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On supersymmetric AdS_3 solutions of Type II

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ABSTRACT: We classify supersymmetric warped $\text{AdS}_3 \times_w M_7$ backgrounds of Type IIA and Type IIB supergravity with non-constant dilaton, generic RR fluxes and magnetic NSNS flux, in terms of a dynamic $\text{SU}(3)$ -structure on M_7 . We illustrate our results by recovering several solutions with various amounts of supersymmetry. The dynamic $\text{SU}(3)$ -structure includes a G_2 -structure as a limiting case, and we show that in Type IIB this is integrable.

KEYWORDS: AdS-CFT Correspondence, Flux compactifications

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1 Introduction

Technically more approachable, but not less physically interesting than their higher-dimensional counterparts, AdS_3/CFT_2 dualities provide a hospitable environment for finding answers to questions on both sides of the holographic correspondence. Conformal field theories in two dimensions, which underlie string theory and are key tools in the description of critical phenomena, feature a highly-constraining infinite-dimensional algebra of conformal transformations that often allows for their exact solution. Gravity in three-dimensional asymptotically anti-de Sitter spacetime provides a toy model for quantum gravity. There is thus a clear motivation for the study of AdS_3 backgrounds of string theory.

Owing to the high dimensionality of the internal space, the problem of exploring the space of AdS_3 backgrounds is challenging. A way forward is to impose a symmetry on the background, in the form of supersymmetry or isometry, at the expense of the size of the subspace of backgrounds one can access, depending on the degree of the symmetry. In the present work we impose the minimal amount of supersymmetry, aiming for a more comprehensive scan of supersymmetric AdS_3 backgrounds of Type II supergravity. We classify warped $AdS_3 \times_w M_7$ backgrounds with non-constant dilaton, generic RR fluxes and magnetic NSNS flux. Minimal supersymmetry equips the internal manifold M_7 with a dynamic $SU(3)$ -structure, due to the existence of two Majorana spinors on M_7 . In the limiting case where the spinors are parallel to each other, the dynamic $SU(3)$ -structure corresponds to a G_2 -structure. We translate the necessary and sufficient conditions for supersymmetry to restrictions on the torsion classes of the $SU(3)$ -structure, and obtain expressions for the supergravity fields in terms of the geometric data. We illustrate our results by recovering several AdS_3 solutions with various amounts of supersymmetry. In Type IIB supergravity we take a closer look at the G_2 -structure limiting case, show that it is integrable, and

reduce the problem of finding a solution to not only the supersymmetry equations but also the equations of motion, to a single geometric equation. Generically, the dual superconformal field theories in two dimensions preserve $\mathcal{N} = (0, 1)$ supersymmetry, and include well-studied families of theories with higher supersymmetry such as those arising from D3-branes wrapped on Riemann surfaces [1–3], whose duals appear in section 5.1.

The rest of this paper is organized as follows. In section 2, we present the supersymmetry equations as a set of equations for a pair of polyforms on M_7 . In section 3, we review G_2 - and $SU(3)$ -structures in seven dimensions, and parameterize the polyforms in terms of the latter. Sections 4 and 5 contain our results for Type IIA and Type IIB supergravity respectively.

2 Supersymmetry equations

We consider bosonic backgrounds of Type II supergravity whose spacetime is a warped product $AdS_3 \times_w M_7$, where M_7 is a seven-dimensional Riemannian manifold. The ten-dimensional metric reads:

$$ds_{10}^2 = e^{2A} ds^2(AdS_3) + ds^2(M_7), \tag{2.1}$$

where A is a function on M_7 . Preserving the symmetries of AdS_3 , the NSNS field-strength H_{10d} , and the sum of the RR field-strengths F_{10d} in the democratic formulation of Type II supergravity [4], are decomposed as

$$H_{10d} = \varkappa \text{vol}(AdS_3) + H, \quad F_{10d} = e^{3A} \text{vol}(AdS_3) \wedge \star_7 \lambda(F) + F. \tag{2.2}$$

The magnetic fluxes H , and

$$\text{Type IIA: } F = \sum_{p=0,2,4,6} F_p, \quad \text{Type IIB: } F = \sum_{p=1,3,5,7} F_p \tag{2.3}$$

are forms on M_7 . The operator λ acts on a p -form F_p as $\lambda(F_p) = (-1)^{\lfloor p/2 \rfloor} F_p$. The RR field-strengths satisfy $d_{H_{10d}} F_{10d} = 0$, which decomposes as

$$d_H(e^{3A} \star_7 \lambda(F)) + \varkappa F = 0, \quad d_H F = 0, \tag{2.4}$$

where $d_H \equiv d - H \wedge$. The first set of equations act as equations of motion for F , and the second one as Bianchi identities.

We also decompose the ten-dimensional supersymmetry parameters, ϵ_1 and ϵ_2 , under $\text{Spin}(1, 2) \times \text{Spin}(7) \subset \text{Spin}(1, 9)$:

$$\epsilon_1 = \zeta \otimes \chi_1 \otimes \begin{pmatrix} 1 \\ -i \end{pmatrix}, \quad \epsilon_2 = \zeta \otimes \chi_2 \otimes \begin{pmatrix} 1 \\ \pm i \end{pmatrix}. \tag{2.5}$$

The upper sign in ϵ_2 corresponds to Type IIA, and the lower sign to Type IIB. χ_1 and χ_2 are Majorana $\text{Spin}(7)$ spinors; ζ is a Majorana $\text{Spin}(1, 2)$ spinor that solves the Killing equation

$$\nabla_\mu \zeta = \frac{1}{2} m \gamma_\mu \zeta, \tag{2.6}$$

where the real constant parameter m is related to the AdS_3 radius L_{AdS_3} as $L_{AdS_3}^2 = 1/m^2$.

The Cliff(1, 9) gamma matrices are decomposed as

$$\Gamma_\mu = e^A \gamma_\mu^{(3)} \otimes \mathbb{I} \otimes \sigma_3, \quad \Gamma_m = \mathbb{I} \otimes \gamma_m \otimes \sigma_1, \quad (2.7)$$

where $\gamma_\mu^{(3)}$ are Cliff(1, 2) gamma matrices, γ_m are Cliff(7) gamma matrices and μ, m are spacetime indices. We choose $\gamma_\mu^{(3)}$ to be real, and γ_m imaginary and antisymmetric. For more details see the appendix of [5].

Necessary and sufficient conditions for supersymmetry are generally given in terms of a set of Killing spinor equations. For AdS₃ backgrounds, these can be rewritten in terms of a pair of bispinors ψ_\pm defined by

$$\chi_1 \otimes \chi_2^t \equiv \psi_+ + i\psi_-. \quad (2.8)$$

Taking into account the Fierz expansion of $\chi_1 \otimes \chi_2^t$, and by mapping anti-symmetric products of gamma matrices to forms, ψ_+ and ψ_- can be treated as polyforms on M_7 , of even and odd degree respectively. The necessary and sufficient conditions for supersymmetry in terms of differential constraints on these polyforms were derived in [6] for Type IIA, and in [5] for Type IIB.

Supersymmetry imposes

$$2mc_- = -c_+ \varkappa, \quad (2.9)$$

where c_\pm are constants defined by

$$c_\pm \equiv e^{\mp A} (\|\chi_1\|^2 \pm \|\chi_2\|^2). \quad (2.10)$$

In what follows we will consider backgrounds with zero electric component for H_{10d} , $\varkappa = 0$, and thus $\|\chi_1\|^2 = \|\chi_2\|^2$. In Type IIB, $\varkappa = 0$ can be set to zero by applying an $SL(2, \mathbb{R})$ duality transformation.¹ In Type IIA, as shown in [6], $\varkappa \neq 0$ leads to zero Romans mass; such AdS₃ backgrounds can thus be studied in M-theory, see [7–9]. Without loss of generality, we set $c_+ = 2$, that is

$$\|\chi_1\|^2 = \|\chi_2\|^2 = e^A. \quad (2.11)$$

Given the above choices, the system of supersymmetry equations then reads:

$$d_H(e^{A-\phi}\psi_\mp) = 0, \quad (2.12a)$$

$$d_H(e^{2A-\phi}\psi_\pm) \mp 2me^{A-\phi}\psi_\mp = \frac{1}{8}e^{3A} \star_7 \lambda(F), \quad (2.12b)$$

$$(\psi_\mp, F)_7 = \mp \frac{m}{2} e^{-\phi} \text{vol}_7. \quad (2.12c)$$

Here, an upper sign applies to Type IIA and a lower one to Type IIB; $(\psi_\mp, F)_7 \equiv (\psi_\mp \wedge \lambda(F))_7$, with $(\cdot)_7$ denoting the projection to the seven-form component.

We can decompose χ_2 in terms of χ_1 as follows:

$$\chi_2 = \sin \theta \chi_1 - i \cos \theta v_m \gamma^m \chi_1, \quad (2.13)$$

¹We thank N. Macpherson for this observation.

where without loss of generality, we take v to be a real one-form of unit norm and restrict $\theta \in [0, \pi/2]$. At the boundary value $\theta = 0$, χ_1 and χ_2 are orthogonal and define a “strict” $SU(3)$ -structure on TM_7 . At the other boundary value $\theta = \pi/2$, χ_1 and χ_2 are parallel and define a G_2 -structure. At intermediate values of θ , the pair (χ_1, χ_2) define a “dynamic” $SU(3)$ -structure on TM_7 , or alternatively a $G_2 \times G_2$ -structure on the generalized tangent bundle $TM_7 \oplus T^*M_7$.

In the next section we review G_2 - and $SU(3)$ -structures in seven dimensions, and parameterize ψ_{\pm} in terms of the latter.

3 G_2 - and $SU(3)$ -structures in seven dimensions

We briefly summarize the mathematical formalism for G_2 - and $SU(3)$ -structures on seven-dimensional Riemannian manifolds that we will use in analyzing the supersymmetry equations (2.12).

A G_2 -structure on a seven-dimensional Riemannian manifold M_7 is defined by a nowhere-vanishing, globally defined three-form φ . Equivalently, a G_2 -structure is defined by a nowhere-vanishing, globally defined Majorana spinor. The three-form φ is constructed as a bilinear of the Majorana spinor as

$$\varphi_{mnp} = -i\chi^t \gamma_{mnp} \chi, \tag{3.1}$$

where χ is taken to have unit norm. The three-form φ is normalized so that

$$\varphi \wedge \star_7 \varphi = 7 \text{vol}_7. \tag{3.2}$$

In the presence of a G_2 -structure, the differential forms on M_7 can be decomposed into irreducible representations of G_2 . In particular, this may be applied to the exterior derivative of the three-form φ and its Hodge dual $\star_7 \varphi$:

$$d\varphi = \tau_0 \star_7 \varphi + 3\tau_1 \wedge \varphi + \star_7 \tau_3, \tag{3.3a}$$

$$d \star_7 \varphi = 4\tau_1 \wedge \star_7 \varphi + \star_7 \tau_2. \tag{3.3b}$$

The k -forms τ_k are the torsion classes of the G_2 -structure. τ_0 transforms in the **1** representation of G_2 , τ_1 in the **7**, τ_2 in the **14**, and τ_3 in the **27**.

An $SU(3)$ -structure on a seven-dimensional Riemannian manifold M_7 is defined by a nowhere-vanishing, globally defined triplet comprising a real one-form v , a real two-form J , and a complex decomposable three-form Ω , subject to the following defining relations:²

$$v \lrcorner J = v \lrcorner \Omega = 0, \quad \Omega \wedge J = 0, \quad \frac{i}{8} \Omega \wedge \bar{\Omega} = \frac{1}{3!} J \wedge J \wedge J. \tag{3.4}$$

²In terms of local coordinates,

$$X \lrcorner \omega_{(k)} \equiv \frac{1}{k-1!} X^n \omega_{nm_1 \dots m_{k-1}} dx^{m_1} \wedge \dots \wedge dx^{m_{k-1}}.$$

Equivalently, an $SU(3)$ -structure is defined by a pair of non-parallel Majorana spinors; see the appendix of [5]. The one-form v foliates M_7 with leaves M_6 . The metric on M_7 is then locally decomposed as

$$ds^2(M_7) = v^2 + ds^2(M_6), \tag{3.5}$$

with an accompanying volume form $\text{vol}_7 \equiv \frac{1}{3!}v \wedge J \wedge J \wedge J$. The exterior derivative can be decomposed as

$$d = v \wedge v \lrcorner d + d_6, \tag{3.6}$$

where d_6 is the exterior derivative on M_6 . k -forms orthogonal to v can be decomposed into primitive (p, q) -forms with respect to J .

The intrinsic torsion of an $SU(3)$ -structure splits into torsion classes, which transform in irreducible representations of $SU(3)$. They parameterize the exterior derivatives of (v, J, Ω) as:

$$dv = RJ + T_1 + \text{Re}(\overline{V}_1 \lrcorner \Omega) + v \wedge W_0, \tag{3.7a}$$

$$dJ = \frac{3}{2}\text{Im}(\overline{W}_1 \Omega) + W_3 + W_4 \wedge J + v \wedge \left(\frac{2}{3}\text{Re}EJ + T_2 + \text{Re}(\overline{V}_2 \lrcorner \Omega) \right), \tag{3.7b}$$

$$d\Omega = W_1 J \wedge J + W_2 \wedge J + \overline{W}_5 \wedge \Omega + v \wedge (E\Omega - 2V_2 \wedge J + S). \tag{3.7c}$$

R is a real scalar, E and W_1 are complex scalars, V_1, V_2 and W_5 are complex $(1, 0)$ -forms, W_0 and W_4 are real one-forms, T_1 and T_2 are real primitive $(1, 1)$ -forms, W_2 is a complex primitive $(1, 1)$ -form, W_3 is a real primitive $(2, 1) + (1, 2)$ -form, and S is a complex primitive $(2, 1)$ -form. R, E and W_1 transform in the $\mathbf{1}$ representation of $SU(3)$, V_1, V_2 and W_5 in the $\mathbf{3}$, W_0 and W_4 in the $\mathbf{3} + \overline{\mathbf{3}}$, T_1, T_2 and W_2 in the $\mathbf{8}$, W_3 in the $\mathbf{6} + \overline{\mathbf{6}}$, and S in the $\mathbf{6}$.

As detailed in [5], the polyforms ψ_{\pm} are parameterized in terms of the $SU(3)$ -structure as

$$\begin{aligned} \psi_+ &= \frac{1}{8}e^A \left[\text{Im}(e^{i\theta} e^{iJ}) + v \wedge \text{Re}(e^{i\theta} \Omega) \right], \\ \psi_- &= \frac{1}{8}e^A \left[v \wedge \text{Re}(e^{i\theta} e^{iJ}) + \text{Im}(e^{i\theta} \Omega) \right], \end{aligned} \tag{3.8}$$

where θ is the angle appearing in (2.13). When $\theta = \pi/2$, the one-form v drops out of the decomposition (2.13) of χ_2 , the spinors (χ_1, χ_2) become parallel, and thus define merely a G_2 -structure rather than an $SU(3)$ -structure. Nevertheless, the above decomposition is still valid: it can be shown that for compact M_7 , existence of a $\text{Spin}(7)$ structure implies existence of an $SU(3)$ -structure [10]. Hence, we may decompose the three-form φ defined by the spinor $\chi_1 = \chi_2$ in terms of an $SU(3)$ -structure (v, J, Ω) , leading to the above result even in this limiting case. The phase $e^{i\theta}$ multiplying Ω can be “absorbed” by a redefinition $e^{i\theta}\Omega \rightarrow \Omega$, and we will apply this redefinition in the following sections.

4 Type IIA

In this section we analyze the Type IIA supersymmetry equations (2.12) (upper sign): by substituting (3.8) in (2.12) (redefining $e^{i\theta}\Omega \rightarrow \Omega$), the necessary and sufficient conditions

for supersymmetry translate to restrictions on the torsion classes of the SU(3)-structure defined by (χ_1, χ_2) on M_7 . Furthermore, the RR and NSNS field-strengths are expressed in terms of the SU(3)-structure data.

The geometry of M_7 and the NSNS field-strength are constrained by equation (2.12a), which yields:³

$$d\left(e^{2A-\phi} \cos \theta v\right) = 0, \quad (4.1a)$$

$$d\left(e^{2A-\phi} (-\sin \theta v \wedge J + \text{Im}\Omega)\right) - e^{2A-\phi} \cos \theta H \wedge v = 0, \quad (4.1b)$$

$$d\left(e^{2A-\phi} \cos \theta v \wedge J^2\right) + 2e^{2A-\phi} H \wedge (-\sin \theta v \wedge J + \text{Im}\Omega) = 0. \quad (4.1c)$$

The RR field-strengths are derived from (2.12b), corresponding to

$$e^{3A} \star_7 F_6 = -d\left(e^{3A-\phi} \sin \theta\right) + 2me^{2A-\phi} \cos \theta v, \quad (4.2a)$$

$$e^{3A} \star_7 F_4 = d\left(e^{3A-\phi} \cos \theta J\right) - e^{3A-\phi} \sin \theta H - 2me^{2A-\phi} \text{Im}\Omega + 2me^{2A-\phi} \sin \theta v \wedge J, \quad (4.2b)$$

$$e^{3A} \star_7 F_2 = -d\left(e^{3A-\phi} \left(v \wedge \text{Re}\Omega - \sin \theta \frac{1}{2} J^2\right)\right) + e^{3A-\phi} \cos \theta H \wedge J - me^{2A-\phi} \cos \theta v \wedge J^2, \quad (4.2c)$$

$$e^{3A} \star_7 F_0 = -d\left(e^{3A-\phi} \cos \theta \frac{1}{3!} J^3\right) - e^{3A-\phi} H \wedge \left(v \wedge \text{Re}\Omega - \sin \theta \frac{1}{2} J^2\right) - me^{2A-\phi} \sin \theta v \wedge \frac{1}{3} J^3. \quad (4.2d)$$

From (4.1), using (3.7), we obtain the following relations for the torsion classes of the SU(3)-structure:

$$0 = R = V_1 = T_1 = \text{Im}W_1 = \text{Im}W_2 = \text{Im}E, \quad (4.3a)$$

$$0 = d_6(2A - \phi)^{(1,0)} + W_5, \quad (4.3b)$$

$$0 = \cos \theta d_6(2A - \phi) - \sin \theta d_6 \theta - \cos \theta W_0. \quad (4.3c)$$

Furthermore, using the decomposition of the NSNS field-strength H with respect to the SU(3)-structure

$$H = H^R \text{Re}\Omega + H^I \text{Im}\Omega + \left(H^{(1,0)} + H^{(0,1)}\right) \wedge J + H^{(2,1)} + H^{(1,2)} + v \wedge \left(H_v^{(1,1)} + H_v^0 J + H_v^{(0,1)} \lrcorner \Omega + H_v^{(1,0)} \lrcorner \bar{\Omega}\right), \quad (4.4)$$

³ $J^2 \equiv J \wedge J$ and $J^3 \equiv J \wedge J \wedge J$.

where $H^{(2,1)}$ and $H_v^{(1,1)}$ are primitive, we obtain

$$2 \cos \theta \operatorname{Re}(H^{(2,1)}) = -\operatorname{Im}S - \sin \theta W_3, \quad (4.5a)$$

$$2 \cos \theta \operatorname{Re}(H^{(1,0)}) = \sin \theta [-W_4 + W_0 - d_6(2A - \phi)] \\ + 2\operatorname{Im}V_2 - \cos \theta d_6 \theta, \quad (4.5b)$$

$$\cos \theta H^I = -\operatorname{Re}E - 2\dot{A} + \dot{\phi} - \frac{3}{2} \sin \theta \operatorname{Re}W_1, \quad (4.5c)$$

$$H^R = 0, \quad (4.5d)$$

$$2 \sin \theta \operatorname{Re}(H^{(1,0)}) + 4\operatorname{Im}(H_v^{(1,0)}) = \cos \theta W_4. \quad (4.5e)$$

Here we have introduced the notation $\dot{f} \equiv v \lrcorner df$ for any function f . Using (4.3) and (4.5), as well as the identities in the appendix of [5] to Hodge dualize, we derive the following expressions for the RR field-strengths:

$$e^\phi F_0 = -\cos \theta (3\dot{A} - \dot{\phi} + 2\operatorname{Re}E) + \sin \theta (3H_v^0 + \dot{\theta} - 2me^{-A}) - 4H^I, \quad (4.6a)$$

$$e^\phi F_2 = X_2 \lrcorner \operatorname{Im}\Omega - \sin \theta T_2 - \cos \theta H_v^{(1,1)} - \operatorname{Re}W_2 - 2v \wedge \operatorname{Im}(Y_2^{(1,0)}) \\ + \left[2\operatorname{Re}W_1 + \cos \theta (2H_v^0 + \dot{\theta} - 2me^{-A}) + \sin \theta \left(3\dot{A} - \dot{\phi} + \frac{4}{3} \operatorname{Re}E \right) \right] J, \quad (4.6b)$$

$$e^\phi F_4 = v \wedge 2\operatorname{Im}(\cos \theta W_3^{(2,1)} - \sin \theta H^{(2,1)}) - v \wedge X_4 \lrcorner J \wedge J \\ + \left(\frac{3}{2} \cos \theta \operatorname{Re}W_1 - 2me^{-A} - \sin \theta H^I \right) v \wedge \operatorname{Re}\Omega \\ + \left[\cos \theta (3\dot{A} - \dot{\phi} + \frac{2}{3} \operatorname{Re}E) + \sin \theta (2me^{-A} - \dot{\theta} - H_v^0) \right] \frac{1}{2} J^2 \\ - (\cos \theta T_2 - \sin \theta H_v^{(1,1)}) \wedge J + \operatorname{Im} \left[(\cos \theta \bar{V}_2 - 2 \sin \theta H_v^{(0,1)}) \wedge \Omega \right] \quad (4.6c)$$

$$e^\phi F_6 = \left[\cos \theta (2me^{-A} - \dot{\theta}) - \sin \theta (3\dot{A} - \dot{\phi}) \right] \frac{1}{3!} J^3 + v \wedge J^2 \wedge \operatorname{Im}(X_6^{(1,0)}), \quad (4.6d)$$

where

$$X_2 \equiv -d_6(3A - \phi) + W_0 - \bar{W}_5 - W_5, \quad (4.7a)$$

$$Y_2 \equiv \sin \theta [d_6(3A - \phi) + 2W_4] + \cos \theta d_6 \theta + 2 \cos \theta (H^{(1,0)} + H^{(0,1)}), \quad (4.7b)$$

$$X_4 \equiv \cos \theta (d_6 A + W_0 + W_4) - \sin \theta (H^{(1,0)} + H^{(0,1)}), \quad (4.7c)$$

$$X_6 \equiv \sin \theta d_6(3A - \phi) + \cos \theta d_6 \theta. \quad (4.7d)$$

Substituting the above expressions in the pairing equation (2.12c) yields the additional scalar constraint:

$$3H_v^0 - 6me^{-A} + 2\dot{\theta} + 6 \cos \theta \operatorname{Re}W_1 - 4 \sin \theta H^I = 0. \quad (4.8)$$

Equations (4.3), (4.5), (4.6), and (4.8) constitute necessary and sufficient conditions for the preservation of supersymmetry.

4.1 Solutions

We now look at solutions of the supersymmetry conditions we have derived. In particular we will recover the $\mathcal{N} = 8$ supersymmetric $\text{AdS}_3 \times S^6$ solution of [6] (realizing the $F(4)$ superalgebra), and the $\mathcal{N} = (4, 0)$ supersymmetric $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ solution of [11]. In addition to the supersymmetry equations, the equations of motion are solved provided that the fluxes satisfy the Bianchi identities (see for example [12]), and this is the case for the solutions below.

$\text{AdS}_3 \times S^6$ with $\mathcal{N} = 8$ supersymmetry. The angle θ , the warp factor A , and the dilaton ϕ satisfy

$$d_6\theta = 0, \quad d_6A = 0, \quad d_6\phi = 0. \quad (4.9)$$

The one-form v is closed — see (4.1a) given (4.9) — and locally a coordinate z can be introduced such that

$$v = \xi(z)dz, \quad (4.10)$$

for a function $\xi(z)$ which can be fixed by a change of coordinate. Following [6], it is fixed to

$$\xi(z) = -\frac{2}{3} \left(\frac{q}{p}\right)^{1/3} e^{-A(z)}, \quad p, q = \text{constants}. \quad (4.11)$$

Accordingly, the metric on M_7 (3.5) reads

$$ds^2(M_7) = \frac{4}{9} \left(\frac{q}{p}\right)^{2/3} e^{-2A(z)} dz^2 + ds^2(M_6), \quad (4.12)$$

and the metric on M_6 is taken to be conformal to the round metric on the six-sphere S^6 :

$$ds^2(M_6) = e^{2Q(z)} ds^2(S^6). \quad (4.13)$$

The non-zero torsion classes of the $\text{SU}(3)$ -structure are

$$\text{Re}W_1 = 2e^{-Q}, \quad \text{Re}E = -\frac{9}{2} \left(\frac{q}{p}\right)^{-1/3} e^A \frac{dQ}{dz}. \quad (4.14)$$

It follows that $\hat{J} \equiv e^{-2Q}J$ and $\hat{\Omega} \equiv e^{-3Q}\Omega$ define a nearly-Kähler structure on S^6 :

$$d\hat{J} = 3\text{Im}\hat{\Omega}, \quad d\hat{\Omega} = 2\hat{J} \wedge \hat{J}. \quad (4.15)$$

Setting $m = 1$ as in [6] the solution is determined by

$$e^{2Q} = \left(\frac{q}{p}\right)^{1/3} \frac{1}{\sqrt{z}}, \quad e^{2A} = \frac{4}{9} \left(\frac{q}{p}\right)^{1/3} \frac{1+z^3}{\sqrt{z}}, \quad e^\phi = q^{-1/6} p^{-5/6} z^{-5/4}, \quad (4.16)$$

and

$$\cos\theta = \frac{2}{3} e^{Q-A}. \quad (4.17)$$

The only non-zero fluxes are F_0 and $F_6 = 5q\text{vol}(S^6)$.

This solution arises as a near-horizon limit of a D2-O8 configuration. The internal space is non-compact, with $z \in [0, \infty]$. Near $z = 0$, there is an O8-plane singularity, and at $z \rightarrow \infty$ a type of D2-brane singularity; see [6] for more details. Flux quantization imposes $2\pi F_0 \in \mathbb{Z}$, and $q/(6\pi^2) \in \mathbb{Z}$.

AdS₃ × S³ × S³ × S¹ with $\mathcal{N} = (4, 0)$ supersymmetry. The AdS₃ × S³ × S³ × S¹ solution of [11, section 4.1] belongs to the class of solutions with strict SU(3)-structure, i.e. $\theta = 0$. The warp factor A , and the dilaton ϕ satisfy

$$d_6 A = 0, \quad d_6 \phi = 0. \quad (4.18)$$

The one-form v is closed and locally is set to $v = e^{\phi_0} e^{3A(\rho)} d\rho$, where ϕ_0 is a constant. The metric on M_7 reads

$$ds^2(M_7) = e^{2\phi_0} e^{6A(\rho)} d\rho^2 + e^{2A(\rho)} \widehat{ds}^2(M_6). \quad (4.19)$$

Furthermore, the torsion classes of the SU(3)-structure are restricted so that

$$d_6 \hat{J} = \frac{3}{2} m \text{Im} \hat{\Omega} + \hat{W}_3, \quad (4.20a)$$

$$d_6 \hat{\Omega} = m \hat{J} \wedge \hat{J}, \quad (4.20b)$$

where $\hat{J} \equiv e^{-2A} J$, $\hat{\Omega} \equiv e^{-3A} \Omega$, and $\hat{W}_3 \equiv e^{-2A} W_3$. The dilaton ϕ and the warp factor A are given by:

$$e^\phi = e^{\phi_0} e^{5A}, \quad e^{-8A} = 2F_0 e^{2\phi_0} \rho + \ell, \quad (4.21)$$

where ℓ is a constant. The only non-zero fluxes are F_0 and

$$F_4 = d\rho \wedge \left(2\text{Im}(\hat{W}_3^{(2,1)}) - \frac{1}{2} m \text{Re} \hat{\Omega} \right), \quad (4.22a)$$

$$F_6 = \frac{2}{3!} m e^{-\phi_0} \hat{J}^3. \quad (4.22b)$$

The Bianchi identity to be satisfied is that of F_4 , $dF_4 = 0$, which yields

$$d\text{Im}(\hat{W}_3^{(2,1)}) - \frac{m^2}{4} \hat{J} \wedge \hat{J} = 0. \quad (4.23)$$

Thus, what remains to be done in order to solve the equations of motion is to find a suitable manifold M_6 admitting an SU(3)-structure with the right torsion classes. On $S^3 \simeq \text{SU}(2)$, a set of left-invariant forms $(\sigma^1, \sigma^2, \sigma^3)$ can be found such that

$$d\sigma^j = \frac{1}{2} \epsilon_{jkl} \sigma^k \wedge \sigma^l, \quad (4.24)$$

where $j, k, l \in \{1, 2, 3\}$, and ϵ^{jkl} is the Levi-Civita symbol. Let σ_a^j be the left-invariant forms on S_a^3 , $a \in \{1, 2\}$, and let

$$\begin{aligned} \zeta^j &\equiv \frac{1}{\sqrt{2}m} (\sigma_2^j + i\sigma_1^j) \\ \hat{J} &= \frac{i}{2} \delta_{ij} \zeta^i \wedge \bar{\zeta}^j, \quad \hat{\Omega} = \frac{1}{3! \sqrt{2}} (1 + i) \epsilon_{jkl} \zeta^j \wedge \zeta^k \wedge \zeta^l. \end{aligned} \quad (4.25)$$

Then $(\hat{J}, \hat{\Omega})$ form an SU(3)-structure on $S_1^3 \times S_2^3$, with corresponding metric

$$\widehat{ds}^2(M_6) = \frac{2}{m^2} \left(ds^2(S_1^3) + ds^2(S_2^3) \right), \quad (4.26)$$

where $ds^2(S_a^3) = \frac{1}{4} \sum_j (\sigma_a^j)^2$. Making use of (4.24), the non-vanishing torsion classes are determined to be

$$\hat{W}_1 = m, \quad \hat{W}_3^{(2,1)} = -m \frac{1+i}{4! \sqrt{2}} \epsilon_{ijk} \zeta^i \wedge \zeta^j \wedge \bar{\zeta}^k, \quad (4.27)$$

with $\hat{W}_3^{(2,1)}$ satisfying

$$d\hat{W}_3^{(2,1)} = \frac{m^2}{4} i \hat{J} \wedge \hat{J}. \quad (4.28)$$

as desired. The fluxes now read:

$$F_4 = \frac{4}{m^2} \left(\text{vol}(S_1^3) + \text{vol}(S_2^3) \right) \wedge d\rho, \quad (4.29a)$$

$$F_6 = \frac{16}{m^5} e^{-\phi_0} \text{vol}(S_1^3) \wedge \text{vol}(S_2^3). \quad (4.29b)$$

The coordinate ρ here is related to the coordinate r in [11] via

$$(2F_0 e^{2\phi_0} \rho + \ell)^{1/2} = \frac{1}{L^4} (F_0 \nu r + c), \quad (4.30)$$

and also $e^{-\phi_0} = qL^5$, where $(L, \nu = \pm 1, c, q)$ are constant parameters in [11].

5 Type IIB

In this section we analyze the Type IIB supersymmetry equations (2.12) (lower sign) in a way similar to that of the analysis of the Type IIA supersymmetry equations in the previous section.

The NSNS sector is constrained by (2.12a), which yields:

$$d \left(e^{2A-\phi} \sin \theta \right) = 0, \quad (5.1a)$$

$$d \left(e^{2A-\phi} \cos \theta J \right) - e^{2A-\phi} \sin \theta H = 0, \quad (5.1b)$$

$$d \left(e^{2A-\phi} \left(v \wedge \text{Re}\Omega - \sin \theta \frac{1}{2} J^2 \right) \right) - e^{2A-\phi} \cos \theta H \wedge J = 0, \quad (5.1c)$$

$$d \left(e^{2A-\phi} \cos \theta J^3 \right) + 3! e^{2A-\phi} H \wedge \left(v \wedge \text{Re}\Omega - \sin \theta \frac{1}{2} J^2 \right) = 0. \quad (5.1d)$$

The RR field-strengths are derived from (2.12b), corresponding to

$$e^{3A} \star_7 F_7 = -2m e^{2A-\phi} \sin \theta, \quad (5.2a)$$

$$e^{3A} \star_7 F_5 = d \left(e^{3A-\phi} \cos \theta v \right) + 2m e^{2A-\phi} \cos \theta J, \quad (5.2b)$$

$$e^{3A} \star_7 F_3 = d \left(e^{3A-\phi} (\sin \theta v \wedge J - \text{Im}\Omega) \right) + e^{3A-\phi} \cos \theta H \wedge v \\ - 2m e^{2A-\phi} \left(v \wedge \text{Re}\Omega - \sin \theta \frac{1}{2} J^2 \right), \quad (5.2c)$$

$$e^{3A} \star_7 F_1 = -d \left(e^{3A-\phi} \cos \theta v \wedge \frac{1}{2} J^2 \right) + e^{3A-\phi} H \wedge (\sin \theta v \wedge J - \text{Im}\Omega) \\ - m e^{2A-\phi} \cos \theta \frac{1}{3} J^3. \quad (5.2d)$$

From (5.1), in addition to $e^{2A-\phi} \sin \theta$ being constant we obtain

$$\begin{aligned} \sin \theta H = & \frac{3}{2} \cos \theta \operatorname{Im}(\overline{W_1} \Omega) + [\cos \theta d_6(2A - \phi) - \sin \theta d_6 \theta + \cos \theta W_4] \wedge J + \cos \theta W_3 \\ & + v \wedge \left[\cos \theta T_2 + \left(\frac{2}{3} \cos \theta \operatorname{Re} E + \cos \theta (2\dot{A} - \dot{\phi}) - \sin \theta \dot{\theta} \right) J + \cos \theta \operatorname{Re}(\overline{V_2} \lrcorner \Omega) \right], \end{aligned} \quad (5.3a)$$

$$\begin{aligned} \cos \theta H = & \cos \theta H^R \operatorname{Re} \Omega + \cos \theta H^I \operatorname{Im} \Omega + (2 \operatorname{Re} V_1 - \sin \theta W_4) \wedge J + 2 \cos \theta \operatorname{Re}(H^{(2,1)}) \\ & + v \wedge \left[-\operatorname{Re} W_2 - \sin \theta T_2 - \left(\frac{2}{3} \sin \theta \operatorname{Re} E + \operatorname{Re} W_1 \right) J \right] \\ & + v \wedge \operatorname{Im} \left[(d_6(2A - \phi) - W_0 + \overline{W_5}) \lrcorner \Omega \right] - \sin \theta v \wedge \operatorname{Re}(\overline{V_2} \lrcorner \Omega), \end{aligned} \quad (5.3b)$$

$$2H^I = \cos \theta (2\dot{A} - \dot{\phi}) - \sin \theta \dot{\theta}. \quad (5.3c)$$

From (5.2), making use of (5.3) and the identities in the appendix of [5] to Hodge dualize, we derive the following expressions for the magnetic RR field-strengths:

$$e^\phi F_1 = - \left(2 \cos \theta m e^{-A} + 4H^R + 3 \cos \theta R \right) v + 2 \operatorname{Im}(X_1^{(1,0)}), \quad (5.4a)$$

$$\begin{aligned} e^\phi F_3 = & \left(-2m e^{-A} - \cos \theta H^R - \operatorname{Im} E + \frac{3}{2} \sin \theta \operatorname{Im} W_1 \right) \operatorname{Im} \Omega \\ & + \left[-2 \operatorname{Im} V_1 - 2 \operatorname{Re} V_2 + 2 \sin \theta \operatorname{Im}(W_0^{(1,0)} - dA^{(1,0)}) \right] \wedge J \\ & + v \wedge (\operatorname{Im} W_2 - \sin \theta T_1) - 2 \left[\operatorname{Im} W_1 - \sin \theta (R + m e^{-A}) \right] v \wedge J \\ & + 2 \cos \theta \operatorname{Im}(H^{(2,1)}) + 2 \sin \theta \operatorname{Im}(W_3^{(2,1)}) - \operatorname{Re} S + X_3 \lrcorner (v \wedge \operatorname{Re} \Omega), \end{aligned} \quad (5.4b)$$

$$\begin{aligned} e^\phi F_5 = & \cos \theta \left(R + 2m e^{-A} \right) v \wedge \frac{1}{2} J^2 - \operatorname{Im}(X_5^{(1,0)}) \wedge J^2 \\ & - \cos \theta v \wedge J \wedge T_1 + 2 \cos \theta v \wedge \operatorname{Re} V_1 \wedge \operatorname{Im} \Omega, \end{aligned} \quad (5.4c)$$

$$e^\phi F_7 = -2m e^{-A} \sin \theta \operatorname{vol}_7, \quad (5.4d)$$

where

$$X_1 \equiv -\cos \theta d_6(A - \phi) + \sin \theta d_6 \theta - \cos \theta W_0 - 8 \operatorname{Im}(H_v^{(1,0)}), \quad (5.5a)$$

$$X_3 \equiv dA + d_6(2A - \phi) + W_5 + \overline{W_5} - 2 \sin \theta \operatorname{Re} V_1, \quad (5.5b)$$

$$X_5 \equiv \cos \theta d_6(3A - \phi) - \cos \theta W_0 - \sin \theta d_6 \theta. \quad (5.5c)$$

Substituting the above expressions in the pairing equation (2.12c) yields the scalar constraint

$$3R + 6m e^{-A} + 4 \cos \theta H^R + 2 \operatorname{Im} E - 6 \sin \theta \operatorname{Im} W_1 = 0. \quad (5.6)$$

Equations (5.1a), (5.3), (5.4), and (5.6) constitute necessary and sufficient conditions for the preservation of supersymmetry.

5.1 Solutions

A family of solutions for the limiting case $\theta = 0$, that is the strict SU(3)-structure case, were examined in [5, section 5]: the internal manifold M_7 is a U(1) fibration over a conformally Kähler base, and they feature a varying axio-dilaton, a primitive (2, 1)-form flux $H + i e^\phi F_3$,

and five-form flux F_5 . The solutions of [1–3, 13, 14], with $\mathcal{N} = (2, 0)$ supersymmetry, belong in this family.

Here, we will examine the other limiting case: G_2 -structure solutions, i.e., solutions with $\theta = \pi/2$. Although equivalent, it turns out to be more convenient to work directly with the G_2 -structure rather than to use the $\theta = \pi/2$ limit of the supersymmetry conditions derived above.

The polyforms ψ_{\pm} are parameterized in terms of the G_2 -structure, defined by the three-form φ , as

$$\psi_+ = \frac{1}{8}e^A(1 - \star_7\varphi), \quad \psi_- = \frac{1}{8}e^A(-\varphi + \text{vol}_7). \quad (5.7)$$

Plugging these expressions into the supersymmetry equations (2.12), and making use of (3.3) leads to the following constraints for the torsion classes

$$\tau_1 = \tau_2 = 0, \quad \tau_0 = -\frac{12}{7}me^{-A}. \quad (5.8)$$

Vanishing of the τ_2 torsion class means that the G_2 -structure is integrable, meaning one can introduce a G_2 Dolbeault cohomology [15]. Furthermore, we obtain

$$d(2A - \phi) = 0, \quad H = 0, \quad (5.9)$$

and

$$\begin{aligned} e^\phi F_3 &= dA \lrcorner \star_7 \varphi + \frac{2}{7}me^{-A}\varphi + \tau_3, \\ e^\phi F_7 &= -2me^A \text{vol}_7, \end{aligned} \quad (5.10)$$

while $F_1 = F_5 = 0$.

Next, we examine the Bianchi identities, which reduce to $dF_3 = 0$. Imposing the Bianchi identities in addition to the supersymmetry conditions yields a solution to the equations of motion. We will work with a rescaled G_2 -structure $\hat{\varphi} = e^{-3A}\varphi$ and corresponding metric $\widehat{ds^2}(M_7) = e^{-2A}ds^2(M_7)$. The rescaled torsion classes are given by

$$\hat{\tau}_0 = e^A\tau_0 = -\frac{12}{7}m, \quad \hat{\tau}_1 = \tau_1 - dA = -dA, \quad \hat{\tau}_2 = e^{-A}\tau_2 = 0, \quad \hat{\tau}_3 = e^{-2A}\tau_3. \quad (5.11)$$

Using these, the Bianchi identities read

$$d\left(\hat{\tau}_3 - \frac{1}{6}\hat{\tau}_0\hat{\varphi} - \hat{\tau}_1 \lrcorner \hat{\star}_7 \hat{\varphi}\right) = 0. \quad (5.12)$$

Thus, the problem of finding a solution to the equations of motion is reduced to this purely geometric condition. Note that (up to constant prefactors), the same condition appears for heterotic backgrounds on G_2 -structure spaces [16, eq. (2.13)].

5.1.1 Examples

Let us now give several examples of solutions to the Bianchi identities, which have been reduced to the constraint (5.12).

First, we consider $M_7 = S^3 \times M_4$, with standard G_2 -structure, trivial warp factor $A = 0$, and where M_4 is any hyper-Kähler manifold [17]. This recovers the near-horizon limit of D1- and D5-branes, with $\mathcal{N} = (4, 4)$ supersymmetry [18]. Let $(\sigma^1, \sigma^2, \sigma^3)$ be the left-invariant one-forms on S^3 satisfying $d\sigma^j = \frac{1}{2}\epsilon^{jkl}\sigma^k \wedge \sigma^l$ and $(\omega^1, \omega^2, \omega^3)$ be the hyper-Kähler structure on M_4 satisfying

$$d\omega^j = 0, \quad \frac{1}{2}\omega_i \wedge \omega_j = \delta_{ij}\text{vol}(M_4). \quad (5.13)$$

Then the G_2 -structure

$$\begin{aligned} 8m^3\hat{\varphi} &= -\text{vol}(S^3) - \frac{1}{2\sqrt{2}}\delta_{ij}\omega^i \wedge \sigma^j \\ 16m^4\hat{\star}_7\hat{\varphi} &= \text{vol}(M_4) + \frac{1}{8\sqrt{2}}\epsilon_{jkl}\omega^j \wedge \sigma^k \wedge \sigma^l \end{aligned} \quad (5.14)$$

has

$$\hat{\tau}_0 = -\frac{12}{7}m, \quad 4m^2\hat{\tau}_3 = -\frac{6}{7}\text{vol}(S^3) + \frac{1}{14\sqrt{2}}\omega^j \wedge \sigma^j, \quad (5.15)$$

and $\hat{\tau}_1 = \hat{\tau}_2 = 0$, which satisfy (5.12).

The next two examples are solutions in the presence of spacetime-filling O5-plane and D5-brane sources, which wrap calibrated three-cycles inside M_7 . The presence of these lead to a source term in the Bianchi identity, $dF_3 = J_4$. This thus modifies the right-hand side of (5.12) such that the sourced Bianchi identities instead reduce to

$$d\left(\hat{\tau}_3 - \frac{1}{6}\hat{\tau}_0\hat{\varphi} - \hat{\tau}_1\lrcorner\hat{\star}_7\hat{\varphi}\right) = J_4. \quad (5.16)$$

The first sourced example is given by the twisted toroidal orbifold $M_7 = T^7/(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2)$: we refer the reader to [19] which we follow closely, as well as [20] for details. Given a set of coordinates y^m on M_7 , we may introduce a twisted frame $\{e^m(y)\}$. In terms of this frame, the three-form determining the G_2 -structure can then be defined as

$$\begin{aligned} \varphi &= e^{127} - e^{347} - e^{567} + e^{136} - e^{235} + e^{145} + e^{246} \\ \star\varphi &= e^{3456} - e^{1256} - e^{1234} + e^{2457} - e^{1467} + e^{2367} + e^{1357}. \end{aligned} \quad (5.17)$$

Generically, the frame satisfies

$$de^m = \frac{1}{2}\tau_{np}^m e^n \wedge e^p. \quad (5.18)$$

This twisting breaks the G_2 -holonomy of the toroidal orbifold by introducing non-vanishing torsion classes τ_0, τ_3 , such that φ is co-closed, but no longer closed. The representation of the \mathbb{Z}_2 -involutions on the coordinates y^m of M_7 , as well as the consistency constraint

$d^2 e^m = 0$, restrict the possible values τ_{np}^m can take. We will restrict our attention to τ_{np}^m being the structure constants of $\text{SO}(p, q) \times \text{U}(1)$ with $p + q = 4$: this comes down to setting

$$\begin{pmatrix} \tau_{45}^1 & \tau_{46}^2 & \tau_{36}^1 & -\tau_{35}^2 \\ \tau_{25}^3 & -\tau_{16}^3 & -\tau_{26}^4 & -\tau_{15}^4 \\ \tau_{24}^6 & \tau_{14}^5 & -\tau_{23}^5 & \tau_{13}^6 \end{pmatrix} = \begin{pmatrix} a_4 \frac{a_1}{a_2} & a_5 \frac{a_1}{a_3} & -a_6 \frac{a_1}{a_2} & a_1 \\ a_4 & -a_5 \frac{a_2}{a_3} & a_6 & a_2 \\ -a_4 \frac{a_3}{a_2} & a_5 & -a_6 \frac{a_3}{a_2} & a_3 \end{pmatrix}, \quad (5.19)$$

with a_i constant and all other τ_{np}^m vanishing. Neither τ_0 nor τ_3 vanishes generically, with

$$\tau_0 = -\frac{2}{7} \left(-a_1 - a_2 + a_3 + a_4 + a_5 - a_6 + \frac{a_1 a_4}{a_2} - \frac{a_1 a_6}{a_2} - \frac{a_3 a_6}{a_2} + \frac{a_1 a_5}{a_3} - \frac{a_2 a_4}{a_3} + \frac{a_2 a_5}{a_3} \right). \quad (5.20)$$

As discussed in [20], setting $A = 0$ leads to solutions with source term J_4 given by

$$J_4 = k_m(a_i) \psi^m, \quad (5.21)$$

with $\psi^m \in \{e^{3456}, e^{1256}, e^{1234}, e^{2457}, e^{1467}, e^{2367}, e^{1357}\}$ and $k_m(a_i)$ dependent on the twisting parameters. Note that the AdS_3 radius is proportional to the torsion class $\tau_0(a_i)$, and hence the twisting parameters a_i are restricted such that $\tau_0 \neq 0$.

The second example of a sourced solution can be obtained by taking $M_7 = H(3, 1)$, the generalized Heisenberg group, as discussed in [21] and recently investigated in the context of three-dimensional heterotic Minkowski backgrounds [22]. Geometrically, $H(3, 1)$ is a nilmanifold, for which a frame can be found satisfying

$$de^m = \begin{cases} 0 & m \neq 7 \\ ae^{12} + be^{34} + ce^{56} & m = 7 \end{cases}, \quad (5.22)$$

with a, b, c non-zero parameters. Again expressing the three-form φ in terms of the frame $\{e^m\}$ as in (5.17), it follows that $\tau_1 = \tau_2 = 0$ and

$$\tau_0 = \frac{2}{7}(a + b + c), \quad (5.23)$$

which restricts the parameters to satisfy $a + b + c \neq 0$ in order to find AdS_3 backgrounds.⁴ Setting $A = 0$, one finds (5.16) is satisfied with calibrated source term J_4 given by

$$J_4 = k_1 e^{1234} + k_2 e^{3456} + k_3 e^{1256} \quad (5.24)$$

with $k_1 = (a + b)^2 + (ab + bc + ca)$ and (a, b, c) cyclically permuted for k_2, k_3 .

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⁴Compared to (5.17), we reparametrize $e^4 \rightarrow -e^4$, $e^5 \leftrightarrow e^6$ to remain consistent with [21].

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