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Third-order Riemann Pulses in Optical Fiber

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Abstract: We report on the generation of third-order Riemann pulses in nonlinear optical fiber, obtained by tailoring the initial pulse in presence of high-order dispersion and Kerr nonlinearity.

Analytical and numerical results show controllable pulse steepening and shock formation. © 2020 The Author(s)

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Simple Riemann waves (RWs), solutions of the Inviscid Burgers' Equation (IBE), are of fundamental importance to study shock formation in different physical frameworks beyond hydrodynamics [1]. Experimental demonstration and control of RW signatures have been recently reported in nonlinear optics, in both the temporal [2-5] and spatial [6-7] domains. Nevertheless, standard RWs are achieved under both weak dispersion/diffraction and strong nonlinearity, and have been only demonstrated in the self-defocusing regime of the nonlinear Schrödinger equation (NLSE). High-order dispersive effects in fibers can affect the RW dynamics by significantly deteriorating their expected temporal profiles. Here, we propose a conceptually new class of optical RWs which can be implemented in nonlinear optical fibers, namely, third-order Riemann pulses (TORPs). Such RW packets arise from the interplay between high-order dispersion and nonlinearity, and can be realized by properly tailoring the phase profile of an input ultrashort light pulse.

Our analysis starts by considering the NLSE that describes pulse propagation in a nonlinear optical fiber with Kerr nonlinearity, considering also both second- and third-order dispersions (SOD and TOD), as follows:

$$i \frac{\partial A}{\partial Z} - \frac{\ddot{\beta}^2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{i\bar{\beta}^3}{6} \frac{\partial^3 A}{\partial T^3} + |A|^2 A = 0. \quad (1)$$

In Eq. (1), $A(T, Z)$ is the dimensionless electric field envelope, while $T = t/T_0$ and $Z = z/L_{NL}$, are the normalized temporal and propagation coordinates, respectively. We scale the pulse duration by T_0 , and its peak power by P_0 . $\ddot{\beta}^2 = \beta_2 / (\gamma P_0 T_0^2)$ and $\bar{\beta}^3 = \beta_3 / (\gamma P_0 T_0^3)$ are the dimensionless SOD and TOD terms, where $\gamma = k_0 n_2$ is the nonlinear Kerr coefficient, k_0 is the vacuum wavenumber, n_0 and n_2 are the linear and nonlinear refractive index, and β_2 and β_3 denote the SOD and TOD coefficients of the fiber. Arbitrary solutions to Eq. (1) can be found by expressing the pulse envelope in a polar form by means of the Madelung transformation:

$$A(T, Z) = \sqrt{\rho(T, Z)} \exp \left[-\frac{i}{\bar{\beta}} \int_{-\infty}^T u(T', Z) dT' \right], \quad (2)$$

with $\rho(T, Z)$ and $u(T, Z)$ being real functions. If the condition $\bar{\beta} \ll 1$ is satisfied, Eq. (1) can be reduced to a semiclassical approximation of the NLSE, known as nonlinear shallow water equation (NSWE) [1]. In general, the NSWE is expressed as a system of two coupled equations, and is characterized by two Riemann invariant solutions. Among them, specific solutions, known as simple RWs, can be obtained by setting one of these two Riemann invariants to be a null value, thus reducing the NSWE system to one equation only. More importantly, the propagation dynamics in the nonlinear optical fiber as governed by Eq. (1) can be efficiently modeled by the IBE, whose solution is expressed in an implicit form as:

$$\rho(T, Z) = \rho_0 \left(T + \frac{\beta^2}{2\bar{\beta}^3} Z - \frac{5}{2} \sqrt{3\rho(z, t)^2 \bar{\beta} Z} \right), \quad (3)$$

where ρ_0 is the initial pulse envelope and $\tilde{\beta} = \ddot{\beta} / \bar{\beta}^2$.

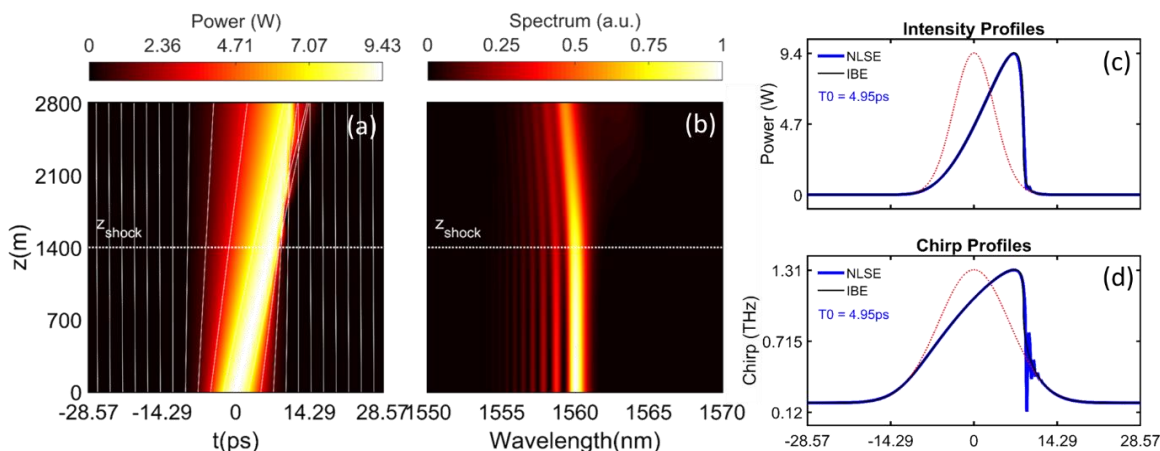


Figure 1: (a) Intensity and (b) spectral plots of NLSE simulations, modeling the evolution of a TORP over 2.8km of dispersion-shifted fiber. The pulse intensity is compared with characteristic lines, obtained analytically from the IBE (white lines), showing shock formation at $z = 1.4$ km (here characteristics lines intersect leading to a multi-valued solution (parameters: Gaussian pulse with 9.42W peak power and 8.24ps duration (FWHM) propagating in a DSF: $\beta_2 = -0.154$ ps².km-1, $\beta_3 = 0.1234$ ps³.km-1 and $\gamma = 0.5$ W-1.km-1 at $\lambda_0 = 1550$ nm). Temporal (c) intensity and (d) chirp profiles are shown at the input and at shock point, comparing IBE predictions (solid black) with NLSE simulations (dashed blue).

From a physical viewpoint, the dynamics of a TORP are associated with a progressive steepening of the pulse envelope, and eventually the formation of a so-called gradient catastrophe (or shock wave formation), where the time derivative of the envelope tends to infinity. The distance at which the shock distance occurs can be analytically calculated by using the method of characteristics, that provides a parametric representation of the energy flow. Equation (3) represents the main result of this research work, mainly associated with the fact that the steepening evolution of the TORPs is uniquely managed by the TOD effect. As a consequence, TORPs exist both in the presence and in the absence of group velocity dispersion (GVD). Unlike standard RWs, already reported in the literature, here the contribution of the GVD only introduces a transversal shifting of the shock along the time axis, thus generating time-shifted RWs [5]. Remarkably, the sign of the GVD does not influence on the steepening process, which means that this new class of Riemann pulses can be also generated for negative values of β_2 , i.e., in the self-focusing regime of the NLSE. From a technological perspective, TORPs can be synthesized by appropriately shaping the initial temporal optical pulse, so that the specific relationship between the instantaneous frequency chirp and the pulse amplitude remains valid in subsequent propagation [2]. To prove our theoretical analysis, we also performed NLSE simulations for a specific case of TORPs, with a Gaussian intensity profile. Numerical results are illustrated in Fig. 1 and are obtained by using the split-step Fourier transform method applied to Eq. (1). The target Riemann profile is obtained by imposing the appropriate phase modulation to a transform-limited Gaussian pulse (8.24 ps FWHM, 9.42 W peak power), according to Eq. (2). During propagation, the pre-chirped Gaussian pulse undergoes a progressive steepening of its leading edge, by maintaining a constant peak intensity, and subsequently reaches a nearly vertical front appearing at the mid-point of the fiber, i.e., $z_{shock} = 1.4$ km, at time $t_{shock} = 8.42$ ps. In the Fourier domain, the spectrum associated with the TORP displays an Airy-like shape, which exhibits a higher intense main lobe located about $\lambda = 10$ nm away from the carrier wavelength (up to the shock distance), as shown in Fig. 1(b). Furthermore, both the intensity and chirp profiles, retrieved from NLSE simulations at the shock distance, display a quasi-perfect matching with IBE predictions [black lines in Figs. 1(c,d)], which is further confirmed by the overlap between the intensity distribution and the IBE characteristic line [white lines in Fig. 1(a)]. Indeed, characteristic lines intersect at the predicted shock distance, to give rise to a multi-valued solution.

This work opens the possibility to achieve an effective control of Riemann pulse dynamics in optical fibers, as currently explored in our experiment. Our finding may prove applicable for other physical systems, where an IBE approximation of the NLSE description provides a powerful tool for studying peculiar nonlinear wave dynamics.

References

- [1] J. Bec and K. Khanin, "Burgers turbulence," *Phys. Rep.* **447**, 1-66 (2007).
- [2] B. Wetzal, *et al.*, "Experimental generation of Riemann waves in optics: a route to shock wave control," *Phys. Rev. Lett.* **117**, 073902 (2016).
- [3] S. Randoux, *et al.*, "Optical random Riemann waves in integrable turbulence," *Phys. Rev. Lett.* **118**, 233901 (2017).
- [4] W. Wan, S. Jia, and J. W. Fleischer, "Dispersive superfluid-like shock waves in nonlinear optics," *Nature Phys.* **3**, 46 (2007).
- [5] S. Wabnitz, "Optical tsunamis: shoaling of shallow water rogue waves in nonlinear fibers with normal dispersion," *J. Opt.* **15**, 064002 (2013).
- [6] G. Marcucci, *et al.*, "Optical spatial shock waves in nonlocal nonlinear media," *Adv. Phys. X* **4**, 1662733 (2019).
- [7] D. Bongiovanni, *et al.*, "Optical generation and control of spatial Riemann waves," *Opt. Lett.* **44**, 3542 (2019).