



# Introduction to Cavitation and Supercavitation

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# Introduction to Cavitation and Supercavitation

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## 1. The Physical Phenomenon

### 1.1. DEFINITION

Cavitation, *i.e.* the appearance of vapour bubbles and pockets inside an initially homogeneous liquid medium, occurs in very different situations. According to the flow configuration (shape and relative motion of the walls limiting the flow field, or physical properties of the liquid), it can take various figures. In order to encompass all possible cases, we propose the following definition :

*Cavitation is the breaking of a liquid medium under excessive stresses.*

That definition makes cavitation relevant to the field and the methods of continuum mechanics. It is convenient for cases in which the liquid is either *still* or *flowing*.

Examples of the first case :

- An oscillating pressure field is applied above a liquid contained in a bowl : if the oscillation amplitude is large enough, bubbles can appear inside the liquid.
- A solid body with sharp edges (a disk, for example) is suddenly accelerated by a shock in still water. Bubbles can appear at the very first instants in the region neighbouring the edges, when the velocity of the liquid particulates are still negligible.

Examples of flowing liquids :

- Flows in venturis or in narrow passages (valves for hydraulic control).
- Flow near the upper side of a wing or a propeller blade.

Here we will be rather concerned with **flowing liquids**.

In order to solve the problem of excessive stresses, we need a *threshold of stress* : beyond such a threshold, the liquid cohesion is no longer ensured. Ideally, the threshold should be determined from physical consideration at a submicroscopic scale. Taking the actual state of the scientific development into account, together with the need of practical solutions for possibly complicated fluid systems, it is useful to call only on fluid properties at the macroscopic scale.

Concerning the stress, it must be specified that only the normal component  $\bar{n}$  of the total stress  $\bar{\tau}$  has to be considered here (recall that  $\bar{n} = -p\bar{v}$  for an incompressible Newtonian fluid, where  $p$  stands for the **absolute pressure**). Indeed we have no experimental result in which cavitation would be produced uniquely by the shearing stress  $\bar{\tau}$ , under a large absolute ambient pressure.

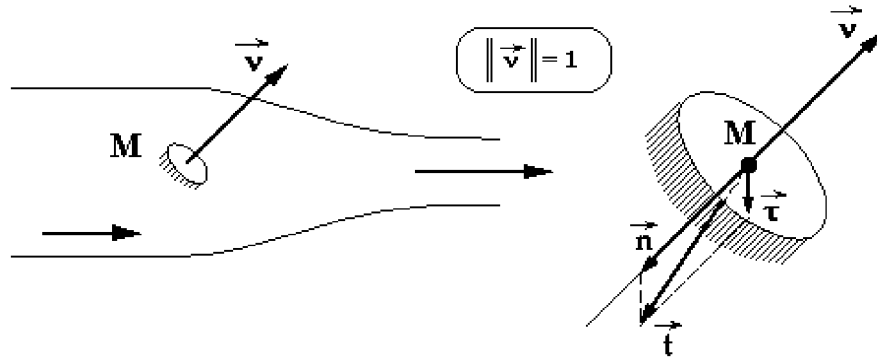


Fig. 1.

For example, the correct filling of a serynge requires that the piston motion is slow : if not, the liquid column is broken and the filling is stopped. That is because, due to the head losses inside the needle, the pressure drops to a value lower than atmospheric pressure, the difference increasing with the liquid velocity, and then with the piston velocity. More, at the serynge inlet, where the flow is a submerged liquid jet, additional turbulent fluctuations of the pressure occur : both mechanisms contribute to lower the pressure, possibly to a value below the liquid vapour pressure, so that vapour is produced. A similar phenomenon can be found in volumetric pumps for fuel injection in engines : head losses and rapid accelerations of the liquid columns result in deep underpressures so that evaporation occurs and only a small part of the chamber is filled with liquid.

## 1.2. THE VAPOUR PRESSURE

The first (and basic) *threshold* at our disposal is given by the thermodynamic diagram of phase changes (see figure 2). Here, we are interested in the curve  $T_rC$  which links the triple point  $T_r$  to the critical point  $C$ , and separates the liquid and vapour domains. Across that curve, reversible transformations *under static or equilibrium conditions*, result in evaporation or condensation of the fluid, at a level  $p_v$  which is a function of the absolute temperature  $T$ . We can obtain the phase change by lowering the pressure at an approximately constant temperature (as it actually happens in real flows) to obtain cavitation. Thus at first sight cavitation appears similar to boiling, except that the driving mechanism is different.

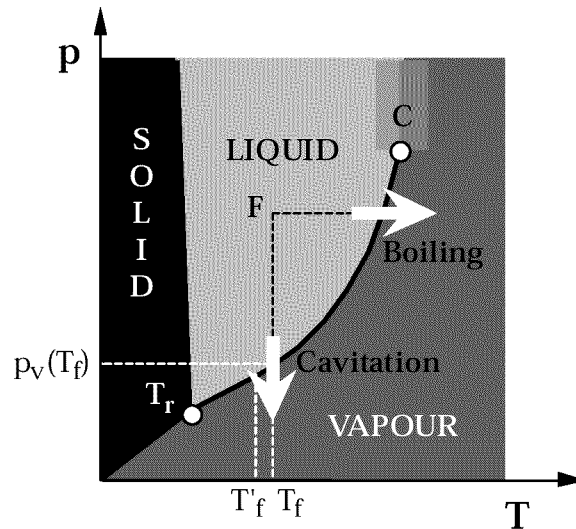


Fig. 2. Diagram for phase changes

In fact, for most of the cases (with cold water, for example), the path in the  $T - p_v$  diagram is practically isothermal : here the driving mechanism is the local pressure controlled by the flow dynamics, and only a very small amount of heat is required to the formation of a significant volume of vapour. However, in some cases the heat transfer needed by vaporisation results in the phase change at a temperature  $T'_f$  lower than the ambient liquid temperature  $T_f$ . The temperature difference  $T_f - T'_f$  is called "*thermal delay*" to cavitation. It is greater when the ambient temperature is closer to the fluid critical temperature. For example, that phenomenon may be important when pumping the cryogenic liquids used in rocket engines.

It must be kept in mind that curve  $p_v(T)$  is not an *absolute limit* between the liquid state and the gaseous state. Deviations from that curve may exist in the case of rapid phase changes. Even in almost static conditions, the phase change may occur at a pressure lower than  $p_v$ . For example, let us consider the so-called Andrews-isotherms in the  $v-p$  diagram, where  $v = 1/\rho$  and  $\rho$  stands for the density (see figure 3). Such curves are well modelled by the Van der Waals law in the liquid and vapour domains. More, the transformation from liquid to vapour on branch **AM** of the Van der Waals curve is possible. On this branch, the liquid is in a metastable equilibrium. We note that this allows the liquid to withstand *negative absolute pressures, i.e. tensions*, without any phase change. Such considerations are necessary for a correct interpretation of some phenomena occurring in industrial situations as well as in laboratory conditions : it may happen that the interfaces between the liquid and vapour domains undergo very rapid evolution laws so that the reversibility conditions are far from being met.

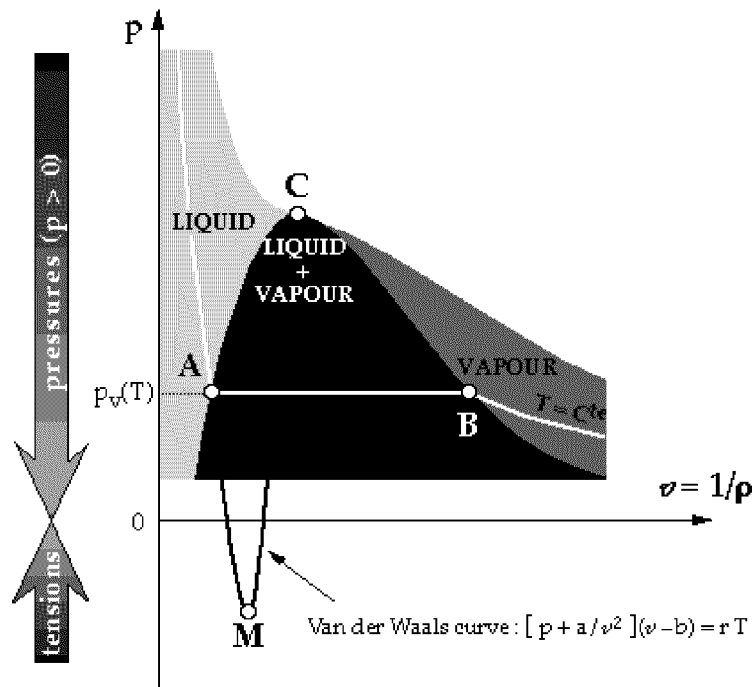


Fig. 3. Andrews-isotherms

In conclusion, it appears that the condition : "*local absolute pressure equal to the vapour pressure at the global temperature of the system*" does not ensure in all cases that cavitation actually appears in the flow. The difference between  $p_v$  and the actual pressure at cavitation inception is called "*static delay*" to cavitation. In some cases, we have also to consider a "*dynamic delay*" which is due to inertial phenomena : a "certain" time is necessary to obtain observable vapour cavities.

### 1.3. THE MAIN PATTERNS OF VAPOUR CAVITIES

Cavitation can take different forms at its inception and in its further development. The main patterns are the following :

#### a. Transient isolated bubbles

These bubbles appear in the region of low pressures as a result of the rapid growing of very small *air nuclei* present in the liquid. They are convected by the main flow and then disappear subsequently when they encounter adverse pressure gradients.

#### b. Attached, or sheet, cavities

Such cavities (or vapour pockets) are often attached at the leading edges of such bodies as blades, foils ... on their upper side, *i.e.* on the low pressure side.

#### c. Cavitating vortices

Cavitation can appear in the low pressure core of vortices in turbulent wakes or, as a more regular pattern, in tip vortices of 3-D wings or propeller blades.

It must be noted that, in cases **b** and **c**, cavitation at its inception is strongly dependent on the basic non-cavitating flow structure. This point will be examined later. In all cases, the development of cavitation tends to disturb and modify this basic flow.

- In some cases, it may happen that some patterns of cavitation are not easily settled in those classes. For example, on upper sides of foils or propeller blades, cavitation figures with a very short life time may appear which are conveyed by the flow as bubbles but are shaped as attached cavities. If needed, it would be possible to give an objective foundation to the classification by appealing the relative velocity of the liquid at interface with respect to the mass centre of the vapour figures : if the velocity is rather normal to the interface, the figure should be considered as a bubble (then attention is mainly paid to the volume variation). On the contrary, if the relative velocity is close to the plane tangent to the interface, the figure should be considered as a cavity ( in the absence of circulation), or as a cavitating vortex (with circulation).

## 2. Cavitation in Hydraulics

### 2.1. CAVITATION REGIMES

For practical purposes, it is useful to distinguish :

- the *limiting regime* between the non-cavitating flow and the cavitating flow ;
- the *regime of developed cavitation*. In laboratory situations, developed cavitation corresponds to a certain permanency and a certain extent of the cavitation figures, relatively to the size of the system under consideration, while for industry it corresponds to a significant fall of the system performances.

That distinction is in relation with the possibility to accept (or not) cavitation in industrial situations. In the case of limiting regime, the *threshold of cavitation inception or cavitation disappearance* is of interest, while in the second case, assuming that this threshold is overstepped, the consequences of cavitation on the operation of the hydraulic system in question focus attention (see § 2.3).

In the case of attached cavities, another distinction may be useful : either *partial cavities*, which re-attach on the foil or the blade, or *supercavities*, the closure of which is downstream the foil, inside the liquid domain.

## 2.2. TYPICAL SITUATIONS FAVOURABLE TO CAVITATION

Some examples of situations favourable to cavitation were previously presented. Here are listed typical situations in which cavitation can appear and grow in a liquid flow :

- The *wall geometry* entails local overvalues of the velocity and then local underpressures in a flow which is globally steady : that is the case for the restriction in the cross-section area of liquid ducts (venturis), or for the curvature imposed to the flow streamlines (bends of pipes, uppersides of blades in propellers and pumps).
- The *shearing* between two neighbouring flows having very different velocities entails *large turbulent fluctuations of the pressure* : that is the case for jets, wakes...
- The strongly *unsteady character* of some flows (*e.g.* water hammer in hydraulic control circuits, in ducts of hydraulic power plants, or in the feeding lines of Diesel engines) results in large values of the temporal terms of the fluid acceleration and then in the production of low pressures at some instants of the flow cycle.
- The *local roughness* of the walls (*e.g.* concrete walls of dam spillways) produces local wakes in which small attached cavities possibly take place.
- The consequence of the *vibratory motion* of the walls (*e.g.* liquid cooling of Diesel engines, standard A.S.T.M.E. erosion device) is the creation of oscillating pressure fields superimposed to the mean pressure field. If the oscillation amplitude is large enough, cavitation can appear at those times when the negative oscillation occurs.
- In mechanical systems such as motors, if water is present in *interstices* due to defects in the joining of neighbouring pieces and if the wall of interstices are moving periodically, cavitation can occur there and erode the walls.
- Finally, attention has to be drawn on the case of solid bodies which are suddenly *accelerated* by a shock in a still liquid field, particularly if they have sharp edges : the liquid acceleration needed for getting round these edges produces low pressures even if the velocities are rather low immediately after the shock.

## 2.3. THE MAIN EFFECTS OF CAVITATION IN HYDRAULICS

If a hydraulic system is designed to operate with an homogeneous liquid, the additional vapour structures due to cavitation can be interpreted at first sight (by analogy with the case of mechanical systems) as mechanical clearances which increase the number of its freedom degrees. The vapour structures are often unstable, at least the ones which are carried along by the flow : when they reach a region of increasing pressure, they collapse violently since their internal pressure is constant and close to the vapour pressure.

That collapse is analogous to shocks by which clearances between neighbouring pieces disappear in mechanical systems. Thus, it is expected that cavitation results in the main following effects :

- *the alteration of the performance* of the system (reduction of the lift and increase in the foil drag, fall of turbomachinery efficiency , least capacity to evacuate water in spillways, energy dissipation...);
- *the appearance of unexpected forces* on the solid structures ;
- *the production of noise and vibrations* ;
- in the case of developed cavitation, if the relative velocities between the liquid and the solid walls are high, the *erosion of these walls*.

Thus, at first glance cavitation appears as a harmful phenomenon which must be avoided. However, in many cases it can be shown that the condition of no cavitation is the most severe among all conditions the designer is faced. It strongly reduces the performances that the given hydraulic system would elsewhere perform. If those performances have to be obtained without any too high financial charge, a "certain" degree of cavitation development should be allowed. Of course, this can be done only if the effects of developed cavitation are known.

**Remark :** Here, we insist on the negative effects of cavitation. However, cavitation has applications in industrial processes which require energy concentration on small surfaces in order to produce there high pressure peaks. For this purpose, cavitation is often obtained from ultrasonic devices by which bubbles are produced and then implode, without costly spending in energy. For example, we quote the following applications :

- *the cleaning of surfaces* by ultrasonics or with cavitating jets,
- the dispersion of particles in a liquid medium,
- *the production of emulsions* ,
- the *electrolytical deposit* (by cavitation, the ion layers which wrap electrodes are broken, accelerating then the deposit process),
- and in the field of medical engineering, the *therapeutic massage* and the *destruction of bacteria*.

### 3. Specific Features of Cavitating Flows

#### 3.1. PRESSURE AND PRESSURE GRADIENT

In non-cavitating flows, the ambient pressure has no effect on the flow dynamics and attention is paid only to the *pressure gradient*. On the contrary, the cavitating flows are primarily dependent on the *ambient pressure* (which, for convenience, is taken here as equal to the absolute pressure, as previously mentioned). It will be shown that by only lowering the pressure at a reference point, cavitation can appear and develop in the flow region where the pressure is the lowest. Thus, considering the pressure value, not only its gradient, is essential in cavitation studies and experiments.

If then one looks for the prediction of *cavitation occurrence* by **theoretical or numerical way**, one has to compare the calculated value of the pressure in a sensitive region of the flow to a critical value (for example the vapour pressure) given by physical experience, which expresses the physical properties of the liquid. The method of calculation depends on the flow configuration :

- For *one-dimensional, steady flows in pipes*, the use of the Bernoulli relation, with due account to head losses, is sufficient to obtain the region of minimum pressure together with the value of this minimum.
- *Steady flows without significant shearing stresses* can be considered as potential flows (e.g. flows around wings, propeller blades...). Classical methods require that firstly the kinematics problem be solved and then the pressure is still given by the Bernoulli relation. In this case, the pressure minimum occurs on the boundary of the flow. Physical experience confirms that theoretical result fairly well.
- The case of *turbulent flows with important shearing stresses* is among the most complicated ones and for this reason has been treated only by an experimental way until a recent past. Now, the progress of computational methods in fluid dynamics allows us to envisage a possible prediction of cavitation inception by this way, at least for the simplest configurations : some encouraging results have recently been obtained in this field.

In that case, the pressure field is controlled by the following Poisson equation :

$$\frac{\Delta p}{\rho} = \frac{1}{2} \Omega^2 - e_{ij} e_{ij}$$

which is obtained by applying the divergence operator to the Navier-Stokes equation. The two terms on the right hand represent the respective contributions of the rotation rate vector  $\vec{\Omega} = \vec{r} \otimes \vec{\nabla}$  and the deformation rate tensor  $e_{ij} = \frac{1}{2} \left( \partial v_i / \partial x_j + \partial v_j / \partial x_i \right)$ . The velocity vector  $\vec{V}$  has components  $v_i$ ,  $i = 1, 2$  or  $3$ . The Poisson equation has to be solved once the velocity field is known. The results of available numerical models, as applied to sheared turbulent flows, exhibit specific features such as the prominent role of the vorticity in the Poisson equation, compared with the deformation rate : the regions of minimum pressure coincide markedly with filaments on which vorticity is concentrated, inside of the flow field. Thus cavitation must appear firstly in those regions, which indeed agrees with physical observation,

- In the case of wings or propeller blades tip vortices, the structure of the flow is well identified and it is possible to use simple models such as the Rankine vortex or the Burger's vortex. Then, the problem reduces to the search for two parameters, the circulation around the vortex and the size of its viscous core.

Of course, the pressure also plays an important role in the case of *developed cavitation*, and this is the source of additional complexity in the modelling of cavitating flows :

- For example, the modelling of *cavities* attached to foils or blades requires a condition on the pressure along the cavity boundary : this modifies the nature of the mathematical problem to solve numerically. On the physical ground, the change in the pressure distribution entails a change in the pressure gradient and then a modification of the boundary layer behaviour.
- When a *large number of bubbles* explode on the upper side of a foil, the initial, non-cavitating, pressure distribution can be deeply modified and the interaction between the basic, non-cavitating flow and the bubbly flow must be taken in account.
- Lastly, the evolution of the *turbulent, cavitating vortices* of a wake cannot be predicted on the usual base of the classical fluid mechanics equations by which mass and intensity conservation of the vortex filaments are expressed : when the core of a vortex filament is changed into vapour, it becomes dependent on the pressure imposed in its vicinity. In other words, cavitation breaks the link (expressed by the relation  $\omega / \delta \ell = k$ , where  $k$  stands for a constant) between the elongation rate and the rotational rate of a rotational filament.



On the **experimental side**, two main difficulties appear with respect to the measurement of the pressure. The first one is of fundamental type, as it appears from numerical simulations of non-cavitating flows : pressure transducers, which can be only flush mounted on the solid frontiers of the flow domain, do not give necessarily a valuable information on the value of the pressure inside of sheared turbulent flows - even although parallel in the mean (Lesieur, 1998). The second difficulty is rather technical : it appears that phenomena connected to bubble collapse have a very small size (less than about 0.1 mm) and a short duration (of the order of the microsecond). Then the transducers must have a very small spatial resolution and a very short rise time. In fact, those requests often are beyond the present technical possibilities. Moreover, in the case of erosion studies, pressures of the order of hundreds of Megapascals, or even Gigapascals, have to be measured : the material resistance of the pressure transducers then becomes a central problem for experimentalists.

In order to study conditions of cavitation inception and physics of developed cavitation, special equipments have been progressively worked out. The most common are made of a vertical closed loop with the test section at the top and a circulating pump at the lower part : that allows to draw benefit from the increase of pressure due to gravity and thus to avoid cavitation in the pump. The absolute pressure at the test section level is adjusted below the atmospheric pressure thanks to a void pump. In some cases, a compressor is used to increase the pressure above the atmospheric pressure. Such test loops are named *Hydrodynamic Tunnels* by analogy with the *Wind Tunnels* used in Aerodynamics.

### 3.2. THE LIQUID-VAPOUR INTERFACES

The cavitating flows, like other two-phases liquid-gas flows, are characterized by the presence of numerous interfaces. But their response to external perturbations, for example an increase in the pressure, can be very different from the liquid-gas flows response. Two-phase flows containing gas bubbles are usually not subject to rapid changes in their mean density (except for the case of shock waves), just because of the non-condensable character of the gas which ensures a kind of global stability to the flow. On the contrary, in cavitating flows, the interfaces are submitted on one side to a pressure practically equal to the vapour pressure and then to a constant pressure. Thus, (as already mentioned previously) they cannot sustain an increase or decrease of the external pressure without rapidly evolving in both shape and size : they are of deeply unstable nature.

It is not possible to place measurement probes inside a cavitating flow because they would be followed by their own cavity which, in general, would disturb the flow. In compensation, if the liquid is transparent, it is possible to visualise interfaces which reflect the light very efficiently. Generally enough, the interfaces can be considered as material surfaces (see the following paragraph) and thus an idea of the flow dynamics can be obtained by taking photographs (with short flash lighting duration, of the order of one microsecond) or by taking rapid films (at a typical rate of ten thousand frames per second).

When considering the exchange of liquid and vapour through the interface, we can

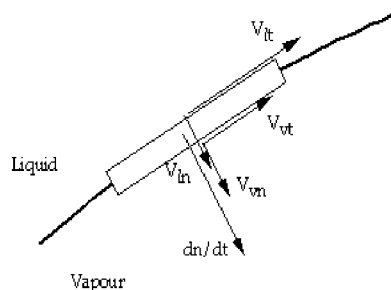


Figure 4

introduce the mass flowrate  $\dot{m}$  (per unit of area) which is proportional to the relative normal velocities of either liquid or vapour. Thus we obtain, using mass conservation (see Fig. 4) :

$$\dot{m} = \rho_\ell \left[ V_{\ell n} - \frac{dn}{dt} \right] = \rho_v \left[ V_{vn} - \frac{dn}{dt} \right]$$

In that equation, indexes  $\ell$  and  $v$  are relative to the liquid and vapour phases respectively, while index  $n$  means that the velocity component normal to the interface must be considered. The symbol  $dn/dt$  is the normal velocity of the interface, the equation of which is :  $F(x_1, x_2, x_3, t) = 0$  . It is related to  $F(x_1, x_2, x_3, t)$  by :

$$\frac{dn}{dt} = - \frac{1}{\|\text{grad}(F)\|} \frac{\partial F}{\partial t} \quad (2)$$

If we neglect the flowrate at interface (what is actually made in most of the cases), we consider that the three normal velocities are equal and the surface  $F = 0$  is a *material surface*, ie a surface which is made of the same fluid particulates at different instants. Two cases are of particular interest :

- For a spherical bubble with a variable radius  $R(t)$ , the interface is defined as  $F(r, t) = r - R(t) = 0$ , which gives  $dn/dt = dR/dt$  and, with a negligible flowrate,  $V_{\ell n} = V_{vn} = dR/dt$ .
- For a steady cavity attached to a wall with a flowing liquid in its vicinity, we have  $\partial F/\partial t = 0$  and then  $\partial n/\partial t = 0$ . If we assume that the mass flowrate is negligible, the normal velocities of the liquid and the vapour near the interface are zero. Then the liquid velocity at interface is reduced to its component  $V_{\ell t}$  tangent to the cavity frontier.

### 3.3. THERMAL EFFECTS

The effects of the liquid temperature on cavitation are diverse :

a- If we consider a system in which the global temperature can evolve under a constant ambient pressure (as for example the case of Diesel engines cooling), the increase of temperature results in a greatest aptitude to cavitate : in the diagram of Figure 2, the distance between the point F and the phase change curve diminishes and thus smaller pressure variations can produce cavitation inception.

b- Now, as said in Section 1.2, the liquid vaporization requires a heat transfer from the bulk of the liquid towards the region of low pressure and the liquid-vapour interface. That results in the temperature difference  $T_f - T'_f$  mentioned in the Figure 2, which is commonly called "*thermal delay*" to cavitation.

In order to estimate that delay, one needs to use the energy equation in addition to the mass conservation and the momentum equations (see annex 1 for the case of a spherical bubble). Then the boundary conditions  $T_f(T)$  and  $p_v(T'_f)$  are unknown quantities. They are linked through the vaporization curve equation and the Clapeyron's relation :

$$L = T \left[ \frac{1}{\rho_v} - \frac{1}{\rho_\ell} \right] \frac{dp_v}{dT}$$

in which  $L$  stands for the latent heat of vaporization.

In general, the thermal delay increases with the global temperature because the densities of the liquid and its vapour become closer. Near the critical point C (Fig. 2), they tend to be equal, while the slope  $dp_v/dT$  of the vaporization curve tends to become very large. As a result, for water,  $L$  decreases slowly on a large range of temperature variations, then it abruptly tends to zero in the close vicinity of the critical temperature.

### 3.4. SOME TYPICAL ORDERS OF MAGNITUDE

The instabilities of the interfaces result in *explosion* or *implosion* (named also *collapse*) phenomena, with large variations in their typical sizes and velocities during very short times. This character makes their scaling, together with their experimental or numerical analysis, rather difficult. However, we can quote some values which are currently encountered in the field of cavitation :

- the collapse of spherical vapour bubbles in water, with radius one centimeter, under an external pressure of one bar : its duration is approximately one millisecond ;
- the final phase of the collapse of bubbles or cavitating vortices : about one microsecond ;
- the normal velocities of interfaces : between some meters/second and some hundreds m/s ;
- the overpressures due to the implosion of vapour structures (bubbles and vortices) : they can reach several thousand bars.

## 4. The Non-Dimensional Parameters

### 4.1. CAVITATION NUMBER $\sigma_v$

Let us consider a hydraulic system liable to cavitate, such as a turbine, a pump, a gate, a foil in a hydrodynamic tunnel... and  $p_r$  the pressure at a conventional reference point  $r$  where the measurement of the pressure is possible. Usually,  $r$  is chosen in a region close to the one where cavitation inception is expected. If  $T_f$  is the operating temperature of the liquid and  $\Delta p$  a pressure difference which characterises the system, the *cavitation number* (which is also called *cavitation parameter*) is the ratio :

$$\sigma_v = \frac{p_r - p_v(T_f)}{\Delta p} \quad (3)$$

For example, in the case of a gate, one takes :

$$\sigma_v = \frac{p_{\text{downstream}} - p_v(T_f)}{p_{\text{upstream}} - p_{\text{downstream}}}$$

while, for a foil placed at a submersion depth  $h$  in a horizontal free surface channel where the pressure on the surface is  $p_0$  and the flow velocity  $U$  :

$$\sigma_v = \frac{p_0 + \rho gh - p_v(T_f)}{\frac{1}{2}\rho U^2}$$

and, for a pump ( $V_p$  stands for the velocity at the periphery of the runner) :

$$\sigma_v = \frac{p_{\text{inlet}} - p_v(T_f)}{\rho V_p^2}$$

It must be noted that the cavitation number is defined from dynamical parameters only and not from the system geometry. Also, in a non-cavitating flow, this non-dimensional parameter *cannot be considered as a similarity parameter* since, in the upper term of the ratio by which  $\sigma_v$  is defined, the difference between  $p_r$  and  $p_v$  has no physical significance for the actual flow : it cannot be obtained by integration of the pressure gradient along a real path. It becomes a similarity parameter only at cavitation inception.

#### 4.2. CAVITATION NUMBER AT INCEPTION, $\sigma_{vi}$

The number  $\sigma_{vi}$  is the value of the parameter  $\sigma_v$  corresponding to inception of cavitation at any point of the flow system. Cavitation appears as a consequence of either the decrease in the pressure at the reference point (*i.e.* the ambient pressure) or the increase in the  $\Delta p$ -value. In some cases, for better experimental convenience (in particular for a best repeatability), number  $\sigma_{vd}$ , corresponding to cavitation disappearance from an initial regime of developed cavitation, is also used.

Operating in *non-cavitating conditions* then requires that the following condition be satisfied :

$$\sigma_v > \sigma_{vi} \quad (4)$$

Threshold  $\sigma_{vi}$  depends on all influence factors usually considered in fluid mechanics : flow geometry, viscosity, gravity, surface tension, turbulence rate, thermal parameters, wall roughness and in addition the content of the liquid in gas (dissolved gas and "free gas", *i.e.* gas nuclei).

In general, a smaller value of  $\sigma_{vi}$  for a given system expresses its better adaptation to the flow. For example, for a circular cylinder with diameter 10 mm in the test section of a hydrodynamic tunnel, one finds about 1.5, while for elliptical cylinders at zero incidence, with chord 80 mm and axe ratios 1/4 and 1/8, the  $\sigma_{vi}$ -values are 0.45 and 0.20 respectively.

When  $\sigma_v$  becomes smaller than  $\sigma_{vi}$ , usually cavitation becomes more and more developed. However, exceptionnally it may happen that after a first development, cavitation finally disappears as the consequence of a further lowering of  $\sigma_{vi}$ .

**NOTE :** In many circumstances, particularly for the numerical modelling of cavitating flows, the following estimate is made for  $\sigma_{vi}$  :

$$\sigma_{vi} = -C_{p_{\min}} \quad (5)$$

where  $C_{p_{\min}}$  is the minimum pressure coefficient (a negative value, in general), and  $C_p$  at a point **M** is defined by the relation :

$$C_p = \frac{p_M - p_r}{\Delta p}$$

In this expression,  $p_r$  still stands for the absolute pressure at the reference point, as in relation (3). Two assumptions underlie that short cut :

- on the one hand, cavitation occurs at the point of minimum pressure
- on the other hand, the threshold of pressure is the vapour pressure.

We know that those assumptions are too restrictive. Thus the estimate resulting from equation (5) must be taken cautiously.

#### 4.3. RELATIVE UNDERPRESSURE OF A CAVITY, $\sigma_c$

If a developed cavity is attached to the upperside of a blade (or if a large amount of bubbles is present on the upperside) one observes that the pressure in the region covered by the cavity is uniform. Calling  $p_c$  its value, one defines the non-dimensional parameter :

$$\sigma_c = \frac{p_r - p_c}{\Delta p}$$

which is the "*relative underpressure of the cavity*". It is a *true similarity parameter*, as the numerator in the ratio expresses an actual pressure difference inside the flow domain. In the numerical modelling of flows with developed cavities, a great use is made of this number. It is often (but unduly) named "cavitation number" : indeed, it plays a large role in dynamics due to its "underpressure" character.

Usually, pressure  $p_c$  is the sum of two partial pressures : the vapour pressure  $p_v$  and a term  $p_g$  which expresses the presence of gas inside the cavity. When this last term is negligible, the number  $\sigma_c$  becomes equal to  $\sigma_v$  (which probably explains the confusion just mentioned).

### 5. Some Historical Aspects

The word "cavitation" appeared in England at the end of the nineteenth century. Before, it seems that the problem of the behaviour of liquids in rotating machinery was suspected by *Torricelli* then *Euler* and *Newton*. In the middle of the nineteenth century, *Donny* and *Berthelot* have measured the *cohesion of liquids*. The negative effect of cavitation on the performances of a ship propeller was firstly noted by *Parsons* (1893) who built the first experimental loop - the first hydrodynamic tunnel - for its study. The cavitation number was introduced by *Thoma* and *Leroux* around the years 1923-1925.

Subsequently, many experiments were carried out in order to study the physical aspects of the phenomenon and examine its effects on industrial systems. Theoretical and numerical approaches were also largely used. Two main ways of research can roughly been distinguished :

- The first way considered the *Bubble dynamics* (*Rayleigh 1917 ; Lamb 1923 ; Cole 1948 ; Blake 1949 ; Plesset 1949...*). The simplicity of the shape of spherical bubbles made their study (either theoretical or experimental) relatively easier. Thus, a large amount of work was devoted to topics related to bubble dynamics.

- The second way related to developed cavities or supercavities and leant on the old *wake theory* (Helmholtz 1868 ; Kirchhoff 1869 ; Levi-Civita 1907 ; Villat 1913 ; Riabouchinski 1920 - those references can be found in Jacob's book : "*Introduction mathématique à la mécanique des Fluides*"). This theory considered wakes as regions where pressure is uniform and which are limited by surfaces on which the tangential velocity is not continuous. It is more suited to cavitating wakes than to monophasic wakes. Later, Tulin (1953) and Wu (1956) made use of linearization in order to adapt this theory to the case of slender bodies such as wings and blades.

Vortical cavitation was not considered as deeply from the theoretical point of view. However, we can quote studies made by Genoux and Chahine (1983) and by Ligneul (1989) concerning torus and tip vortices respectively.

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## Annex I to Section 3.3

### The spherical bubble growing with thermal delay : setting of equations

We assume that a spherical bubble, initially in equilibrium, is compelled to grow under the effect of a sudden decrease of the ambient pressure. The liquid is assumed to be non viscous and weightless. Inside the bubble, the volume is saturated with vapour only. There the temperature is  $T_b(t)$ , ( $t$  stands for the time), the pressure is  $p_v(T_b)$ , while the bubble radius is  $R(t)$ . Note that those quantities are the main unknown of interest.

The liquid domain is described thanks to the distance  $r$ ,  $r \geq R(t)$ , the temperature  $T(r,t)$ , the pressure  $p(r,t)$ , the liquid density  $\rho(r,t)$  and the radial velocity  $u_r(r,t)$ . Far from the bubble center, the temperature  $T(\infty, t)$  is  $T_\infty$  the pressure  $P(\infty, t)$  is  $P_\infty$  ( $p_v(T_\infty)$ ), the density is  $\rho_\infty$  and the velocity is zero.

At the bubble interface the temperature and the pressure have to meet the following conditions :

$$T(R,t) = T_b(t),$$

$$p(R,t) = p_v(T_b) = p_v(t).$$

Initially ( $t = 0$ ), the conditions are :  $T(r,0) = T_\infty$ ,  $p(r,0) = p_v(T_\infty)$ ,  $\rho(r,0) = \rho_\infty$ ,  $u_r(r,0) = 0$ ,  $R(0) = R_0$ .

Taking the spherical symmetry into account, the equations of the problem are the following ones :

- Mass conservation

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho r^2 u_r) = 0$$

- Momentum equation

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial r}$$

- Liquid state equation (here we assume a barotropic behaviour, as given for example by the Tait's equation)

$$f(p, \rho) = 0 \text{ or } \frac{p + A}{P_0 + A} = \left( \frac{\rho}{\rho_0} \right)^n, \text{ with } A \cong 3000 \text{ bars}, n \cong 7,15 \text{ for water.}$$

- Energy equation

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{dT}{dt}$$

In the last equation, the work of the pressure is neglected, as the compressibility of the liquid is very small. In the same way, the thermal diffusivity  $\alpha$ ,  $\alpha = \frac{\lambda}{\rho c_p}$ , is assumed to be a constant ( $\lambda$  stands for the thermal conductivity of the liquid and  $c_p$  for its heat capacity).

Finally, it is necessary also to consider the following quantities :

- The heat flux  $\phi$  at the distance  $r$  :  $\phi(r, t) = -4\pi\lambda r^2 \frac{\partial T}{\partial r}$

- The mass exchange  $\dot{m}$  at the interface :  $\dot{m}(R, t) = (T_b) \left[ u_r(R, t) - \frac{dR}{dt} \right]$

with  $\dot{m}(R, t) = \phi(R, t)/L(T_b)$

-  $L$  is the latent heat of vaporization which has to meet the Clapeyron's equation :

$$L = T_b \left[ \frac{1}{\rho_v} - \frac{1}{\rho_\ell} \right] \frac{dp_v}{dT}$$

## Annex 2 to Section 3.3

### Vapour Bubble Collapse (Rayleigh problem)

Here we suppose that a vapour bubble, initially at equilibrium, is submitted suddenly to an increase of the ambient pressure from the value  $p_v$  to the final value  $P_\infty, P_\infty > p_v$ . All thermal aspects are neglected and the liquid is assumed to be incompressible. The mass conservation equation reduces to  $\text{div} \vec{V} = 0$ , which gives :

$$u_r(r, t) = \dot{R} \frac{R^2}{r^2}$$

in which we put  $\dot{R} = \frac{dR}{dt} = U$ . Then the momentum equation of Annex 1 can be written :

$$\ddot{R} \frac{R^2}{r^2} + 2\dot{R}^2 \left( \frac{R}{r^2} - \frac{R^4}{r^5} \right) = -\frac{1}{\rho} \frac{\partial p}{\partial r}$$

and after integration, the equation for the pressure is :

$$\frac{p(r, t) - P_\infty}{\rho} = \ddot{R} \frac{R^2}{r} + 2\dot{R}^2 \left( \frac{R}{r} - \frac{R^4}{4r^4} \right)$$



That equation can be rewritten at the interface,  $r = R$ , and then integrated, taking into account the fact that the pressure at infinity is assumed to be constant. One obtains :

$$\rho \dot{R}^2 R^3 = -\frac{2}{3}(P_\infty - p_v)[R^3 - R_0^3]$$

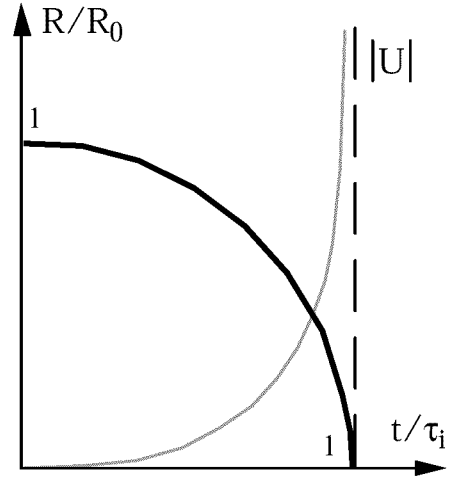
From that differential equation the relation  $R(t)$  can be obtained.

The radius becomes zero after the time  $\tau_i$ , which is called "*Rayleigh time*" :

$$\tau_i = -\sqrt{\frac{3\rho}{2(P_\infty - p_v)}} \int_{R_0}^0 \frac{dR}{\sqrt{\frac{R_0^3}{R^3} - 1}} \cong 0.915 R_0 \sqrt{\frac{\rho}{P_\infty - p_v}}$$

The behaviour of  $R(t)$  and the interface velocity  $U(t)$  is shown on the figure.

While the mean value of the velocity is  $R_0/\tau_i$ ,  $U$  tends to an infinite value at the end of the collapse, the singularity intensity being approximately  $1.12 R_0/\tau_i (R_0/R)^{3/2}$ . In fact, compressibility effects are present at the end of the collapse, which tend to weaken the strength of the singularity.



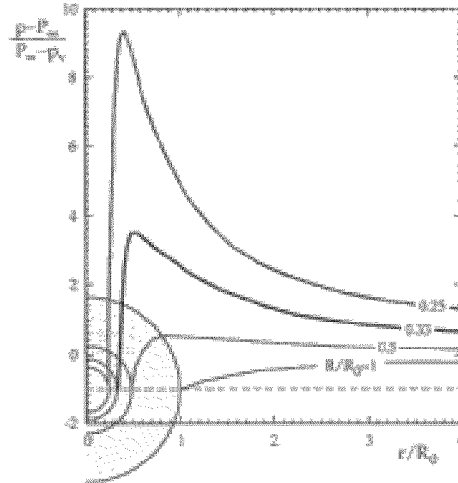
*Behaviour of  $R(t)$  and  $U(t)$  during the bubble collapse*

The pressure equation, together with the equation which gives the velocity  $\dot{R}$ , allows expressing the pressure field as :

$$\frac{p(r, t) - P_\infty}{P_\infty - P_v} = \frac{R}{3r} \left( \frac{R_0^3}{R^3} - 4 \right) - \frac{R^4}{3r^4} \left( \frac{R_0^3}{R^3} - 1 \right)$$

The following figure shows the evolution of the pressure distribution at different instants. Attention must be paid to the kind of pressure wave which comes from infinity to the neighbouring of the bubble. That phenomenon is due to the high unsteadiness of the flow, although inertia and pressure forces only are considered here. The pressure wave is still found with more evolved models, although with some modifications.

In the case of water, it is found that at the instant when  $R/R_0$  is equal to  $1/20$ , the interface velocity is 720 m/s (i.e. approximately the half of the sound speed), while the pressure maximum is 1260 bars if the pressure difference  $P_\infty - p_v$  is one bar.



*Evolution of the pressure field*

From the physical point of view, the violent behaviour of the bubble collapse results from two main facts :

- On one hand, the pressure inside the bubble is constant and does not bring any resistance to the liquid motion.

- On the other hand, the conservation of the liquid volume, through the spherical symmetry, tends to concentrate the liquid motion on a smaller and smaller region.

It must be noted that the Rayleigh time is found in various experimental situations, for order of magnitude of  $R_0$  between the micrometer and the meter, and for large ranges of the overpressure  $P_\infty - p_v$ .