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1	Tracer transport within abyssal mixing layers
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ABSTRACT

Mixing layers near sloped topography in the abyss are thought to play a 12 critical role in the global overturning circulation. Yet the behavior of passive 13 tracers within sloping boundary layer systems has received little attention, 14 despite the extensive use of tracer observations to understand abyssal circu-15 lation. Here, we investigate the behavior of a passive tracer released near a 16 sloping boundary within a flow governed by one-dimensional boundary layer 17 theory. The spreading rate of the tracer across isopycnals is influenced by a 18 number of factors including the bottom-intensification of mixing, the dipole 19 of upwelling (in the boundary layer) and downwelling (in the outer mixing 20 layer) and along-isopycnal diffusion. For isolated near-boundary tracer re-21 leases the bulk diffusivity, proportional to the rate of increase of the variance 22 of the tracer distribution in buoyancy space, is much less than what would be 23 expected from averaging the diapycnal diffusivity over the tracer patch. This 24 stems from the presence of the bottom boundary that prevents tracer diffu-25 sion through it. Furthermore, when along-isopycnal diffusion is weak, the 26 boundary tends to drive the tracer up the slope toward less dense fluid on av-27 erage due to asymmetries between boundary layer and interior flows. With 28 strong along-isopycnal diffusion this upslope movement is reduced, while at 29 the same time the average diapycnal spreading rate is increased due to a re-30 duced influence of the bottom boundary. These results have implications for 31 what can be learned about the characteristics of mixing near sloping bound-32 aries from past and future tracer-release experiments. 33

34 1. Introduction

The transport of tracers within the ocean plays an important role not only in ocean dynamics, thermodynamics and biogeochemistry, but also as a method with which to observe the ocean, and infer circulation properties. Because, by its nature, tracer transport integrates over both spatial and temporal scales, it allows us to measure the large-scale, integral impact of a range of smaller scale processes that are difficult to observe directly. However, in order to correctly interpret and use tracer measurements, a good understanding of tracer transport and its relation to circulation properties is needed.

Tracer transport has been particularly useful for understanding the dynamics of the ocean's deep 42 overturning circulation, where small-scale turbulent mixing plays a key role (Watson and Ledwell 43 2000). Since the pioneering study of Munk (1966) and the realization that mixing in the interior 44 is generally weak (e.g. Ledwell et al. 1993), mixing near the ocean's boundaries has been thought 45 to play a critical role in closing the global diapycnal circulation (Armi 1978; Ivey 1987b; Wunsch 46 1970; Phillips 1970; Thorpe 1987; Garrett 1991). Observations suggest that turbulent mixing 47 is bottom intensified due to internal wave breaking within the stratified fluid above the boundary 48 layer (Toole et al. 1994; Polzin et al. 1997; Ledwell et al. 2000; St. Laurent et al. 2012; Waterhouse 49 et al. 2014). The bottom-intensification of mixing within so called "abyssal mixing layers" implies 50 a downward mass transport across isopycnals in what some authors have termed the Stratified 51 Mixing Layer (SML, McDougall and Ferrari 2017). In order to balance the formation of dense 52 waters at high-latitudes there must therefore be a somewhat larger upwelling transport within 53 thin bottom boundary layers (BBLs) along the sloping seafloor where the turbulent buoyancy flux 54 converges (de Lavergne et al. 2016, 2017; Ferrari et al. 2016; McDougall and Ferrari 2017). The 55 requirement for net upwelling and the near-compensation between the net diapycnal transports 56

in the BBL and SML imply a complex balance between factors such as topographic geometry 57 (McDougall and Ferrari 2017; Holmes et al. 2018), variations in stratification (Ferrari et al. 2016; 58 Callies and Ferrari 2018; Banyte et al. 2018) and the lateral structure in the intensity of turbulent 59 mixing near the ocean floor (Kunze 2017a,b). How this balance is achieved at both global and 60 regional scales remains an open question that observations of tracer behavior may help to answer. 61 Field tracer release experiments (TREs) have provided many insights into the dynamics of di-62 apycnal ocean circulation. TREs have shown that mixing away from boundaries is weak (Ledwell 63 et al. 1993; Ledwell 1998), and highlighted the importance of strong boundary mixing for closing 64 basin-scale budgets (Goudsmit et al. 1997; Inall 2009; Ledwell and Bratkovich 1995). The Brazil 65 Basin TRE (BBTRE, Ledwell et al. 2000) in particular demonstrated the presence of intensified 66 mixing in the abyss above the rough seafloor of the Mid-Atlantic Ridge, corroborating microstruc-67 ture measurements (Polzin et al. 1997). The BBTRE tracer was released well above the ocean 68 bottom, and its centroid showed a tendency to descend across isopycnals in the eastern basin con-69 sistent with the expected diapycnal downwelling in the SML. However, the observations close to 70 the boundary suggested that in ridge canyons the tracer moves upslope, likely toward less dense 71 waters. These observations pointed to the importance of the near boundary region, though BBTRE 72 was not designed to study this region. 73

The importance of intense boundary mixing was further highlighted by the Diapycnal and Isopycnal Mixing Experiment in the Southern Ocean (DIMES, Ledwell et al. 2011; Watson et al. 2013). DIMES also exposed a common discrepancy whereby diffusivities inferred from tracer measurements often exceed those estimated from microstructure surveys. These discrepancies are often attributed to sampling issues (e.g. Wüest et al. 1996; Voet et al. 2015) and can be reconciled if the full temporal and spatial distribution of the tracer is taken into account (Mashayek et al. 2017). Studies such as Mashayek et al. (2017) and Ledwell et al. (2000) highlight the importance of tracer-boundary interactions, but do not discuss the details of tracer transport within any particular
boundary-driven flow. Idealized studies have examined near-boundary tracer dispersion above a
horizontal boundary (e.g. Saffman 1962; Csanady 1969). Here we examine a similar problem near
a sloping boundary.

Our tracer study will be conducted in the context of one-dimensional boundary layer theory (e.g. 85 Wunsch 1970; Phillips et al. 1986; Thorpe 1987; Garrett 1990). The theory considers the one-86 dimensional problem of flow over a uniform slope driven by an isotropic diffusivity. To satisfy 87 the no-flux boundary condition, isopycnals slope down as they approach the boundary, leading to 88 a buoyancy-driven upslope flow in a weakly-stratified BBL where friction is important (Garrett 89 et al. 1993). When the diffusivity is bottom-intensified a corresponding downslope flow appears 90 in the outer portion of the abyssal mixing layer (the SML). Recently, Callies (2018) has shown that 91 for realistic abyssal ocean parameters boundary layer theory predicts a much weaker stratification 92 than typically observed. He suggests that restratification by submesoscale eddies, generated by 93 baroclinic instability of the resulting flow field (Wenegrat et al. 2018), is necessary to maintain the 94 stratification and therefore permit significant near-boundary water-mass transformation. 95

Eddies, along with tides, intrusions and other processes, can also drive strong along-isopycnal tracer transports (e.g. Ivey 1987a; McPhee-Shaw 2006; Wain and Rehmann 2010; Winters 2015; Dell and Pratt 2015). This along-isopycnal exchange (in this article captured at first-order by an along-isopycnal tracer diffusivity) provides one means in which the strong boundary mixing can communicate with the interior.

In this article we will aim to address the following questions; 1) what processes within abyssal mixing layers are most important for determining tracer transport? 2) Will a tracer be transported toward lighter (upslope) or heavier (downslope) density classes, on average, when released near

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a sloping boundary? 3) What can be inferred about the properties of mixing and circulation from
 bulk measurements of the tracer cloud dispersion?

To approach these questions we analyze the behavior of a passive tracer released within a flow 106 governed by one-dimensional boundary layer theory (described in Section 2). The tracer evolves 107 in two dimensions due to the effects of diapycnal diffusion, isopycnal diffusion and advection 108 (Section 3). We introduce an analytical framework based around the tracer moments in buoyancy 109 space (the tracer center-of-mass and variance) to understand the contribution of different processes 110 to tracer dispersion (Sections 4-6). We find that the presence of the boundary can slow the rate 111 of diapycnal tracer dispersion below that expected from averaging the diapycnal diffusivity over 112 the tracer patch. Due to asymmetries between the upslope BBL and downslope SML flows (the 113 BBL upwelling being strong and narrow compared to the more diffuse SML downwelling), the 114 tracer tends to move upslope on average. The extent of upslope movement and tracer dispersion 115 depends on the initial release position of the tracer, as well as a number of other factors such as the 116 decay scale of bottom-intensified mixing and along-isopycnal diffusion (Section 7). Our results 117 have implications not only for our understanding of tracer behavior near the ocean floor (Section 118 9), but also for what can be learned about ocean dynamics from field TREs (Section 8). 119

2. Boundary layer theory

The idealized two-dimensional tracer release experiments will be performed within a flow governed by one-dimensional boundary layer theory. In this section we briefly review the key results required for our tracer study. For a more detailed derivation the reader is referred to Garrett (1990, 1991) and Callies (2018). We consider steady flow above a bottom with uniform slope tan θ and use a coordinate system aligned with the bottom (i.e. *z* is bottom-normal with origin at the boundary, *y* is upslope and *x* is along-slope, Fig. 1). Everything is uniform in the upslope and along-slope

directions except pressure and buoyancy (and in the next section the tracer). Buoyancy is charac-terized by a constant far-field vertical stratification N^2 ,

$$\frac{\partial b}{\partial z} \to N^2 \cos \theta, \quad \text{as} \quad z \to \infty \tag{1}$$

$$\frac{\partial b}{\partial y} = N^2 \sin \theta$$
, all z, (2)

where b is the buoyancy field relative to a reference density ρ_0 . The steady, rotating equations of motion for this system are (e.g. Garrett et al. 1993),

$$-fV\cos\theta = \frac{d}{dz}\left(v_u\frac{dU}{dz}\right),\tag{3}$$

$$fU\cos\theta = -\frac{1}{\rho_0}\frac{\partial P}{\partial y} + b\sin\theta + \frac{d}{dz}\left(v_v\frac{dV}{dz}\right),\tag{4}$$

$$0 = -\frac{1}{\rho_0} \frac{\partial P}{\partial z} + b\cos\theta, \tag{5}$$

$$N^{2}V\sin\theta = \frac{d}{dz}\left(\kappa\frac{\partial b}{\partial z}\right),\tag{6}$$

where U(z), V(z) are the along-slope and upslope velocities, $v_u(z), v_v(z)$ are along-slope and ups-lope eddy viscosities (which may differ for reasons discussed shortly), $\kappa(z)$ is an eddy diffusivity, P(y,z) is the pressure field and f is the (vertical) Coriolis parameter. These equations can be combined into (e.g. Garrett 1991),

$$\frac{d^2}{dz^2} \left(v_v \frac{d^2 \Psi}{dz^2} \right) + \left(\frac{f^2 \cos^2 \theta}{v_u} + \frac{N^2 \sin^2 \theta}{\kappa} \right) \Psi = N^2 \sin \theta \cos \theta + \frac{f^2 \cos^2 \theta}{v_u} \kappa_{\infty} \cot \theta, \quad (7)$$

where κ_{∞} is the far-field diffusivity and the scalar streamfunction $\Psi(z)$ is given by $\frac{d\Psi}{dz} = V$ with $\Psi(0) = 0$ and,

$$\Psi \to \kappa_{\infty} \cot \theta$$
, as $z \to \infty$. (8)

For constant diffusion, $v_u(z) = v_{u0}$, $v_v(z) = v_{v0}$, $\kappa(z) = \kappa_0$ (= κ_{∞}), the solution with no-slip boundary conditions is characterized by a BBL of reduced stratification and upslope flow (Wunsch 1970; Thorpe 1987, dashed black lines in Fig. 2). There is an along-slope flow in the interior in the

direction opposite to Kelvin wave propagation, or upwelling-favorable in a bottom-Ekman sense (Garrett et al. 1993). The BBL width is $\mathscr{O}(q_0^{-1})$ where,

$$q_0^4 = \frac{N^2 \sin^2 \theta}{4P r_{\nu 0} \kappa_0^2} \left(1 + (SP r_{u 0})^{-1} \right), \tag{9}$$

¹⁵³ $S^{-1} = f^2 \cos^2 \theta / N^2 \sin^2 \theta$ is the inverse slope Burger number and $Pr_{u0} = v_{u0} / \kappa_0$, $Pr_{v0} = v_{v0} / \kappa_0$ ¹⁵⁴ are Prandtl numbers. For typical abyssal parameters of $N^2 = 10^{-6} \text{s}^{-2}$, $f = 10^{-4} \text{s}^{-1}$, $v_{u0} = v_{v0} =$ ¹⁵⁵ $\kappa_0 = 10^{-3} \text{m}^2 \text{s}^{-1}$, $\tan \theta = 1/400$ then $S^{-1} = 1600$ is large and the BBL thickness corresponds to ¹⁵⁶ an Ekman layer thickness of $q_0^{-1} = 4.5\text{m}$.

¹⁵⁷ In this article we will instead focus on the case where the diffusivity is bottom intensified with ¹⁵⁸ form,

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$$\kappa = \kappa_{\infty} + (\kappa_0 - \kappa_{\infty})e^{-z/d}, \qquad (10)$$

where κ_0 is the diffusivity near the boundary and *d* is a decay scale. For the parameter space of interest here the BBL thickness q_0^{-1} is much smaller than the diffusivity decay scale *d*. Following Callies (2018), for $q_0 d >> 1$ we can construct an approximate analytic solution for the bottomintensified case by patching together a solution in the BBL, where the mixing coefficients can be assumed to be constant and equal to their near-boundary values v_{u0} , v_{v0} and κ_0 , to a solution in the interior, where the influence of friction through the fourth derivative term in Eq. (7) can be neglected. This procedure is presented in Appendix A.

¹⁶⁷ However, as shown by Callies (2018) (see Eqs. (68) and (71) in Appendix A), for the typi-¹⁶⁸ cal large inverse slope Burger number S^{-1} and order one Prandtl number regime in the abyssal ¹⁶⁹ ocean such a solution predicts very weak stratification over the abyssal mixing layer. Callies ¹⁷⁰ (2018) attributes this weak stratification to the lack of representation of baroclinic instability and ¹⁷¹ its associated eddy-driven restratification in the one-dimensional system. A realistically stratified ¹⁷² one-dimensional solution requires small values of the parameter $(SPr_{u0})^{-1}$ (see Appendix A for

more details) and can therefore be recovered using a large Pr_{u0} . Such a choice of large vertical 173 momentum mixing in the along-slope momentum equation can be physically interpreted as a pa-174 rameterization for restratification by baroclinic eddies based on the thickness-weighted average 175 formalism (Rhines and Young 1982; Greatbatch and Lamb 1990; Gent and McWilliams 1990; 176 Gent et al. 1995). For simplicity, and to avoid the need to resolve eddies or include an explicit 177 Gent and McWilliams (1990) type parameterization, throughout most of this article we will con-178 sider the limit $(SPr_{u0})^{-1} \rightarrow 0$ obtained for large Pr_{u0} . This choice assumes that eddies maintain 179 the stratification in the SML at its far-field value¹. In Section 7c we briefly discuss the impact of 180 reduced stratification in the SML through a non-zero $(SPr_{u0})^{-1}$. 181

¹⁸² Note that the representation of eddy-driven restratification through enhanced vertical momentum ¹⁸³ mixing should have less impact on the BBL solution, which is determined by non-geostrophic fric-¹⁸⁴ tional turbulent boundary layer physics. The eddy-driven restratification and frictional boundary ¹⁸⁵ layer processes can be conveniently isolated (at least in our two-dimensional context) by enhancing ¹⁸⁶ only the along-slope viscosity. Thus we maintain the choice $Pr_{\nu 0} = 1$ in the upslope momentum ¹⁸⁷ equation.

With these parameter choices, the full one-dimensional solution is given by (taking the limit $(SPr_{u0})^{-1} \rightarrow 0$ in Eqs. (68) and (71) in Appendix A),

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$$\Psi = \kappa \cot \theta \left(1 - e^{-q_0 z} \left(\cos q_0 z + \sin q_0 z \right) \right), \tag{11}$$

$$\frac{\partial b}{\partial z} = N^2 \cos \theta \left(1 - e^{-q_0 z} \left(\cos q_0 z + \sin q_0 z \right) \right).$$
(12)

¹⁹³ As for a constant κ solution there is a BBL of thickness $\sim q_0^{-1}$ with weak stratification and upslope ¹⁹⁴ flow (solid lines with circles in Fig. 2)². Outside the BBL the vertical stratification is equal to the

above.

¹This parameter choice is equivalent to using non-rotating boundary layer theory due to the parameter dependence $(SPr_{u0})^{-1} \sim f^2/v_{u0}$.

²Note that due to the choice $(SPr_{u0})^{-1} \rightarrow 0$ the BBL thickness is modestly increased to $q_0^{-1} \sim 28$ m for the typical abyssal parameters considered

¹⁹⁵ far-field value N^2 and there is a weak downslope flow which largely compensates the upwelling ¹⁹⁶ within the BBL (the imbalance between SML and BBL transports is governed by Eq. (8)). These ¹⁹⁷ compensating upwelling and downwelling flows are equivalent to the water-mass transformation ¹⁹⁸ dipole discussed recently by a number of authors (e.g. de Lavergne et al. 2016; Ferrari et al. 2016; ¹⁹⁹ McDougall and Ferrari 2017). It is this flow that will be used to advect and diffuse a passive tracer ²⁰⁰ in a two-dimensional *y*-*z* plane, as discussed in the next section.

3. Two-dimensional tracer dispersion in a one-dimensional slope flow

²⁰² a. The tracer conservation equation

We set up a two-dimensional tracer advection-diffusion problem in the one-dimensional boundary layer flow discussed in the previous section. That is, we look for the distribution of a tracer $\mathcal{C}(y,z,t)$ (where \mathcal{C} has concentration units of tracer m⁻²) which varies in the upslope and slopenormal directions (Fig. 1) given an initial distribution $\mathcal{C}(y,z,0)$ and a tracer conservation equation,

$$\frac{\partial \mathcal{C}}{\partial t} = -\nabla \cdot \boldsymbol{F}_{\mathcal{C}},\tag{13}$$

²⁰⁸ where the tracer flux,

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$$F_{\mathcal{C}} = V\mathcal{C} - \kappa \nabla \mathcal{C} - A_H K_I \cdot \nabla \mathcal{C}.$$
⁽¹⁴⁾

 F_{C} has three-components; an advective flux associated with the velocity V = (V(z), 0) from boundary layer theory (V(z) is given by the *z* derivative of Eq. (11)) and diffusive fluxes associated with a small-scale isotropic diffusivity $\kappa(z)$ and an along-isopycnal diffusivity A_{H} . The along-isopycnal diffusion is specified using the symmetric second rank tensor (Redi 1982),

$$K_{I} = \frac{1}{|\nabla b|^{2}} \begin{bmatrix} b_{z}^{2} & -b_{y}b_{z} \\ -b_{y}b_{z} & b_{y}^{2} \end{bmatrix},$$
(15)

where the subscript on *b* indicates differentiation. The coefficients of K_I are determined using the buoyancy gradients $b_z = B_z + b'_z = N^2 \cos \theta + b'_z$ and $b_y = B_y = N^2 \sin \theta$ split into background (B_z and B_y) and perturbation components where from Eq. (12),

$$b'_{z} = -N^{2}\cos\theta e^{-q_{0}z}(\cos q_{0}z + \sin q_{0}z).$$
(16)

This formulation of the tracer equation is consistent with boundary layer theory (i.e. if *b* replaces \mathcal{C} then Eq. (13) reduces to Eq. (6)).

b. Numerical model setup

The tracer conservation Eq. (13) cannot be solved analytically in the general case (although 222 we will consider simplified cases that can be solved analytically) and so we resort to numerical 223 simulations. We use the spectral code Dedalus (Burns et al. 2019, http://dedalus-project.org/) in 224 the rotated y, z coordinates (Fig. 1) with periodic boundary conditions in y. We consider a control 225 parameter set with a y-domain length $L_y = 1500$ km, a slope of $\alpha = 1/400$ (roughly the western 226 side of the Mid-Atlantic Ridge) and a z-domain height of $L_z = 3000$ m. The far-field stratification 227 $N^2 = 10^{-6} \text{m}^2 \text{s}^{-1}$. The diffusivity will either be bottom-intensified with an exponential profile 228 [Eq. (10)] or constant, with base parameters d = 500m, $\kappa_0 = 10^{-3}$ m²s⁻¹ and $\kappa_{\infty} = 10^{-5}$ m²s⁻¹. 229 For those simulations with no along-isopycnal diffusion ($A_H = 0$) we use 384 Fourier modes in y 230 and 192 Chebyshev modes in z, corresponding to an average $\Delta y \sim 4$ km and an average $\Delta z = 15$ m. 231 The Chebyshev basis allows finer resolution of the small-scale z gradients in the BBL (see Fig. 232 2). Eq. (13) is solved implicitly in time using a time step of 8 days. However, because of small-233 scale variations in the along-isopycnal tensor coefficients [Eq. (15)] near the boundary, those 234 simulations with non-zero along-isopycnal diffusion were instead performed with 576 modes in 235 y and 768 modes in z and a time step of 4 days. The simulations match analytic results almost 236

perfectly where such results are obtainable (e.g. Section 5). The results are robust to numerical choices, as confirmed by doubling the number of modes in z and running with a time step four times smaller, which gave almost identical results (not shown).

4. Bulk diffusivity and tracer moments

The simplest and most common method to quantify the overall tracer dispersion rate in different coordinates is through the rate of increase of the variance of the tracer distribution. A Gaussian distribution of tracer spreading in one-dimension (*s*) due to a constant diffusivity κ evolves as,

$$\mathcal{C}(s,t) = A \frac{\sigma_0}{\sigma} e^{-(s-\mu)^2/2\sigma^2},\tag{17}$$

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$$\sigma^2(t) = \sigma_0^2 + 2\kappa t, \tag{18}$$

where *t* is time, *A* is a constant, σ^2 is the variance with initial value σ_0^2 and μ is the centroid or center-of-mass. The variance increases linearly with time at a rate of 2 κ . Thus for any tracer distribution, which may be spreading due to complex advection-diffusion processes in multiple dimensions, we can define an equivalent or *bulk diffusivity* in any dimension *s* in terms of the rate of increase of the variance σ_s (also see Wüest et al. 1996; Goudsmit et al. 1997),

$$\kappa_{bk}^s = \frac{1}{2} \frac{\partial \sigma_s^2}{\partial t}.$$
(19)

Here, by defining a domain averaging operator over the dimension s,

$$\langle * \rangle^s \equiv \int_{-\infty}^{\infty} * \, ds, \tag{20}$$

we can write both the variance σ_s and the center-of-mass μ_s in terms of the *s*-moments of the tracer distribution,

$$\mu_s \equiv \frac{\langle sC \rangle^s}{\langle C \rangle^s},\tag{21}$$

$$\sigma_s^2 \equiv \frac{\langle s^2 \mathcal{C} \rangle^s}{\langle \mathcal{C} \rangle^s} - \mu_s^2, \tag{22}$$

where $\langle sC \rangle^s$ and $\langle s^2C \rangle^s$ are the first and second moments, and $\langle C \rangle^s = A\sqrt{2\pi}\sigma_s$ is the zeroth moment that quantifies the (conserved) total amount of tracer. The moments provide a useful framework that can be used to understand aspects of the tracer dispersion analytically (e.g., Saffman 1962; Young et al. 1982). They will be used in later sections to derive expressions for the evolution of the tracer center-of-mass and bulk diffusivity in terms of the various advective and diffusive tracer fluxes.

²⁶⁶ A similar calculation can also be applied in buoyancy space, to obtain a *bulk diapycnal diffusivity* ²⁶⁷ κ_{bk}^{b} (or just κ_{bk}). In this case a mean stratification profile N^{2} , which in our two-dimensional context ²⁶⁸ will be the interior stratification which (in the limit $(SPr_{u0})^{-1} \rightarrow 0$) is independent of height and ²⁶⁹ time, is also required in order to convert the spreading rate into the units of a diffusivity. We define,

$$\kappa_{bk} \equiv \frac{1}{N^4} \frac{1}{2} \frac{\partial \sigma_b^2}{\partial t}$$
(23)

²⁷¹ where,

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$$\mu_b = \frac{\langle b\mathcal{C} \rangle}{\langle \mathcal{C} \rangle},\tag{24}$$

$$\sigma_b^2 = \frac{\langle b^2 \mathcal{C} \rangle}{\langle \mathcal{C} \rangle} - \mu_b^2, \tag{25}$$

and the averaging operation in our general two-dimensional context occurs over the full tracer distribution in the two spatial dimensions y and z,

$$\langle * \rangle \equiv \langle \langle * \rangle^{y} \rangle^{z} = \int_{0}^{\infty} \int_{-\infty}^{\infty} * dy dz.$$
(26)

²⁷⁹ The bulk diffusivity [Eq. (23)] will be used to characterize the overall rate of spreading of the ²⁸⁰ tracer across buoyancy surfaces. However, it should be emphasized that κ_{bk} should not necessarily ²⁸¹ be interpreted directly as a diffusivity, as such an interpretation masks the potentially complex ²⁸² processes that are leading to the spreading of that tracer distribution in the first place (here, two-²⁸³ dimensional advection and diffusion). Comparisons of κ_{bk} to the actual isotropic diffusivity κ will prove useful, as is commonly done for field TREs where microstructure measurements are also available (e.g. Ledwell et al. 2000; Watson et al. 2013; Mashayek et al. 2017). We will also compare κ_{bk} to the diffusivity calculated using a one-dimensional model commonly applied to field data (Ledwell and Watson 1991; Ledwell 1998, see Appendix B).

5. Boundary restriction: tracer behavior in *z*

We begin by examining the behavior of the tracer in the slope-normal coordinate *z*. Integrating the two-dimensional tracer conservation equation [Eq. (13)] across all *y* yields,

$$\frac{\partial \langle \mathcal{C} \rangle^{y}}{\partial t} = \frac{\partial}{\partial z} \left(K^{z} \frac{\partial \langle \mathcal{C} \rangle^{y}}{\partial z} \right), \tag{27}$$

where $K^z = \kappa + A_H b_y^2 / |\nabla b|^2$ and the boundary conditions are $K^z \frac{\partial \langle C \rangle^y}{\partial z} = 0$ at z = 0 and $\langle C \rangle^y \to 0$ as 292 $z \rightarrow \infty$. Thus in the slope-normal coordinate z the tracer diffuses according to a one-dimensional 293 diffusion equation that does not depend on the upslope velocity V. We are not aware of any 294 analytic solutions to Eq. (27) for the case where κ is exponential, A_H is non-zero and there is a 295 boundary at z = 0 (see Zamani and Bombardelli 2014, and references therein). However, when K^z 296 is a constant (where here for simplicity we set $K^{z} = \kappa$ with κ constant) a solution is easy to obtain 297 and, despite the simple governing equation, shows some interesting behavior as a consequence of 298 the boundary at z = 0. 299

300 *a. Constant diffusivity*

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³⁰¹ When κ is constant and $A_H = 0$ the tracer evolution for an initial Gaussian distribution with ³⁰² center-of-mass at $z = \mu_0$ and spread σ_0 (provided the initial tracer distribution is isolated from the boundary, $\sigma_0 \ll \mu_0$ is given by the sum of two Gaussians centered at $z = \mu_0$ and $z = -\mu_0$,

$$\langle \mathcal{C} \rangle^{y}(z,t) = A \frac{\sigma_0}{\sigma} \left(e^{-(z-\mu_0)^2/2\sigma^2} + e^{-(z+\mu_0)^2/2\sigma^2} \right),$$
 (28)

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where
$$\sigma^2(t) = \sigma_0^2 + 2\kappa t.$$
 (29)

³⁰⁷ Due to the symmetry about z = 0, this solution satisfies the no flux boundary condition at z = 0, ³⁰⁸ while the individual Gaussians satisfy the one-dimensional diffusion equation without a boundary. ³⁰⁹ This analytic solution matches the numerical two-dimensional solution averaged in *y* (shown at ³¹⁰ three different times by the orange curves in Fig. 3a).

The center-of-mass, variance and bulk diffusivity derived from the *z*-moments of this solution (defined for z > 0 using Eq. (26) for the domain average) are,

$$\mu_{z} = \frac{\langle z\mathcal{C} \rangle}{\langle \mathcal{C} \rangle} = \frac{2\sigma}{\sqrt{2\pi}} e^{-\mu_{0}^{2}/2\sigma^{2}} + \mu_{0} \operatorname{Erf}(\frac{\mu_{0}}{\sqrt{2\sigma^{2}}})$$
(30)

(31)

$$\sigma_{z}^{2}=rac{\langle z^{2}\mathcal{C}
angle}{\langle \mathcal{C}
angle}=\mu_{0}^{2}+\sigma^{2}-\mu_{z}^{2}$$

$$\kappa_{bk}^{z} = \frac{1}{2} \frac{\partial \sigma_{z}^{2}}{\partial t}$$

$$= \kappa \left(1 - \frac{2}{\pi} e^{-\mu_0^2/\sigma^2} - \frac{2\mu_0}{\sqrt{2\pi\sigma^2}} \operatorname{Erf}(\frac{\mu_0}{\sqrt{2\sigma^2}}) e^{-\mu_0^2/2\sigma^2} \right)$$
(32)

At early times, when $\mu_z >> \sigma$ the center-of-mass is stationary $\mu_z = \mu_0$, $\sigma_z^2 = \sigma^2$ and the bulk 318 diffusivity is equal to the actual diffusivity $\kappa_{bk}^z = \kappa$. However, as time increases, namely once 319 the tracer encounters the boundary such that $\sigma \sim \mu_z$, the center-of-mass moves away from the 320 boundary, eventually increasing like the square-root of time $\mu_z \sim \sqrt{t}$ (orange line in Fig. 3b). As 321 a result, the rate of increase of variance σ_z reduces (due to the μ_z^2 term in Eq. (31), orange line in 322 Fig. 3c), and the bulk diffusivity is reduced below κ (compare orange solid and dashed lines in 323 Fig. 3c). Hence the effect of the boundary is to reduce the bulk diffusivity by limiting downward 324 tracer spreading. In the limit of long time, 325

 $\kappa_{bk}^{z} \to (1 - \frac{2}{\pi})\kappa \approx 0.36\kappa, \quad \text{as} \quad t \to \infty,$ (33)

meaning that the bulk diffusivity is reduced by more than a factor of two. This is perhaps surprising, since it may be thought that the boundary prevents the spreading of half of the Gaussian tracer distribution. However, once the tracer encounters the boundary it no longer spreads like a half-Gaussian because its 'reflection spreads into the region z > 0, resulting in an effective accumulation of tracer near the boundary.

332 b. Bottom-intensified diffusivity

³³³ When instead the diffusivity is bottom intensified ($\kappa_{\infty} \neq \kappa_0$, with $A_H = 0$) the solution cannot be ³³⁴ obtained analytically. However, the influence of the boundary on the tracer diffusion can still be ³³⁵ understood by deriving equations for the evolution of the *z*-moments directly from the tracer con-³³⁶ servation equation (this technique will also be used in later sections to analyze the tracer behavior ³³⁷ in buoyancy coordinates). Multiplying Eq. (27) by *z* and integrating over the domain,

$$\langle \mathcal{C} \rangle \frac{\partial \mu_z}{\partial t} = \frac{\partial \langle z \mathcal{C} \rangle}{\partial t} = \langle z \frac{\partial}{\partial z} (\kappa \mathcal{C}_z) \rangle, \tag{34}$$

$$= \langle \frac{\partial}{\partial z} (z \kappa C_z) \rangle + \langle -\kappa C_z \rangle \tag{35}$$

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$$=\langle -\kappa \mathcal{C}_{z} \rangle, \tag{36}$$

where in the second line we have used the chain rule and the third line we have used the boundary condition at z = 0 to eliminate the first term. Eq. (36) simply states that the tracer center-of-mass will move according to the domain-averaged tracer flux. Furthermore, using the chain rule to move the *z*-derivative from C to κ and the boundary condition to rewrite $\langle \partial_z(\kappa C) \rangle$ in terms of the tracer concentration on the boundary yields,

$$\langle \mathcal{C} \rangle \frac{\partial \mu_z}{\partial t} = \kappa_0 \int_{-\infty}^{\infty} \mathcal{C}(y, 0, t) dy + \langle \kappa_z \mathcal{C} \rangle.$$
(37)

The first term in Eq. (37) exposes the "boundary effect" discussed above: the boundary prevents the tracer flux in the negative *z* direction (once tracer accumulates there) and thus the flux in the ³⁵⁰ positive *z* direction dominates, driving the center-of-mass away from the boundary. A *z*-dependent ³⁵¹ diffusivity can also drive net tracer movement through the second term in Eq. (37). For a bottom-³⁵² intensified diffusivity, this term enhances the downward tracer diffusion below the initial tracer ³⁵³ patch driving the center-of-mass toward the boundary (dotted blue line in Fig. 3b). However, this ³⁵⁴ downward motion is transient and the boundary effect dominates once enough tracer accumulates ³⁵⁵ on the boundary (solid blue line in Fig. 3b).

An equation for the second z tracer moment can be derived using a procedure analogous to that employed for Eq. (36),

$$\frac{\partial \langle z^2 \mathcal{C} \rangle}{\partial t} = 2 \langle -\kappa \mathcal{C}_z z \rangle.$$
(38)

³⁶⁰ The bulk diffusivity or rate of change of the variance is then,

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$$\kappa_{bk}^{z} = \frac{\langle -\kappa C_{z}(z - \mu_{z}) \rangle}{\langle C \rangle},\tag{39}$$

I.e. κ_{bk}^{z} is given by the domain-averaged tracer fluxes weighted by their distance from the centerof-mass $z - \mu_{z}$. Furthermore, by shifting the z-derivative from the C to κ , and absorbing the factor $(z - \mu_{z})$,

$$\kappa_{bk}^{z} = \frac{\langle \kappa \mathcal{C} \rangle}{\langle \mathcal{C} \rangle} - \frac{\mu_{z} \kappa_{0}}{\langle \mathcal{C} \rangle} \int_{-\infty}^{\infty} \mathcal{C}(y,0,t) dy + \frac{\langle (z-\mu_{z}) \mathcal{C} \kappa_{z} \rangle}{\langle \mathcal{C} \rangle}.$$
(40)

Eq. (40) clearly shows that the bulk diffusivity κ_{bk}^z is not what would be expected from simply averaging the isotropic diffusivity κ over the tracer patch (or the *in-situ diffusivity* $\langle \kappa C \rangle / \langle C \rangle$, dashed lines in Fig. 3c, also see Mashayek et al. 2017). Instead, the boundary significantly *reduces* this spreading rate, once tracer encounters the boundary (compare solid and dashed lines in Fig. 3c). This occurs because the boundary prevents tracer spreading in the negative *z* direction, and constricts the tracer to remain closer to its center-of-mass [Eq. (39)]. On the other hand, the vertical diffusivity gradient, when present, drives a modest enhancement of the bulk diffusivity through the last term in Eq. (40) (dotted blue line in Fig. 3c). It should be noted that the terms in Eq. (40) [and Eq. (37)] are not independent; each term affects the tracer distribution C on which they are all dependent.

6. Diapycnal spreading: tracer behavior in *b*

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Though the upslope velocity V does not influence the spreading of the tracer in the slope normal 379 direction z, it does affect the tracer spreading rate in the upslope direction y and across buoyancy 380 surfaces. The two-dimensional evolution of tracer patches released at $z_0 = d/2 = 250$ m in flows 381 with constant (Fig. 4a-c) and bottom-intensified (Fig. 4d-f) diffusivities clearly show the impacts 382 of the boundary layer flow. With a constant diffusivity there is only upslope flow within the BBL. 383 This flow drives tracer up the slope after which it diffuses vertically out of the BBL, forming 384 a characteristic upslope tracer tongue (Fig. 4a-c). The center-of-mass (with upslope position 385 $\mu_y = \langle y C \rangle / \langle C \rangle$) moves up the slope, at a rate determined by the tracer-weighted velocity, 386

$$\frac{\partial \mu_y}{\partial t} = \frac{\langle VC \rangle}{\langle C \rangle},\tag{41}$$

³⁸⁹ (obtained by multiplying Eq. (13) by *y* and integrating over the domain), and slightly away from ³⁹⁰ the boundary (μ_z increases, see closed circles in Fig. 4a-c). Thus the advective tracer transport in ³⁹¹ the BBL is significant despite its thinness. The expanding tracer patch continues to supply tracer to ³⁹² the BBL through downward diffusion, accounting for the large-amount of tracer advected upward ³⁹³ within the BBL.

When the diffusivity is instead bottom-intensified (Fig. 4d-f), the upslope spreading of tracer is reduced due to both the restriction of diffusion to the region near the boundary and the presence of downslope transport in the SML (compare closed circles in Figs. 4c and 4f). Due to the shear in the SML velocity, the expanding interior tracer patch acquires a tilt. The evolution of the tracer concentrations binned into buoyancy classes also indicates a strong upward net motion of the tracer distribution, in both cases (orange and blue distributions in Fig. 5a), reflected in the upward motion of the center-of-mass (solid lines in Fig. 5b). In the following subsections we develop a framework based on the tracer moments to evaluate the contributions of various advective and diffusive tracer fluxes to the diapycnal tracer spreading.

a. Advective and diffusive tracer fluxes and the buoyancy budget constraint

Following the same procedure used in Section 5 for the one-dimensional case, we obtain the following equations for the buoyancy-moments of the tracer,

$$\frac{\partial \langle b\mathcal{C} \rangle}{\partial t} = \langle F_{\mathcal{C}} \cdot \nabla b \rangle, \tag{42}$$

$$\frac{\partial \langle b^2 \mathcal{C} \rangle}{\partial t} = 2 \langle b \mathbf{F}_{\mathcal{C}} \cdot \nabla b \rangle, \tag{43}$$

(44)

$$\kappa_{bk} = rac{1}{N^2 \langle \mathcal{C}
angle} \langle (b-\mu_b) oldsymbol{F}_{\mathcal{C}} \cdot
abla b
angle,$$

where $\mu_b = \langle bC \rangle / \langle C \rangle$. Eq. (42) shows that the tracer center-of-mass moves across isopycnals if the domain-averaged diapycnal tracer flux is non-zero. The rate of change of the tracer variance in buoyancy space, proportional to κ_{bk} [Eq. (44)], is positive if the diapycnal fluxes move tracer on average away from the tracer center-of-mass μ_b . Noting that the along-isopycnal flux drops out yields contributions from diapycnal advection and isotropic diffusion,

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$$\frac{\partial \langle b\mathcal{C} \rangle}{\partial t} = \langle \mathcal{C} \boldsymbol{V} \cdot \nabla b \rangle - \langle \kappa \nabla \mathcal{C} \cdot \nabla b \rangle, \tag{45}$$

$$\kappa_{bk} = \frac{1}{N^4 \langle \mathcal{C} \rangle} \langle (b - \mu_b) \mathcal{C} \mathbf{V} \cdot \nabla b \rangle - \frac{1}{N^4 \langle \mathcal{C} \rangle} \langle (b - \mu_b) \kappa \nabla \mathcal{C} \cdot \nabla b \rangle.$$
(46)

Eqs. (45)-(46) can be further simplified by noting that from the buoyancy equation (vector form of Eq. (6)),

$$\boldsymbol{V} \cdot \nabla \boldsymbol{b} = -\nabla \cdot (-\kappa \nabla \boldsymbol{b}). \tag{47}$$

Multiplying by C, integrating over the domain and using the chain rule and boundary conditions,

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 $\langle \mathcal{C} \boldsymbol{V} \cdot \nabla b \rangle = -\langle \boldsymbol{\kappa} \nabla \mathcal{C} \cdot \nabla b \rangle. \tag{48}$

Thus the effects of advection and diffusion on the center-of-mass are identical such that Eq. (45) can be written,

$$\frac{\partial \langle b\mathcal{C} \rangle}{\partial t} = -2 \langle \kappa \nabla \mathcal{C} \cdot \nabla b \rangle. \tag{49}$$

This surprising result, that advection and diffusion both drive tracer on average in the same direction (despite their influence on buoyancy being exactly opposite), comes about because the tracer is localized. This localization implies that the tracer gradient ∇C must change sign, weighting the diapycnal diffusive flux such that tracer is diffused diapycnally on average in the same direction as it is advected diapycnally.

Similarly, multiplying Eq. (47) by
$$C(b - \mu_b)$$
 and integrating over the domain yields,

$$\langle (b-\mu_b)\mathcal{C}\boldsymbol{V}\cdot\nabla b\rangle = \langle (b-\mu_b)\mathcal{C}\nabla\cdot(\kappa\nabla b)\rangle,\tag{50}$$

$$= -\langle \kappa \nabla b \cdot \nabla \left(\mathcal{C}(b - \mu_b) \right) \rangle, \tag{51}$$

$$= -\langle (b - \mu_b) \kappa \nabla b \cdot \nabla \mathcal{C} \rangle - \langle \kappa \mathcal{C} | \nabla b |^2 \rangle.$$
(52)

⁴³⁶ Using Eq. (52), Eq. (46) for the bulk diffusivity can be rewritten as,

$$\kappa_{bk} = -\frac{2}{N^4 \langle \mathcal{C} \rangle} \langle \kappa \nabla b \cdot \nabla \left(\mathcal{C}(b - \mu_b) \right) \rangle + \frac{1}{\langle \mathcal{C} \rangle} \langle \kappa \mathcal{C} \frac{|\nabla b|^2}{N^4} \rangle.$$
(53)

If the tracer is mostly outside of the BBL where $|\nabla b|^2 \approx N^4$ then the additional second term is the in-situ diffusivity $\langle \kappa C \rangle / \langle C \rangle$. Eqs. (42)-(53) hold generally for any tracer when integrated over a region with zero sources or boundary tracer fluxes. Eqs. (49) and (53) also hold even if the buoyancy field is changing in time.

⁴⁴² b. Boundary and diffusivity gradient contributions to center-of-mass motion

We now split the buoyancy gradient field from boundary layer theory [Eqs. (1) and (2)] into its background $\nabla B = (B_y, B_z) = N^2(\sin\theta, \cos\theta)$ and perturbation $\nabla b' = (0, b'_z)$ [Eq. (16)] components. Then, using the chain rule and the boundary conditions, the center-of-mass tendency [Eq. (45)] can be expressed as,

$$\langle \mathcal{C} \rangle \frac{\partial \mu_b}{\partial t} = B_y \langle V \mathcal{C} \rangle + \kappa_0 B_z \int_{-\infty}^{\infty} \mathcal{C}(y,0,t) dy + B_z \langle \mathcal{C} \kappa_z \rangle - \langle \kappa \mathcal{C}_z b_z' \rangle, \tag{54}$$

exposing the influence of the boundary (second term on the RHS, which will also be referred to as the "boundary effect") and the diffusivity gradient (third term on the RHS) discussed earlier in the one-dimensional context (Eq. (37), except here multiplied by B_z). The final term is associated with variations in the buoyancy gradient in the BBL and is negligible in our context (not shown). The first advective term, as discussed above, is equal to the sum of the three other diffusive terms. We can go even further by identifying the advective tracer fluxes in the BBL and SML with the boundary and diffusivity gradient terms respectively.

455 c. BBL and SML contributions to center-of-mass motion

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Recall that the approximate analytic solution derived from one-dimensional boundary layer theory is obtained by combining two solutions (see Appendix A); one for the SML where the diffusivity varies (in the limit $(SPr_{u0})^{-1} \rightarrow 0$, Eqs. (67) and (68)), and one for the BBL where the diffusivity is constant and equal to κ_0 [Eq. (70)]. Therefore,

$$V_{BBL} \cdot \nabla b = \kappa_0 \nabla^2 b, \tag{55}$$

where $V_{BBL} = (\partial \Psi^{BBL} / \partial z) \hat{y}$. Multiplying by C and integrating as for the derivation of Eq. (48),

$$\langle \mathcal{C} \boldsymbol{V}_{BBL} \cdot \nabla b \rangle = -\kappa_0 \langle \nabla \mathcal{C} \cdot \nabla b \rangle = \kappa_0 B_z \int_{-\infty}^{\infty} \mathcal{C}(y,0,t) dy - \kappa_0 \langle \mathcal{C}_z b_z' \rangle.$$
(56)

Thus, the influence of diapycnal tracer advection in the BBL on μ_b is equivalent to the influence of the boundary term plus a minor correction due to the BBL buoyancy perturbation. In turn, this implies that tracer diapycnal advection in the SML is equivalent to the diffusivity gradient term plus an even smaller correction,

$$\langle \mathcal{C} \boldsymbol{V}_{SML} \cdot \nabla b \rangle = \boldsymbol{B}_{\boldsymbol{z}} \langle \mathcal{C} \boldsymbol{\kappa}_{\boldsymbol{z}} \rangle - \langle (\boldsymbol{\kappa} - \boldsymbol{\kappa}_0) \mathcal{C}_{\boldsymbol{z}} \boldsymbol{b}_{\boldsymbol{z}}' \rangle.$$
(57)

⁴⁶⁸ Both of these results match intuition: the boundary effect drives the tracer center-of-mass upward ⁴⁶⁹ in *b* whenever there is tracer within the BBL (by limiting tracer diffusion toward denser fluid), as ⁴⁷⁰ does the upslope BBL diapycnal flow. Similarly, the diffusivity gradient drives the tracer center-⁴⁷¹ of-mass toward denser fluid (κ_z is negative), as does the downslope SML diapycnal flow.

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For a constant diffusivity there is no SML flow or diffusivity gradient, and so the center-of-mass 472 moves toward lighter water due solely to the influence of BBL advection and the boundary effect, 473 which contribute equally (orange dashed line in Fig. 5b, which when multiplied by 2 provides the 474 full center-of-mass evolution μ_b , the solid line). In the bottom-intensified case, both the diffusivity 475 gradient and the downslope SML transport drive the tracer center-of-mass downward initially 476 (dotted blue line in Fig. 5b, which again should be multiplied by 2 to account for influence of 477 the SML transport), but the boundary effect and advection within the BBL quickly dominates 478 driving the tracer toward less dense fluid on average (solid blue line in Fig. 5b). 479

480 *d. BBL* and *SML* contributions to the bulk diffusivity

Following the same procedures as for the center-of-mass, the bulk diffusivity [Eq. (46)] can be split into contributions from a number of different processes,

$$\kappa_{bk} = \frac{\langle \kappa C \rangle}{\langle C \rangle} + \frac{1}{N^4 \langle C \rangle} \left[B_y \langle (b - \mu_b) V C \rangle + \kappa_0 B_z \int_{-\infty}^{\infty} C(y, 0, t) (b(y, 0) - \mu_b) dy + B_z \langle C(b - \mu_b) \kappa_z \rangle \right]$$

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$$+\kappa_{0}B_{z}\int_{-\infty}^{\infty}\mathcal{C}(y,0,t)(b(y,0)-\mu_{b})dy+B_{z}\langle\mathcal{C}(b-\mu_{b})\kappa_{z}\rangle$$
$$-\langle(b-\mu_{b})\kappa\mathcal{C}_{z}b_{z}'\rangle+B_{z}\langle\mathcal{C}\kappa b_{z}'\rangle\bigg|.$$
(58)

As for the simple one-dimensional case considered in Section 5, this equation clearly shows that 487 the bulk diffusivity characterizing the overall spreading rate of tracer across isopycnals is not 488 simply equal to the tracer-weighted isotropic diffusivity or in-situ diffusivity (first term on the 489 RHS, compare solid with dashed lines in Fig. 5c). Instead, there are contributions from advection, 490 from the accumulation of tracer on the boundary, from the vertical gradient in the diffusivity and 491 from the BBL buoyancy perturbation (these last two terms in Eq. (58) related to b'_z are negligible, 492 less than $5 \times 10^{-5} \text{m}^2 \text{s}^{-1}$, in all cases considered here). Once again, the advective term can be 493 split into contributions from the BBL and SML, which identify with the diffusive boundary term 494 and the diffusivity gradient term in Eq. (58) respectively³. Eq. (58) provides a useful diagnostic 495 for examining the tracer dispersion. However, it should be noted that the terms are not mutually 496 independent as they all affect and depend on the tracer concentration C. 497

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$$(b-\mu_b)\mathcal{C}\boldsymbol{V}_{BBL}\cdot\nabla b\rangle = \kappa_0\langle (b-\mu_b)\mathcal{C}\nabla^2 b\rangle$$
(59)

$$= -\kappa_0 \langle \nabla \left(\mathcal{C}(b - \mu_b) \right) \cdot \nabla b \rangle \tag{60}$$

$$=\kappa_0 B_z \int_{-\infty}^{\infty} \mathcal{C}(y,0,t) (b(y,0,t) - \mu_b) dy - \kappa_0 \langle \frac{\partial}{\partial z} \left(\mathcal{C}(b-\mu_b) \right) b_z' \rangle, \tag{61}$$

where we have once again used the chain rule and the boundary conditions. The last term is a minor correction dependent on b'_z . In turn Eq. (61) implies that the influence of SML advection is equal to the diffusivity gradient effect (plus another minor correction dependent on b'_z).

³Using Eq. (55) in the advective flux term in Eq. (46),

In the case of a constant diffusivity (orange line in Fig. 5c) the main factor influencing the spreading rate of the tracer across isopycnals, apart from the in-situ diffusivity, is the boundary effect (and equivalently BBL advection). While the influence of the boundary on the center-ofmass motion is always to push the center-of-mass toward less dense fluid (the second RHS term in Eq. (54) is always positive), its influence on the tracer dispersion rate κ_{bk} (second term inside the square brackets in Eq. (58)) can have either sign due to the $(b - \mu_b)$ weighting factor. In fact, this term can be further manipulated to yield,

$$\frac{\kappa_0 B_z}{N^4 \langle \mathcal{C} \rangle} \left(\int_{-\infty}^{\infty} \mathcal{C}(0) b(0) dy - \mu_b \int_{-\infty}^{\infty} \mathcal{C}(0) dy \right) = \frac{\kappa_0 B_z}{N^4 \langle \mathcal{C} \rangle} \langle \mathcal{C}(0) \rangle^y \left(\mu_b^0 - \mu_b \right), \tag{62}$$

where $\mathcal{C}(0)$ and b(0) are shorthand for the tracer and buoyancy on the boundary $\mathcal{C}(y,0,t)$ and 506 b(y,0), and $\mu_b^0 = \langle b(0)\mathcal{C}(0) \rangle^y / \langle \mathcal{C}(0) \rangle^y$ is the centroid buoyancy of the tracer on the boundary. 507 Thus the boundary effect will generally reduce κ_{bk} if μ_b^0 is at a denser level than μ_b (the usual case, 508 compare closed circles and crosses in Fig. 4)⁴. Its magnitude depends on the separation between 509 μ_b^0 and μ_b and the amount of tracer on the boundary $\langle C(0) \rangle^y$. Once again, this boundary effect 510 is equivalent to the influence of BBL advection and can also be understood from this perspective; 511 BBL advection will generally reduce the diapycnal spreading rate as it moves more tracer toward 512 the center-of-mass than away from the center-of-mass (e.g. see Fig. 4a,d). 513

The reduction in the bulk diffusivity κ_{bk} below the in-situ diffusivity (compare dashed and solid lines in Fig. 5c) due to the boundary effect occurs as soon as the tracer encounters the boundary. The behavior of the constant and bottom-intensified diffusivity cases are similar, apart from the magnitude of the initial, and in-situ, diffusivities (compare blue and orange lines in Fig. 5c). This is because the influence of SML advection and the diffusivity gradient are weak (dotted blue line in Fig. 5c). These terms act to drive a modest enhancement of the diapycnal spreading rate as

⁴Of course bizarre tracer distributions where the tracer is in contact with the boundary only above the tracer center-of-mass μ_b are possible, if unlikely, in which case the boundary effect would enhance κ_{bt} .

the diapycnal SML velocity is *divergent* around the tracer patch due to the positive curvature in κ (since N^2 is constant). It is the curvature of κ that matters, despite the appearance of only the first-derivative of κ in Eq. (58), because of the weighting factor $(b - \mu_b)$.

523 e. Summary

To summarize, the main effect highlighted by these near-boundary point-release tracer experi-524 ments is a strong reduction in the diapycnal spreading rate, below that which would be expected 525 from in-situ measurements of the isotropic diffusivity averaged over the tracer patch⁵, due to the 526 presence of the boundary which limits tracer diffusion toward denser water. Accompanying this 527 reduced spreading rate is a tendency for the overall tracer patch, quantified by its center-of-mass, 528 to move up the slope toward less dense fluid. We also showed that due to the buoyancy budget 529 constraint the advective and diffusive fluxes have equivalent influences on the tracer dispersion 530 [Eqs. (49) and (53)], as do their individual BBL and SML contributions. We will discuss these 531 results in the context of field TREs in Section 8. However, in the next section we first examine the 532 sensitivity of the tracer dispersion to several other parameters. 533

7. Sensitivity to other parameters

535 a. Release Location

In the previous two sections we showed that the presence of the boundary significantly reduces the diapycnal spreading rate of the tracer, and alters the tracer center-of-mass motion. The timescale over which these effects appear depends on where the tracer is released. When released further from the boundary (e.g. at $z_0 = 3d/4$, Fig. 6a-c) the tracer initially spreads according

⁵Note that this conclusion is not altered if alternative methods of estimating the in-situ diffusivity are considered. For example, the *tracergradient* weighted diffusivity $\langle \kappa | C_z | \rangle / \langle | C_z | \rangle$ gives a similar estimate (not shown) consistent with Mashayek et al. (2017).

to the isotropic diffusivity, with enhanced downward diffusion due to bottom-intensification and the downslope SML flow (orange solid line in Fig. 7b). There is an initial slight increase in the bulk diffusivity (solid orange line in Fig. 7c before day ~ 250), due to the fact that the SML flow is divergent (dotted orange line in Fig. 7c, which represents the equivalent influence of the diffusivity gradient). However, once the tracer encounters the boundary the diapycnal spreading rate is significantly reduced.

The reduction in the diapycnal spreading rate occurs earlier for tracers released closer to the 546 boundary (e.g. at $z_0 = 62.5$ m, Figs. 6d-f and blue lines in Fig. 7). While tracers released very 547 close to the boundary visually appear to experience more overall dispersion (compare Figs. 6f and 548 6c), most of this dispersion manifests in the lateral direction (despite the absence of any along-549 isopycnal diffusion) with less spreading across buoyancy surfaces (compare solid blue and orange 550 lines in Fig. 7c). Furthermore, when released close to the boundary the tracer experiences a large 551 upward movement of both its center-of-mass (blue line in Fig. 7b) and overall distribution, such 552 that most of the tracer lies at buoyancies well above its initial buoyancy after 320 days (blue lines 553 in Fig. 7a). This net movement toward less dense fluid is a consequence of the upslope BBL flow 554 and the asymmetric distribution of tracer between the BBL and SML. The latter asymmetry is due 555 to the boundary, which tends to trap tracer in the BBL. The net upward movement of tracer is 556 strongest when the tracer is released next to the boundary, while tracers released away from the 557 boundary in the SML experience a net downward movement (Fig. 8). Field TREs conducted very 558 close to a sloping boundary may be expected to show a similar net diapycnal upward movement of 559 tracer (e.g. Inall 2009). However, in Section 7e we show that the introduction of along-isopycnal 560 diffusion can limit this upward movement. 561

562 b. Topographic Slope

Though the position of the initial release point of the tracer clearly impacts the initial behavior 563 of the tracer, the slope of the boundary does not in these point release experiments. Altering the 564 slope of the boundary has a number of effects on the boundary layer flow. Firstly, it influences the 565 thickness of the BBL q_0^{-1} , with steeper slopes having thinner BBLs (a relatively minor effect due 566 to the fourth-power exponent in Eq. (9), also see Callies and Ferrari 2018). Secondly, it alters the 567 strength of the upslope and downslope boundary layer flows (linearly, due to the factor of $\cot \theta$ in 568 Eq. (11), also compare orange and blue solid lines in Fig. 2a and see Dell and Pratt 2015). The 569 change in the upslope and downslope flow strength arises because the local diapycnal advective 570 flux of buoyancy must remain the same (as the diffusion has not changed), but the separation of 571 buoyancy surfaces in the direction parallel to the boundary changes. Comparing tracer releases 572 performed over a slope of 1/100 versus 1/400 shows significant visual differences in the rate of 573 tracer dispersal (Fig. 9). However, the rate of tracer spreading across buoyancy surfaces is in fact 574 almost identical (not shown), because the magnitude of the diapycnal velocity does not change, 575 being set by diffusion. Instead, the spacing of buoyancy surfaces in the y direction $(B_y = N^2 \sin \theta)$ 576 increases in proportion to the decrease in upslope flow ($V \sim \cot \theta$), such that the total advective 577 flux of tracer $\langle CV \cdot \nabla b \rangle = B_v \langle VC \rangle$ remains the same. However, this result only holds for cases 578 where the tracer is released at a single point. When the tracer is instead released as a horizontal 579 layer, a case considered in Appendix C, the slope of the boundary does influence the spreading 580 rate as it influences the width of the SML and thus the extent to which the tracer is exposed to 581 strong boundary mixing. 582

583 c. Reduced mixing layer stratification

Throughout most of this article we have taken the limit $SPr_{\mu0}^{-1} \rightarrow 0$, where eddies are assumed 584 to flatten isopycnals in the SML and maintain the stratification there. However, observations from 585 the Brazil Basin suggest that isopycnals are often sloped in the active mixing layer and the strat-586 ification is reduced (St. Laurent et al. 2001; Ledwell et al. 2000). Callies (2018) found that the 587 observed Brazil Basin stratification was well fitted by the one-dimensional boundary layer solu-588 tion for a Prandtl number of 230, yielding $SPr_{\mu0}^{-1} \approx 2$. This corresponds to a three-fold reduction 589 in the SML stratification⁶, with a similar reduction in the upslope transport Ψ (Eq. (71) in Ap-590 pendix A)⁷. This reduction in upslope transport reduces the upslope spreading of tracer (compare 591 Figs. 9d-f and 9g-i) and the net movement of tracer toward lighter fluid (as the upslope buoyancy 592 gradient b_y remains the same). The rate of increase of the variance of the tracer in buoyancy space 593 also reduces. However, the bulk diffusivity includes a stratification normalization factor [Eq. (23)] 594 which, as the tracer does not extend beyond the SML, in this case should be chosen as the SML 595 stratification $N^2/(1 + SPr_{u0}^{-1})$. With this normalization factor the bulk diffusivity and its contri-596 butions from the boundary effect, the diffusivity gradient and BBL and SML advection remain 597 almost compared with the $SPr_{u0}^{-1} = 0$ case (not shown). This implies that the slope of the isopyc-598 nals (independent of the change in N^2) has only a minor impact on the tracer dispersion. However, 599 this may no longer be true for tracer clouds that extend across the entire SML into the far-field. 600

d. Decay scale of bottom-intensified mixing

The tracer dispersion is also sensitive to the decay scale used for the isotropic diffusivity [d in Eq. (10)]. When the tracer is released at the same distance from the boundary but this d scale is

⁶Note that all our tracer experiments are conducted within the SML and do not extend into the far-field, defined by the distance $d \log(\kappa_0/\kappa_{\infty})$

⁽Callies 2018) where the stratification does eventually return to N^2 .

⁷There is also a small factor of $3^{1/4}$ modification to the BBL scale q_0 (Eq. (9)).

reduced to 200m (a value that may be more representative of the Brazil Basin, St. Laurent et al. 604 2001; Callies 2018) then the tracer remains restricted closer to the boundary and the net upslope 605 movement of the tracer toward lighter buoyancy is reduced (compare Figs. 9j-1 with 9a-c). The 606 reduction in the center-of-mass movement in buoyancy space is due to the increase in the vertical 607 gradient of the isotropic diffusivity κ_{z} , or equivalently an increase in the SML transport, that 608 drives more tracer downward (compare blue and orange dotted and dashed lines in Fig. 10b). This 609 increase in the vertical gradient of the diffusivity also acts to enhance the tracer spreading rate in 610 buoyancy space such that the bulk diffusivity does not reduce as much as would be expected from 611 the reduced in-situ diffusivity (compare orange and blue lines in Fig. 10c). 612

e. Influence of along-isopycnal diffusion

Finally we consider the impact of along-isopycnal diffusion on the tracer behavior. Large along-614 isopycnal diffusion may be expected in abyssal mixing layers due to processes such as intrusions 615 (e.g. McPhee-Shaw 2006) or baroclinic instability setup by boundary mixing (e.g. Callies 2018; 616 Wenegrat et al. 2018). Along-isopycnal diffusion can rapidly mix tracer between the SML and 617 BBL, an effect which is clear when comparing TREs with large ($A_H = 100 \text{m}^2 \text{s}^{-1}$, Fig. 11d-f) and 618 small ($A_H = 10m^2s^{-1}$, Fig. 11a-c) along-isopycnal diffusivities. The along-isopycnal diffusivity 619 also appears to influence the spreading of the tracer in buoyancy space, despite the inability of 620 along-isopycnal diffusion to directly flux tracer across isopycnals (Fig. 12). At early times, the 621 enhanced along-isopycnal diffusion limits the initial downward motion of the center-of-mass by 622 mixing tracer more rapidly into the BBL, enhancing the boundary effect (compare orange and 623 blue solid and dashed lines in Fig. 12b before day 500). However, at later times along-isopycnal 624 diffusion limits the motion of the center-of-mass up the slope toward less dense fluid as the tracer 625 is mixed between the regions of upslope and downslope motion in the BBL and SML respectively. 626

In terms of the framework introduced in Section 6, the limited upslope motion of the center-ofmass arises because of a reduction in the influence of BBL advection (or equivalently the boundary effect, compare dashed lines in Fig. 12b after day 1000), as there is less tracer in contact with the boundary.

Similarly, the reduction of the influence of the boundary, or BBL advection, on the tracer distri-631 bution under strong along-isopycnal diffusion results in an enhanced rate of diapycnal spreading 632 as quantified by κ_{bk} (compare solid blue and orange lines in Fig. 12c). The magnitude of the 633 boundary effect responsible for the reduction in κ_{bk} is dependent on two factors [Eq. (62)]; 1) the 634 amount of tracer on the boundary $\langle \mathcal{C}(0) \rangle^y$ and 2) the buoyancy spacing between the total tracer 635 center-of-mass μ_b and that on the boundary μ_b^0 . Large along-isopycnal diffusion decreases both 636 of these factors by moving tracer away from the boundary on average and by homogenizing the 637 tracer between the boundary and SML such that $\mu_b^0 - \mu_b$ is reduced (compare positions of closed 638 circles and crosses in Fig. 11). The second of these two factors generally dominates (not shown). 639 A secondary effect of along-isopycnal diffusion is to reduce the in-situ diffusivity averaged 640 over the tracer patch (compare dashed blue and orange lines in Fig. 12c) by driving a net lateral 641 movement of tracer toward the weakly mixing interior. This net lateral movement toward the far-642 field with along-isopycnal diffusion is once again due to the presence of the boundary that prevents 643 tracer transport through it. 644

The results of these experiments are summarized in Fig. 13, where three measures of the timeaveraged or cumulative diffusivity up to day 800 are shown for a range of along-isopycnal diffusivities⁸. For all values of A_H the total diapycnal spreading rate of tracer across-isopycnals, as quantified by the bulk diffusivity, is smaller than the in-situ diffusivity (compare circles and crosses

⁸We do not include a similar summary figure for the dependence of the center-of-mass motion on A_H as this depends strongly on the time-scale of interest (compare blue and orange solid lines in Fig. 12b).

in Fig. 13). As along-isopycnal diffusion increases the diapycnal spreading rate increases and the 649 in-situ diffusivity decreases. The enhancement of diapycnal tracer fluxes by along-isopycnal diffu-650 sion (which has no influence on buoyancy) is a salient result which stems from the presence of the 651 boundary and interactions between the tracer gradients and diapycnal diffusion. Finally, a more 652 sophisticated estimate of the tracer derived diffusivity discussed in Appendix B, based on a least-653 squares fit to the results of a one-dimensional advection-diffusion equation, gives similar results 654 to the bulk diffusivity (compare triangles and circles in Fig. 13), meaning that if this estimate was 655 interpreted as a measure of the in-situ diffusivity it would be an under-estimate. 656

8. Relation to field experiments

Our study has focused on tracer behavior in the immediate boundary region, as opposed to past 658 field TREs such as BBTRE and DIMES. Nevertheless, it is useful to discuss our results in the 659 context of these past experiments. The BBTRE tracer (Ledwell et al. 2000) was released along 660 a target isopycnal that sat about 1000m above the bottom of a fracture zone trough. Much of 661 the tracer cloud thus lay in the outer SML, although because of topographic variability some 662 tracer was located much closer to the boundary (Fig. 4 of Ledwell et al. 2000). The initial tracer 663 dispersion was well modeled with a one-dimensional advection-diffusion equation and showed a 664 tendency to move downward toward denser water (Fig. 2a of Ledwell et al. 2000), consistent with 665 our results for bottom-intensified mixing (e.g. Fig. 6a-c). A smaller portion of the tracer was 666 also drawn eastward and more strongly downward toward the MAR boundary. Once significant 667 amounts of tracer came into contact with the boundary Ledwell et al. (2000) point out that the 668 tracer tended to get mixed back toward lighter density levels, consistent with the boundary effect 669 presented in this paper. During this later time period Ledwell et al. (2000) do not attempt to use 670 a one-dimensional model for these later periods of BBTRE, as it would likely underestimate the 671

diffusivity. We have applied such a one-dimensional model to our idealized experiments and found 672 that it provides a similar estimate of the diapycnal tracer spreading rate to the bulk diffusivity κ_{bk} 673 (see Appendix B), and therefore indeed underestimates the in-situ diffusivity $\langle \kappa C \rangle / \langle C \rangle$. In our 674 case this underestimate is roughly a factor of 3 (Fig. A3). However, in the BBTRE case (all else 675 being equal) we would expect a more accurate result as a larger proportion of the tracer lies in 676 the interior where the boundary has less impact. In Appendix C we examine tracers released as a 677 strip extending further into the interior where we indeed find that the boundary has less influence. 678 However, we also find that a one-dimensional model has some difficulty modeling this case due to 679 the spatial variation in the isotropic diffusivity across the tracer patch. 680

⁶⁸¹ Our results have less immediate applicability to DIMES, given the topographic complexity and ⁶⁸² strong mean flows characterizing the Southern Ocean. In particular, we note that the order-of-⁶⁸³ magnitude mismatch between diffusivities estimated from the DIMES tracer and microstructure ⁶⁸⁴ observations cannot be explained using the physics discussed here. The boundary effect discussed ⁶⁸⁵ in this study would suggest that in-situ estimates should exceed those obtained from the tracer ⁶⁸⁶ measurements rather than the other way around, and by less than an order-of-magnitude.

⁶⁶⁷ While neither BBTRE or DIMES were specifically designed to study near boundary tracer dis-⁶⁶⁸ persion, future experiments are planned with this aim. The relations derived in this article may help ⁶⁶⁹ in the analysis of these experiments. Firstly, the realization that there is some equivalence between ⁶⁹⁰ the effects of diapycnal advection and diffusion on the tracer moments in buoyancy space [e.g. ⁶⁹¹ Eqs. (49) and (53), which hold generally providing boundary tracer fluxes and interior sources can ⁶⁹² be eliminated] may allow some calculations to be simplified. For example, if the change in the ⁶⁹³ center-of-mass density of the tracer patch $\Delta \langle bC \rangle$ over a specified time period is measured, then by time-integrating Eq. (49), using the chain rule and eliminating the boundary term,

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$$\overline{\langle \mathcal{C} \, \nabla \cdot (\kappa \nabla b) \rangle} = \frac{\Delta \langle bC \rangle}{2},\tag{63}$$

where the over line indicates a time-integral. Eq. (63) allows the convergence of the diffusive 696 buoyancy flux, or net buoyancy source Db/Dt, averaged over the tracer patch to be easily obtained. 697 Secondly, if detailed information is available (perhaps as part of a series of surveys of the initial 698 spreading of tracer) then it may be possible to utilize some of our relations to better infer properties 699 of the underlying turbulent diffusivity. For example, if a structure $f(\tilde{z})$ of the turbulent diffusivity 700 as a function of distance above bottom \tilde{z} was assumed (such that $\kappa(\tilde{z}) = \kappa_0 f(\tilde{z})$ with f(0) = 1), then 701 substitution into Eq. (58) and some rearrangement (where we have also, for the point of argument, 702 ignored the terms dependent on b'_z , and removed the advective term in favor of multiplying the 703 diffusive terms by 2 as shown in Eq. (53)) yields, 704

$$\kappa_{0} \approx \langle \mathcal{C} \rangle \kappa_{bk} \left\{ \langle f\mathcal{C} \rangle + 2N^{-2} \cos \theta \left[\langle \mathcal{C}(0) \left(b(0) - \mu_{b} \right) \rangle^{y} + \langle \mathcal{C}(b - \mu_{b}) f_{z} \rangle \right] \right\}^{-1}.$$
(64)

⁷⁰⁶ If the three-dimensional tracer moments $\langle bC \rangle$, $\langle b^2C \rangle$, κ_{bk} , along with the shape function cor-⁷⁰⁷ relations $\langle fC \rangle$, $\langle C(b - \mu_b) f_z \rangle$ and the concentrations of tracer near the boundary $\langle C(0) \rangle^y$ and ⁷⁰⁸ $\langle C(0)b(0) \rangle^y$ can be measured, then Eq. (64) provides an estimate of the peak near-bottom diffu-⁷⁰⁹ sivity κ_0 taking into account the boundary and κ_z effects. Nevertheless, it remains unclear whether ⁷¹⁰ these time- and space-averaged tracer correlations can be estimated with sufficient accuracy given ⁷¹¹ sampling limitations in the field.

Finally, it may also be possible to use the knowledge and relations gained from this study to build an intermediate complexity prognostic model of the tracer dispersion that is simplified relative to the full 2D or 3D problem but takes into account some of the boundary effects neglected by the 1D advection-diffusion model considered in Appendix B. A more comprehensive inverse model along these lines may allow better estimates of the properties of the small-scale turbulent diffusivity to ⁷¹⁷ be obtained from the sparsely sampled tracer data. The development of such a model is, however,
 ⁷¹⁸ outside the scope of this article.

719 9. Summary

We have examined the behavior of a passive tracer released near a sloping boundary within a flow governed by one-dimensional boundary layer theory. Results can be summarized as follows:

For isolated near-boundary tracer releases the presence of the *boundary reduces the net di- apycnal tracer spreading rate* below that which would be expected from averaging the in-situ
 diffusivity over the tracer patch (Figs. 5c and 13).

2. The dipole of diapycnal flow, upward in the BBL and downward in the SML, also influences 725 the tracer dispersion. In particular, when the tracer is released close to the boundary it tends 726 to move upslope toward less dense fluid on average (e.g. Fig. 4d-f) due to the asymmetric 727 distribution of diapycnal flow between the BBL and SML (the SML flow being spread over 728 a much wider region). However, the extent of this upslope motion depends on the proximity 729 of the tracer release point to the boundary (Fig. 6) with tracers released further from the 730 boundary in the outer SML experiencing a downward net diapycnal motion (as summarized 731 by Fig. 8). 732

As a consequence of the advection-diffusion balance in the buoyancy equation, there is an
 equivalence between the diffusive and advective tracer fluxes; they both contribute equally to
 the domain-averaged diapycnal tracer flux [Eq. (48)]. Further, the slowing of the diapycnal
 tracer spreading rate due to the presence of the boundary is equivalent to the influence of tracer
 advection within the BBL, while the modest enhancement in tracer spreading associated with

the divergent diapycnal flow in the SML is equivalent to the diffusive influence of the gradient
 in the isotropic diffusivity.

⁷⁴⁰ 4. For point release experiments the boundary slope has little effect on the diapycnal spreading
⁷⁴¹ rate.

- The introduction of reduced stratification and sloping isopycnals in the SML (Callies 2018)
 reduces the rate at which tracer spreads across isopycnals. However, as a diffusivity measures
 the rate of tracer spreading in physical space the bulk diffusivity does not change.
- 6. When the decay scale of bottom-intensified mixing is decreased then the overall tracer spreading across isopycnals decreases. However, this decrease is not as much as would be expected
 from the change in in-situ diffusivity because the SML transport becomes more divergent.
- 7. Along-isopycnal diffusion enhances the diapycnal tracer spreading rate for near-boundary point releases by reducing the damping influence of the boundary and BBL advection (Fig. 13). Along-isopycnal diffusion also drives more tracer away from the boundary to where mixing is weak, reducing the in-situ diffusivity averaged over the tracer patch.

While the differences in diffusivity arising from the additional boundary effects exposed in this 752 study are generally less than an order of magnitude, our results nevertheless highlight the complex-753 ity of tracer behavior near sloping boundaries. Additional complexities present in the observational 754 context likely make matters worse. Variations in topographic slope, mixing intensity and stratifi-755 cation are likely to drive additional tracer transports. Submeso- to large-scale currents will also 756 drive interactions between the tracer patch and different boundaries at a range of depths, buoyan-757 cies and times. It is possible that averaging over these complexities will result in smooth Gaussian 758 tracer spreading across buoyancy surfaces with a well-defined central diffusivity. However, our 759 results suggest that relating this tracer-derived diffusivity to the small-scale turbulent diffusivity 760

⁷⁶¹ important for buoyancy and mass transport must be done with care. In Section 8 we discussed
 ⁷⁶² several avenues that may help with the analysis of future near-boundary TREs.

Further modeling work is also needed to assess the impact of additional complexities. For exam-763 ple, a large *net* up- or down-slope mass transport associated with upslope variations in topography, 764 stratification or mixing may influence the tracer behavior. The role of intrusions of boundary layer 765 fluid into the interior (e.g., Gloor et al. 2000) is another area that deserves further attention. In this 766 article, the influence of such intrusions on tracer transport was captured at first-order through an 767 along-isopycnal diffusivity. Such an approximation may be appropriate for the statistical average 768 tracer behavior, but is less applicable to a tracer released, for example, at a particular phase of a 769 passing eddy. Finally, the relationship between the along-isopycnal diffusion and the eddy-driven 770 overturning required to maintain a stratified SML in rotating boundary layer theory (e.g. Callies 771 2018), and whether this influences the tracer transport, is worthy of further exploration. A deep 772 understanding of these processes is needed in order to represent them appropriately in general 773 circulation models. 774

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780 Appendix A: The approximate analytic solution

Here we present the derivation of the approximate analytic solution to one-dimensional boundary layer theory with a bottom-intensified diffusivity [Eq. (10)] following Callies (2018) (note that our derivation differs from Callies (2018) through the use of distinct viscosities in the *x* and *y* ⁷⁸⁴ directions). We non-dimensionalize the boundary layer equations using,

$$\kappa = \kappa_0 \hat{\kappa}, \quad v_u = v_{u0} \hat{\kappa}, \quad v_v = v_{v0} \hat{\kappa}, \quad z = d \hat{z}, \quad \Psi = \kappa_0 \cot \theta \hat{\Psi}, \tag{65}$$

where κ_0 , v_{u0} and v_{v0} represent the near-boundary diffusivity and viscosity. Eq. (7) then becomes,

$$\frac{d^2}{d\hat{z}^2} \left(\hat{\kappa} \frac{d^2 \hat{\Psi}}{d\hat{z}^2}\right) + 4(dq_0)^4 \left(\frac{\hat{\Psi}}{\hat{\kappa}} - \frac{1 + (SPr_{u0})^{-1} \frac{r}{\hat{\kappa}}}{1 + (SPr_{u0})^{-1}}\right) = 0, \tag{66}$$

where $r = \kappa_{\infty}/\kappa_0$, q_0 is the BBL width parameter based on the mixing coefficients near the boundary [Eq. (9)] and we have assumed constant Prandtl numbers Pr_{u0} and Pr_{v0} . For reasonable deep ocean parameters $q_0^{-1} \sim 4.5$ m (or $q_0^{-1} = 28$ m for small $(SPr_{u0})^{-1}$ where q_0 is determined by upslope friction only, see Eq. (9)). Therefore, with $d \sim 500$ m, $dq_0 \sim 20$ the fourth derivative term in Eq. (66) can be ignored outside the thin BBL. Following Garrett (2001), a solution outside the BBL is therefore (restoring the dimensions),

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$$\Psi^{SML} = \frac{\cot\theta}{1 + (SPr_{u0})^{-1}} \left(\kappa + (SPr_{u0})^{-1}\kappa_{\infty}\right)$$
(67)

$$\frac{\partial b}{\partial z}^{SML} = N^2 \frac{\cos \theta}{1 + (SPr_{u0})^{-1}} \left(1 + (SPr_{u0})^{-1} \frac{\kappa_{\infty}}{\kappa}\right)$$
(68)

⁷⁹⁷ For large $(SPr_{u0})^{-1}$, this gives much reduced stratification over the entire SML until κ approaches ⁷⁹⁸ κ_{∞} (Callies 2018).

Eqs. (67) and (68) provide a solution in the SML, but do not satisfy the boundary conditions $\Psi = 0$ and $\partial \Psi / \partial z = 0$ at z = 0. To satisfy these boundary conditions we introduce an inner BBL solution where the fourth derivative term in Eq. (66) becomes important. For this inner solution we can make the assumption that the mixing coefficients are constant $\kappa \sim \kappa_0$, $v_u \sim v_{u0}$, $v_v \sim v_{v0}$. With $\Psi = \Psi^{BBL} + \Psi^{SML}$, where Ψ^{SML} is assumed constant in the BBL, substitution into Eq. (7) yields,

$$\frac{d^4 \Psi^{BBL}}{dz^4} + 4q_0^4 \Psi^{BBL} = 0, ag{69}$$

The solution that satisfies the boundary condition $\Psi^{BBL}(0) = -\Psi^{SML}(0)$ and $\partial \Psi^{BBL}/\partial z(0) = 0$ (neglecting the small $\partial \Psi^{SML}/\partial z$ at z = 0) is given by,

$$\Psi^{BBL} = -\frac{\cot\theta}{1 + (SPr_{u0})^{-1}} \left(\kappa_0 + (SPr_{u0})^{-1}\kappa_\infty\right) e^{-q_0 z} \left(\cos q_0 z + \sin q_0 z\right),\tag{70}$$

so that the full approximate solution is,

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⁸¹⁰
$$\Psi = \frac{\cot\theta}{1 + (SPr_{u0})^{-1}} \left(\kappa + (SPr_{u0})^{-1} \kappa_{\infty} \right) \left(1 - e^{-q_0 z} \left(\cos q_0 z + \sin q_0 z \right) \right), \tag{71}$$

where we have also used the fact that $\kappa_0 e^{-q_0 z} (\cos q_0 z + \sin q_0 z) \approx \kappa e^{-q_0 z} (\cos q_0 z + \sin q_0 z)$. In the limit $(SPr_{u0})^{-1} \rightarrow 0$ Eq. (71) reduces to Eq. (11), which will be used to determine the upslope velocity $V = \partial \Psi / \partial z$ and buoyancy field $b_z = N^2 \sin \theta \Psi / \kappa$ for the 2D tracer release experiments.

⁸¹⁴ Appendix B: A one-dimensional model of the tracer dispersion across isopycnals

In this appendix we apply a one-dimensional model, commonly used in field TREs (e.g. Ledwell and Watson 1991; Ledwell 1998) to the evolution of the tracer distribution in buoyancy space. The tracer evolution is modeled using the one-dimensional equation,

$$\frac{\partial \overline{C}}{\partial t} + (\overline{w} - \overline{\kappa}_h) \frac{\partial \overline{C}}{\partial h} = \overline{\kappa} \frac{\partial^2 \overline{C}}{\partial h^2}, \tag{72}$$

where the over line denotes an area-average on isopycnals at a given height *h* above a target buoyancy surface. A mean stratification profile is used to convert between *h* and *b*. The areaaveraged diffusivity is assumed to be a linear function of h, $\overline{\kappa} = \overline{\kappa}_0 + h\overline{\kappa}_h$. Following Ledwell and Watson (1991) and Ledwell (1998), we discretize Eq. (72) using a forward-in-time, centered-inspace discretization and minimize the sum of squares,

$$\sum_{h} \left(\mathcal{C}_n^{obs} - L^n \mathcal{C}_0 \right), \tag{73}$$

with respect to the three parameters $\overline{\kappa}_0$, \overline{w} and $\overline{\kappa}_h$. The sum is performed over the discretized height *h*. Here C_n^{obs} is the distribution of tracer in buoyancy space from the full 2D simulation at ⁸²⁷ a given time t_n , L is a matrix operator representing the centered-in-space spatial discretization of ⁸²⁸ Eq. (72) and C_0 is the initial tracer distribution⁹.

The model matches the evolution of the buoyancy tracer profiles from the full two-dimensional 829 simulations very well for both constant (not shown) and bottom-intensified (compare solid lines 830 and open circles in Fig. A1a) diffusivity cases. The least-squares fit provides an estimate of a 831 single diffusivity and its linear gradient over the full evolution of the tracer up to a given time. 832 This diffusivity, at the center-of-mass position of the fitted tracer distribution, compares well with 833 the time-averaged bulk diffusivity measured from the tracer moments (compare solid blue and 834 orange lines in Fig. A1b). In particular, the fitting method reproduces the rapid initial decrease in 835 the tracer spreading rate below that expected from the in-situ diffusivity due to the presence of the 836 boundary¹⁰. 837

The least-squares fit also provides estimates of a mean diapycnal velocity \overline{w} and diffusivity gradient (solid and dashed lines in Fig. A1c respectively). The fit predicts negative values for \overline{w} and $\overline{\kappa_h}$ at initial times before the tracer contacts the boundary, consistent with what would be expected within the SML. However, at later times $\overline{\kappa}_h$ is predicted as positive. Integrating Eq. (72) over all *h* shows that the rate of change of the center-of-mass μ_h and the one-dimensional bulk diffusivity in the one-dimensional model are given by,

$$\frac{\partial \mu_h}{\partial t} = \overline{w} + \overline{\kappa}_h \tag{74}$$

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$$\kappa_{bk}^{1D} = \frac{1}{2} \frac{\partial \sigma_h^2}{\partial t} = \frac{\langle \overline{\kappa} \mathcal{C} \rangle^h}{\langle \mathcal{C} \rangle} = \overline{\kappa}_h \mu_h + \overline{\kappa}_0 \tag{75}$$

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⁹We use a non-linear Levenberg-Marquardt least-squares minimization algorithm (Moré 1978). The spatial discretization uses a regular height grid with 100-points converted from buoyancy space using the far-field stratification N^2 . The time step is 1 day. ¹⁰However, if the presence of a boundary *at a particular value of b* was known, then this could be included in the boundary conditions of the

one-dimensional model and a better estimate of the in-situ diffusivity could be obtained. In a field TRE boundaries are likely present at varying depths and buoyancies over the history of the experiment, and thus this knowledge is unlikely.

Thus, the positive \overline{w} and $\overline{\kappa}_h$ at later times are present in order to drive the center-of-mass toward 847 less dense fluid [Eq. (74)]. In the full two-dimensional case this center-of-mass movement is 848 achieved by the boundary effect and BBL advection, that are not captured by the one-dimensional 849 model [they do not appear in Eq. (74)]. Further, in the one-dimensional model the bulk diffu-850 sivity is equal to the one-dimensional in-situ diffusivity $\langle \overline{\kappa} \mathcal{C} \rangle^h / \langle \mathcal{C} \rangle$, which in turn is equal to the 851 diffusivity at the tracer center-of-mass position μ_h [Eq. (75)]. Comparing Eq. (75) to Eq. (58), 852 the boundary effect, the advective term and the diffusivity gradient term in Eq. (58) all drop 853 out because \overline{w} and κ_h are constants and can be removed from the averaging leaving the factor 854 $\langle (h-\mu_h)\mathcal{C}\rangle = 0$. Thus, the one-dimensional model, while reproducing the evolution of the tracer 855 variance in buoyancy space, assigns all of that spreading rate to the *in-situ* diffusivity $\overline{\kappa}$. Because 856 we found the spreading rate to be less than expected from the in-situ diffusivity, the 1D method 857 would thus yield a much reduced in-situ diffusivity compared to that actually acting on the tracer 858 $\langle \kappa C \rangle / \langle C \rangle$. 859

Appendix C: Tracer released as a layer

We have investigated the spreading of a tracer released at a single location near the boundary, within the actively mixing layer. Different behavior may be expected in the case where a tracer is released outside the mixing layer, is homogenized along-isopycnals, and only later interacts with the boundary. In this appendix we examine an idealized example of such a case where the tracer is released as a layer spread horizontally and centered around a particular buoyancy surface (see Fig. A2: the tracer is released within the dashed white box). To emphasize the boundary effects we release the tracer as a strip extending only across the SML.

Enhanced vertical diffusion of the tracer patch occurs in the region close to the boundary (Fig. A2c). The near-boundary tracer spreads further up than down the slope as a consequence of the

⁸⁷⁰ BBL flow as for the point-release experiments (e.g. Fig. 4). However, the center-of-mass motion ⁸⁷¹ is much weaker here, because the majority of the tracer patch experiences only weak diffusion ⁸⁷² (solid lines in Fig. A3b).

When a larger proportion of tracer is located further from the boundary the bulk diffusivity 873 is more comparable to the in-situ diffusivity (compare dashed and solid lines in Fig. A3c, and 874 compare to the point-release cases in Fig. 5). This is because the damping effect of the boundary 875 depends on the ratio of tracer near the boundary versus away from the boundary, $\langle C(0) \rangle^{y} / \langle C \rangle$ [see 876 Eq. (62)]. As the initial tracer patch is extended further into the far-field the in-situ and bulk 877 diffusivities weaken and converge (compare orange and blue lines in Fig. A3c). The addition of 878 along-isopycnal diffusion also drives more tracer into the far-field where mixing is weak, acting 879 to reduce both the in-situ and bulk diffusivities (Fig. A4). This contrasts with the near-boundary 880 point-release experiments where along-isopycnal diffusion enhanced the diapycnal spreading rate 881 by reducing the boundary effect (Fig. 13). In the layer cases the boundary effect is already weak 882 and so the enhancement of tracer movement away from the boundary dominates. 883

Finally, when the tracer is released as a layer the tracer profiles in buoyancy space evolve in a 884 manner which is not as well captured by the one-dimensional advection-diffusion equation con-885 sidered in Appendix B (compare solid and dotted lines in Fig. A3a), compared to the point-release 886 cases (e.g. Fig. A1a). The tracer buoyancy profiles in the layer case exhibit more rapid spreading 887 of the tails of the distribution and slower spreading of the core of the distribution (associated with 888 the near-boundary and weak-diffusion far-field regions respectively, Fig. A2). The 1D advection-889 diffusion equation with constant parameters \overline{w} , $\overline{\kappa}_h$ and κ_0 [e.g. Eq. (72)] cannot capture this 890 behavior, as the constant velocity and linear diffusivity can drive the development of skewness 891 but not of kurtosis in the distribution. It is possible that a 1D advection-diffusion equation with a 892 diffusivity that was quadratically dependent on h, for example, may be able to capture this behav-893

ior. However, such a model is not supported by the setting. Due to this difficulty the diffusivity 894 predicted by the one-dimensional method does not provide as good an estimate of the overall 895 spreading rate of the tracer, predicting a weaker value (compare triangles to open circles in Fig. 896 A4). It is possible that a similar problem may manifest in field experiments when a tracer encoun-897 ters a boundary where mixing is enhanced, or in any situation where the tracer patch spreads over 898 two regions with very different mixing characteristics. Providing that the tracer is sampled in both 899 regions, then this effect can be identified from the non-standard behavior of the tracer distribu-900 tions in buoyancy space. However, if the near-boundary regions where diffusion is strong are not 901 sampled then the diffusivity may be underestimated. 902

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A schematic illustrating the two-dimensional slope-normal coordinate system used in this Fig. 1. 1042 article. The boundary has a constant slope of tan θ . The coordinate system is aligned with the 1043 slope such that z is normal to the slope, y points up the slope and x is along-slope. The density 1044 field is characterized by a constant stratification N^2 within the SML, while isopycnals slope 1045 down within the weakly stratified BBL. One-dimensional boundary layer theory yields an 1046 upslope velocity profile V(z) that is characterized by strong upwelling across isopycnals in 1047 the BBL with weaker downwelling across isopycnals in the SML. The tracer C is released 1048 at some point in the domain and evolves in the two spatial dimensions y, z and in time t. 1049

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1160 1161 1162 1163 1164 1165 1166	Fig. A4.	A summary of the time-averaged (or cumulative) diffusivity between days 0 and 800 from all the layer-release experiments with a slope of $1/200$ and where the layer has an initial width of 200km, or extends out to a distance of $2d = 1000$ m above the bottom. Shown are the time-averaged bulk (open circles) and in-situ (crosses) diffusivities as a function of the along-isopycnal diffusivity A_H . Also shown (black triangles) is the diffusivity estimated from a three-parameter least-squares fit to a 1D advection-diffusion equation (Ledwell and Watson 1991; Ledwell 1998, see Appendix B).	68



FIG. 1. A schematic illustrating the two-dimensional slope-normal coordinate system used in this article. The boundary has a constant slope of tan θ . The coordinate system is aligned with the slope such that *z* is normal to the slope, *y* points up the slope and *x* is along-slope. The density field is characterized by a constant stratification N^2 within the SML, while isopycnals slope down within the weakly stratified BBL. One-dimensional boundary layer theory yields an upslope velocity profile V(z) that is characterized by strong upwelling across isopycnals in the BBL with weaker downwelling across isopycnals in the SML. The tracer *C* is released at some point in the domain and evolves in the two spatial dimensions *y*, *z* and in time *t*.



FIG. 2. Upslope velocity *V* (left) and buoyancy frequency (right) $\frac{\partial b}{\partial z}$ from the approximate analytic solution [Eq. (11)] for a constant diffusivity $\kappa = \kappa_0$ (dashed black) and an exponentially-decaying isotropic diffusivity $\kappa = \kappa_{\infty} + (\kappa_0 - \kappa_{\infty})e^{-z/d}$ (colored). Parameters are $N^2 = 10^{-6}s^{-2}$, $\kappa_0 = 10^{-3}m^2s^{-1}$, $\kappa_{\infty} = 10^{-5}m^2s^{-1}$ and d = 500m. The orange curves and the constant κ case use a slope of $\tan \theta = 1/400$, where the BBL width $q_0^{-1} = 28.3m$, while the blue curves use a slope of 1/100, where $q_0^{-1} = 14.1m$. Stratification in the constant κ case is identical to that in the exponential case. The open circles indicate the effective vertical grid resolution of the Dedalus simulations for 192 Chebyshev modes.



FIG. 3. Behavior of the tracer in z for a constant diffusivity $\kappa = \kappa_0$ (orange lines) and an exponential diffusivity 1181 (blue lines). (a) The tracer distribution as a function of z integrated in y, $\langle C \rangle^{y}$, at days 0, 48 and 192. (b) Time 1182 series of the center of mass μ_z (solid lines) and the time-integrated contribution of the diffusivity gradient to 1183 center-of-mass changes (dotted lines, obtained by time-integrating the second term on the RHS of Eq. (37), and 1184 adding the initial center-of-mass $\mu_z(0)$). Note that the contribution of the boundary term (first term on RHS of 1185 Eq. (37)) accounts for the increase in μ_z beyond the effect of the diffusivity gradient. (c) The bulk diffusivity 1186 κ_{bk}^{z} from the tracer variance (solid lines), the in-situ diffusivity (dashed lines) and the contribution to κ_{bk}^{z} from 1187 the diffusivity gradient (dotted lines). Note again that the boundary term (second term in Eq. (40)) is not shown 1188 and accounts for the down-turn in κ_{bk}^{z} . 1189



FIG. 4. The evolution of a tracer patch released $z_0 = d/2 = 250$ m above the bottom (open white circle) with (a-c) a constant diffusivity and (d-f) a bottom-intensified diffusivity. Tracer concentrations are shown at days (a,d) 400, (b,e) 800 and (c,f) 1600. The position of the tracer center-of-mass (μ_y , μ_z) is shown with a solid circle and the position of the center-of-mass of tracer located only on the boundary is shown with a cross. Note that the aspect ratio is strongly exaggerated.



FIG. 5. Buoyancy space tracer behavior for the two runs considered in Fig. 4 with constant diffusivity (orange 1195 lines) and exponential diffusivity (blue lines). (a) The tracer distribution as a function of b at days 0, 192 and 1196 1600. (b) Time series of the center of mass μ_b (solid lines), the contribution of the diffusivity gradient to center-1197 of-mass trends (dotted lines) and the contribution of BBL advection to center-of-mass trends (dashed lines). 1198 As discussed in Section 6a, advection and diffusion each contribute half of the total trend, with the boundary 1199 term and BBL advection also making equal contributions. (c) Time series of the bulk diffusivity (solid lines), 1200 in-situ diffusivity (dashed lines) and the contribution of the diffusivity gradient to the bulk diffusivity (dotted 1201 line). As discussed in Section 6d, the influence of SML advection is equivalent to the diffusivity gradient effect 1202 (dotted line), while BBL advection and the boundary effect account, in equal parts, for the reduction in the bulk 1203 diffusivity well below the in-situ diffusivity. 1204



FIG. 6. The evolution of tracer patches released (a-c) $z_0 = 3d/4 = 375$ m and (d-f) $z_0 = d/8 = 62.5$ m above the bottom (open white circle) with a bottom-intensified isotropic diffusivity and no along-isopycnal diffusivity. The position of the tracer center-of-mass (μ_y, μ_z) is shown with a closed white circle and the center-of-mass on the boundary is shown with a cross.



FIG. 7. Buoyancy space tracer behavior for the two runs considered in Fig. 6, where the tracer is released $z_0 = 3d/4 = 375m$ (orange lines) and $z_0 = d/8 = 62.5m$ (blue lines) above the bottom. (a) The tracer distribution as a function of buoyancy at days 0, 192 and 800. (b) Time series of the center of mass μ_b (solid lines), the contribution of the diffusivity gradient to center-of-mass trends (dotted lines) and the contribution of BBL advection to center-of-mass trends (dashed lines). (c) Time series of the bulk diffusivity (solid lines), in-situ diffusivity (dashed lines) and the contribution of the diffusivity gradient to the bulk diffusivity (dotted lines).



FIG. 8. The change in the tracer center-of-mass in buoyancy space from its initial buoyancy $\Delta \mu_b = \mu_b(t) - \mu_b(0)$ at day 800 as a function of the initial release height of the tracer above the bottom boundary.



FIG. 9. (a-c) The evolution of tracer patches released $z_0 = d/2 = 250$ m above the bottom (open white circle) as for Fig. 4d-f (with different axis limits) with a slope of 1/400, d = 500m and $SPr_{u0}^{-1} = 0$. (d-f) As for a-c except with a slope of 1/100. (g-i) As for a-c except with the vertical stratification in the SML reduced by a factor of three through the choice $SPr_{u0}^{-1} = 2$. (j-l) As for a-c except with the diffusivity decay scale reduced to d = 200m. Plots are shown at day (a,d,g,j) 400, (b,e,h,k) 800 and (c,f,i,l) 1600. The position of the tracer center-of-mass (μ_y, μ_z) is shown with a solid white circle and the center-of-mass on the boundary is shown with a cross.



FIG. 10. Buoyancy space tracer behavior for the runs considered in Fig. 9a-c and Fig. 9j-1 where the tracer is released 250m above the bottom and the vertical decay scale of the diffusivity is d = 500m (orange lines) and d = 200m (blue lines). (a) The tracer distribution as a function of buoyancy at days 0, 192 and 1600. (b) Time series of the center of mass μ_b (solid lines), the contribution of the diffusivity gradient to center-of-mass trends (dotted lines) and the contribution of BBL advection to center-of-mass trends (dashed lines). (c) Time series of the bulk diffusivity (solid lines), in-situ diffusivity (dashed lines) and the contribution of the diffusivity gradient to the bulk diffusivity (dotted lines).



FIG. 11. The evolution of tracer patches released $z_0 = d/2 = 250$ m above the bottom (open white circle) with an exponential diffusivity for two values of along-isopycnal diffusivity (a-c) $A_H = 10\text{m}^2\text{s}^{-1}$ and (d-f) $A_H = 100\text{m}^2\text{s}^{-1}$ at day (a,d) 400, (b,e) 800 and (c,f) 1600. The position of the tracer center-of-mass (μ_y, μ_z) is shown with a solid white circle and the center-of-mass on the boundary is shown with a cross.



FIG. 12. Buoyancy-space tracer behavior for the two runs considered in Fig. 11 with along-isopycnal diffusivity $A_H = 10m^2s^{-1}$ (orange lines) and $A_H = 100m^2s^{-1}$ (blue lines). (a) The tracer distribution as a function of buoyancy at days 0, 192 and 1600. (b) Time series of the center of mass μ_b (solid lines), the contribution of the diffusivity gradient to center-of-mass trends (dotted lines) and the contribution of BBL advection to center-ofmass trends (dashed lines). (c) Time series of the instantaneous bulk diffusivity (solid lines), in-situ diffusivity (dashed lines) and the contribution of the diffusivity gradient to the bulk diffusivity (dotted lines).



FIG. 13. A summary of the time-averaged (or cumulative) diffusivity between days 0 and 800 from all the point-release experiments with a slope of 1/400, bottom-intensified mixing and a release point at $z_0 = d/2$. Shown are the time-averaged bulk (open circles) and in-situ (crosses) diffusivities as a function of the alongisopycnal diffusivity A_H . Also shown (black triangles) is the diffusivity estimated from a three-parameter leastsquares fit to a 1D advection-diffusion equation (Ledwell and Watson 1991; Ledwell 1998, see Appendix B).







Fig. A2. Tracer distributions for experiments where the initial tracer distribution is a narrow Gaussian centered about a particular buoyancy (whose initial extent is indicated by the dashed white box), the along-isopycnal diffusivity is $A_H = 10m^2s^{-1}$ and the slope is 1/200. The initial width of the tracer distribution extends out to a distance (a-c) 2d = 1000m above the bottom, or a horizontal distance of 200km from the boundary and (d-f) 3d = 1500m above the bottom, or a horizontal distance of 300km from the boundary. Distributions are shown at day (a,d) 400, (b,e) 800 and (c,f) 1600. The position of the tracer center-of-mass (μ_y, μ_z) is shown with a white circle and the center-of-mass on the boundary is shown with a cross.



Fig. A3. Buoyancy-space tracer behavior for the cases considered in Fig. A2. (a) The tracer distribution as a function of buoyancy at days 192 and 1600. Also shown are the predicted tracer distributions from the one-dimensional least-squares fit discussed in Appendix B (dotted lines). (b) Time series of the center of mass μ_b (solid lines), the contribution of the diffusivity gradient to center-of-mass trends (dotted lines) and the contribution of BBL advection to center-of-mass trends (dashed lines). (c) Time series of the instantaneous bulk diffusivity (solid lines), in-situ diffusivity (dashed lines) and the contribution of the diffusivity gradient to the bulk diffusivity (dotted lines).



Fig. A4. A summary of the time-averaged (or cumulative) diffusivity between days 0 and 800 from all the layer-release experiments with a slope of 1/200 and where the layer has an initial width of 200km, or extends out to a distance of 2d = 1000m above the bottom. Shown are the time-averaged bulk (open circles) and in-situ (crosses) diffusivities as a function of the along-isopycnal diffusivity A_H . Also shown (black triangles) is the diffusivity estimated from a three-parameter least-squares fit to a 1D advection-diffusion equation (Ledwell and Watson 1991; Ledwell 1998, see Appendix B).