11)*x(5)*(x(5)+2*(x(2)+x(3)+x(4)+c))*x(8)+Rocu*kcu*pi^3*x(11)*x(5)*(x(5)+2*
13
14 %vol(1)=VR+VS+Vsupra+Vcu; % total weight
15 vol(1)=Vsupra+Vcu; % activ weight
16
17
18 end
```

# Design Optimization with Matlab Toolbox

**Problem Setup and Results**

Solver: **ga - Genetic Algorithm**

Problem

Fitness function: **@objective**

Number of variables: **11**

Constraints:

Linear inequalities: A:  b:

Linear equalities: Aeq:  beq:

Bounds: Lower:  Upper:

Nonlinear constraint function: **@constraint**

Integer variable indices: **1**

Run solver and view results

Use random states from previous run

**Start** **Pause** **Stop**

Current iteration: **766** **Clear Results**

Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 2.051210e-16

**Objective function value: 46.237592163394886**

Optimization terminated: average change in the penalty fitness value less than options.FunctionTolerance and constraint violation is less than options.ConstraintTolerance.

**Final point:**

1	2	3	4	5	6	7	8	9	10	11
10	39.774	0	0.871	0.85	0	0	22.946	0.999	0.275	0.786

**Options**

**Population**

Population type: **Double vector**

Population size:  Use default: max(min(10\*numberOfVariables, 100), 40)  
 Specify: **1000**

Creation function: **Uniform**

Initial population:  Use default: []  
 Specify:

Initial scores:  Use default: []  
 Specify:

Initial range:  Use default: []  
 Specify:

**Fitness scaling**

Scaling function: **Rank**

**Selection**

Selection function: **Stochastic uniform**

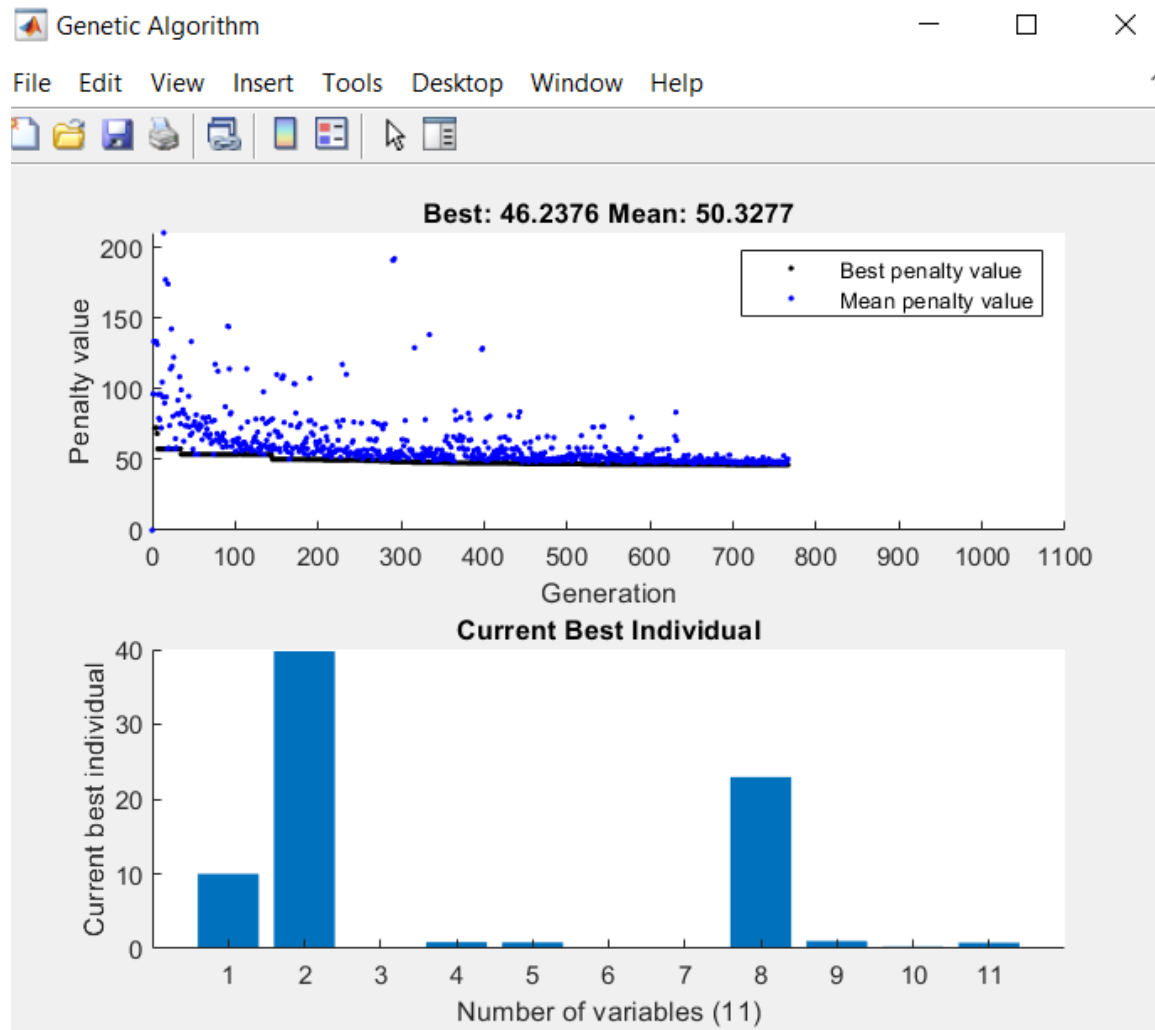
**Reproduction**

Elite count:  Use default: 0.05\*max(min(10\*numberOfVariables, 100), 40)  
 Specify:

Crossover fraction:  Use default: 0.8

◆ To reduce the mass, the algorithm tends to maximize the radius and the pole pair number (small end-winding), we obtain 46kg → 65kW/kg! (active mass)

# Design Optimization with Matlab Toolbox



# Design Optimization with Matlab Toolbox

- ◆ If we have a constraint on the radius, we can impose  $R_1 = 25\text{cm}$  (for example)

Variable	Number		Lower	Upper
p	Pole-pairs number (integer)	x(1)	1	10
$R_1$	Inner radius of the rotor (cm)	x(2)	<b>25</b>	<b>25</b>
a	Thickness rotor yoke (cm)	x(3)	0	0
b	Thickness HTS winding (cm)	x(4)	0.5	2
d	Thickness stator winding (cm)	x(5)	0.5	5
e	2 <sup>nd</sup> air-gap thickness (cm)	x(6)	0	0
f	Thickness stator yoke (cm)	x(7)	0	0
L	Axial length (cm)	x(8)	20	50
$\alpha$	Outer opening HTS coils	x(9)	0	1
$\beta$	Inner opening HTS coils	x(10)	0.1	1
v	Opening of the stator coils	x(11)	0.5	1

# Design Optimization with Matlab Toolbox

**Problem Setup and Results**

Solver: **ga - Genetic Algorithm**

Problem

Fitness function: **@objective**

Number of variables: **11**

Constraints:

Linear inequalities: A:  b:

Linear equalities: Aeq:  beq:

Bounds: Lower:  Upper:

Nonlinear constraint function: **@constraint**

Integer variable indices: **1**

Run solver and view results

Use random states from previous run

**Start** **Pause** **Stop**

Current iteration: **448** **Clear Results**

Optimization running

Objective function value: **52.47960334612428**

Optimization terminated: average change in the penalty fitness value less than options.FunctionTolerance and constraint violation is less than options.ConstraintTolerance.

Final point:

1	2	3	4	5	6	7	8	9	10	11
6	25	0	1.202	1.422	0	0	25.9	1	0.259	0.75

**Options**

**Population**

Population type: **Double vector**

Population size:  Use default: max(min(10\*numberOfVariables, 100), 40)  
 Specify:

Creation function: **Uniform**

Initial population:  Use default: []  
 Specify:

Initial scores:  Use default: []  
 Specify:

Initial range:  Use default: []  
 Specify:

**Fitness scaling**

Scaling function: **Rank**

**Selection**

Selection function: **Stochastic uniform**

**Reproduction**

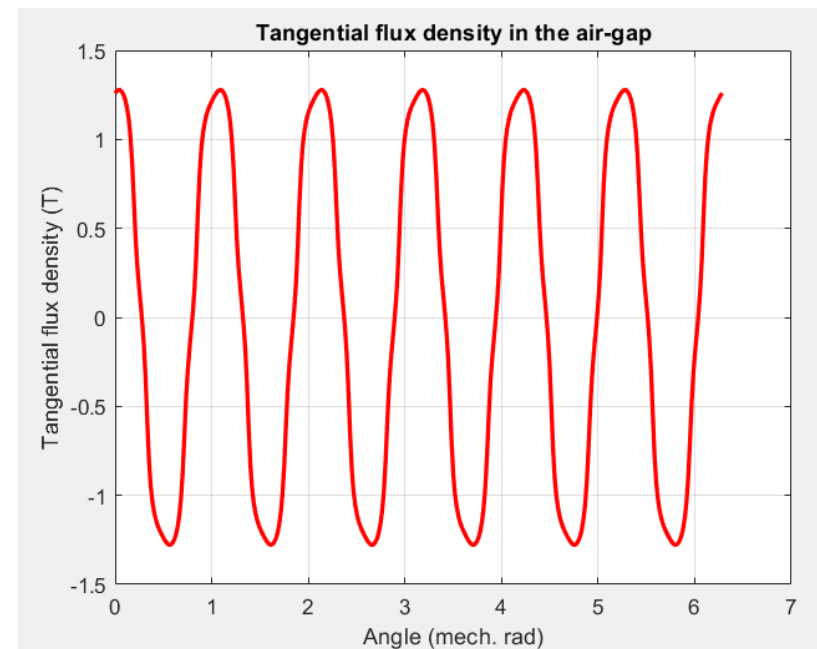
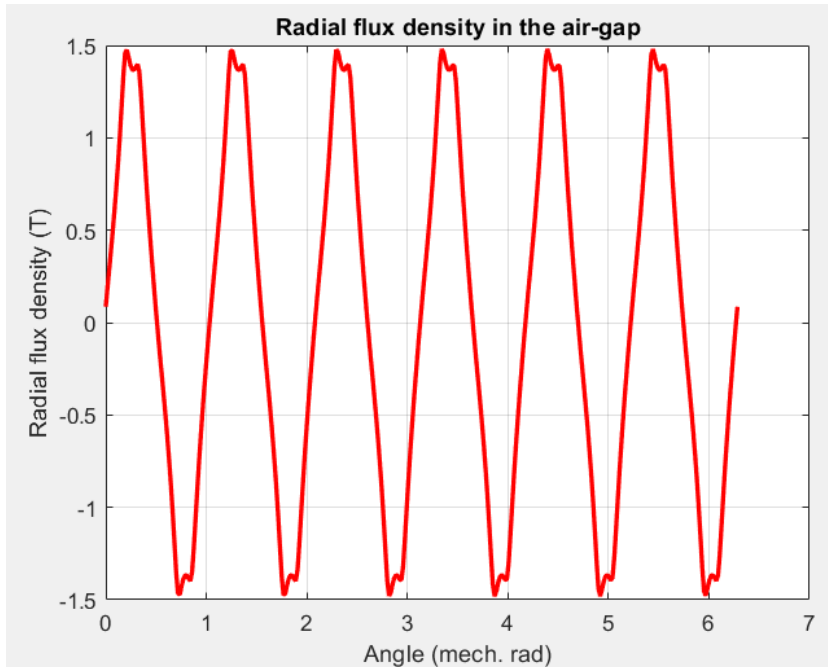
Elite count:  Use default: 0.05\*max(min(10\*numberOfVariables, 100), 40)  
 Specify:

Crossover fraction:  Use default: 0.8

◆ This increases the axial length... we obtain 52.5kg → 57 kW/kg! (active mass)

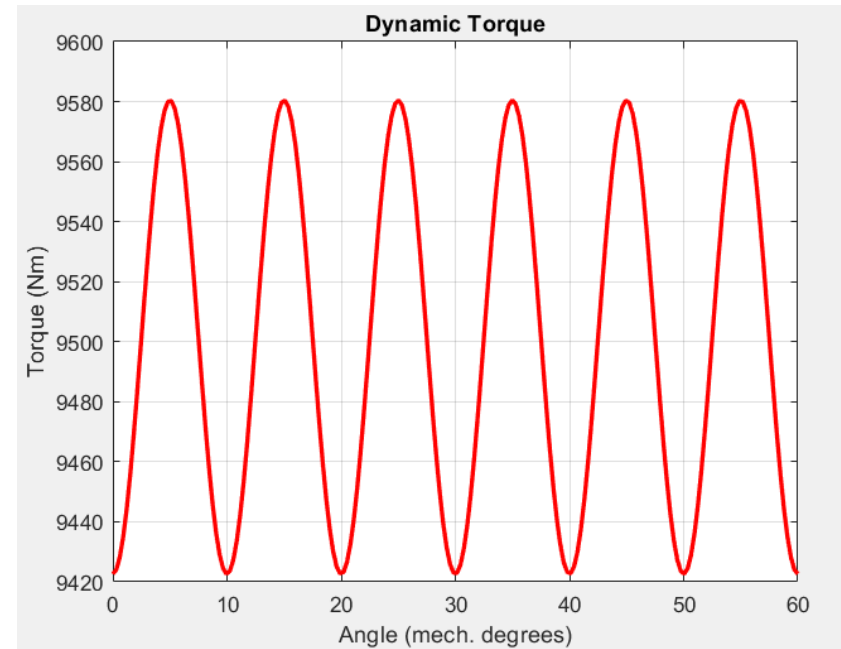
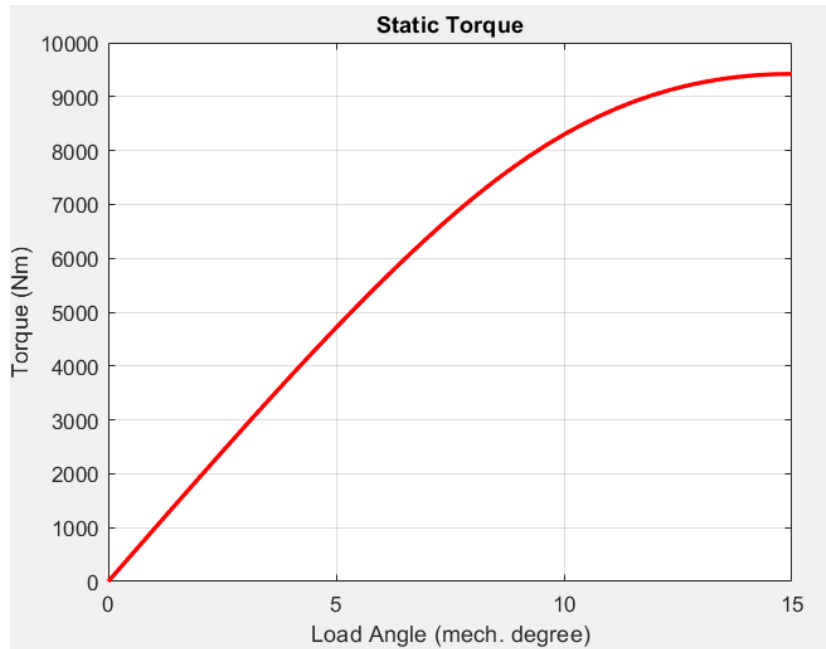
# Design Optimization with Matlab Toolbox

## Results: Flux density in the air-gap



# Design Optimization with Matlab Toolbox

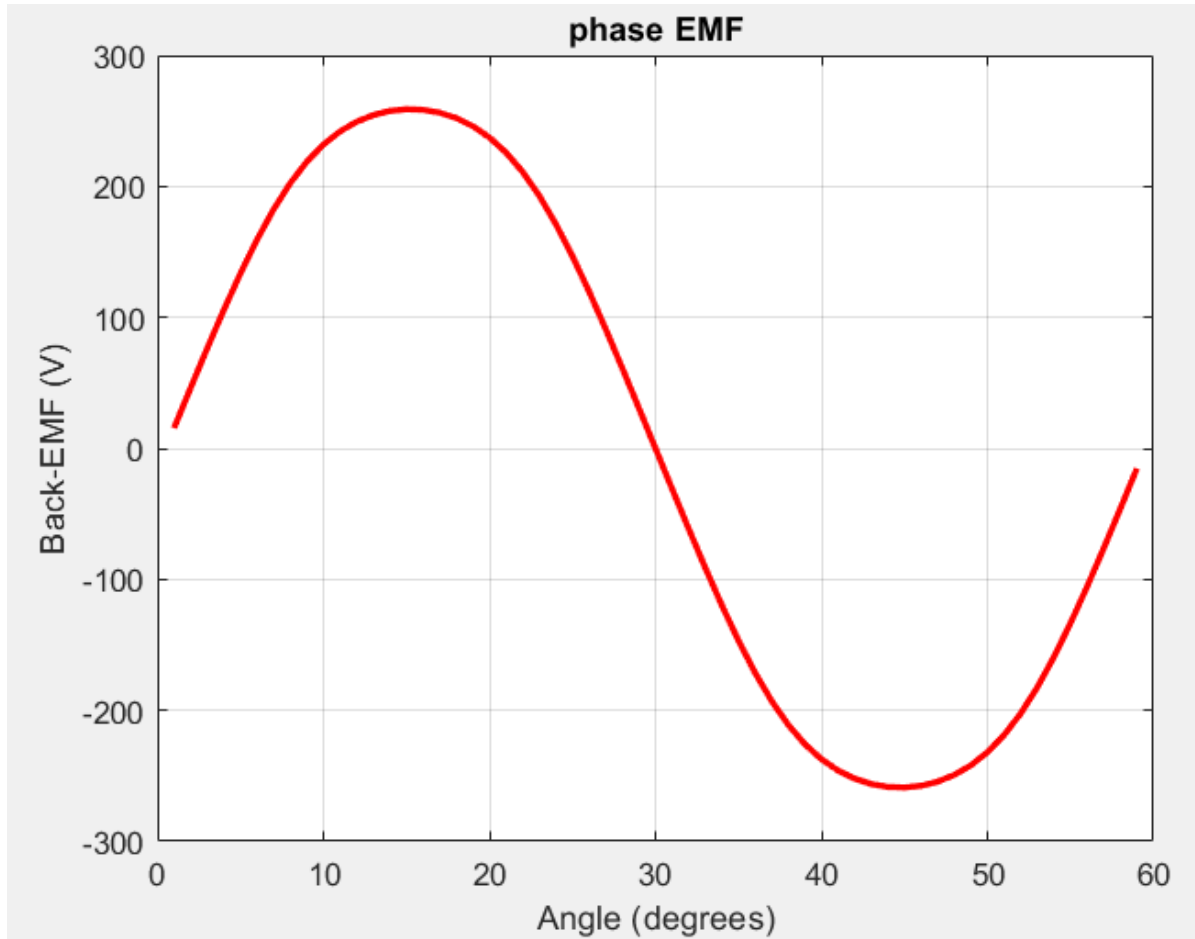
## Results: Static and dynamic torque



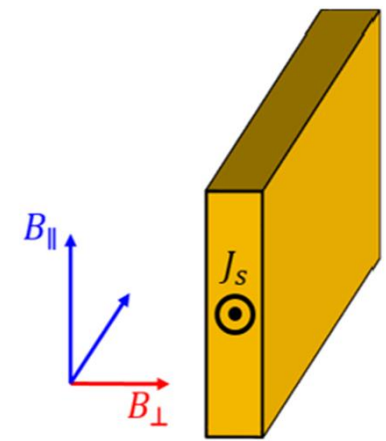
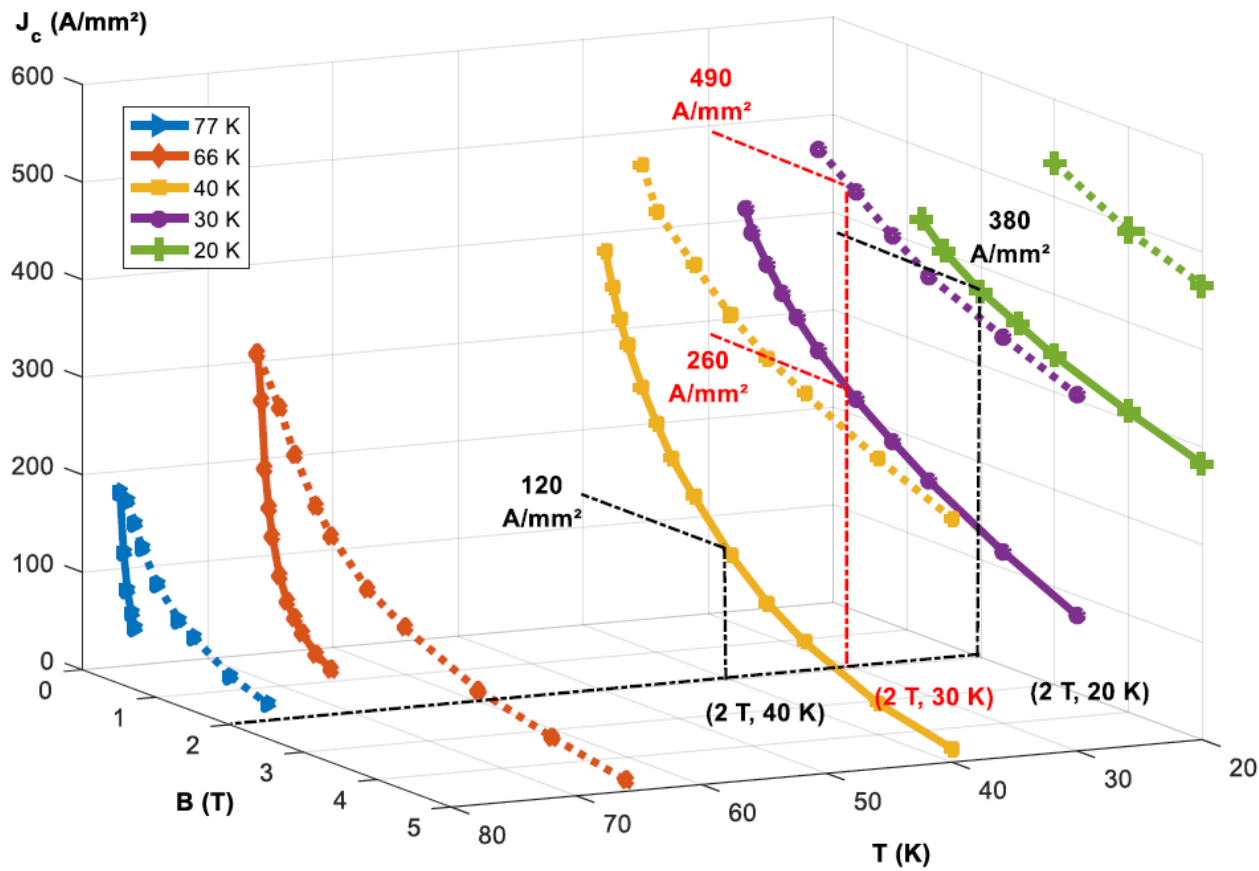


# Design Optimization with Matlab Toolbox

## Results: Back EMF (1 conductor/slot)



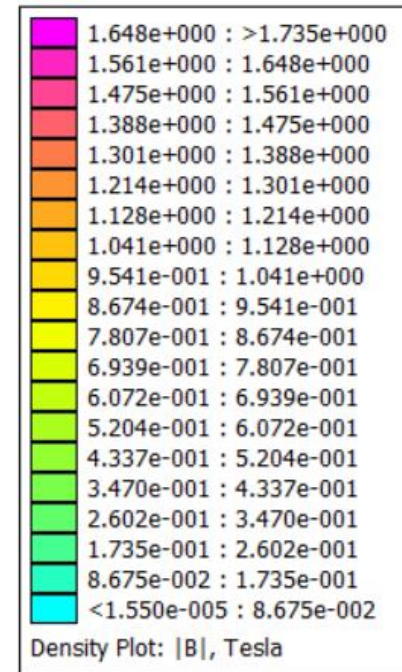
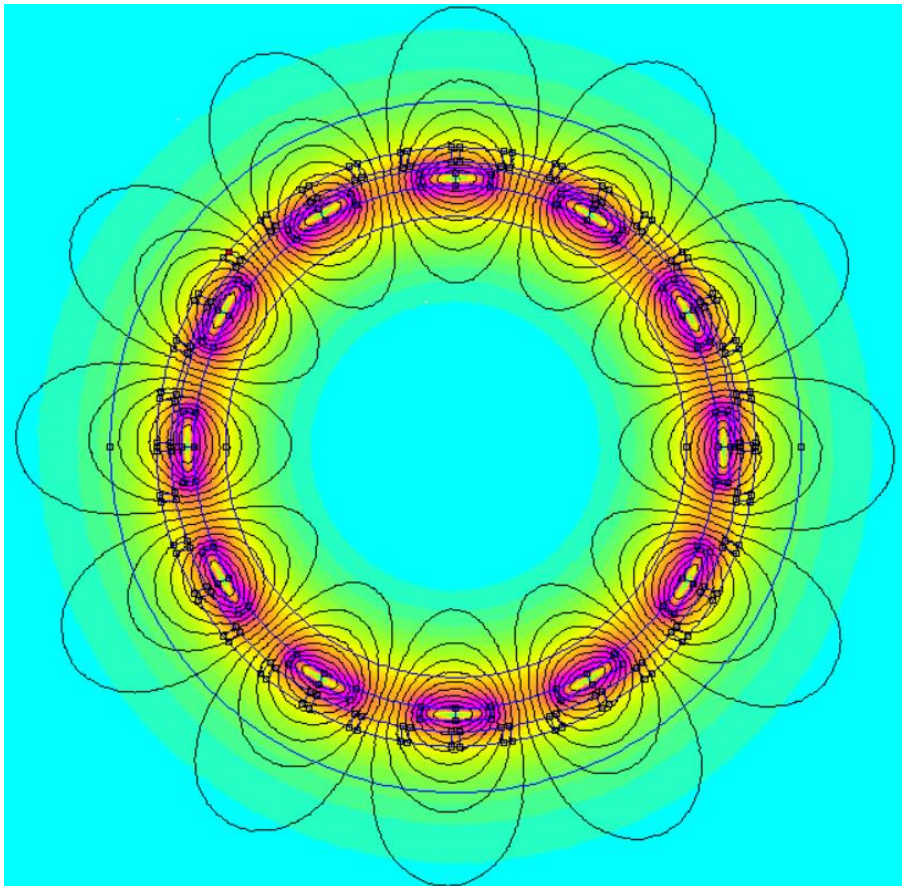
# HTS winding verification for the critical current density (needs to be done....)



◆ This should be include in the design procedure..... Perhaps the maximal constraint is on the end-winding (3D model.....)

# HTS winding verification for the critical current density (needs to be done....)

◆ B on HTS wire is never greater than 1.65T; Under 30K  $J_c = 300\text{A}/\text{mm}^2$  can be correct ....



# Multi-objective Design Optimization

gamultiobj.m

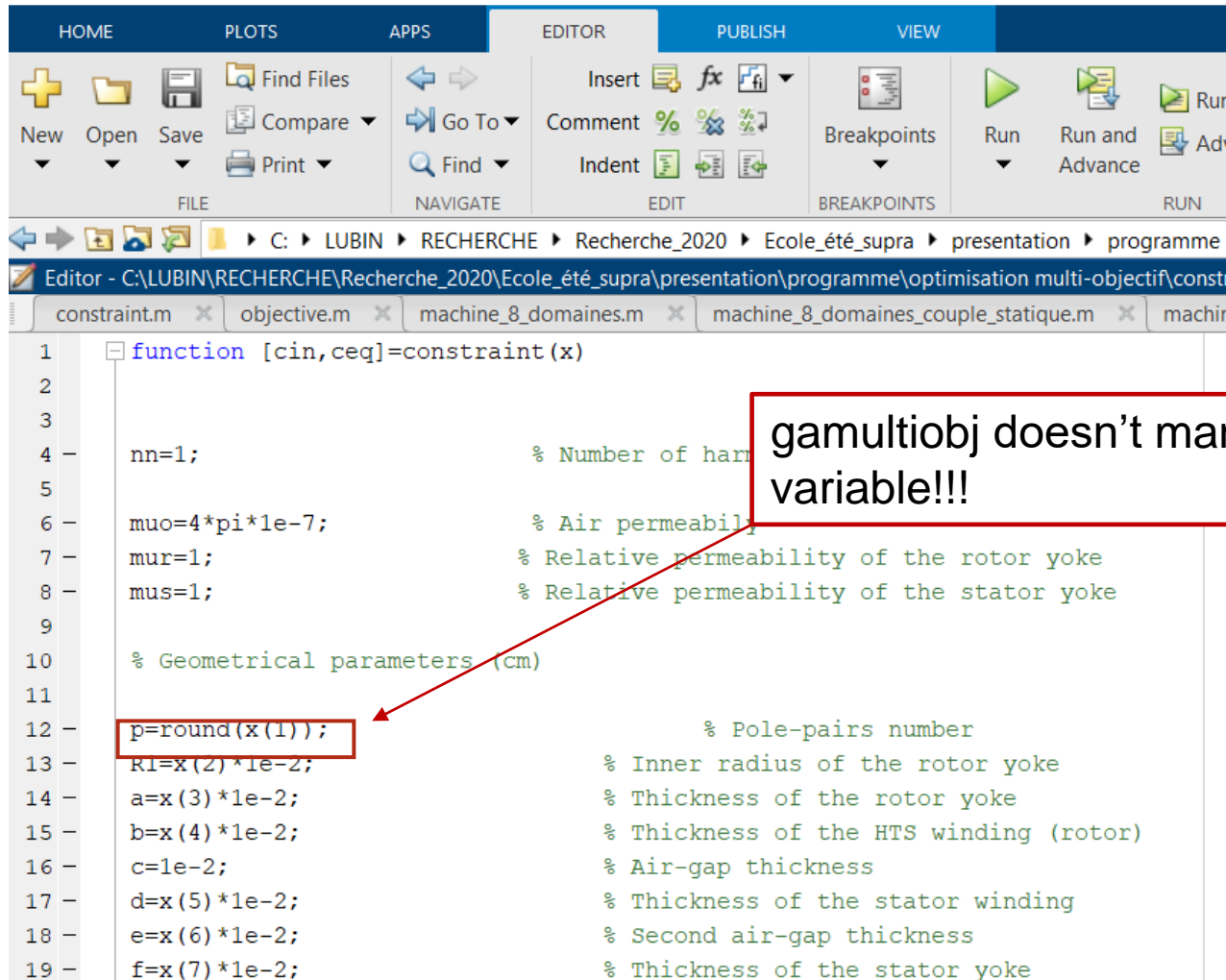
- ◆ We have to define two Matlab function functions: constraint.m and objective.m  
constraint.m: return the value of the constraint (torque)  
Objective.m: return the value of the objectives
- ◆ We have to choose an optimization method: multi objective Genetic Algorithm (**gamultiobj.m**)

**Two antagonist objectives: minimize the mass and minimize  $R_2$**

Variable	Number		Lower	Upper
p	Pole-pairs number (integer)	x(1)	1	10
$R_1$	Inner radius of the rotor (cm)	x(2)	10	40
a	Thickness rotor yoke (cm)	x(3)	0	0
b	Thickness HTS winding (cm)	x(4)	0.5	2
d	Thickness stator winding (cm)	x(5)	0.5	5
e	2 <sup>nd</sup> air-gap thickness (cm)	x(6)	0	0
f	Thickness stator yoke (cm)	x(7)	0	0
L	Axial length (cm)	x(8)	20	50
$\alpha$	Outer opening HTS coils	x(9)	0	1
$\beta$	Inner opening HTS coils	x(10)	0.1	1
v	Opening of the stator coils	x(11)	0.5	1

# Multi-objective Design Optimization

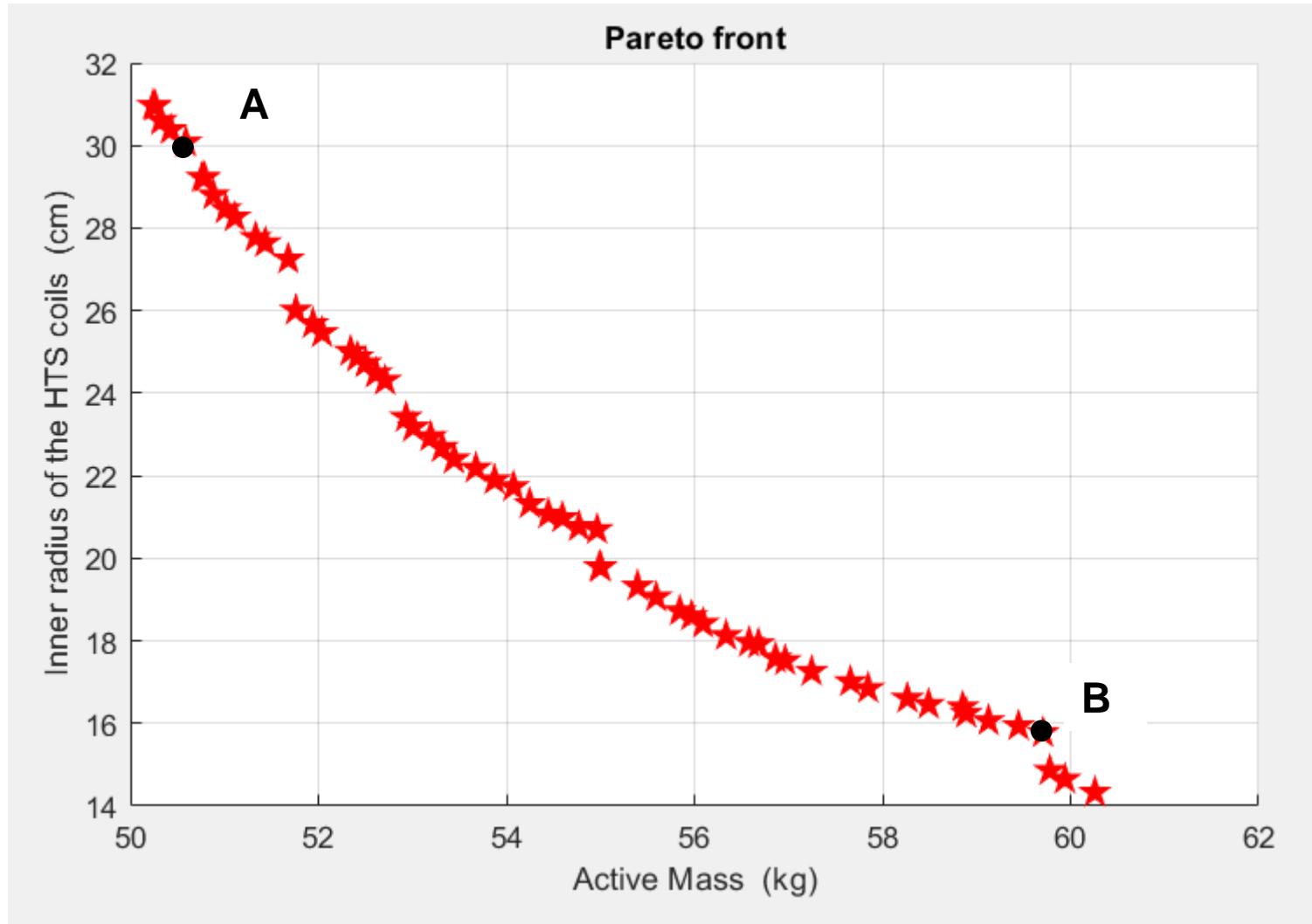
MATLAB R2019b - academic use



```
1 function [cin,ceq]=constraint(x)
2
3
4 nn=1; % Number of harmonics
5
6 muo=4*pi*1e-7; % Air permeability
7 mur=1; % Relative permeability of the rotor yoke
8 mus=1; % Relative permeability of the stator yoke
9
10 % Geometrical parameters (cm)
11
12 p=round(x(1)); % Pole-pairs number
13 R1=x(2)*1e-2; % Inner radius of the rotor yoke
14 a=x(3)*1e-2; % Thickness of the rotor yoke
15 b=x(4)*1e-2; % Thickness of the HTS winding (rotor)
16 c=1e-2; % Air-gap thickness
17 d=x(5)*1e-2; % Thickness of the stator winding
18 e=x(6)*1e-2; % Second air-gap thickness
19 f=x(7)*1e-2; % Thickness of the stator yoke
```

gamultiobj doesn't manage the integer variable!!!

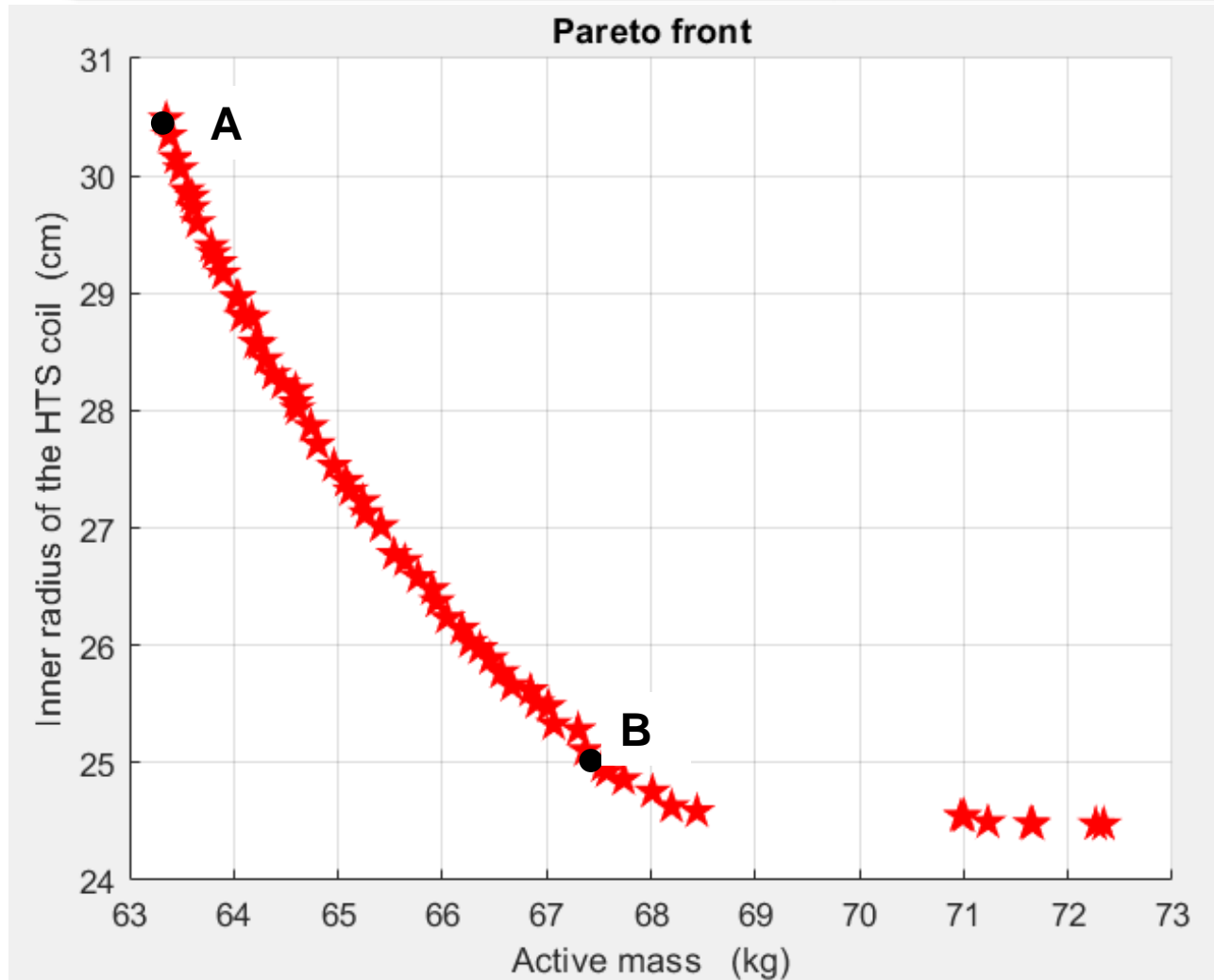
# Multi-objective Design Optimization: Pareto front ( $J_c = 300 \text{ A/mm}^2$ )



# Multi-objective Design Optimization: Results A and B

Variable	Number	Lower	Upper	A	B
p	Pole-pairs number (integer)	1	10	<b>8</b>	<b>4</b>
$R_1$	Inner radius of the rotor (cm)	10	40	<b>30.1</b>	<b>15.8</b>
a	Thickness rotor yoke (cm)	0	0	<b>0</b>	<b>0</b>
b	Thickness HTS winding (cm)	0.5	2	<b>1.2</b>	<b>2.1</b>
d	Thickness stator winding (cm)	0.5	5	<b>1.17</b>	<b>1.84</b>
e	2 <sup>nd</sup> air-gap thickness (cm)	0	0	<b>0</b>	<b>0</b>
f	Thickness stator yoke (cm)	0	0	<b>0</b>	<b>0</b>
L	Axial length (cm)	20	50	<b>30.2</b>	<b>33.3</b>
$\alpha$	Outer opening HTS coils	0	1	<b>0.99</b>	<b>0.99</b>
$\beta$	Inner opening HTS coils	0.1	1	<b>0.413</b>	<b>0.427</b>
v	Opening of the stator coils	0.5	1	<b>0.652</b>	<b>0.747</b>
Mass	Activ mass, windings (kg)			<b>50.6</b>	<b>59.7</b>
Rext	External radius (cm)			<b>33.4</b>	<b>20.7</b>
PTM	Power to Mass ratio (kW/kg)			<b>59.3</b>	<b>50.2</b>

# Multi-objective Design Optimization: Pareto front ( $J_c = 200 \text{ A/mm}^2$ )



Variable	A	B
p	7	7
$R_1$	30.5	24.9
a	0	0
b	1.25	1.49
d	1.45	1.45
e	0	0
f	0	0
L	24.7	28.5
$\alpha$	0.99	0.99
$\beta$	0.298	0.305
v	0.783	0.948
Mass	63.4	67.6
$R_{ext}$	34.2	28.9
PTM	47.3	44.4

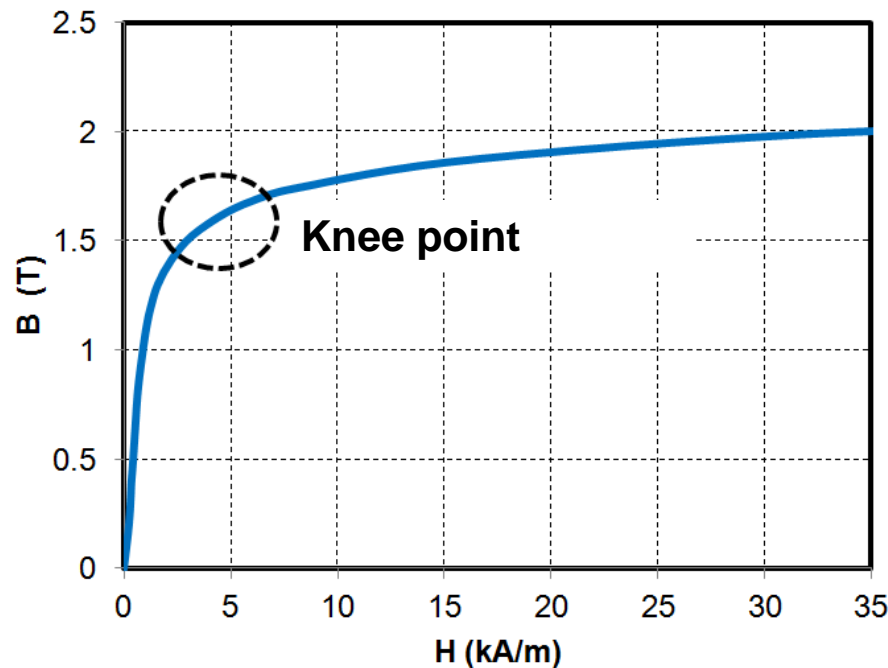
- ◆  $J_c=300\text{A/mm}^2$  and  $R_2 = 25\text{cm} \rightarrow M = 52.5\text{kg}$ , PTM = 57 kW/kg
- ◆  $J_c=200\text{A/mm}^2$  and  $R_2 = 25\text{cm} \rightarrow M = 67.6\text{kg}$ , PTM = 44 kW/kg



# Design Optimization with iron yoke

We have to limit the maximal value of the flux density in the stator and rotor yokes around the knee point of the B(H) characteristic (we have the analytical expression of the flux density in the yokes), if not the model will be not valid.....

Supplementary constraint :  $B_{\text{yoke}} < B_{\text{knee}}$  (around 1,6T)



**To do..... Further results are expected  
Is iron necessary if PTM is the objective, not sure????**



**THANK YOU !**