

A Framework for Reasoning about Dynamic Axioms in Description Logics

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Bartosz Bednarczyk¹ Stéphane Demri² Alessio Mansutti²

¹Computational Logic Group, TU Dresden & Institute of Computer Science, University of Wrocław

²LSV, CNRS, ENS Paris-Saclay, Université Paris-Saclay

Knowledge representation in Description Logics

ALC concepts: $C, C' ::= \top \mid A \mid \neg C \mid C \sqcap C' \mid \exists r.C$

General concept inclusion (GCI)

$$\mathcal{T}: C_1 \sqsubseteq C_2$$

Assertions

$$\mathcal{A}: C(a), r(a, b)$$

Ontology
(\mathcal{T}, \mathcal{A})

De

update

Incorporating updates in DLs is mainly done by updating ABoxes

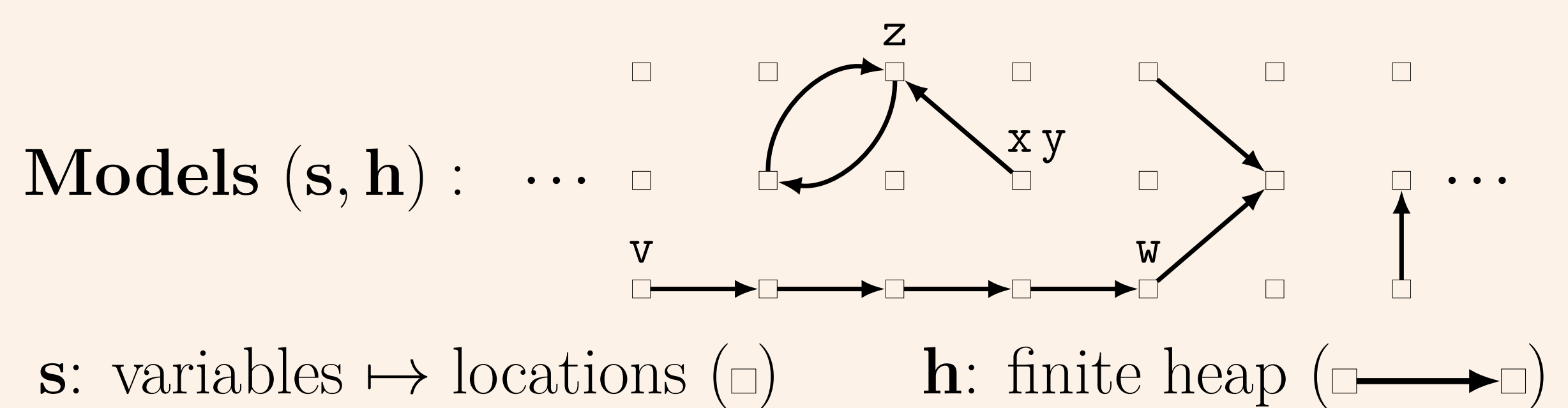
- Liu et al. Updating Description Logic ABoxes, KR'06
- De Giacomo et al. On the update of DL ontologies at the instance level, AAAI'06
- Drescher et al. Putting ABox updates into action, FroCoS'09

What if interpretations change?

How to specify the evolution of the satisfaction of GCIs or assertions when interpretations change?

Reasoning on Programs with Separation Logics

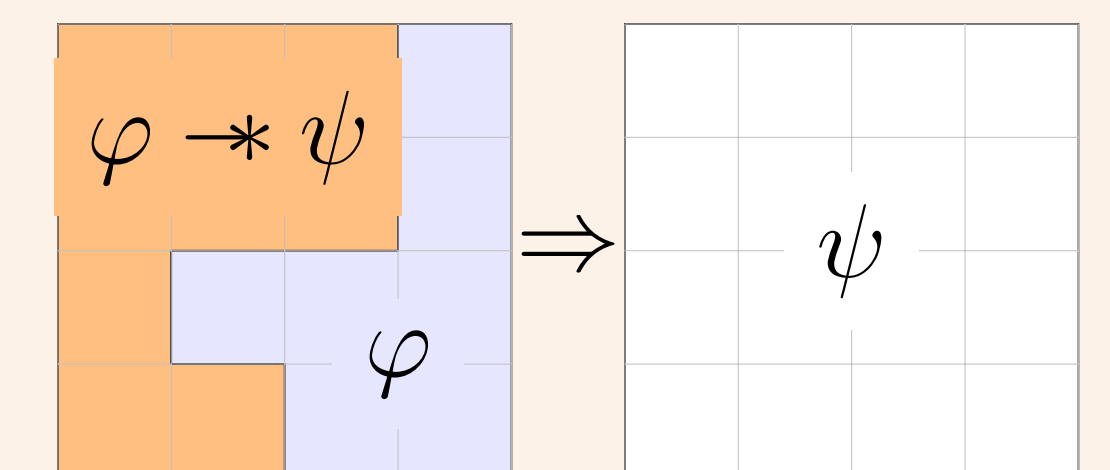
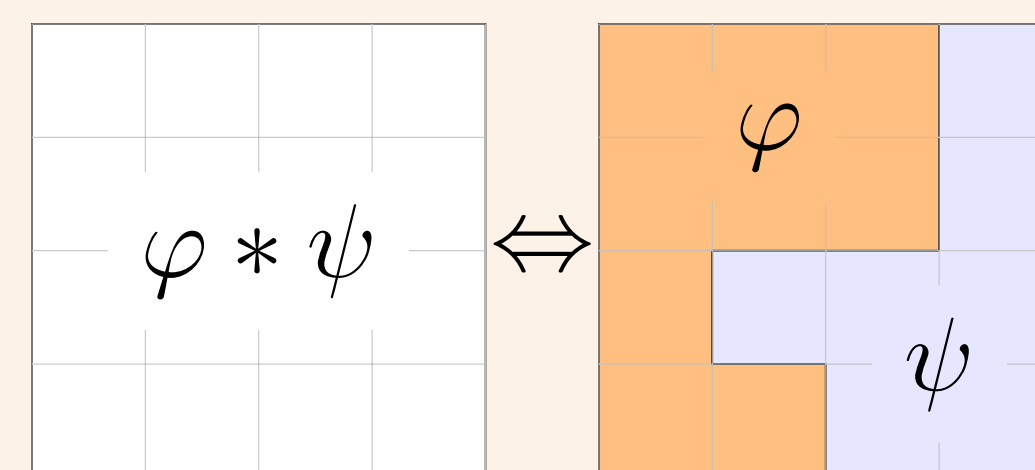
Separation logic [J. Reynolds, P. O'Hearn et al.] reasons about programs with dynamic data structures.



Se

se

$*$ and \ast to reason modularly about the memory



$(s, h) \models \varphi * \psi$ iff h can be split into h_1 and h_2 so that $(s, h_1) \models \varphi$ and $(s, h_2) \models \psi$

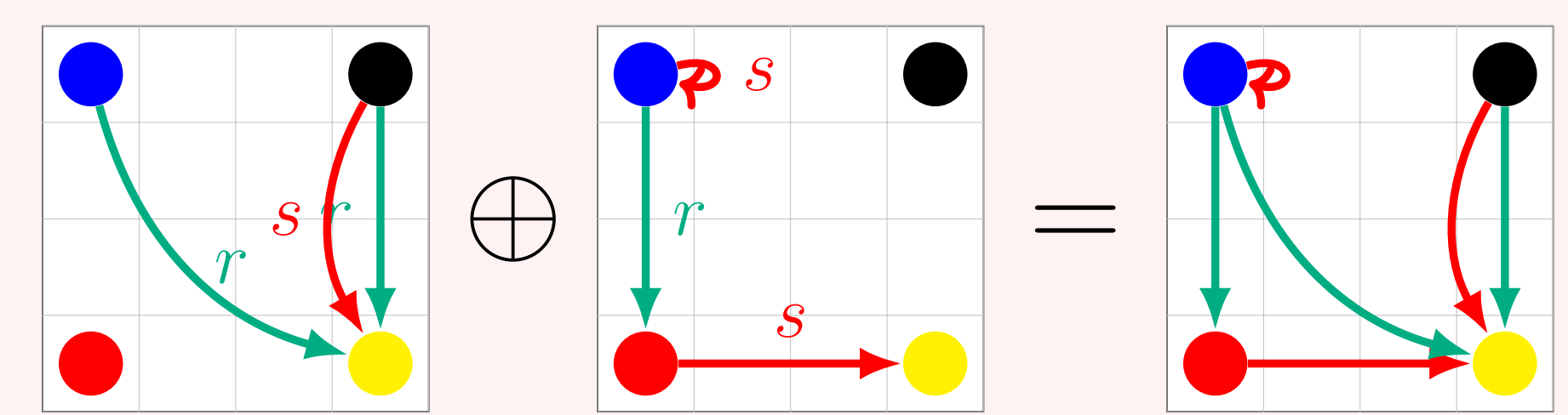
$(s, h) \models \varphi \ast \psi$ iff for every h_1 disjoint from h , if $(s, h_1) \models \varphi$ then $(s, h + h_1) \models \psi$

Our Framework: Description Logics enriched with Dynamic Axioms

We enrich the class \mathbb{I} of interpretations with a **composition operator**

$\oplus : \mathbb{I} \times \mathbb{I} \rightarrow \mathbb{I}$ that is *partial*, associative and commutative.

In the paper, \oplus performs disjoint (de)composition of roles.



Our framework instantiates the bunched logic introduced in [P. O'Hearn, D. Pym. The logic of bunched implications, BSL'99].

Reasoning about update

ontology with connective

se

Dynamic Axioms (DA): $\mathcal{U}, \mathcal{V} ::= \top \mid C(a) \mid r(a, b) \mid C \sqsubseteq D \mid \mathcal{U} * \mathcal{V} \mid \mathcal{U} \oplus \mathcal{V} \mid \neg \mathcal{U} \mid \mathcal{U} \sqcap \mathcal{V}$

standard assertions and GCIs

Boolean operations on dynamic axioms

• $\mathcal{I} \models \mathcal{U}_1 * \mathcal{U}_2$ iff there are $\mathcal{I}_1, \mathcal{I}_2$ such that $\mathcal{I} = \mathcal{I}_1 \oplus \mathcal{I}_2$, $\mathcal{I}_1 \models \mathcal{U}_1$ and $\mathcal{I}_2 \models \mathcal{U}_2$

• $\mathcal{I} \models \mathcal{U}_1 \oplus \mathcal{U}_2$ iff there is \mathcal{J} such that $\mathcal{I} \oplus \mathcal{J}$ is defined, $\mathcal{J} \models \mathcal{U}_1$ and $\mathcal{I} \oplus \mathcal{J} \models \mathcal{U}_2$ (\oplus is a kind of dual of \ast)

Enriching ontology

dynamic axioms

– Ontologies $(\mathcal{T}, \mathcal{A}, \mathcal{D})$ have now a third component, a finite set of DAs.

⚠ Ontologies with *positive* (i.e. negation-free) DAs for \mathcal{EL} can be inconsistent, e.g. $r(a, b) \sqcap (r(a, b) \oplus \top)$.

The Consistency Problem with Dynamic Axioms: Main Results

Logic \ Dynamic axioms	Positive DAs	DAs
\mathcal{EL} ($C ::= \top \mid A \mid C \sqcap C \mid \exists r.C$)	PTime (new proof system)	Undecidable (reduction from \mathcal{ALC} + role inclusions $r_1 \circ \dots \circ r_n \sqsubseteq s$)
\mathcal{ALC}	EXPTIME-complete (translation into \mathcal{ALCOB})	