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# MUSIC STRUCTURE REPRESENTATION: A FORMAL MODEL

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## ABSTRACT

In the present work we introduce a general formal (Object Oriented) model for the representation of musical structure information, taking into account some common feature of the information vehiculed by music analysis. The model is suitable to represent many different kinds of musical analytical entities. As an example of both musical and mathematical-computational relevance, we introduce the D.Lewin's GIS theory. A GIS is equivalent to a particular kind of group action over a set: we show that other related structures can be treated in a similar manner. We conclude with the prototype implementation of these ideas in MX, an XML format for music information.

## 1. INTRODUCTION

Music representation in XML is a field in evolution. There are many proposals for a possible standard. MX's peculiarity is its multilayered structure (cfr. [3], [5], [6], [7]). One of these layers is devoted to music structure information, i.e. compositional and analytical aspects of musical information. The present work constitutes a theoretical survey to individuate formal entities for possible representations in the format and contains the sketch of a general model which could suggest some idea to extend its current implementation.

Analytical information encoding into a format containing music like mx, could open some different possibilities. We will imagine here some of these application. A music-analysis tool based on mx format would give an analyst a graphical interface to edit his own analysis, not only by visual, but also by aural means. The same capability (to aurally show the elements of the analysis), could make an mx-based tool a strongly popularizing tool, not only from an educational point of view. A software based on the mx format would allow to listen (in several audio versions) to parts of pieces described in the analyses, to directly follow the comments of the analyst on the score while music involved by these is executed. To have more than one analysis of the same piece would provide an effective instrument for comparative analysis. Furthermore, the possibility to compare different analyses could have still wider effects. Let's imagine as an example to have access to sources in a big database, containing varied analyses that could also be generated automatically, as an example, at the submission of the file in the database. By means of

searches on analysis-data in the files, one could recognize similarities on structural level between pieces analyzed by common features. An opportune agent could lead surveyings on a specific feature, searching recurrences in order to determine affinities.

The way in which analytical information could be stored is the subject of the next section.

## 2. MUSICAL STRUCTURE

The first problem is to define what is musical structure. If we consider the approaches to music analysis, we'll find each one to have its own tools, methods and results. Each analysis reveals a specific structure. The problem could become then: how to model music analyses. We are not interested here in each motivation for a specific methodology. We'll limit ourselves to observe some common features.

### 2.1. Basic common structure of analyses

A very basic structure (of an analysis, not of a piece!), common to almost all analytical approaches (e.g. Forte and set-theory [4], Nattiez and Paradigmatic analysis [12], automated analysis conducted in ways like Meudic's [10]) is the *segmentation* of a piece. In each school of music analysis, segmentation has its own specificities in term of when and how it is conducted. Very interesting theoretical problems arises around segmentation, intended as the process by which a piece is segmented, but a discussion here is beyond the scope of the present work. We will call here segmentation the *result* of the process. So we will define a segmentation of a piece, in a very general sense, as a *collection of segments*. A segment will be a *set of references to events* in a music piece.

When the process of segmentation is done and a collection of segments is obtained, the next step we want to concentrate on is the relating the segments one another. This is done accordingly the specific methodology in action. Now we simply notice the *presence of relations between segments*.

Segmentation, segments and relations between segments are the very basic structure contained in MX format to represent music analyses. We will intend this structure as the formal shadow of an analysis of a piece of music.

## 2.2. A formal (object oriented) model for specific analysis representation

To discover similarities between segments of music we need a particular musical sensibility. We need to concentrate to particular music features. In order to define relation between segments we need a way to define these features. First of all we must decide which musical feature we are interested in. Many approaches associate feature vectors to segments (like in [1]). We think that the more general concept of *object* could offer a more powerful possibility: in an object oriented perspective, one will be able to define his own *feature object class* in order to model his own music attitude. In a limit case a feature object could be just a textual description (not so naturally formalizable within a vector). So, if we have a segmentation  $\sigma$  we can imagine an arrow into an abstract FeatureObject  $\Omega$  class:

$$\begin{aligned} f : \Sigma &\longrightarrow \Omega \\ \sigma &\longmapsto \omega \end{aligned} \quad (1)$$

Depending on the particular decisions for a concrete derived class of objects, let's call it  $S$  (Space of objects), a part of an analysis will be based on a new association:

$$\begin{aligned} f_{my} : \Sigma &\longrightarrow S \\ \sigma &\longmapsto s \end{aligned} \quad (2)$$

Now we can predicate something on the segments  $\sigma \in \Sigma$  in our particular point of view (one could say *frame*) by means of the featureObjects  $s \in S$ . In this perspective we can relate segments by defining relations between our objects associated to them. In certain cases there will be formal rules to establish such relations. In certain cases there will be *methods* to say if a segment is related to another and in which way it is.

The next two sections show a formal mathematical model (of music analytical relevance) which could constitute an example of a possible instantiation for feature objects and relations. There will be shown that in these models relations and objects have particular algebraic structures. This suggests the possibility (beyond the scope of this paper, but constituting a trace for further developments) to take advantage from these structures for further investigation.

## 3. GENERALIZED INTERVAL SYSTEMS

We're going to introduce a structure proposed by the american music-theorist D.Lewin ([9]). This structure is based on the concept of interval in an extended manner. With **GIS** it's possible to represent not only pitch-related, but also rhythmic musical objects and their relationships. We want to emphasize the idea that the choice of a particular **GIS** lets the modelling of particular features of music. A **GIS** will be a candidate model for a class of objects to put in the S-level introduced in the previous section.

First of all we proceed with the formal definition of **GIS** as given by Lewin. Then we will give an alternative characterization of a **GIS** which makes use of the concept

of a simply transitive action over a set<sup>1</sup>. This will permit us to generalize the model in order to represent a more general family of algebraic musical structures.

**Definition 3.1.** A **GIS** is defined as  $(S, \mathbf{IVLS}, \mathbf{int})$ , being:

- $S$  ('space'), a *set*
- **IVLS** (intervals), a *group*
- **int** an *application*

$$\mathbf{int} : S \times S \longrightarrow \mathbf{IVLS}$$

satisfying the following conditions:

$$(A) \quad \forall r, s, t \in S$$

$$\mathbf{int}(r, s) \mathbf{int}(s, t) = \mathbf{int}(r, t) \quad (3)$$

$$(B) \quad \forall s \in S, i \in \mathbf{IVLS} \exists ! t \in S :$$

$$\mathbf{int}(s, t) = i \quad (4)$$

**Definition 3.2.** We will say *commutative* a **GIS** which intervals group is abelian.

Given a **GIS**, we'll be able to describe *particular musical features*: those represented by the 'space' of musical objects (fixing a 'space'  $S$  is a sort of concentrating on a particular musical dimension). We'll also be able to *calculate* - via **IVLS** - an *interval* between two musical objects (elements of  $S$ ). In a certain way choosing a **GIS** is like modelling a particular analytical attitude.

Some very simple examples.

*1 Example.* Let the music space be the any musical scale steps. Given two steps  $s, t \in S$ , let  $\mathbf{int}(s, t)$  be the number of steps between the two. For instance let's take into account the diatonic scale. We have

$$\mathbf{int}(C_4, D_4) = 1$$

$$\mathbf{int}(C_4, A_3) = -2$$

If we move by two steps (e.g. from  $C_4$  to  $E_4$ ), then by two more steps (e.g. from  $E_4$  to  $G_4$ ) we'll have done  $2+2=4$  degrees of the scale:

$$\mathbf{int}(C_4, E_4) = 2$$

$$\mathbf{int}(E_4, G_4) = 2$$

$$\mathbf{int}(C_4, G_4) = 4$$

The 'space'  $S$  so described, with the additive group of relative integers  $\mathbf{Z}$  and the application **int** is a first, trivial example of a **GIS**. In fact: **int** satisfies (A). It satisfies also (B): given any step  $s$  of the scale  $S$  and any  $n \in \mathbf{Z}$  one can find a step moving by  $n$  steps of the scale such that

$$\mathbf{int}(firstnote, foundnote) = n$$

<sup>1</sup> See [15] for a punctual formalization of the equivalence between the two definitions

2 *Example.* Another example of **GIS** is the space of pitch classes of equal temperament. We forget the octave of a pitch and concentrate uniquely on its class. We can represent the pitch classes disposed around a 'clock', a well known representation.  $int(s, t)$  will be the number of 'minutes' in clockwise sense to from the 'minute'  $s$  to the 'minute'  $t$ . It's easy to see that

*Proposition 3.3.*  $(S, (\mathbf{Z}_{12}, +), int)$  is a **GIS**.

*Remark 3.4.*  $S$  is the model of set-theoretical analysis.

Now some examples of rhythmic **GIS**

3 *Example.* Let  $S$  be a succession of equidistant temporal points (imagine a pulsing uniform 'tactus'). We can represent it with  $\mathbf{Z}$ . The interval between two of such points will be the number of 'tactus' between them. Hence we'll have:

$$IVLS = (\mathbf{Z}, +)$$

$int : (s, t) \mapsto$  number of temporal units between  $s$  and  $t$

i.e.

$$int : (s, t) \mapsto t - s$$

4 *Example.* Another rhythmic space is the set of time spans. An interval will be the ratio between two spans (we perceive a relation between two couples of spans if their ratios coincides). We will then consider

$$S = \mathbf{Q}$$

$$IVLS = (\mathbf{Q}^*, \cdot)$$

$$int : (x, y) \mapsto \frac{y}{x}$$

As we said at the beginning of this section, it is possible to define the **GIS** structure in the following alternative way.

**Definition 3.5** (of **GIS**, alternative). A **GIS** is  $(S, IVLS, \tau)$ , where  $S$  is a set, **IVLS** a group and  $\tau$  a simply transitive action of **IVLS** over  $S$ .

#### 4. HARMONIC OBJECTS: KLANG AND THEIR TRANSFORMATIONS

In this section we will describe a new family of objects that is interesting from many points of view. The theory discussed here is introduced by D.Lewin in his book. He was inspired by the work of the music-theorist H.Riemann. The original purpose was to find a valid alternative to music analysis with roman numbers<sup>2</sup>.

We found very interesting relations between the model presented here and the entities automatically recognized by the algorithm suggested in [14]. In particular we think that the objects we're going to present, are a suitable formal model for those entities.

At the end of this section we will demonstrate that this Riemannian model is an example of a generalization of

<sup>2</sup> For an interesting introductory overview about the evolution of harmony notation see the book of Nicolas Cook[2]

the **GIS** structure, not only an example of an exception. This suggested us to the possibility to take into account a more general family of structures. We are going to show how these are again well defined mathematical structures.

#### 4.1. Definitions

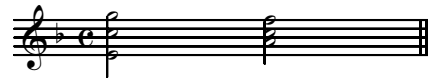
**Definition 4.1.** A *Klang* is an ordered couple

$$(p, \sigma) \in \mathbf{Z}_{12} \times \mathbf{Z}_2$$

**Definition 4.2.** We will call  $K$  the set of *Klangs*

*Remark 4.3.* In the musical semantic a *Klang*  $(p, \sigma)$  is an harmonic object of fixed tonic (represented by  $p$ ) and modality (major, minor, represented by  $\sigma$ ). We will have a very intuitive notation of the modality of a *Klang* if we will consider  $\mathbf{Z}_2$  in multiplicative notation (+1 for a major mode, -1 for a minor one).

We can now define some particular transformation of the set of *Klangs*  $K$ . Consider the following example: a C major harmony followed by a F Major one. We will



**Figure 1.** C Major, F Major

express what one could write V-I using roman numbers notation (the first chord has the role of the dominant with respect to the second one thought as the tonic), by

$$(C, Major)DOM = (F Major)$$

The meaning is that

C Major becomes the dominant of F Major

and, using the suggested formalism, we will write

$$(0, +1)DOM = (5, +1)$$

where the pitch class 'C' is represented by  $0 \in \mathbf{Z}_{12}$  and 'F' by 5. In this way one can formalize what is perceived music is the variation of a tonal reference. It's interesting to observe that with this formula we do represent the same information contained in the expression 'V-I' but not only: we also have the explicit information about which are the tonics and modalities of the considered harmonies. We will consider bijective transformations of  $K$  in  $K$  and we will call them *Klang* transformations. In general, given

$$Transf : K \longrightarrow K$$

we will write  $(p, \rho)Transf = (q, \sigma)$  (we use right notation for historical motivations and to simplify intuition). The meaning is: the harmony  $(p, \rho)$  gets the role given by  $Transf$  in the next harmony  $(q, \sigma)$ .

## 4.2. Transformation examples

We described informally the *DOM* transformation. We can give a formal definition. Following the musical semantic we can say that with *DOM* if we start with a major (minor) tonality, we obtain a new major (minor) tonality having its tonic 5 semitones upper with respect to the original tonic. It's possible to formalize this in the following way:

**Definition 4.4** (formal, of *DOM* trans $f$ ).

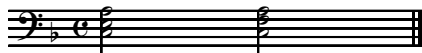
$$\begin{aligned} \text{DOM} : K &\longrightarrow K \\ (p, \sigma) &\longmapsto (p + 5, \sigma) \end{aligned} \quad (5)$$

*Remark 4.5.* Here and in this section we will use + and - followed by numbers to mean the result of the action  $\tau$  of on  $\mathbf{Z}_{12}$  (we can consider the cartesian product introduced in the definition of Klang as a particular GIS). We do not enter in more formal details here, but we want to emphasize that this is not only a stenography and that the calculations that will follow are formally justifiable.

*Remark 4.6.* *DOM* is a bijection of  $K$  in itself.<sup>3</sup>

What kind of musical sensibility do we model with this kind of transformation? What do we understand of a composition if we can remark *DOM* transformations in the harmonic texture? A composition ruled only by *DOM* could contain any tonality of a fixed mode. We can model by *DOM* harmonic movements to neighbour tonality (in tonal music). We can also think that a only-*DOM*-sensible ear could be able to model macro-areas of tonal influence, without going into deeper details about every single chord structure in a single area.

Let's now consider a finer grained harmonic sensibility, introducing a new transformation of Klang:



**Figure 2.** A minor, F Major

**Definition 4.7.** We will call *MED* a Klang transformation that describes an harmony  $(p, \rho)$  becoming *mediant* of a new one  $(q, \sigma)$  (*III - I* or *III $\flat$  - I*). Starting from a major harmony we get to a minor one having its tonic 3 semitones 'lower'.

$$\begin{aligned} \text{MED} : K &\longrightarrow K \\ (p, \rho) &\longmapsto \left( \begin{cases} p - 3 & \text{if } \rho = +1 \\ p - 4 & \text{if } \rho = -1 \end{cases}, \rho * (-1) \right) \end{aligned}$$

*Remark 4.8.* *MED* is a bijection

<sup>3</sup> It's injective:  $\forall k_1 = (p_1, \sigma_1), k_2 = (p_2, \sigma_2) \in K$  we have  $(k_1)\text{DOM} = (k_2)\text{DOM} \Leftrightarrow (p_1 + 5, \sigma_1) = (p_2 + 5, \sigma_2) \Leftrightarrow k_1 = k_2$ .

It's surjective:  $\forall k_1 \in K \exists k_2 \in K$  so that  $(k_1)\text{DOM} = k_2$ . One can consider  $p_2 := p_1 - 5$  and  $\sigma_2 := \sigma_1$ .

Let's now go back to the concept of a *harmony ruled by repeated applications of DOM*. We can formalize this concept considering  $\langle \text{DOM} \rangle$ , the group generated by the *DOM* transformation<sup>4</sup>. We will write  $\text{DOM}^n$  to indicate repeated compositions of *DOM* with itself  $n$  times.  $\text{DOM}^{-1}$  will be the inverse of *DOM*.

We said that *it's possible to reach with DOM any tonality of given modality*. To formalize this concept we need some more precise definition. It's trivial that

**Proposition 4.9.**  $\langle \text{DOM} \rangle$  acts on  $K$ .

Let's remind the following definitions and propositions (cfr.[8])

**Definition.** Let  $G$  be a group acting on a set  $X$  (that's equivalent to say that  $X$  is a  $G$ -set). We define *orbit* of a an element  $x \in X$  the subset of  $X$

$$G \cdot x = \{g \cdot x | g \in G\}$$

**Proposition.** *The orbits of a G-set coincide or are disjoint. So a G-set is decomposable as the union of disjointed subsets.*

**Proposition.** *Given a G-set X, the relation  $\sim$  on X*

$$x \sim y \Leftrightarrow \exists g \in G : g \cdot x = y$$

*is an equivalence relation.*

**Definition.**  $X/G$  is the set of the equivalence classes of  $\sim$  (quotient of  $X$  mod  $G$ ).

We can now enunciate the following

**Proposition 4.10.** *the orbit of an element  $(p, +1)$  (major harmony) of the  $\langle \text{DOM} \rangle$ -set  $K$ , is the set of all and only those elements of the form  $(\_, +1)$ , i.e. all the major harmonies. (the same happens for the minor harmonies). The set {major harmonies, minor harmonies} is the quotient set  $K / \langle \text{DOM} \rangle$  that is:*

$$K / \langle \text{DOM} \rangle \simeq \mathbf{Z}_2$$

Consider now the following musical fact: the mediant of the mediant of any given harmony (major or minor) is it's dominant. Algebraically we should have  $\text{MED}^2 = \text{DOM}$ . In fact:

$$\begin{aligned} ((p, -1)\text{MED})\text{MED} &= (p - 3, +1)\text{MED} \\ &= (p - 3 - 4, -1) \\ &= (p - 7, -1) \\ &= (p + 5, -1) \end{aligned}$$

$$\begin{aligned} ((p, +1)\text{MED})\text{MED} &= (p - 4, -1)\text{MED} \\ &= (p - 4 - 3, +1) \\ &= (p - 7, +1) \\ &= (p + 5, +1) \end{aligned}$$

<sup>4</sup> Note that this is a group with respect to composition of transformations (bijections) of  $K$  to  $K$ , being a subgroup of all the bijections of  $K$  in itself.

(we used the fact that  $5 \cong 7 \pmod{12}$ ). And  $\langle DOM \rangle$  is a subgroup of  $\langle MED \rangle$ .

What about the behavior of  $MED$ ? We have the following

**Proposition 4.11.** *We have<sup>5</sup>*

$$\forall k \in K \text{ orbit}_{\langle MED \rangle}(k) = K$$

i.e.  $K / \langle MED \rangle \simeq \{\cdot\}$

*Proof.* First of all let's try to define  $MED^n$ .

Given  $k = (p, \sigma) \in K$ , we'll call  $(q, \rho)$  the Klang  $(p, \sigma)MED$

The modality (second component) of a Klang change sign for odd values of  $n$ , it doesn't for even values. That is  $\rho = \sigma \cdot (-1)^n$ .

What happens to the tonic (first component of the Klang)?

Let  $n$  be even.

For  $\sigma = +1$  we have  $q = p - 3 - 4 - 3 - \dots - 3 - 4 \pmod{12}$

For  $\sigma = -1$  We have  $q = p - 4 - 3 - 4 - \dots - 4 - 3 \pmod{12}$

Hence, in both cases,

$$q = p - \frac{n}{2} \cdot 7 \pmod{12}$$

Let  $n$  be odd.

For  $\sigma = +1$  we have  $q = p - 3 - 4 - \dots - 4 - 3 \pmod{12}$ .

So in this case

$$q = p - \frac{n-1}{2} \cdot 7 - 3 \pmod{12}$$

For  $\sigma = -1$  we have  $q = p - 4 - 3 - \dots - 3 - 4 \pmod{12}$ .

So in this case

$$q = p - \frac{n-1}{2} \cdot 7 - 4 \pmod{12}$$

How then acts  $\langle MED \rangle$  on the set  $K$ ? Musically reasoning we should expect the orbit of any  $k \in K$  to be the whole  $K$ . So, we wonder if the following proposition is true

**Proposition.** *Given  $(p, \sigma) \in K$*

$$\forall (q, \rho) \in K \exists n \in \mathbf{N} \text{ so that } (p, \sigma)MED^n = (q, \rho)$$

Remark that calculating  $n$  correspond to the musical question: how many mediant harmony modulations do we need to go from any given harmony to another? This question is answered solving the last equation with respect to  $n$ .

We can proceed in the following way (solution algorithm).

The parity of  $n$  is determined by the condition  $\sigma \cdot (-1)^n = \rho$ .

$\sigma, \rho$  of the same sign:  $n$  is even.

Depending on the parity of  $n$  and on  $\sigma$  we will consider one of the three pieces of  $MED^n$ .

<sup>5</sup> We intend with  $\text{orbit}_{\langle MED \rangle}(k)$  the orbit of  $k$  as an element of the  $\langle MED \rangle$ -set  $K$ , that is  $\langle MED \rangle \cdot k$

We'll have three cases:

n	$\sigma$	equation in $\mathbf{Z}_{12}$
even	+1 or -1	$7h = r \ (h := \frac{n}{2}, r := p - q)$
odd	+1	$7h = r \ (h := \frac{n-1}{2}, r := p - q - 3)$
odd	-1	$7h = r \ (h := \frac{n-1}{2}, r := p - q - 4)$

Every equation has only one solution in  $\mathbf{Z}_{12}$  because 7 and 12 are coprime. The proposition follows.  $\square$

Let's now introduce a third transformation of particular musical interest. We are taking into account the harmonic movement from an harmony to the one having the same tonic, but opposed modality (an example is the well-known phenomenon of the 'terza piccarda': when a piece in minor mode ends with a chord of the same tonic, but major mode).



**Figure 3.** D minor, D Major

Proceeding formally:

**Definition 4.12** (of  $PAR$  transformation).

$$PAR : K \longrightarrow K$$

$$(p, \sigma) \longmapsto (p, -\sigma) \quad (6)$$

*Remark 4.13.*  $PAR$  is a bijection. (It's trivial)

As usual let  $\langle PAR \rangle$  be the group generated by  $PAR$ .

*Remark 4.14.*  $PAR^2 = 1$ .

Consider now the group generated by  $MED$  and  $PAR$ . We immediately note that

*Exercise.* the group  $\langle MED, PAR \rangle$  is not abelian.

Counter-example:

$$((0, +1)MED)PAR = (9, -1)PAR = (9, +1)$$

(i.e. C Major becomes MEDiant of A minor that becomes PARallel of A major)

$$((0, +1)PAR)MED = (0, -1)MED = (8, +1)$$

(i.e. C Major becomes PARallel of C minor that becomes MEDiant of A flat Major)

What about the orbit of an element  $k \in K$ ? We have:

**Proposition 4.15.**

$$\forall k := (p, \sigma) \in K \text{ orbit}_{\langle PAR \rangle}(k) = \{(p, \sigma), (p, -\sigma)\}$$

i.e.  $K / \langle PAR \rangle \simeq \mathbf{Z}_2$

$\langle MED, PAR \rangle$  contains  $MED$ , hence it's clear that the orbit of an element will coincide with the whole  $K$ , and that  $\langle MED \rangle \subseteq \langle MED, PAR \rangle$ .

It's not difficult to prove that

**Proposition 4.16.**  $\langle MED \rangle \neq \langle MED, PAR \rangle$ .

*Proof.* We have to show that  $PAR \notin \langle MED \rangle$ , i.e. there is no value of  $n$  such that  $MED^n = PAR$ . If there were a solution what would it be? For instance let's pose the following harmonic question: is there a value of  $n \in \mathbf{N}$  for which C Maj becomes  $MED^n$  of C min? Formally:

$$(0, +1)MED^n = (0, -1) \quad (7)$$

Let's follow the procedure of the proof in the previous note. Modes of harmonies in 7 are opposed, so  $n$  will be odd. The starting mode is +1, hence  $7h = r$  (where  $h := \frac{n-1}{2}$  and  $r := p - q - 3 = 0 - 0 - 3 = -3$ ) i.e.

$$7h = -3$$

which (unique) solution is for  $h = 3$ , so  $n = 7$ .

Then we have  $(0, +1)MED^7 = (0, -1)$ . Let's consider the detailed definition of  $MED^7$ :

$$(p, \sigma)MED^7 = \left( \begin{cases} p & \text{if } \sigma = +1 \\ p - 1 & \text{if } \sigma = -1 \end{cases}, \sigma \cdot (-1) \right)$$

Hence  $MED^7 \neq PAR$ . But it's the only element of  $\langle MED \rangle$  that maps C Maj to C min, and so we proved that  $PAR \notin \langle MED \rangle$ , and so

$$\langle MED \rangle \subset \langle MED, PAR \rangle$$

□

### 4.3. Conclusions: generalizing the GIS structure

Summarizing. We exhibited a family of musical entities, which reduce harmonic information to single element of a set. We showed how harmonic facts can be algebraically interpreted. We showed that Klangs and particular transformation groups (of musical meaning), constitute more general structures than GIS. In which way, exactly? Let's go into deeper details.

We note that

**Proposition 4.17.** *The action (application of transformations) of the group  $\langle MED \rangle$  on  $K$  is simply transitive.*

*Proof.* In fact  $\langle MED \rangle \cdot k = K$  (4.11). □

Hence

**Proposition 4.18.**  *$K, \langle MED \rangle$ , action is a GIS (commutative).*

We discovered in 4.16 that also the orbit  $\langle MED, PAR \rangle \cdot k$  is the whole  $K$ . But is the action of  $\langle MED, PAR \rangle$  simply transitive? The existence in the axiom ?? is guaranteed by the orbit coincident to  $K$ . On the other hand the solution of 7 in proposition 4.16 falsifies the unicity in the same axiom.

Let's remember at this point the following

**Definition.** The action of a group on a set  $X$  is *transitive* iff

$$\forall x, y \in X \exists g \in G : x \cdot g = y$$

Then we can say that

**Proposition 4.19.** *The action of  $\langle MED, PAR \rangle$  on  $K$  is transitive.*

*Proof.* The existence of an element of the group which maps an element of the set in another one, for any choice of the elements, is ensured by the presence of  $MED$  and by the proposition 4.11. □

Finally, let's consider the actions of  $\langle PAR \rangle$  and  $\langle DOM \rangle$  on  $K$ . These are not even transitive (4.15 and 4.15).

In conclusion, we characterized some musical entities, considered in conjunction with some transformations, which are or not GIS, depending on the family of transformations. By way of compensation we can say that when the axioms of GIS are not respected, we can anyhow find very similar algebraical structure. Weakening the axioms we can still model these musical structure with group actions over sets.

We want to conclude this formal part of the work emphasizing that the characterization of the entities described in terms of group actions over sets will let us to define, in a uniform way, verification rules for binary relations between elements of a (finite) G-set, relations that emerge from the G-set structure. Given a G-set  $X$ , being  $g \in G$ , two elements  $x, y \in X$  will be g-related iff  $action(x, g) = y$ .

## 5. IMPLEMENTATION

We conclude with the sketch of the implementation in the structural layer of MX. For a general description of MX format see [7], [5].

### 5.1. Representing more analyses

```
<!ELEMENT structural
(themes?, CPN*, analysis*)>
```

We added to the structural element the capacity to contain zero or more analyses as sub-elements. In this way it will be possible to contain more different analytical viewpoints. We want to emphasize that this is in line with the general philosophy of MX: many different points of view on the same represented entity (the same happens in the audio, performance and notational levels).

```
<!ELEMENT analysis
(segmentation,
relationships?,
featureObjectRelationships?)
>
<!ATTLIST analysis
id ID #REQUIRED
```

```

author CDATA #IMPLIED
description CDATA #IMPLIED
>

```

An analysis is made up by a segmentation, (optionally) a set of relations between segments and (optionally) a set of relations between featureObjects that one will be able to use when defining relations between segments.

## 5.2. Representing the segmentation

```

<!ELEMENT segmentation (segment+)>
<!ATTLIST segmentation
  id ID #REQUIRED
  description CDATA "null"
  method CDATA "null"
>

```

A segmentation is made up of one or more segments. A segmentation is identified by an unique ID in the MX document. A segmentation is described by a generic parameter ('null' by default). A segmentation is described by an indication of which method has been used to generate it ('null' by default).

```

<!ELEMENT segment
(segmentevent+,
featureObject*)>
<!ATTLIST segment
  id ID #REQUIRED
>

```

A segment is made up of a set of events in the music piece. A segment is identified by an unique ID in the MX document. A segment contains zero or more feature objects, so one can predicate in many ways on the same segment of an analysis.

```

<!ELEMENT segmentevent EMPTY>
<!ATTLIST segmentevent
  id_ref IDREF #REQUIRED
>

```

An event of the piece of music is identified by a reference to an event in the element of the spine of the MX document (cfr.[7]).

We want to emphasize that *a segment can be any collection of musical events* (in one or more voices) and that segments can overlap (as in the analyses of Lewin) or not (as in other analyses). In our model there is no limitation, so we can support any approach to segmentation.

## 5.3. FeatureObject implementation

Feature objects will be assigned to a segment. Now the problem is to let custom implementation of featureObject classes. To address this problem we follow an *extensible content model* pattern (see [13]). So, we have a basic structure of the element featureObject, defined with some parameter entities that will allow the extensions.

```

<!ENTITY
  \%addedFeatureObjectClasses
  "">
<!ENTITY
  \%supportedFeatureObjectClasses
  "simpleDescription
  \%addedFeatureObjectClasses;"
>

<!ELEMENT featureObject
  (\%supportedFeatureObjectClasses)>
<!ATTLIST featureObject
  id ID #IMPLIED
  name CDATA #REQUIRED
>

<!ELEMENT
  simpleDescription
  (#PCDATA)>

```

A feature object is identified by an unique ID in the MX document and will have (optionally) a name. A feature object will contain an object of one of the supported classes. The standard model contains a trivial featureObject class simpleDescription which content is a simple text.

### 5.3.1. Adding a custom featureObject class

If we want to extend the model to represent our custom feature object we have to create our own definition of a class and introduce the name of the new class in the parameter entity defined in the dtd. For instance what follows is a Klang implementation<sup>6</sup>. We chose to represent pitch class and modality of a Klang as attributes (attribute models part of the internal state of an object

```

<!DOCTYPE mx SYSTEM "mx.dtd"[
<!ENTITY
  \%addedFeatureObjectClasses
  "|Klang">
<!ELEMENT Klang EMPTY>
<!ATTLIST Klang
pc (0|1|2|3|4|5|6|7|8|9|10|11)
  #REQUIRED
mod (-1|+1) "+1"
  #REQUIRED
implementingClass CDATA
  #IMPLIED
>
]>
...
<featureObject id="497"
  name="klang c-">
  <Klang pc="0" mod="-1"/>
</featureObject>
...

```

<sup>6</sup> The optional attribute implementingClass could contain the name of an implementing class written in some programming language. This could be useful in an environment involving an application.



## 5.4. Representing relations between segments

As we said in 2.2, we want to relate segments by means of relations between feature objects associated to them.

In our model a relationship is a sub-element of the container relationships, which is part of an analysis.

```
<!ELEMENT relationship EMPTY>
<!ATTLIST relationship
  id ID #REQUIRED
  description CDATA #IMPLIED
  segmentA_id IDREF #REQUIRED
  segmentB_id IDREF #REQUIRED
  featureObjectA_id IDREF
    #IMPLIED
  featureObjectB_id IDREF
    #IMPLIED
  featureObjectRelationship IDREF
    #IMPLIED
>
```

A relation between segments is identified by an unique identifier in the mx document.

A relation between segments is defined by the references to the two segments A and B involved.

One can also specify which featureObject to chose from the feature objects associated to segments A and B. When the objects are specified, it is possible to specify also in which kind of relation A and B are <sup>7</sup>.

Now we introduce a prototypical definition of feature-Object relations.

```
<!ELEMENT
  featureObjectRelationship
  (verRule)>
<!ATTLIST featureObjectRelationship
id ID #IMPLIED
>
<!ELEMENT verRule (#PCDATA)>
```

A relation of featureObjects is identified by an unique identifier in the MX document (that can be referenced in the definition of a relationship between segments). It is defined by a sub-element which can contain the rule to check if the relation is verified. The idea is to let the user to describe this rule in terms of a syntax that a parser will understand. We make an example in a java-jsp expression language-like syntax[11].

If we want to introduce an implementation of the DOM transformation of Klang we will add the following lines to the document.

```
<!DOCTYPE mx SYSTEM "mx.dtd" [
  ...
  <!ENTITY
verRule_KlangMED
```

<sup>7</sup> Note that it is possible to have more than one relation between A and B. For instance, if we have a G-set  $X$  with an action that is not simply transitive, we can have the same objects (i.e. set elements  $x_1, x_2 \in X$ ) in more than one relation (if there exist two distinct group-elements  $g, h \in G$  such that  $action(x_1, g) = x_2$  and  $action(x_1, h) = x_2$ ).

```
"((k1.mod eq 1)&&(k1.pc-3 eq k2.pc))
||
((k1.mod eq(-1))&&(k1.pc-4 eq k2.pc))"
>
]>
...
<featureObjectRelationship id="1045">
<verRule>&verRule_KlangMED;</verRule>
</featureObjectRelationship/>
...

```

## 6. CONCLUSIONS

Let's try to sketch a possible not so sci-fi implementation of the ideas here exposed.

Let's imagine a musical database containing mx-encoded files. Now, the system could proceed to examine the harmonic successions of the pieces and to write in the file the obtained analysis, defining at the same time, automatically, the tie between the recognized chords and the segments of the piece which such chords are attributed. Imagine segmentations like those obtained by the algorithm described by S.C.Sapp(cfr. [14])<sup>8</sup>. We will have segments with keys associated to them (key regions). But keys are representable by Klang objects.

At the end of the process, chain of relations between successive harmonies would be exposed and saved in the xml file. An agent which we asked to carry out a search of pieces harmonically similar to others could try (e.g. using approximate string matching techniques) to search pieces containing chains of relations with a sufficient number of correspondences. The presence of an algebraic structure could inform an intelligent agent to use algorithms which could make use of this structure.

The encoding of the analytical information obtained by the work of complex algorithms, and the possibility to take advantage of it later, through more simple algorithms, would remarkably speed up some different tasks of information retrieval.

Future developements will be the refinement of the implementation here proposed and the examination of other existing models from the perspective of objects here described.

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<sup>8</sup> The algorithm operates by different granularities. Segmentations of different granularity could be separated from the others in the file.

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